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Disclosure, Competition, and Learning from Asset Prices*

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Abstract

This paper studies the classic information-sharing problem in a duopoly setting in which firms learn information from a financial market. By disclosing information, a firm incurs a proprietary cost of losing competitive advantage to its rival firm but benefits from learning from a more informative asset market. Firms' disclosure decisions can exhibit strategic complementarity, which is strong enough to support both a disclosure equilibrium and a nondisclosure equilibrium. Allowing minimal learning from asset prices dramatically changes firms' disclosure behaviors: without learning from prices, firms do not disclose at all; but with minimal learning from prices, firms can almost fully disclose their information. Learning from asset prices benefits firms, consumers, and liquidity traders, but harms financial speculators.

Keywords: Disclosure, proprietary cost, learning from prices, multiplicity, welfare.

JEL Classifications: D61; G14; M41

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1. Introduction

Information sharing among oligopoly firms has been a contentious topic in the antitrust field and has received substantial attention from academics and regulations on trade associations. The literature shows whether firms want to voluntarily disclose information depends upon the nature of competition (Cournot or Bertrand) and the nature of information (common value or private value). Firms compete in quantities in Cournot settings and they compete in prices in Bertrand settings. Common-value information represents shocks affecting all firms (e.g., a common demand shock), while private-value information represents shocks affecting each firm separately (e.g., idiosyncratic cost shocks). The literature finds that firms choose to withhold information in settings of Cournot/common-value and Bertrand/private-value, while they choose to share information completely in settings of Cournot/private-value and Bertrand/common-value.[1]

However, there is an important feature of real-world economies that is missing in this line of research, namely that firms often learn new information from financial markets and use this information to guide their production decisions. The archetypal examples of these financial markets include the stock market and the commodity futures market.[2] Going back at least to Hayek (1945), researchers argue that asset prices are a useful source of information for real decisions. Asset prices aggregate different pieces of information from various traders who trade in financial markets for their own profit motives. The trading process and the information aggregation are

[1] See, for example, Gal-Or (1986), Darrough (1993), Raith (1996), Vives (1984, 2008), and more recently, Bagnoli and Watts (2015) and Arya, Mittendorf, and Yoon (2016).

[2] For instance, Fama and Miller (1972, p. 335) note: "at any point in time market prices of securities provide accurate signals for resource allocation; that is, firms can make production-investment decisions...." Black (1976, p. 174–176) wrote: "futures prices provide a wealth of valuable information for those who produce, store, and use commodities. Looking at futures prices for various transaction months, participants in this market can decide on the best times to plant, harvest, buy for storage, sell from storage, or process the commodity...The big benefit from futures markets is the side effect: the fact that participants in the futures markets can make production, storage, and processing decisions by looking at the pattern of futures prices, even if they don't take positions in that market."

expected to be affected by disclosure of firms. The question is then whether and how incorporating this realistic feature of learning from asset prices affects the information-sharing incentives of firms and the equilibrium disclosure policies in oligopoly settings.

In this paper, I develop a model to study these questions. My model builds on the standard information-sharing duopoly setting with demand uncertainty and Cournot competition (e.g., Vives, 1984; Gal-Or, 1985; Darrough, 1993), where two firms first unilaterally decide whether to reveal their signals about product demand, and then, after receiving signals and possibly revealing them, compete in production quantities. In the standard setting without learning from asset prices, no information sharing is the dominant strategy for firms and forms the unique Nash equilibrium. This is because disclosure reveals strategic information to competitors and reduces the disclosing firm's competitive advantage, which is referred to as the "proprietary cost" (Darrough, 1993) or "competitive disadvantage cost"/"loss of competitive advantage" (Bhattacharya and Ritter, 1983; Foster, 1986). For instance, high demand of the disclosing firm may be indicative of high demand for competitors (i.e., "a rising tide lifts all boats"), which encourages competitors to expand their production, eroding the disclosing firm's profits.

I extend the standard setting by introducing a financial market, or more specifically, a futures market. The futures contract is on the commodity produced by the two competing firms. Financial speculators, such as hedge funds or commodity index traders, trade the futures contracts (against liquidity traders) based on their private information about the later product demand. This information is aggregated into the futures price. It is natural that the speculators' information and the firms' information are not identical and thus, firms look into the futures price to extract new information possessed by speculators to guide real production decisions; that is, firms learn information from asset prices.

In this extended setting, firms face the following trade-off in deciding on their disclosure policies. The negative effect of disclosure is still the proprietary cost iden-

tified in the previous literature (e.g., Vives, 1984; Gal-Or, 1985; Darrough, 1993). The positive effect of disclosure comes from a more informative asset price that improves firms' learning quality. Specifically, the payoff on the futures contract is driven by three pieces of demand shocks, which are observed respectively by the two firms and financial speculators. So, publicly releasing the private information of firms reduces the uncertainty faced by financial speculators. This encourages risk-averse speculators to trade more aggressively on their private information. In consequence, the futures price will aggregate more of speculators' private information, benefiting firms' learning from the asset price. Each firm weighs this benefit of improved learning from the asset price against the proprietary cost to determine its optimal disclosure policy.

There are two types of equilibrium in my setting: a nondisclosure equilibrium, in which firms do not disclose any information; and a partial disclosure equilibrium, in which firms voluntarily disclose their private information with added noises. This result arises in sharp contrast to the literature on Cournot/common-value duopoly settings which shows that firms never disclose their private information about market demand (e.g., Gal-Or, 1985; Darrough, 1993). In my setting, the nondisclosure equilibrium is more likely to prevail only when financial speculators know less information and when the financial market features less noise trading. This is because under both conditions, firms have a weaker incentive to learn from the financial market. When speculators know little information, firms do not have much to learn from speculators via the asset price. When there is little noise trading in the financial market, the asset price has already aggregated speculators' information very well and thus, the scope to improve price informativeness via disclosure is small.

I show that firms' disclosure decisions can be a strategic complement. Complementarity arises when there is a lot noise trading in the financial market. If this complementarity is sufficiently strong, both a disclosure equilibrium and a nondisclosure equilibrium can be supported. This multiplicity result also runs in sharp contrast to the information-sharing literature which shows that there always exists a unique

equilibrium. When multiplicity arises, both firms are better off on the disclosure equilibrium than on the nondisclosure equilibrium for two reasons. First, disclosure of each firm directly benefits its rival by releasing new information about product demand. Second, disclosure of both firms reduces the uncertainty faced by speculators who in turn trade more aggressively on their information. This makes the asset price more informative, thereby benefiting both firms. Taken together, it is in the firms' interests to coordinate on the disclosure equilibrium.

The most striking result in my analysis is that allowing minimal learning of firms from the asset price can dramatically change firms' equilibrium disclosure behavior. Specifically, as mentioned before, in a standard setting without learning from prices, firms do not disclose at all. That is, the equilibrium disclosure precision is zero in a setting in which the size of noise trading is infinity. Now consider a setting with learning from prices and suppose that there is a lot noise trading so that multiple equilibria arise. As argued above, firms prefer to coordinate on the disclosure equilibrium. It can be shown that as the size of noise trading diverges to infinity, firms' disclosure precision also diverges to infinity. Thus, there is a discontinuity of disclosure policy at infinitely large noise trading. Intuitively, when the noise trading at the financial market is infinity, firms cannot at all learn from the asset price and so the benefit of disclosure disappears, leading to the nondisclosure equilibrium. However, when the noise trading is finite (although large) so that firms can learn from the asset price, they coordinate on a very aggressive disclosure equilibrium to improve the informativeness of asset prices, which is beneficial for both firms.

Finally, I examine the welfare effect of allowing firms to learn from the asset price. Relative to a setting without learning from asset prices, in a setting with learning from prices, firms, consumers, and liquidity traders are better off, and only financial traders are worse off. Allowing firms to learn from asset prices benefits firms and consumers both directly and indirectly. First, because firms have an extra signal (which is the asset price), they make more informed decisions, benefiting both

firms and consumers. Second, in a setting with learning from prices, firms are more likely to disclose information, and the extra disclosed information also benefits firms and consumers. Liquidity traders benefit mainly from the extra disclosure, which improves market liquidity and therefore lowers liquidity traders' transaction costs. Financial speculators lose also because of the extra disclosure that limits the benefits of speculators in trading the risky futures contracts.

1.1. Related Literature

This paper is broadly related to two strands of literature. First, it contributes to the classic literature on information sharing of firms in oligopoly (e.g., Gal-Or, 1986; Darrough, 1993; Raith, 1996; Vives, 1984, 2008). As mentioned before, this literature shows that firms choose not to disclose at all in settings of Cournot/common-value and Bertrand/private-value, while they choose to share information completely in settings of Cournot/private-value and Bertrand/common-value.

My paper builds on a Cournot/common-value setting which features the proprietary cost. My analysis extends the canon of existing studies to include the realistic feature that firms learn information from asset prices. This extension generates two novel insights. First, firms either choose not to disclose information at all, or to disclose information to the public, and if they disclose, they only disclose information partially. This differs from the literature which finds that firms do not disclose in a Cournot/common-value setting. Second, in the presence of learning from asset prices, firms' disclosure decisions can be a strategic complement, which gives rise to multiple equilibria. This also differs from the unique nondisclosure equilibrium identified in the standard setting. When multiplicity arises in my setting, it is more likely for firms to coordinate on the disclosure equilibrium, and on this coordinated disclosure equilibrium, the disclosure precision goes to infinity as the noise in the financial market becomes extremely volatile. This shows that adding minimal learning from prices can dramatically change the equilibrium disclosure behavior of firms.

The second related strand of literature is the literature on the real effect of a financial market, where trading and prices in a financial market affects production decisions, which in turn affect the traded asset's cash flows. This effect is known as the "feedback effect," which is reviewed by Bond, Edmans, and Goldstein (2012). Several papers provide supporting empirical evidence; see, e.g., Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Frésard (2014).

A few recent papers study the effect of disclosure in contexts that feature a feedback effect and the like. Gao and Liang (2013) show that disclosure crowds out private-information production, which reduces price informativeness and harms managers' learning and investments. Banerjee, Davis, and Gondhi (2017) show that public information can lower price efficiency by encouraging traders choose to acquire non-fundamental information exclusively. Han, Tang, and Yang (2016) show that disclosure attracts noise trading that reduces price informativeness and harms managers' learning quality. Amador and Weill (2010) show that releasing public information about monetary and/or productivity shocks can reduce welfare through reducing the informational efficiency of the good price system. Goldstein and Yang (2018) show that disclosure can be either good or bad, depending on whether disclosure is about the dimension about which the firm already knows. In contrast, in my paper, disclosure benefits rather than harms firms via the feedback effect, and the cost of disclosure is endogenously generated from losing a competitive advantage (the proprietary cost) that is unique to the oligopoly setting. In addition, almost all the existing studies are conducted in a one-firm setting, while my analysis features multiple firms, and this multi-firm feature generates coordination disclosure motives among firms, which leads to the possibility of multiple disclosure equilibria.

The positive effect of disclosure in my paper is related to the "residual risk effect" in Bond and Goldstein (2015) and the "uncertainty reduction effect" in Goldstein and Yang (2015). That is, releasing information about shocks that are unknown to traders reduces the uncertainty faced by traders. Since traders are risk averse, the re-

duction in risk incentivizes them to trade more on their information. In consequence, the price will aggregate more of traders' private information, benefiting the firms' learning from the asset price. My analysis incorporates this uncertainty reduction effect into a standard information-sharing duopoly setting that has so far ignored the realistic feature that firms can extract information from financial markets to guide production decisions. This extension has yielded many novel insights. For instance, unlike in the standard information-sharing setting where firms hide information, in my setting, firms can disclose information, and in some cases, multiple equilibria can be supported. My analysis also shows that allowing minimal learning from asset prices can dramatically change firms' disclosure behavior.

2. The Model

I consider a standard information-sharing duopoly setting (e.g., Vives, 1984; Gal-Or, 1985; Darrough, 1993), which is extended with a financial market, or more specifically, with a futures market on the commodity produced by two competitive firms. There are three dates, $t = 0, 1$, and 2. On date 0, two competing firms, firm A and firm B, simultaneously decide on their disclosure policies. On date 1, financial speculators and liquidity traders trade commodity futures. Financial speculators are endowed with private information about the later demand for the firms' products, which is aggregated into the equilibrium futures price. Firms make inference on this information from the futures price to guide their production decisions. On date 2, the product market opens and the product price is determined.

2.1. Consumers: Demand for Products

The date-2 demand for firms' products is generated by a representative consumer who maximizes consumer surplus,

$$\mathcal{C}\left(Q, \theta_A, \theta_B, \delta\right) = \mathcal{U}\left(Q, \theta_A, \theta_B, \delta\right) - pQ, \tag{1}$$

where Q is the amount of good purchased from the firms and p is the product price. In (1), $\mathcal{U}(Q, \theta_A, \theta_B, \delta)$ captures the consumer's intrinsic utility from consuming the products, while the term pQ is the cost of purchasing the products. Following the literature (e.g., Singh and Vives, 1984), I specify a quasi-linear intrinsic utility function as follows:

$$\mathcal{U}(Q, \theta_A, \theta_B, \delta) = (m + \theta_A + \theta_B + \delta) Q - \frac{Q^2}{2}. \tag{2}$$

Parameter m is a positive constant, which captures the size of the product market. Variables θ_A, θ_B, and δ are three mutuality independent demand shocks that are normally distributed; that is, $\theta_A \sim N\left(0, \tau_\theta^{-1}\right)$, $\theta_B \sim N\left(0, \tau_\theta^{-1}\right)$, and $\delta \sim N\left(0, \tau_\delta^{-1}\right)$ (with $\tau_\theta > 0$ and $\tau_\delta > 0$). These three demand shocks are observed by firm A, firm B, and financial speculators, respectively.

The representative consumer chooses product quantity Q to maximize her preference (1) taking the product price p as given. This maximization problem leads to the following standard linear inverse demand function for firms' products:

$$p = (m + \theta_A + \theta_B + \delta) - Q. \tag{3}$$

For the sake of simplicity, I have assumed that both firms produce identical products. Alternatively, I can assume that firms produce differentiated products and the results do not change under this alternative assumption.

2.2. Firms: Information Disclosure and Goods Production

The two firms make two decisions in the economy, a disclosure-policy decision on date 0 and a goods-production decision on date 1. Their production decisions determine the supply of products in the product market. Following Darrough (1993), I assume that on date 0, firms A and B respectively observe demand shocks θ_A and θ_B.[3] Firms

[3] As mentioned in the Introduction, the information-sharing literature shows that the concern that disclosure reveals strategic information to competitors and reduces the disclosing firm's competitive advantage—i.e., the "proprietary cost" (Darrough, 1993) or "competitive disadvantage cost" (Foster, 1986)—arise in Cournot/common-value and Bertrand/private-value settings. Therefore, I assume that firms face demand uncertainty (common value) and engage in Cournot competition to capture the proprietary cost induced by disclosure.

precommit themselves in advance to a particular disclosure policy ex ante before they receive their private information. Such a commitment may be coordinated and enforced by trade associations or regulatory agencies such as the FASB or the SEC. Firm A discloses a noisier version of θ_A to the public in the form of

$$x = \theta_A + \eta,$$

where $\eta \sim N\left(0, \tau_\eta^{-1}\right)$ (with $\tau_\eta \in [0, \infty]$) and η is independent of all other shocks. Similarly, firm B discloses a nosier version of θ_B in the form of

$$y = \theta_B + \xi,$$

where $\xi \sim N\left(0, \tau_\xi^{-1}\right)$ (with $\tau_\xi \in [0, \infty]$) and ξ is independent of all other shocks.

The random variables η and ξ are the noises added respectively by the two firms in their disclosed signals. The precision levels τ_η and τ_ξ are chosen by the firms at the beginning of date 0 to maximize their unconditional expected profits. In particular, I allow τ_η and τ_ξ to take values of 0 and ∞, which correspond respectively to the case in which the firms do not disclose (i.e., disclose with infinite noise) and to the case in which the firms disclose their private information perfectly (i.e., disclose without noise). In the literature, these two values are the only possible equilibrium choices (see the survey by Vives (2008)). By contrast, I will show that in the presence of learning from asset prices, firms can choose to disclose their information imperfectly (i.e., $\tau_\eta \in (0, \infty)$ and $\tau_\xi \in (0, \infty)$).

On date 1, firms make production decisions to maximize profits based on private and public information. As mentioned above, firm A's private information is θ_A and firm B's private information is θ_B. There are three pieces of public information: public disclosure x released by firm A, public disclosure y released by firm B, and the price f of a financial asset. The innovation of this paper is that firms extract information from the asset price f to guide their production decisions. As discussed in the Introduction, the literature labels this feature as the "feedback effect," whereby trading and prices in financial markets affect real investment decisions, which in turn affect the financial asset's payoffs (see the review article by Bond, Edmans, and

Goldstein (2012)). In my setup, I assume that the financial asset is a futures contract on the commodity produced by the two firms, and I will discuss its price formation in the next subsection.

I normalize the marginal cost of production as 0. As known in the literature, this normalization does not affect the results. Under this normalization, firm i's profit is

$$\Pi_i\left(q_i, q_j, \theta_A, \theta_B, \delta\right) = pq_i = \left(m + \theta_A + \theta_B + \delta - q_j\right) q_i - q_i^2, \tag{4}$$

for $i, j \in \{A, B\}$ and $i \neq j$. Variables q_i and q_j are respectively the amount of goods produced by the two firms. The second equality in (4) follows from the inverse demand function (3) and $Q = q_A + q_B$. Thus, the optimal date-1 production q_i^* of firm i is determined by

$$\max_{q_i} E\left[\Pi_i\left(q_i, q_j^*, \theta_A, \theta_B, \delta\right) \middle| \theta_i, x, y, f\right],$$

where $E\left[\cdot \middle| \theta_i, x, y, f\right]$ is the conditional expectation operator and q_j^* refers to firm j's optimal production, which is taken as given in firm i's production decision problem.

The optimal date-0 disclosure decision τ_η^* of firm A is determined by

$$\max_{\tau_\eta} E\left[\Pi_A\left(q_A^*, q_B^*, \theta_A, \theta_B, \delta\right)\right].$$

Similarly, the optimal date-0 disclosure decision τ_ξ^* of firm B is determined by

$$\max_{\tau_\xi} E\left[\Pi_B\left(q_A^*, q_B^*, \theta_A, \theta_B, \delta\right)\right].$$

When making the disclosure policy choice, each firm takes the other firm's disclosure policy as given and also takes into account how its own disclosure affects the optimal production decisions of both firms in the product market.

2.3. Financial Market

On date 1, a financial market opens. There are two tradable assets: a futures contract and a risk-free asset. I normalize the net risk-free rate as 0. The payoff on the futures contract is the date-2 product spot price p. Each unit of futures contract is traded at an endogenous price f. The total supply of futures contracts is 0.

There are two groups of market participants: financial speculators and liquidity

traders. Liquidity traders represent random transient demands in the futures market and they as a group demand u units of the commodity futures, where $u \sim N(0, \tau_u^{-1})$ with $\tau_u \in (0, \infty)$. As usual, liquidity traders, also known as "noise traders," provide the randomness (noise) necessary to make the rational expectations equilibrium partially revealing. I do not endogenize the behavior of liquidity traders; rather, I view them as individuals who are trading to invest new cash flows or to liquidate assets to meet unexpected consumption needs.

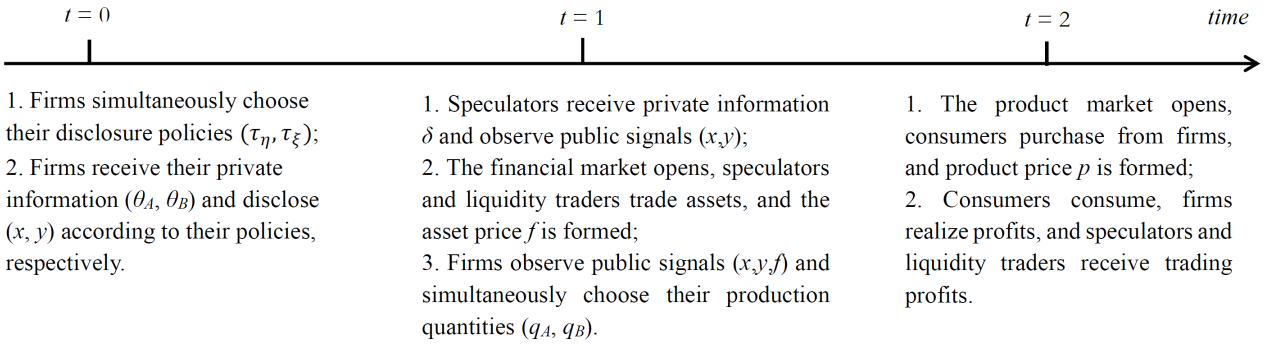
There is a continuum $[0, 1]$ of financial speculators who derive expected utility only from their date-2 wealth. They have constant absolute risk aversion (CARA) utility functions with a common coefficient of risk aversion $\gamma > 0$. Speculators are endowed with cash only, and for simplicity I suppose that their endowment is 0. These traders can be interpreted as hedge funds or commodity index traders.[4] Financial speculators privately observe demand shock δ and thus their trading injects this information into the futures price f.

Three remarks are in order. First, for simplicity, I have assumed that the private information δ of speculators is independent of the private information θ_A and θ_B of firms. This assumption is not crucial for driving the result. What matters is that speculators as a group own some information which is new to firms, so that firms learn information from the asset price (which is the key feature in the literature on feedback effects). Second, I have assumed that speculators observe identical information. A more realistic view is that they own disperse information (potentially very coarse) which is aggregated into the price, leading to a very valuable signal to firms (e.g., Hayek (1945)). I do not take this alternative view for the sake of analytical tractability,[5] and the current setup is sufficient for modeling the feature that firms

[4] According to Cheng and Xiong (2014, p. 424), "(o)ver the past decade, there has been a large inflow of investment capital from a class of investors, so-called commodity index traders (CITs), also known as index speculators."

[5] Specifically, the current setup allows me to first analytically compute the product market equilibrium, which is then inserted into the speculators' demand function and the market-clearing condition to compute the financial market equilibrium. By contrast, in a setting with diverse signals, I have to simultaneously solve the product market equilibrium and the financial market equilibrium.

Figure 1: Model Timeline



learn from asset prices. Third, firms do not participate in the futures market, which allows me to isolate the informational role of asset prices in driving the results.

2.4. Timeline

The order of events is described in Figure 1. On date 0, firms simultaneously choose their disclosure policies, τ_η and τ_ξ. Firms then receive their private information (θ_A, θ_B) and disclose public signals (x, y) according to their policies, respectively. On date 1, financial speculators observe private information δ and public information (x, y), and trade futures against liquidity traders, which determines the asset price f. Firms observe public information (x, y) and the asset price f, and simultaneously choose their production quantities. On date 2, the product market opens, consumers purchase from firms, and product price p is formed. Finally, consumers consume, firms realize profits, and speculators and liquidity traders receive trading profits.

3. The Equilibrium

Following the literature, I consider symmetric equilibrium in which both firms choose the same disclosure policy (i.e., $\tau_\eta^* = \tau_\xi^*$). As Gal-Or (1985, p. 330) argued,

"(s)ymmetric equilibrium is a reasonable solution concept for this model since all firms face the same technology and observe signals of the same precision."

Definition 1. *An equilibrium consists of date-1 disclosure policies of firms* $\left(\tau_\eta^*, \tau_\xi^*\right)$*, date-1 production policies of firms* $q_A(\theta_A, x, y, f)$ *and* $q_B(\theta_B, x, y, f)$*, a date-1 trading strategy of speculators* $D(\delta, x, y, f)$*, a date-1 futures price function* $f(\delta, x, y, u)$*, and a date-2 spot price function* $p(\theta_A, \theta_B, \delta, x, y, f)$*, such that:*

(a) Disclosure policies $\left(\tau_\eta^*, \tau_\xi^*\right)$ *form a Nash equilibrium, i.e.,*

$$\begin{aligned}\tau_\eta^* &= \arg\max_{\tau_\eta} E\left[\Pi_A\left(q_A(\theta_A, x, y, f), q_B(\theta_B, x, y, f), \theta_A, \theta_B, \delta\right)\right],\\ \tau_\xi^* &= \arg\max_{\tau_\xi} E\left[\Pi_B\left(q_A(\theta_A, x, y, f), q_B(\theta_B, x, y, f), \theta_A, \theta_B, \delta\right)\right];\end{aligned}$$

(b) Trading strategy $D(\delta, x, y, f)$ *and futures price function* $f(\delta, x, y, u)$ *form a noisy rational expectations equilibrium in the financial market, i.e.,*

$$\begin{aligned}D(\delta, x, y, f) &= \arg\max_{D} E\left[-e^{-\gamma D[p(\theta_A, \theta_B, \delta, x, y, f) - f(\delta, x, y, u)]} \middle| \delta, x, y, f\right],\\ D(\delta, x, y, f) + u &= 0;\end{aligned}$$

(c) Production policies $q_A(\theta_A, x, y, f)$ *and* $q_B(\theta_B, x, y, f)$ *form a Bayesian-Nash equilibrium in the product market, i.e.,*

$$\begin{aligned}q_A(\theta_A, x, y, f) &= \arg\max_{q_A} E\left[\Pi_A\left(q_A, q_B(\theta_B, x, y, f), \theta_A, \theta_B, \delta\right) \middle| \theta_A, x, y, f\right],\\ q_B(\theta_B, x, y, f) &= \arg\max_{q_B} E\left[\Pi_B\left(q_A(\theta_A, x, y, f), q_B, \theta_A, \theta_B, \delta\right) \middle| \theta_B, x, y, f\right];\ \textit{and}\end{aligned}$$

(d) The spot price $p(\theta_A, \theta_B, \delta, x, y, f)$ *clears the product market, i.e.,*

$$q_A(\theta_A, x, y, f) + q_B(\theta_B, x, y, f) = (m + \theta_A + \theta_B + \delta) - p(\theta_A, \theta_B, \delta, x, y, f).$$

A linear equilibrium is an equilibrium in which policy functions and price functions are linear.

Depending on whether firms disclose information in equilibrium, there are two types of equilibrium as defined below.

Definition 2. *If* $\tau_\eta^* = \tau_\xi^* = 0$*, then the equilibrium is referred to as the "nondisclosure equilibrium." If* $\tau_\eta^* = \tau_\xi^* > 0$*, then the equilibrium is referred to as a "disclosure equilibrium."*

Before formally characterizing the equilibrium, I first analyze a benchmark setting in which firms do not learn from a financial market.

3.1. A Benchmark Setting without Feedback Effects

If I shut down the feature that firms learn information from the asset price f, the model degenerates to a standard information-sharing setting with demand shocks and Cournot competition. As well-known in the literature (e.g., Gal-Or, 1985; Darrough, 1993), concealing information is a dominant strategy, so that both firms choose not to disclose information in equilibrium. This is because disclosure reveals strategic information to competitors, thereby reducing the disclosing firm's competitive advantage. I summarize the equilibrium of this benchmark setting in the following proposition, where I label variables with superscript "$\varnothing$" to indicate that in this setting, firms do not extract information from an asset price. The proof is standard and hence omitted.

Proposition 1. (No Learning from Asset Prices) *In a setting where firms do not learn information from a financial market, there exists a unique linear Bayesian-Nash equilibrium in the product market for given disclosure policies* (τ_η, τ_ξ)*, in which*

$$\begin{aligned} q_A^{\varnothing} &= \frac{m}{3} + \frac{1}{2}\theta_A - \frac{\tau_\eta}{6\left(\tau_\theta + \tau_\eta\right)}x + \frac{\tau_\xi}{3\left(\tau_\theta + \tau_\xi\right)}y, \\ q_B^{\varnothing} &= \frac{m}{3} + \frac{1}{2}\theta_B + \frac{\tau_\eta}{3\left(\tau_\theta + \tau_\eta\right)}x - \frac{\tau_\xi}{6\left(\tau_\theta + \tau_\xi\right)}y, \end{aligned}$$

and on date 0, no firm chooses to disclose information, i.e., $\tau_\eta^{\varnothing} = \tau_\xi^{\varnothing} = 0$.

In the following two subsections, I will derive the equilibrium in a setting where firms learn information from the financial market. There will be two main results that differ from Proposition 1. First, firms may choose to disclose information on date 0, i.e., $\tau_\eta^* = \tau_\xi^* > 0$ for some parameters. Second, there may exist multiple equilibria due to the coordination motives across firms, that is, it is possible that both $\tau_\eta^* = \tau_\xi^* = 0$ and $\tau_\eta^* = \tau_\xi^* > 0$ can be supported as an equilibrium.

3.2. Equilibria in Product Market and Financial Market

Following the literature (e.g., Gal-Or, 1985; Darrough, 1993), I consider linear Bayesian-Nash equilibria in the product market. That is, the production policies of firms A and B are linear in their information variables as follows:

$$q_A^* = a_0 + a_\theta \theta_A + a_x x + a_y y + a_f f, \tag{5}$$

$$q_B^* = b_0 + b_\theta \theta_B + b_x x + b_y y + b_f f, \tag{6}$$

where the a coefficients and the b coefficients are endogenous constants.

The optimal productions q_A^* and q_B^* are determined respectively by the first-order conditions (FOCs) of the profit-maximization problems in Part (c) of Definition 1 (the second-order conditions (SOCs) are always satisfied),

$$q_A^* = \frac{E\left(m + \theta_A + \theta_B + \delta - q_B^* | \theta_A, x, y, f\right)}{2}, \tag{7}$$

$$q_B^* = \frac{E\left(m + \theta_A + \theta_B + \delta - q_A^* | \theta_B, x, y, f\right)}{2}. \tag{8}$$

A Bayesian-Nash equilibrium requires that the above implied policy functions (7)–(8) agree with the conjectured policy functions (5)–(6). In doing so, one needs to express out the conditional moments in (7)–(8), namely to figure out how each firm uses both private and public information (in particular, the asset price f) to forecast later demand shocks and its opponent's production.

Take firm A as an example. Inserting the conjectured production policy (6) of firm B into the FOC (7) of firm A's profit-maximization problem yields

$$q_A^* = \frac{m + \theta_A + E\left(\delta | \theta_A, x, y, f\right) + \left(1 - b_\theta\right) E\left(\theta_B | \theta_A, x, y, f\right) - \left(b_0 + b_x x + b_y y + b_f f\right)}{2}. \tag{9}$$

So, firm A needs to forecast two variables, θ_B and δ. The idea is that the public signal y disclosed by firm B is useful for predicting θ_B, while the asset price f, together with public disclosure x and y, is useful for predicting δ, because the trading of speculators injects information on δ into the futures price f. I now turn to the futures market to figure out how firms extract information from the asset price f.

Solving the speculators' utility-maximization problem in Part (b) of Definition 1

gives rise to their demand function under CARA preference,

$$D\left(\delta,x,y,f\right)=\frac{E\left(p|\delta,x,y,f\right)-f}{\gamma Var\left(p|\delta,x,y,f\right)}, \tag{10}$$

where $E\left(\cdot|\,\delta,x,y,f\right)$ and $Var\left(\cdot|\,\delta,x,y,f\right)$ are the conditional expectation and variance, respectively. Inserting the conjectured policy functions (5)–(6) into the market-clearing condition of product market in Part (d) of Definition 1 yields

$$\begin{aligned} p &= \left(1-a_\theta\right)\theta_A+\left(1-b_\theta\right)\theta_B \\ &\quad +\left(m-a_0-b_0\right)+\delta-\left(a_x+b_x\right)x-\left(a_y+b_y\right)y-\left(a_f+b_f\right)f. \end{aligned} \tag{11}$$

Since speculators observe $\{\delta,x,y,f\}$, they only need to forecast $\left(1-a_\theta\right)\theta_A+\left(1-b_\theta\right)\theta_B$ in the above expression of p. In doing so, speculators use public information x to predict θ_A and public information y to predict θ_B. Applying Bayes' rule to compute $E\left(p|\delta,x,y,f\right)$ and $Var\left(p|\delta,x,y,f\right)$, which are in turn inserted into demand function (10) and the market-clearing condition of the futures market, $D\left(\delta,x,y,f\right)+u=0$, I derive the futures price function as follows:

$$\begin{aligned} f &= \frac{m-a_0-b_0}{a_f+b_f+1}+\frac{\delta}{a_f+b_f+1} \\ &\quad +\frac{\frac{\left(1-a_\theta\right)\tau_\eta}{\tau_\theta+\tau_\eta}-\left(a_x+b_x\right)}{a_f+b_f+1}x+\frac{\frac{\left(1-b_\theta\right)\tau_\xi}{\tau_\theta+\tau_\xi}-\left(a_y+b_y\right)}{a_f+b_f+1}y \\ &\quad +\frac{\gamma\left[\frac{\left(1-a_\theta\right)^2}{\tau_\theta+\tau_\eta}+\frac{\left(1-b_\theta\right)^2}{\tau_\theta+\tau_\xi}\right]}{a_f+b_f+1}u. \end{aligned} \tag{12}$$

Thus, to firm A, the futures price f is equivalent to the following signal in predicting demand shock δ:

$$\begin{aligned} s &\equiv \left(a_f+b_f+1\right)f-\left(m-a_0-b_0\right) \\ &\quad -\left[\frac{\left(1-a_\theta\right)\tau_\eta}{\tau_\theta+\tau_\eta}-\left(a_x+b_x\right)\right]x-\left[\frac{\left(1-b_\theta\right)\tau_\xi}{\tau_\theta+\tau_\xi}-\left(a_y+b_y\right)\right]y \\ &= \delta+\gamma\left[\frac{\left(1-a_\theta\right)^2}{\tau_\theta+\tau_\eta}+\frac{\left(1-b_\theta\right)^2}{\tau_\theta+\tau_\xi}\right]u, \end{aligned} \tag{13}$$

which has an endogenous precision level of

$$\tau_s=\frac{\tau_u}{\gamma^2\left[\frac{\left(1-a_\theta\right)^2}{\tau_\theta+\tau_\eta}+\frac{\left(1-b_\theta\right)^2}{\tau_\theta+\tau_\xi}\right]^2}. \tag{14}$$

The signal s formalizes the fact that firms learn information about δ from the asset

price f, and its precision τ_s captures the informational content in the asset price. I follow the literature and refer to variable τ_s as "price informativeness."

Firm A's information set $\{\theta_A, x, y, f\}$ is equivalent to $\{\theta_A, x, y, s\}$, among which y and s are respectively useful for predicting demand shocks θ_B and δ. Applying Bayes' rule to compute $E(\delta|\theta_A, x, y, f) = E(\delta|s)$ and $E(\theta_B|\theta_A, x, y, f) = E(\theta_B|y)$ and combining with the expression of s in (13), I can express q_A^* in (9) as functions of (θ_A, x, y, f). Comparing this expression with the conjectured policy in (5), I can form five conditions in terms of the unknown a coefficients and b coefficients. Conducting a similar analysis for firm B leads to another five conditions in terms of a's and b's. Solving this system of ten equations yields the values of a's and b's. Finally, inserting the values of a's and b's into equations (11) and (12) gives rise to the spot price function and the futures price function, respectively.

Proposition 2. (Product and Futures Markets) *For any disclosure polices (τ_η, τ_ξ), there exists a unique linear Bayesian-Nash equilibrium in the product market, in which*

$$\begin{aligned} q_A^* &= a_0 + a_\theta \theta_A + a_x x + a_y y + a_f f, \\ q_B^* &= b_0 + b_\theta \theta_B + b_x x + b_y y + b_f f, \end{aligned}$$

where

$$\begin{aligned} a_0 &= b_0 = \frac{\tau_\delta}{\tau_s + 3\tau_\delta} m, a_\theta = b_\theta = \frac{1}{2}, \\ a_x &= -\frac{\tau_s + \tau_\delta}{2(\tau_s + 3\tau_\delta)} \frac{\tau_\eta}{\tau_\theta + \tau_\eta}, b_x = \frac{\tau_\delta}{\tau_s + 3\tau_\delta} \frac{\tau_\eta}{\tau_\theta + \tau_\eta}, \\ a_y &= \frac{\tau_\delta}{\tau_s + 3\tau_\delta} \frac{\tau_\xi}{\tau_\theta + \tau_\xi}, b_y = -\frac{\tau_s + \tau_\delta}{2(\tau_s + 3\tau_\delta)} \frac{\tau_\xi}{\tau_\theta + \tau_\xi}, \\ a_f &= b_f = \frac{\tau_s}{\tau_s + 3\tau_\delta}, \end{aligned}$$

and

$$\tau_s = \frac{\tau_u}{\gamma^2 \left(\frac{1}{4(\tau_\theta + \tau_\eta)} + \frac{1}{4(\tau_\theta + \tau_\xi)} \right)^2}.$$

The date-2 spot price function is

$$\begin{aligned} p &= \frac{\tau_s+\tau_\delta}{\tau_s+3\tau_\delta}m+\frac{1}{2}\theta_A+\frac{1}{2}\theta_B+\delta \\ &\quad -\frac{\tau_\delta-\tau_s}{2\left(\tau_s+3\tau_\delta\right)}\frac{\tau_\eta}{\tau_\theta+\tau_\eta}x-\frac{\tau_\delta-\tau_s}{2\left(\tau_s+3\tau_\delta\right)}\frac{\tau_\xi}{\left(\tau_\theta+\tau_\xi\right)}y-\frac{2\tau_s}{\tau_s+3\tau_\delta}f. \end{aligned}$$

The date-1 futures price function is

$$f=\frac{1}{3}m+\frac{\tau_s+3\tau_\delta}{3\left(\tau_s+\tau_\delta\right)}\delta+\frac{\tau_\eta}{3\left(\tau_\theta+\tau_\eta\right)}x+\frac{\tau_\xi}{3\left(\tau_\theta+\tau_\xi\right)}y+\frac{\tau_s+3\tau_\delta}{3\left(\tau_s+\tau_\delta\right)}\sqrt{\frac{\tau_u}{\tau_s}}u.$$

By the expression of τ_s in Proposition 2, disclosing information improves firms' learning quality from the asset price. Intuitively, demand shocks θ_A and θ_B in the spot price p in (11) are the uncertainty exposed to speculators when they trade futures contracts. Releasing information about these two shocks reduces the uncertainty faced by speculators. Being risk averse, speculators then trade more aggressively on their own private information δ, thereby injecting more information on δ into the futures price f. This effect shares a similar flavor as the "residual risk effect" in Bond and Goldstein (2015) and the "uncertainty reduction effect" in Goldstein and Yang (2015).

Corollary 1. (Price Informativeness) *Disclosure of firms improves the informational content of the asset price. That is,* $\frac{\partial\tau_s}{\partial\tau_\eta}>0$ *and* $\frac{\partial\tau_s}{\partial\tau_\xi}>0$.

3.3. Equilibrium Disclosure Policy

3.3.1. Profit Function

At the beginning of date 0, firms choose disclosure policies to maximize unconditional expected profits. Again, take firm A as an example. Using the FOC of firm A's profit-maximization problem in Part (c) of Definition 1 and the equilibrium production policy in Proposition 2, I can compute firm A's expected profit as follows:

$$E\Pi_A\left(\tau_\eta,\tau_\xi\right)=\underbrace{\frac{m^2}{9}}_{\text{market size}}+\underbrace{\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}}_{\text{proprietary cost}}+\underbrace{\frac{\tau_\xi}{9\tau_\theta\left(\tau_\theta+\tau_\xi\right)}}_{\text{disclosure by firm }B}+\underbrace{\frac{\tau_s}{9\tau_\delta\left(\tau_s+\tau_\delta\right)}}_{\text{learning from prices}}. \quad (15)$$

Here, I explicitly express $E\Pi_A$ as functions of disclosure precision (τ_η,τ_ξ) to emphasize the dependence of expected profit on disclosure policies. Firm A chooses its optimal

disclosure policy τ_η^* to maximize $E\Pi_A\left(\tau_\eta, \tau_\xi^*\right)$, taking as given the optimal disclosure τ_ξ^* of firm B.

There are four terms that go into firm A's expected profit in (15). The first term $\frac{m^2}{9}$ is simply the size of the product market. Disclosure has no effect on this term. The second term $\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta(\tau_\theta+\tau_\eta)}$ captures the "proprietary cost" (Darrough, 1993) or "competitive disadvantage cost" (Foster, 1986), whereby disclosing private information reduces the disclosing firm's competitive advantage. Disclosure harms firm A's profits via this second term; that is, $\frac{\partial}{\partial\tau_\eta}\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta(\tau_\theta+\tau_\eta)} < 0$. The third term $\frac{\tau_\xi}{9\tau_\theta\left(\tau_\theta+\tau_\xi\right)}$ captures the benefit from observing the public signal disclosed by the competing firm B, which is determined by firm B's disclosure precision τ_ξ and so independent of firm A's disclosure precision τ_η. The last term $\frac{\tau_s}{9\tau_\delta(\tau_s+\tau_\delta)}$ represents the benefit from learning from the asset price f. Disclosure benefits firm A via this last term. That is, $\frac{\partial}{\partial\tau_\eta}\frac{\tau_s}{9\tau_\delta(\tau_s+\tau_\delta)} = \frac{1}{9(\tau_s+\tau_\delta)^2}\frac{\partial\tau_s}{\partial\tau_\eta} > 0$, since $\frac{\partial\tau_s}{\partial\tau_\eta} > 0$ by Corollary 1.

In sum, the trade-off faced by firm A in the disclosure choice can be captured by the following FOC:

$$\frac{\partial E\Pi_A\left(\tau_\eta, \tau_\xi\right)}{\partial\tau_\eta} = \underbrace{-\frac{5}{36\left(\tau_\theta+\tau_\eta\right)^2}}_{<0,\text{ proprietary cost}} + \underbrace{\frac{1}{9\left(\tau_s+\tau_\delta\right)^2}\frac{\partial\tau_s}{\partial\tau_\eta}}_{>0,\text{ learning from prices}}. \tag{16}$$

That is, disclosing private information harms firm A via the proprietary cost but benefits firm A via improving price informativeness.

3.3.2. Disclosure Policy Characterization

The equilibrium disclosure policies $\left(\tau_\eta^*, \tau_\xi^*\right)$ form a Nash equilibrium. That is,

$$\tau_\eta^* = \arg\max_{\tau_\eta} E\Pi_A\left(\tau_\eta, \tau_\xi^*\right) \text{ and } \tau_\xi^* = \arg\max_{\tau_\xi} E\Pi_B\left(\tau_\eta^*, \tau_\xi\right),$$

where firm A's profit function $E\Pi_A\left(\tau_\eta, \tau_\xi\right)$ is given by equation (15) and firm B's profit function $E\Pi_B\left(\tau_\eta, \tau_\xi\right)$ is defined similarly. There are two types of disclosure policies in a symmetric equilibrium: (1) a "nondisclosure equilibrium," where both firms do not disclose information (i.e., $\tau_\eta^* = \tau_\xi^* = 0$); and (2) a "disclosure equilibrium," where both firms disclose information (i.e., $\tau_\eta^* = \tau_\xi^* > 0$). The following two

theorems respectively characterize these two types of equilibrium.

Theorem 1. (Nondisclosure Equilibrium) *In equilibrium, both firms choose not to disclose information (i.e., $\tau_\eta^* = \tau_\xi^* = 0$) if and only if one of the following two sets conditions holds:*

$$\begin{aligned} \textit{(a)}\ 48\gamma^2\tau_u\tau_\theta^3 &\leq 5\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta + 16\tau_u\tau_\theta^2\right), \\ 28\gamma^2\tau_u\tau_\theta^3 &\leq 5\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta + 8\tau_u\tau_\theta^2\right), \\ 16\gamma^2\tau_u\tau_\theta^3 &\leq 5\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)^2; \end{aligned}$$

or

$$\begin{aligned} &\textit{(b)}\ 48\gamma^2\tau_u\tau_\theta^3 \leq 5\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta + 16\tau_u\tau_\theta^2\right), \\ &28\gamma^2\tau_u\tau_\theta^3 > 5\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta + 8\tau_u\tau_\theta^2\right), \\ &2\gamma^2\tau_\theta\left(5\gamma^2\tau_\delta^2 + 2\tau_u\tau_\theta^3 + 20\tau_u\tau_\theta^2\tau_\delta\right) \leq 25\tau_\delta\left(\gamma^2\tau_\delta + 4\tau_u\tau_\theta^2\right)^2. \end{aligned}$$

Theorem 2. (Disclosure Equilibrium) *A disclosure equilibrium $\left(\tau_\eta^*, \tau_\xi^*\right) \in \mathbb{R}_{++}^2$ (with $\tau_\xi^* = \tau_\eta^*$) is characterized by the following three conditions:*

(a) (FOC) $\tau_\eta^* > 0$ is a solution to the fourth order polynomial,

$$F\left(\tau_\eta^*\right) \equiv F_4\tau_\eta^{*4} + F_3\tau_\eta^{*3} + F_2\tau_\eta^{*2} + F_1\tau_\eta^* + F_0 = 0;$$

(b) (SOC) τ_η^* satisfies the second-order condition,

$$S\left(\tau_\eta^*\right) \equiv S_6\tau_\eta^{*6} + S_5\tau_\eta^{*5} + S_4\tau_\eta^{*4} + S_3\tau_\eta^{*3} + S_2\tau_\eta^{*2} + S_1\tau_\eta^* + S_0 \leq 0;$$

(c) (Global maximum) τ_η^* is a global maximum of $E\Pi_A\left(\tau_\eta, \tau_\xi^*\right)$, that is,

$$E\Pi_A\left(\tau_\eta^*, \tau_\xi^*\right) \geq E\Pi_A\left(\tau_\eta, \tau_\xi^*\right), \text{ for } \tau_\eta \in \{0, \infty, \hat{\tau}_\eta\},$$

where $\hat{\tau}_\eta$ is the positive roots of the fourth order polynomial:

$$G\left(\tau_\eta^*\right) \equiv G_4\hat{\tau}_\eta^4 + G_3\hat{\tau}_\eta^3 + G_2\hat{\tau}_\eta^2 + G_1\hat{\tau}_\eta + G_0 = 0.$$

The F-coefficients, S-coefficients, and G-coefficients are given in the appendix.

Theorem 1 characterizes the conditions that support the nondisclosure equilibrium. Theorem 2 characterizes a disclosure equilibrium in three conditions in the form of polynomials of the disclosure policy τ_η^*. The first two conditions respectively correspond to the first and second order conditions, while the last condition ensures

that the optimal disclosure maximizes ex ante expected profits globally, rather than only locally. Theorems 1 and 2 suggest the following four-step algorithm to compute all the linear symmetric equilibria:

Step 1: Employ Theorem 1 to check whether the nondisclosure equilibrium is supported.

Step 2: Compute all the positive roots τ_η^* of the fourth order polynomial in Part (a) of Theorem 2 to serve as candidates of disclosure equilibria.

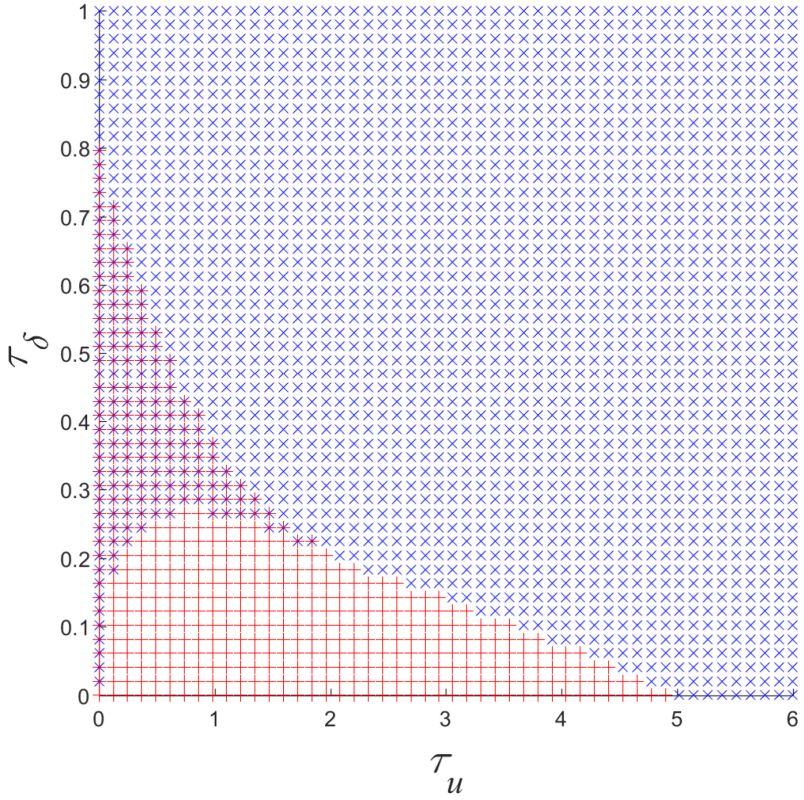
Step 3: For each root τ_η^* computed in Step 2, check whether the SOC in Part (b) of Theorem 2 is satisfied. Retain those roots that satisfy the SOC.

Step 4: For each value retained in Step 3, check whether the condition in Part (c) of Theorem 2 is satisfied. If yes, then it is a disclosure equilibrium; otherwise, it is not.

Figure 2 plots the regimes of equilibrium types in the parameter space of (τ_u, τ_δ) when $\tau_\theta = 1$ and $\gamma = 5$. I use "x" to indicate the nondisclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* = 0$) and "+" to indicate a disclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* > 0$). Hence, in the area marked with "*", both a disclosure equilibrium and the nondisclosure equilibrium are supported. Two observations emerge from Figure 2, both of which are unique to a setting with learning from asset prices.

First, unlike a standard setting with demand uncertainty and Cournot competition in which no disclosure forms a dominant strategy for firms (e.g., Gal-Or, 1985, 1986; Darrough, 1993; Vives, 1984, 2008), introducing learning from asset prices causes firms to disclose information in some cases and not to disclose in other cases. Firms are more likely to withhold information only when τ_δ or τ_u are sufficiently high. When τ_δ is high (i.e., $Var(\delta)$ is low), speculators know little new information so that the value of learning from asset prices is low and hence firms choose not to disclose because of the proprietary-cost concern as in the standard setting. When τ_u is high (i.e., $Var(u)$ is low), there is little noise trading in the financial market and thus, the

Figure 2: Disclosure and Nondisclosure Equilibria



This figure plots the regions of disclosure and nondisclosure equilibria in the parameter space of (τ_u, τ_δ). Parameter τ_u denotes the precision of noise trading in the financial market, and parameter τ_δ is the precision of fiancial speculators' private information. I have set $\tau_\theta = 1$ and $\gamma = 5$. I use "x" to indicate the nondisclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* = 0$) and "+" to indicate a disclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* > 0$). In the area marked with "*", both a disclosure equilibrium and the nondisclosure equilibrium are supported.

market is already very efficient in communicating speculators' information to firms. Again, in this case, the value of learning from prices is low and the only equilibrium is the nondisclosure equilibrium.

Proposition 3. (Nondisclosure) *When τ_u or τ_δ is sufficiently high, the nondisclosure equilibrium prevails as the unique linear symmetric equilibrium.*

The second observation emerging from Figure 2 is that multiple equilibria can be supported. That is, when τ_u and τ_δ are relatively small, both the nondisclosure

equilibrium and a disclosure equilibrium can be supported. This is also different from the standard setting where the nondisclosure equilibrium prevails as the unique equilibrium. The multiplicity of equilibrium is generated by the coordination motivates among firms, which are explored in detail in the next subsection.

3.4. Disclosure in a Very Noisy Financial Market

3.4.1. Complementarity and Multiplicity

Now suppose that there is a lot of noise trading in the financial market (i.e., τ_u is low and so $Var(u)$ is high). The following theorem characterizes the equilibrium for these economies with a noisy financial market.

Theorem 3. (Multiplicity) *Suppose that τ_u is sufficiently low. Then:*
(a) If $4\tau_\theta \geq 5\tau_\delta$, there are two symmetric linear equilibria:

$$\tau_\eta^* = \tau_\xi^* = 0 \text{ and } \tau_\eta^* = \tau_\xi^* = \frac{\gamma^2}{5\tau_u} + o(1),$$

where $o(1)$ is a term that converges to zero as $\tau_u \to 0$.
(b) If $4\tau_\theta < 5\tau_\delta$, there exists a unique symmetric linear equilibrium, which is the nondisclosure equilibrium.

Theorem 3 shows that with a very noisy financial market, multiplicity arises if and only if

$$4\tau_\theta \geq 5\tau_\delta \Longleftrightarrow \frac{Var(\delta)}{Var(\theta_A + \theta_B + \delta)} \geq 38.46\%.$$

That is, multiple equilibria are supported if and only if the financial market knows more than 38.46% of the total demand shock. This condition sounds likely to hold in reality, given that the market aggregates information from a large number of market participants (although many of them are noise traders).

On the qualitative side, Theorem 3 says that multiplicity is more likely to arise when speculators know more information that is useful to firms (i.e., $Var(\delta)$ is relatively large). This multiplicity is driven by a strategic complementarity in the disclosure decisions of firms. Specifically, recall that in the profit expression (15), the

benefit of disclosing information comes from the fact that firms learn from the asset price. When there is a lot noise trading in the market, the scope to improve price informativeness via disclosure is large; it is particularly helpful for both firms to disclose information to reduce the uncertainty faced by speculators, which in turn encourages speculators to trade more aggressively on their private information δ. When this complementarity is sufficiently strong, both disclosure and nondisclosure equilibria are supported.

Proposition 4. (Complementarity) *When there is a lot noise trading in the financial market, there is strategic complementarity in disclosure decisions. That is,* $\frac{\partial^2 E\Pi_A}{\partial \tau_\eta \partial \tau_\xi} > 0$ *and* $\frac{\partial^2 E\Pi_B}{\partial \tau_\eta \partial \tau_\xi} > 0$ *for sufficiently low values of* τ_u.

3.4.2. Shaping Price Informativeness by Coordinated Disclosure

When the size of noise trading is infinitely large, both firms choose not to disclose in equilibrium. That is, $\tau_\eta^* = \tau_\xi^* = 0$ when $\tau_u = 0$. This is because firms do not learn from asset prices when the financial market is populated with infinitely many liquidity traders (and thus the economy degenerates to the standard setting without learning from asset prices).

Now suppose that τ_u is small but positive, so that there is very minimal learning of firms from the financial market. As Part (a) of Theorem 3 shows, when τ_u is small but positive, a disclosure equilibrium is supported provided $4\tau_\theta \geq 5\tau_\delta$. In addition, as $\tau_u \to 0$, the optimal disclosure policy τ_η^* diverges to infinity on the disclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* = \frac{\gamma^2}{5\tau_u} + o(1) \to \infty$ as $\tau_u \to 0$). In addition, this disclosure equilibrium is a preferred equilibrium from the perspective of firms: both firms are better off on the disclosure equilibrium than on the nondisclosure equilibrium. This is because on the disclosure equilibrium firms make more informed decisions after equipped with more public information (the additional public information disclosed by both firms and the more informative asset price). In this sense, the disclosure equilibrium is more likely to be selected by firms. Thus, adding minimal learning

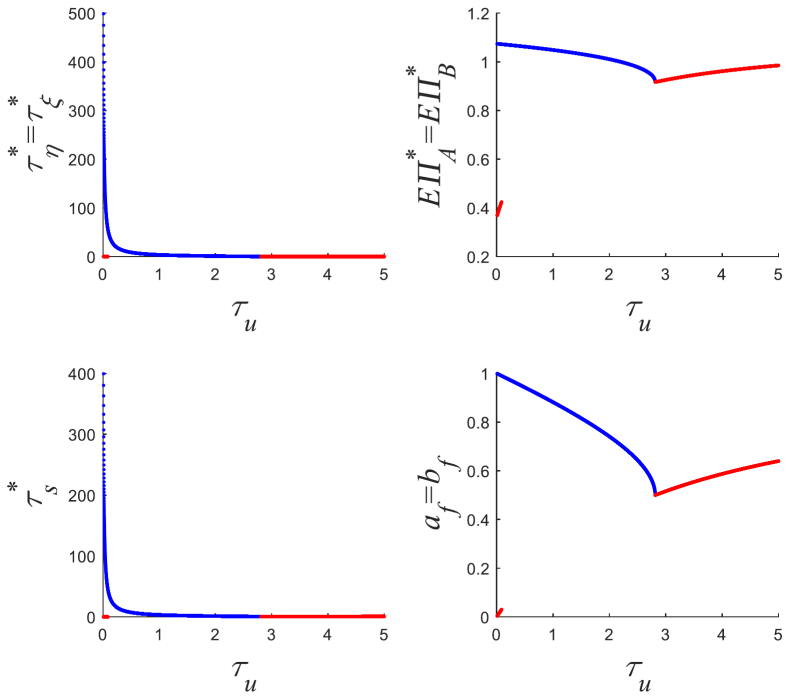
from prices dramatically changes the firms' disclosure behavior: without learning from prices, firms do not disclose information at all; in contrast, with minimal learning from prices, firms may coordinate to disclose their information almost perfectly.

On the disclosure equilibrium, firms choose to disclose more information when there is more noise trading (i.e., τ_η^* increases as τ_u decreases). So, firms effectively coordinate to disclose information to offset the negative effect of added noise trading on price informativeness. In addition, the increased noise trading helps firms to coordinate since when the size of noise trading is large, the marginal effect of coordinated disclosure on price informativeness is large. As a result, as τ_u decreases, price informativeness τ_s^* increases, and firms' production policies rely more on asset prices, i.e., both a_f and b_f increase.

Proposition 5. (Coordinated Disclosure and Price Informativeness)
(a) When $\tau_u = 0$, the unique symmetric linear equilibrium is the nondisclosure equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* = 0$). When $\tau_u \to 0$ and when $4\tau_\theta \geq 5\tau_\delta$, there are two symmetric linear equilibria: $\tau_\eta^* = \tau_\xi^* = 0$ and $\tau_\eta^* = \tau_\xi^* = \frac{\gamma^2}{5\tau_u} + o(1)$; and firms are better off on the disclosure equilibrium than on the nondisclosure equilibrium.
(b) Suppose $4\tau_\theta \geq 5\tau_\delta$. On the disclosure equilibrium, as τ_u decreases toward 0, disclosure policies τ_η^* and τ_ξ^* gradually increase toward ∞, and investment-price sensitivities a_f and b_f gradually increase toward 1.

Figure 3 graphically illustrates Proposition 5 for the parameter configuration $\tau_\theta = 1, \tau_\delta = 0.15$, and $\gamma = 5$. In Figure 3, I plot the equilibrium disclosure policies ($\tau_\eta^* = \tau_\xi^*$), firms' expected profits ($E\Pi_A^* = E\Pi_B^*$), price informativeness (τ_s^*), and investment-price sensitivities ($a_f = b_f$) against the precision τ_u of noise trading in the financial market. The nondisclosure equilibrium is in red while the disclosure equilibrium is in blue. Indeed, for sufficiently small values of τ_u, there are two equilibria: the nondisclosure equilibrium $\tau_\eta^* = \tau_\xi^* = 0$ and a disclosure equilibrium $\tau_\eta^* = \tau_\xi^* > 0$. Firms are better off on the disclosure equilibrium than on the nondisclosure equilibrium. On the disclosure equilibrium, as τ_u gradually decreases to 0, both disclosure

Figure 3: Disclosure and Multiplicity



This figure plots the equilibrium disclosure policies ($\tau_\eta^* = \tau_\xi^*$), firms' expected profits ($E\Pi_A^* = E\Pi_B^*$), price informativeness (τ_s^*), and investment-price sensitivity ($a_f = b_f$) against the precision τ_u of noise trading in the financial market. The nondisclosure equilibrium is plotted in red and the disclosure equilbrium is plotted in in blue. The other parameters are: $\tau_\theta = 1$, $\tau_\delta = 0.15$, and $\gamma = 5$.

policies τ_η^* and τ_ξ^* and price informativeness τ_s^* diverge to ∞, and investment-price sensitivities a_f and b_f increase to 1.

Proposition 5 relates to but differs from Goldstein and Huang (2017). Goldstein and Huang (2017) also highlight the possibility of a discontinuity of the real effect of asset prices in parameters affecting price informativeness. Specifically, they show that the asset price becomes almost uninformative when either the noise trading is extremely volatile or the speculators have very imprecise information. However, the real effects, measured by the sensitivity of investments to asset prices (i.e., a_f and b_f in the notation of this paper), are trivial in the former case but are significant in the latter case. In contrast, in my setting, as the noise trading becomes extremely volatile,

firms optimally respond by disclosing extremely precise information, which thereby causes the asset price to become extremely informative, leading to a significant real effect (investment-price sensitivity), as opposed to the trivial effect in Goldstein and Huang (2017).

4. Welfare Effects of Financial Market Feedback

In this section, I examine the normative implications of allowing firms to learn from asset prices. Again, I use the setting described in Section 3.1. as a benchmark, in which firms make production decisions without observing the futures price f. Specifically, I modify the order of date-1 events in the benchmark setting as follows: firms observe public signals (x, y) and simultaneously choose their production quantities; then speculators receive private information δ, observe public signals (x, y), and trade futures against liquidity traders, which determines the asset price f.

I still use "$\varnothing$" and "*" to respectively label the equilibrium variables in the benchmark setting without learning from asset prices and in the main setting with learning from prices. The welfare variables of interest are: the expected profits of firms, $E\Pi_A$ and $E\Pi_B$; the expected consumer surplus, CS; the certainty equivalent of financial speculator, CE_S; and the expected trading revenue of liquidity traders, ER_L.

Allowing firms to learn from asset prices benefits all agents except for financial speculators. That is, $E\Pi_A^* \geq E\Pi_A^{\varnothing}$, $E\Pi_B^* \geq E\Pi_B^{\varnothing}$, $CS^* \geq CS^{\varnothing}$, $ER_L^* \geq ER_L^{\varnothing}$, and $CE_S^* \leq CE_S^{\varnothing}$. It is intuitive that both firms and consumers benefit from the financial feedback effect, since in the setting with learning from prices, firms make more informed decisions by accessing to more information (the public information (x, y) disclosed by both firms and the futures price f).

Following Grossman and Stiglitz (1980), I compute the ex-ante certainty equivalent of financial traders as follows:

$$CE_S = \frac{1}{2\gamma} \log \left[1 + \frac{\gamma^2}{2\tau_u \left(\tau_\theta + \tau_\eta \right)} \right]. \tag{17}$$

So, public disclosure of firms harms speculators (i.e., $\frac{\partial CE_S}{\partial \tau_\eta} < 0$). This is because releasing public information about later commodity demand shocks brings the asset price closer to its fundamental, which therefore limits the benefit of speculators in trading a risky asset. This idea shares a similar spirit as the well-known "Hirshleifer effect" (Hirshleifer, 1971; see also Kurlat and Veldkamp (2015) and Goldstein and Yang (2017) for related discussions). Given that firms sometimes choose to disclose information in a setting with learning from prices and that they always choose to withhold information in the benchmark setting without learning from prices, speculators are weakly worse off in the former than in the latter settings; that is, $CE_S^* \leq CE_S^\varnothing$ because $\tau_\eta^* \geq \tau_\eta^\varnothing = 0$.

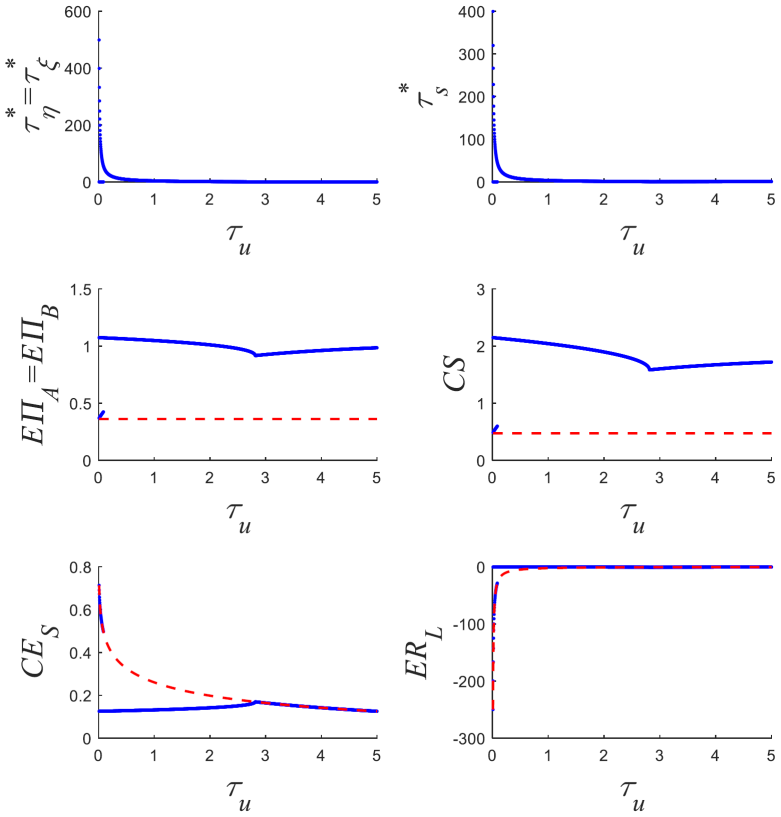
I have not formally modeled the behavior of liquidity traders and thus it is impossible to conduct a complete welfare analysis on this type of traders. Instead, I follow the microstructure literature and compute their expected trading revenue to proxy for the welfare of liquidity traders. This captures the idea that liquidity traders are better off if they can realize their hedging or liquidity needs at a lower expected opportunity cost (see Easley, O'Hara, and Yang (2016) for more discussions). Intuitively, the revenue that liquidity traders receive from buying u shares is

$$ER_L = E\left[(p - f)\, u\right] = -\frac{\gamma}{2\left(\tau_\theta + \tau_\eta\right)\tau_u} < 0. \tag{18}$$

Therefore, ER_L is the negative of the expected opportunity cost associated with a trade of u shares and thus, it is positively related to liquidity traders' welfare to the extent that their exogenous hedging or liquidity needs are largely exogenous.

Disclosure improves the welfare of liquidity traders (i.e., $\frac{\partial ER_L}{\partial \tau_\eta} > 0$ in equation (18)). This is because disclosing public information improves market liquidity. Intuitively, more precise public information implies that there is less uncertainty about the asset value and so speculators trade more aggressively against liquidity traders. As a result, changes in liquidity trading are absorbed with a smaller price change, which benefits liquidity traders. Given that there is weakly more public disclosure in a setting with learning from prices than in a setting without, liquidity traders are

Figure 4: Welfare Effects of Learning from Asset Prices



The top two panels plot the equilibrium disclosure policies ($\tau_\eta^* = \tau_\xi^*$) and price informativeness (τ_s^*) against the precision τ_u of noise trading in the financial market. The other panels respectively plot firms' expected profits $E\Pi_A$ and $E\Pi_B$, expected consumer surplus CS, certainty equivalent CE_S of speculators, and expected trading revenue ER_L of liquidity traders. The solid curves correspond to the setting in which firms learn information from asset prices, while the dashed curves correspond to the benchmark setting in which firms do not learn information from asset prices. The other parameters are: $m = 1$, $\tau_\theta = 1$, $\tau_\delta = 0.15$, and $\gamma = 5$.

weakly better off in the former than in the latter settings. That is, $ER_L^* \geq ER_L^\varnothing$ by $\tau_\eta^* \geq \tau_\eta^\varnothing = 0$.

Theorem 4. (Welfare) *Learning from asset prices benefits firms, consumers, and liquidity traders, but weakly harms financial speculators. That is,* $E\Pi_A^* > E\Pi_A^\varnothing$, $E\Pi_B^* > E\Pi_B^\varnothing$, $CS^* > CS^\varnothing$, $ER_L^* \geq ER_L^\varnothing$, *and* $CE_S^* \leq CE_S^\varnothing$.

Figure 4 graphically illustrates Theorem 4 for the parameter configuration $m = 1$,

$\tau_\theta = 1$, $\tau_\delta = 0.15$, and $\gamma = 5$. In the top two panels, I plot the equilibrium disclosure policies ($\tau_\eta^* = \tau_\xi^*$) and price informativeness (τ_s^*) against the precision τ_u of noise trading in the financial market. In the other four panels, I respectively plot firms' expected profits $E\Pi_A$ and $E\Pi_B$, expected consumer surplus CS, certainty equivalent CE_S of speculators, and expected trading revenue ER_L of liquidity traders in a setting with learning from prices (in blue solid curves) and in the benchmark setting without learning from prices (in red dashed curves). Indeed, allowing firms to learn from asset prices improves the welfare of both firms and consumers no matter on the disclosure equilibrium or on the nondisclosure equilibrium. For financial market participants, as long as firms choose to disclose information in equilibrium, speculators are worse off and liquidity traders are better off in a setting with learning from prices.

5. Conclusion

I study the classic information-sharing problem in a duopoly setting with demand uncertainty and Cournot competition. My setup is a hybrid of Gal-Or (1985) and Grossman and Stiglitz (1980) and incorporates the realistic feature that firms learn information from a financial market, as highlighted by Hayek (1945). Disclosure improves price informativeness via reducing the uncertainty faced by financial speculators and thus, disclosure causes firms to face a trade-off between incurring the proprietary cost and improving learning quality from asset prices. As a result, firms may optimally choose to disclose information in a setting with learning from asset prices, which differs from the standard setting where firms always withhold information. In addition, firms' disclosure decisions can be a strategic complement. When this complementarity is sufficiently strong, both a disclosure equilibrium and a nondisclosure equilibrium are supported, and the nondisclosure equilibrium are more preferred by firms. My analysis shows that adding minimal learning from asset prices dramatically changes the firms' disclosure behavior: without learning from prices, firms do

not disclose information at all; in contrast, with minimal learning from prices, firms may coordinate to disclose their information almost perfectly. Finally, I show that relative to a setting in which firms do not learn from prices, in a setting in which firms do learn from prices, firms, consumers, and liquidity traders are better off, while financial speculators are weakly worse off. Overall, my analysis highlights the importance of incorporating the feature of learning from asset prices in understanding firms' disclosure behavior.

Appendix: Proofs

Proof of Proposition 2

After expressing q_A^* in (9) as functions of (θ_A, x, y, f) and comparing with the conjectured policy in (5), I obtain the following five conditions in terms of the unknown a coefficients and b coefficients:

$$\begin{aligned}
2a_0 &= m - \frac{\tau_s}{\tau_\delta + \tau_s}(m - a_0 - b_0) - b_0, \\
2a_\theta &= 1, \\
2a_x &= -\frac{\tau_s}{\tau_\delta + \tau_s}\left[\frac{(1-a_\theta)\tau_\eta}{\tau_\theta + \tau_\eta} - (a_x + b_x)\right] - b_x, \\
2a_y &= -\frac{\tau_s}{\tau_\delta + \tau_s}\left[\frac{(1-b_\theta)\tau_\xi}{\tau_\theta + \tau_\xi} - (a_y + b_y)\right] + \frac{(1-b_\theta)\tau_\xi}{\tau_\theta + \tau_\xi} - b_y, \\
2a_f &= \frac{\tau_s}{\tau_\delta + \tau_s}(a_f + b_f + 1) - b_f.
\end{aligned}$$

Conducting a similar analysis for firm B leads to the following additional five equations:

$$\begin{aligned}
2b_0 &= m - \frac{\tau_s}{\tau_\delta + \tau_s}(m - a_0 - b_0) - a_0, \\
2b_\theta &= 1, \\
2b_x &= -\frac{\tau_s}{\tau_\delta + \tau_s}\left[\frac{(1-a_\theta)\tau_\eta}{\tau_\theta + \tau_\eta} - (a_x + b_x)\right] + \frac{(1-a_\theta)\tau_\eta}{\tau_\theta + \tau_\eta} - a_x, \\
2b_y &= -\frac{\tau_s}{\tau_\delta + \tau_s}\left[\frac{(1-b_\theta)\tau_\xi}{\tau_\theta + \tau_\xi} - (a_y + b_y)\right] - a_y, \\
2b_f &= \frac{\tau_s}{\tau_\delta + \tau_s}(a_f + b_f + 1) - a_f.
\end{aligned}$$

Solving the above system yields the expressions of a's and b's in Proposition 2. The expressions of τ_s, p, and f in Proposition 2 are obtained by plugging a's and b's respectively into equations (14), (11), and (12).

Proof of Corollary 1

By the expression of τ_s in Proposition 2, direct computations show

$$\begin{aligned}\frac{\partial \tau_s}{\partial \tau_\eta} &= \frac{32\tau_u\left(\tau_\theta+\tau_\eta\right)\left(\tau_\theta+\tau_\xi\right)^3}{\gamma^2\left(2\tau_\theta+\tau_\xi+\tau_\eta\right)^3}>0,\\ \frac{\partial \tau_s}{\partial \tau_\xi} &= \frac{32\tau_u\left(\tau_\theta+\tau_\xi\right)\left(\tau_\theta+\tau_\eta\right)^3}{\gamma^2\left(2\tau_\theta+\tau_\xi+\tau_\eta\right)^3}>0.\end{aligned}$$

Proof of Theorem 1

Nondisclosure is an equilibrium if and only if $\tau_\eta^*=0$ is the best response to $\tau_\xi^*=0$, i.e., if and only if

$$E\Pi_A\left(0,0\right)\geq\max_{\tau_\eta}E\Pi_A\left(\tau_\eta,0\right).$$

By the expression of τ_s in Proposition 2 and the expression of expected profit $E\Pi_A\left(\tau_\eta,\tau_\xi\right)$ in (15), direct computations show that

$$E\Pi_A\left(0,0\right)-E\Pi_A\left(\tau_\eta,0\right)\geq 0\Longleftrightarrow H\left(\tau_\eta\right)\equiv H_2\tau_\eta^2+H_1\tau_\eta+H_0\leq 0,$$

where

$$\begin{aligned}H_2 &= 48\gamma^2\tau_u\tau_\theta^3-5\left(\gamma^2\tau_\delta+16\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta+4\tau_u\tau_\theta^2\right),\\ H_1 &= 4\tau_\theta\left[28\gamma^2\tau_u\tau_\theta^3-5\left(\gamma^2\tau_\delta+4\tau_u\tau_\theta^2\right)\left(\gamma^2\tau_\delta+8\tau_u\tau_\theta^2\right)\right],\\ H_0 &= 4\tau_\theta^2\left[16\gamma^2\tau_u\tau_\theta^3-5\left(\gamma^2\tau_\delta+4\tau_u\tau_\theta^2\right)^2\right].\end{aligned}$$

Thus, nondisclosure is an equilibrium if and only if

$$H\left(\tau_\eta\right)\leq 0,\forall\tau_\eta\geq 0. \tag{A1}$$

Clearly, a necessary condition for (A1) to hold is $H_0\leq 0$. Now suppose $H_0\leq 0$ and discuss the possible values of H_2 and H_1 to check when condition (A1) holds.

If $H_2>0$, then $H\left(\tau_\eta\right)>0$ for sufficiently large τ_η, so that condition (A1) is violated.

If $H_2=0$, then $H\left(\tau_\eta\right)$ becomes linear, and condition (A1) holds if and only if $H_1\leq 0$.

Suppose $H_2 < 0$. If in addition, $H_1 \leq 0$, then the range of $\tau_\eta > 0$ lies on the right branch of $H(\tau_\eta)$ and thus condition (A1) holds. If $H_1 > 0$, then condition (A1) holds if and only if the discriminant of $H(\tau_\eta)$ is nonpositive (i.e., if and only if $H_1^2 - 4H_2H_0 \leq 0$).

To summarize, (A1) holds if and only if one of the following two sets of conditions holds:

$$\{H_2 \leq 0, H_1 \leq 0, H_0 \leq 0\} \text{ or } \left\{H_2 < 0, H_1 > 0, H_1^2 - 4H_2H_0 \leq 0\right\},$$

which are respectively the two sets of conditions in Theorem 1.

Proof of Theorem 2

A symmetric disclosure equilibrium requires that $\tau_\eta^* > 0$ is the best response to $\tau_\xi^* = \tau_\eta^* > 0$. That is,

$$\tau_\eta^* = \arg\max_{\tau_\eta} E\Pi_A\left(\tau_\eta, \tau_\eta^*\right).$$

I characterize the value of τ_η^* in three steps. First, I use the FOC to find the candidates for τ_η^*. Second, I use the SOC to ensure that τ_η^* is a local maximum of $E\Pi_A\left(\tau_\eta, \tau_\eta^*\right)$. Third, I compare $E\Pi_A\left(\tau_\eta^*, \tau_\eta^*\right)$ with the other extreme values of $E\Pi_A\left(\tau_\eta, \tau_\eta^*\right)$ to ensure that τ_η^* is a global maximum of $E\Pi_A\left(\tau_\eta, \tau_\eta^*\right)$.

For the FOC, direct computations show

$$\left.\frac{\partial E\Pi_A(\tau_\eta, \tau_\xi)}{\partial \tau_\eta}\right|_{\tau_\xi = \tau_\eta} = 0 \Longleftrightarrow F(\tau_\eta) \equiv F_4\tau_\eta^4 + F_3\tau_\eta^3 + F_2\tau_\eta^2 + F_1\tau_\eta + F_0 = 0,$$

where

$$\begin{aligned}
F_4 &= -80\tau_u^2, F_3 = 16\tau_u\left(\gamma^2 - 20\tau_u\tau_\theta\right), \\
F_2 &= 8\tau_u\left(6\gamma^2\tau_\theta - 5\gamma^2\tau_\delta - 60\tau_u\tau_\theta^2\right), \\
F_1 &= 16\tau_u\tau_\theta\left(3\gamma^2\tau_\theta - 5\gamma^2\tau_\delta - 20\tau_u\tau_\theta^2\right), \\
F_0 &= 16\gamma^2\tau_u\tau_\theta^3 - 5\gamma^4\tau_\delta^2 - 40\gamma^2\tau_u\tau_\theta^2\tau_\delta - 80\tau_u^2\tau_\theta^4.
\end{aligned}$$

Thus, any candidate disclosure policy $\tau_\eta^* > 0$ must satisfy $F\left(\tau_\eta^*\right) = 0$.

For the SOC, direct computations show

$$\left.\frac{\partial^2 E\Pi_A(\tau_\eta, \tau_\xi)}{\partial \tau_\eta^2}\right|_{\tau_\xi = \tau_\eta} \leq 0 \Longleftrightarrow S(\tau_\eta) \equiv S_6\tau_\eta^6 + S_5\tau_\eta^5 + S_4\tau_\eta^4 + S_3\tau_\eta^3 + S_2\tau_\eta^2 + S_1\tau_\eta + S_0 \leq 0,$$

where

$$\begin{aligned}
S_6 &= 320\tau_u^3, S_5 = -80\tau_u^2\left(\gamma^2 - 24\tau_u\tau_\theta\right), \\
S_4 &= -80\tau_u^2\left(5\gamma^2\tau_\theta - 3\gamma^2\tau_\delta - 60\tau_u\tau_\theta^2\right), \\
S_3 &= -4\tau_u\left(\gamma^4\tau_\delta + 200\gamma^2\tau_u\tau_\theta^2 - 1600\tau_u^2\tau_\theta^3 - 240\gamma^2\tau_u\tau_\theta\tau_\delta\right), \\
S_2 &= 4\tau_u\left(15\gamma^4\tau_\delta^2 - 3\gamma^4\tau_\theta\tau_\delta + 1200\tau_u^2\tau_\theta^4 + 360\gamma^2\tau_u\tau_\theta^2\tau_\delta - 200\gamma^2\tau_u\tau_\theta^3\right), \\
S_1 &= 4\tau_u\tau_\theta\left(30\gamma^4\tau_\delta^2 - 3\gamma^4\tau_\theta\tau_\delta + 480\tau_u^2\tau_\theta^4 - 100\gamma^2\tau_u\tau_\theta^3 + 240\gamma^2\tau_u\tau_\theta^2\tau_\delta\right), \\
S_0 &= 5\gamma^6\tau_\delta^3 - 4\gamma^4\tau_u\tau_\theta^3\tau_\delta + 60\gamma^4\tau_u\tau_\theta^2\tau_\delta^2 - 80\gamma^2\tau_u^2\tau_\theta^5 + 240\gamma^2\tau_u^2\tau_\theta^4\tau_\delta + 320\tau_u^3\tau_\theta^6.
\end{aligned}$$

Hence, any candidate disclosure policy $\tau_\eta^* > 0$ must satisfy $S\left(\tau_\eta^*\right) \leq 0$.

Finally, fixing $\tau_\xi = \tau_\eta^*$, I can find the interior extreme values of $E\Pi_A\left(\tau_\eta, \tau_\xi\right)$ by setting its FOC at zero, that is,

$$\frac{\partial E\Pi_A\left(\tau_\eta, \tau_\xi\right)}{\partial \tau_\eta} = 0 \Longleftrightarrow G\left(\tau_\eta\right) \equiv G_4\tau_\eta^4 + G_3\tau_\eta^3 + G_2\tau_\eta^2 + G_1\tau_\eta + G_0 = 0,$$

where

$$\begin{aligned}
G_4 &= 5\gamma^4\tau_\delta^2 + 1280\tau_u^2\tau_\theta^4 + 1280\tau_u^2\tau_\xi^4 - 128\gamma^2\tau_u\tau_\theta^3 \\
&-128\gamma^2\tau_u\tau_\xi^3 + 5120\tau_u^2\tau_\theta\tau_\xi^3 + 5120\tau_u^2\tau_\theta^3\tau_\xi \\
&+7680\tau_u^2\tau_\theta^2\tau_\xi^2 + 160\gamma^2\tau_u\tau_\theta^2\tau_\delta - 384\gamma^2\tau_u\tau_\theta\tau_\xi^2 \\
&-384\gamma^2\tau_u\tau_\theta^2\tau_\xi + 160\gamma^2\tau_u\tau_\delta\tau_\xi^2 + 320\gamma^2\tau_u\tau_\theta\tau_\delta\tau_\xi,
\end{aligned}$$

$$G_3 = 4\begin{pmatrix}
1280\tau_u^2\tau_\theta^5 - 160\gamma^2\tau_u\tau_\theta^4 - 32\gamma^2\tau_u\tau_\xi^4 \\
+10\gamma^4\tau_\theta\tau_\delta^2 + 5\gamma^4\tau_\delta^2\tau_\xi + 1280\tau_u^2\tau_\theta\tau_\xi^4 + 5120\tau_u^2\tau_\theta^4\tau_\xi \\
+5120\tau_u^2\tau_\theta^2\tau_\xi^3 + 7680\tau_u^2\tau_\theta^3\tau_\xi^2 - 576\gamma^2\tau_u\tau_\theta^2\tau_\xi^2 \\
+240\gamma^2\tau_u\tau_\theta^3\tau_\delta - 256\gamma^2\tau_u\tau_\theta\tau_\xi^3 - 512\gamma^2\tau_u\tau_\theta^3\tau_\xi \\
+80\gamma^2\tau_u\tau_\delta\tau_\xi^3 + 400\gamma^2\tau_u\tau_\theta\tau_\delta\tau_\xi^2 + 560\gamma^2\tau_u\tau_\theta^2\tau_\delta\tau_\xi
\end{pmatrix},$$

$$G_2 = 2\begin{pmatrix}
3840\tau_u^2\tau_\theta^6 - 576\gamma^2\tau_u\tau_\theta^5 + 15\,360\tau_u^2\tau_\theta^5\tau_\xi + 60\gamma^4\tau_\theta^2\tau_\delta^2 \\
+15\gamma^4\tau_\delta^2\tau_\xi^2 + 3840\tau_u^2\tau_\theta^2\tau_\xi^4 + 15\,360\tau_u^2\tau_\theta^3\tau_\xi^3 + 23\,040\tau_u^2\tau_\theta^4\tau_\xi^2 \\
-1152\gamma^2\tau_u\tau_\theta^2\tau_\xi^3 - 2304\gamma^2\tau_u\tau_\theta^3\tau_\xi^2 + 1040\gamma^2\tau_u\tau_\theta^4\tau_\delta \\
-192\gamma^2\tau_u\tau_\theta\tau_\xi^4 - 1920\gamma^2\tau_u\tau_\theta^4\tau_\xi + 80\gamma^2\tau_u\tau_\delta\tau_\xi^4 \\
+60\gamma^4\tau_\theta\tau_\delta^2\tau_\xi + 2400\gamma^2\tau_u\tau_\theta^2\tau_\delta\tau_\xi^2 + 800\gamma^2\tau_u\tau_\theta\tau_\delta\tau_\xi^3 + 2720\gamma^2\tau_u\tau_\theta^3\tau_\delta\tau_\xi
\end{pmatrix},$$

$$G_1 = 4\begin{pmatrix} 1280\tau_u^2\tau_\theta^7 - 224\gamma^2\tau_u\tau_\theta^6 + 5120\tau_u^2\tau_\theta^6\tau_\xi + 40\gamma^4\tau_\theta^3\tau_\delta^2 \\ +5\gamma^4\tau_\delta^2\tau_\xi^3 + 1280\tau_u^2\tau_\theta^3\tau_\xi^4 + 5120\tau_u^2\tau_\theta^4\tau_\xi^3 + 7680\tau_u^2\tau_\theta^5\tau_\xi^2 \\ -96\gamma^2\tau_u\tau_\theta^2\tau_\xi^4 - 512\gamma^2\tau_u\tau_\theta^3\tau_\xi^3 - 960\gamma^2\tau_u\tau_\theta^4\tau_\xi^2 + 30\gamma^4\tau_\theta\tau_\delta^2\tau_\xi^2 \\ +60\gamma^4\tau_\theta^2\tau_\delta^2\tau_\xi + 480\gamma^2\tau_u\tau_\theta^5\tau_\delta - 768\gamma^2\tau_u\tau_\theta^5\tau_\xi + 560\gamma^2\tau_u\tau_\theta^2\tau_\delta\tau_\xi^3 \\ +1360\gamma^2\tau_u\tau_\theta^3\tau_\delta\tau_\xi^2 + 80\gamma^2\tau_u\tau_\theta\tau_\delta\tau_\xi^4 + 1360\gamma^2\tau_u\tau_\theta^4\tau_\delta\tau_\xi \end{pmatrix},$$

$$\begin{aligned} G_0 &= 1280\tau_u^2\tau_\theta^8 - 256\gamma^2\tau_u\tau_\theta^7 + 5120\tau_u^2\tau_\theta^7\tau_\xi + 80\gamma^4\tau_\theta^4\tau_\delta^2 + 5\gamma^4\tau_\delta^2\tau_\xi^4 \\ &+1280\tau_u^2\tau_\theta^4\tau_\xi^4 + 5120\tau_u^2\tau_\theta^5\tau_\xi^3 + 7680\tau_u^2\tau_\theta^6\tau_\xi^2 - 128\gamma^2\tau_u\tau_\theta^3\tau_\xi^4 \\ &-640\gamma^2\tau_u\tau_\theta^4\tau_\xi^3 - 1152\gamma^2\tau_u\tau_\theta^5\tau_\xi^2 + 40\gamma^4\tau_\theta\tau_\delta^2\tau_\xi^3 + 160\gamma^4\tau_\theta^3\tau_\delta^2\tau_\xi \\ &+120\gamma^4\tau_\theta^2\tau_\delta^2\tau_\xi^2 + 640\gamma^2\tau_u\tau_\theta^6\tau_\delta - 896\gamma^2\tau_u\tau_\theta^6\tau_\xi \\ &+160\gamma^2\tau_u\tau_\theta^2\tau_\delta\tau_\xi^4 + 960\gamma^2\tau_u\tau_\theta^3\tau_\delta\tau_\xi^3 + 2080\gamma^2\tau_u\tau_\theta^4\tau_\delta\tau_\xi^2 + 1920\gamma^2\tau_u\tau_\theta^5\tau_\delta\tau_\xi, \end{aligned}$$

with $\tau_\xi = \tau_\eta^*$.

The extreme values of $E\Pi_A(\tau_\eta, \tau_\xi)$ include (1) the positive roots of $G(\tau_\eta) = 0$; and (2) the two boundaries $\tau_\eta = 0$ and $\tau_\eta = \infty$.

Proof of Proposition 3

When τ_u is large

Fix the other parameters and let $\tau_u \to \infty$. Condition (a) in Theorem 1 is satisfied and thus nondisclosure is an equilibrium.

Condition (a) in Theorem 2 is violated, because all the F coefficients are negative for sufficiently large values of τ_u, which implies $F(\tau_\eta) < 0$ for all $\tau_\eta > 0$. So, there are no disclosure equilibria.

When τ_δ is large

Fix the other parameters and let $\tau_\delta \to \infty$. Condition (a) in Theorem 1 is satisfied and thus nondisclosure is an equilibrium.

Now check condition (a) in Theorem 2. Suppose this condition is satisfied for some $\tau_\eta^* > 0$. If τ_η^* is finite as $\tau_\delta \to \infty$, then $F\left(\tau_\eta^*\right) \propto 8\tau_u\left(-5\gamma^2\tau_\delta\right)\tau_\eta^{*2} + 16\tau_u\tau_\theta\left(-5\gamma^2\tau_\delta\right)\tau_\eta^* - 5\gamma^4\tau_\delta^2 < 0$, which violates the condition of $F\left(\tau_\eta^*\right) = 0$. If τ_η^* diverges to ∞ as $\tau_\delta \to \infty$, then $F_4\tau_\eta^{*4} + F_3\tau_\eta^3 = \left(-80\tau_u^2\right)\tau_\eta^4 + 16\tau_u\left(\gamma^2 - 20\tau_u\tau_\theta\right)\tau_\eta^3 \propto \left(-80\tau_u^2\right)\tau_\eta^4 < 0$

and $F_2\tau_\eta^{*2} + F_1\tau_\eta^* + F_0 < 0$, and so $F\left(\tau_\eta^*\right) < 0$, which again violates the condition of $F\left(\tau_\eta^*\right) = 0$. Hence, there are no disclosure equilibria.

Proof of Theorem 3

Fix $(\gamma, \tau_\theta, \tau_\delta)$ and let $\tau_u \to 0$. Condition (a) of Theorem 1 is satisfied for small values of τ_u and thus the nondisclosure equilibrium is supported. The key is to characterize the disclosure equilibrium. I conduct this characterization in four steps. First, I use the FOC in Part (a) of Theorem 2 to compute all the candidates for a disclosure equilibrium. It turns out that there are two possible values of disclosure policy τ_η^*, which I label as τ_η^{large} and τ_η^{small}, respectively. Second, I employ the SOC in Part (b) of Theorem 2 to rule out candidate τ_η^{small} and retain the other candidate τ_η^{large}. Third, I compare $E\Pi_A\left(0, \tau_\eta^{large}\right)$ with $E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right)$ to show that under condition $4\tau_\theta < 5\tau_\delta$, the unique equilibrium is the nondisclosure equilibrium (i.e., Part (b) of Theorem 3). Lastly, I show that if $4\tau_\theta \geq 5\tau_\delta$, then $\tau_\eta^* = \tau_\xi^* = \tau_\eta^{large}$ is supported as a disclosure equilibrium (i.e., Part (a) of Theorem 3).

Compute disclosure equilibrium candidates

A disclosure equilibrium requires $F(\tau_\eta^*) = 0$ in Part (a) of Theorem 2. I can rewrite this equation as follows:

$$-80\left(\tau_\theta + \tau_\eta^*\right)^4 \tau_u^2 + 8\gamma^2\left(\tau_\theta + \tau_\eta\right)^2\left(2\tau_\theta - 5\tau_\delta + 2\tau_\eta^*\right)\tau_u = 5\gamma^4\tau_\delta^2. \tag{A2}$$

Now consider the process of $\tau_u \to 0$ and examine the order of τ_η^*. Clearly, τ_η^* must diverge to ∞ as $\tau_u \to 0$, because if τ_η^* converges to a finite value, then the left-hand-side (LHS) of equation (A2) converges to 0, which cannot maintain equation (A2).

The highest order of the LHS of equation (A2) is $-80\tau_\eta^{*4}\tau_u^2 + 16\gamma^2\tau_\eta^{*3}\tau_u$. Thus, by equation (A2),

$$-80\tau_\eta^{*4}\tau_u^2 + 16\gamma^2\tau_\eta^{*3}\tau_u \propto 5\gamma^4\tau_\delta^2, \tag{A3}$$

where $\propto$ means that the LHS has the same order as the right-hand-side (RHS). Equation (A3) determines the order of τ_η^*.

Given that the RHS of (A3) is positive and that only the term $16\gamma^2\tau_\eta^{*3}\tau_u$ in the LHS of (A3) is positive, there are two possibilities. First, $-80\tau_\eta^{*4}\tau_u^2$ has a lower order than $16\gamma^2\tau_\eta^{*3}\tau_u$, i.e., $-80\tau_\eta^{*4}\tau_u^2 = o\left(16\gamma^2\tau_\eta^{*3}\tau_u\right)$, where the notation $X_2 = o(X_1)$ means $\lim_{\tau_u\to 0}\frac{X_2}{X_1} = 0$. Second, $-80\tau_\eta^{*4}\tau_u^2$ has the same order as $16\gamma^2\tau_\eta^{*3}\tau_u$, i.e., $-80\tau_\eta^{*4}\tau_u^2 = O\left(16\gamma^2\tau_\eta^{*3}\tau_u\right)$, where the notation $X_2 = O(X_1)$ means $\frac{X_2}{X_1}$ converges to a finite constant as $\tau_u \to 0$.

Case 1. $-80\tau_\eta^{*4}\tau_u^2 = o\left(16\gamma^2\tau_\eta^{*3}\tau_u\right)$

By equation (A3),

$$16\gamma^2\tau_\eta^{*3}\tau_u = 5\gamma^4\tau_\delta^2 + o(1) \Rightarrow \tau_\eta^* = \sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{16}\frac{1}{\tau_u}} + o\left(\sqrt[3]{\frac{1}{\tau_u}}\right).$$

I denote this candidate disclosure policy as τ_η^{small}.

Case 2. $-80\tau_\eta^{*4}\tau_u^2 = O\left(16\gamma^2\tau_\eta^{*3}\tau_u\right)$

In this case, τ_η^* diverges at the order of $\frac{1}{\tau_u}$, that is, $\tau_u\tau_\eta^*$ converges to a finite value as $\tau_u \to 0$. By equation (A3),

$$\begin{aligned} -80\tau_\eta^{*4}\tau_u^2 + 16\gamma^2\tau_\eta^{*3}\tau_u &= 5\gamma^4\tau_\delta^2 = O(1) \Rightarrow \\ 16\tau_u\tau_\eta^*\left(\gamma^2 - 5\tau_u\tau_\eta^*\right)\tau_\eta^{*2} &= O(1). \end{aligned}$$

Note that $16\tau_u\tau_\eta^* = O(1)$ and $\tau_\eta^{*2} = O\left(\frac{1}{\tau_u^2}\right)$, and thus

$$\gamma^2 - 5\tau_u\tau_\eta^* = O\left(\frac{1}{\tau_\eta^{*2}}\right) \Rightarrow 5\tau_u\tau_\eta^* = \gamma^2 + O\left(\frac{1}{\tau_\eta^{*2}}\right) \Rightarrow \tau_\eta^* = \frac{\gamma^2}{5\tau_u} + O(\tau_u).$$

Hence, the other candidate is:

$$\tau_\eta^* = \frac{\gamma^2}{5\tau_u} + o(1),$$

which is labeled as τ_η^{large}, where the superscript "large" follows from $\frac{\gamma^2}{5\tau_u} > \sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{16}\frac{1}{\tau_u}}$ for small values of τ_u.

Check the SOC

Inserting the candidate disclosure policy $\tau_\eta^{small} = \sqrt[3]{\frac{5\gamma^2\tau_\delta^2}{16}\frac{1}{\tau_u}} + o\left(\sqrt[3]{\frac{1}{\tau_u}}\right)$ into the SOC in Part (b) of Theorem 2 and keeping the highest order, I compute $S\left(\tau_\eta^{small}\right) \propto \frac{15}{4}\gamma^6\tau_\delta^3 > 0$. That is, the SOC is violated and thus τ_η^{small} cannot be supported as a disclosure equilibrium.

Similarly, for the other candidate policy $\tau_\eta^{large} = \frac{\gamma^2}{5\tau_u} + o(1)$, I can compute

$S\left(\tau_\eta^{large}\right) \propto -\frac{16}{3125}\frac{\gamma^{12}}{\tau_u^3} < 0$, which means that τ_η^{large} is a local maximum for function $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$.

In sum, the value of τ_η^{large} serves as the only candidate for a disclosure equilibrium.

Compare $E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right)$ ***with*** $E\Pi_A\left(0, \tau_\eta^{large}\right)$ ***(Proof of Part (b))***

By the profit expression (15) and using $\tau_\eta^{large} = \frac{\gamma^2}{5\tau_u} + o(1)$, I can show:

$$E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right) < E\Pi_A\left(0, \tau_\eta^{large}\right) \Longleftrightarrow$$

$$\left(-200\,000\tau_\theta^6\right)\tau_u^5 - 20\,000\gamma^2\tau_\theta^4\left(6\tau_\theta + 5\tau_\delta\right)\tau_u^4 - 500\gamma^4\tau_\theta^2\left(44\tau_\theta^2 + 25\tau_\delta^2 + 100\tau_\theta\tau_\delta\right)\tau_u^3$$

$$-100\gamma^6\tau_\theta\left(4\tau_\theta^2 + 25\tau_\delta^2 + 85\tau_\theta\tau_\delta\right)\tau_u^2 + 5\gamma^8\left(48\tau_\theta^2 - 25\tau_\delta^2 - 120\tau_\theta\tau_\delta\right)\tau_u + 4\gamma^{10}\left(4\tau_\theta - 5\tau_\delta\right) < 0.$$

For sufficiently small τ_u,

$$E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right) < E\Pi_A\left(0, \tau_\eta^{large}\right) \Longleftrightarrow 4\tau_\theta < 5\tau_\delta.$$

Thus, if $4\tau_\theta < 5\tau_\delta$, τ_η^{large} does not form a global maximum for function $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$, and hence τ_η^{large} cannot be supported as a disclosure equilibrium. Given that τ_η^{large} is the only disclosure equilibrium candidate, there is no disclosure equilibrium when $4\tau_\theta < 5\tau_\delta$ and τ_u is sufficiently small.

Proof of Part (a)

Now suppose $4\tau_\theta \geq 5\tau_\delta$, so that $E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right) > E\Pi_A\left(0, \tau_\eta^{large}\right)$ for sufficiently small τ_u. I then examine the shape of $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$ and show that τ_η^{large} forms a global maximum of $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$. Using Part (c) of Theorem 2 and the expression of $\tau_\eta^{large} = \frac{\gamma^2}{5\tau_u} + o(1)$, I can show that the FOC of $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$ has the same sign as

$$A\left(\tau_\eta\right) = A_4\tau_\eta^4 + A_3\tau_\eta^3 + A_2\tau_\eta^2 + A_1\tau_\eta + A_0,$$

where

$$\begin{aligned}
A_4 &= -1280\tau_u^2, A_3 = 128\tau_u\left(\gamma^2 - 40\tau_u\tau_\theta\right), \\
A_2 &= 32\tau_u\left(12\gamma^2\tau_\theta - 5\gamma^2\tau_\delta - 240\tau_u\tau_\theta^2\right), \\
A_1 &= 64\tau_u\tau_\theta\left(6\gamma^2\tau_\theta - 5\gamma^2\tau_\delta - 80\tau_u\tau_\theta^2\right), \\
A_0 &= -\left(5\gamma^4\tau_\delta^2 + 1280\tau_u^2\tau_\theta^4 - 128\gamma^2\tau_u\tau_\theta^3 + 160\gamma^2\tau_u\tau_\theta^2\tau_\delta\right).
\end{aligned}$$

Thus, for sufficiently small τ_u, if $4\tau_\theta \geq 5\tau_\delta$, then $A_4 < 0, A_3 > 0, A_2 > 0, A_1 > 0$, and $A_0 < 0$.

Taking derivative of $A(\tau_\eta)$ yields:

$$A'(\tau_\eta) = 4A_4\tau_\eta^3 + 3A_3\tau_\eta^2 + 2A_2\tau_\eta + A_1.$$

Given $4A_4 < 0, 3A_3 > 0, 2A_2 > 0$, and $A_1 > 0$, it must be the case that $A'(0) > 0$ and $A'(\infty) < 0$ and that $A'(\tau_\eta)$ changes signs only once (by Descartes' "rule of signs"). Hence, $A(\tau_\eta)$ first increases and then decreases. Given that $A(\tau_\eta)$ is negative at small and large values of τ_η and that τ_η^{large} is a local maximum for function $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$ (i.e., $A\left(\tau_\eta^{large} - \varepsilon\right) > 0$ for sufficiently small ε), $A(\tau_\eta)$ crosses zero twice, which corresponds to two local extreme values of τ_η. Recall that $A(\tau_\eta)$ has the same sign as the FOC of $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$, function $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$ must first decrease, then increase, and finally decrease. Thus, the two local maximum values are 0 and τ_η^{large}. Given that $E\Pi_A\left(\tau_\eta^{large}, \tau_\eta^{large}\right) > E\Pi_A\left(0, \tau_\eta^{large}\right)$ (under the condition $4\tau_\theta \geq 5\tau_\delta$), it is clear that τ_η^{large} forms a global maximum of $E\Pi_A\left(\cdot, \tau_\eta^{large}\right)$, which implies that τ_η^{large} is supported as a disclosure equilibrium.

Proof of Proposition 4

By the FOC (16) in firm A's disclosure decision problem,

$$\frac{\partial^2 E\Pi_A}{\partial\tau_\eta\partial\tau_\xi} = \frac{\partial}{\partial\tau_\xi}\left[\frac{1}{9\left(\tau_s + \tau_\delta\right)^2}\frac{\partial\tau_s}{\partial\tau_\eta}\right].$$

Using the expression of τ_s in Proposition 2, I can show that

$$\frac{\partial}{\partial\tau_\xi}\left[\frac{1}{9\left(\tau_s + \tau_\delta\right)^2}\frac{\partial\tau_s}{\partial\tau_\eta}\right] \propto -16\left(\tau_\theta + \tau_\eta\right)^2\left(\tau_\theta + \tau_\xi\right)^2\tau_u + 3\gamma^2\tau_\delta\left(2\tau_\theta + \tau_\xi + \tau_\eta\right)^2.$$

Hence, when τ_u is sufficiently small, $\frac{\partial^2 E\Pi_A}{\partial\tau_\eta\partial\tau_\xi} > 0$. Given symmetry, $\frac{\partial^2 E\Pi_B}{\partial\tau_\eta\partial\tau_\xi} > 0$.

Proof of Proposition 5

Proof of Part (a)

When $\tau_u = 0$, price informativeness τ_s is equal to 0, and so the profit expression in equation (15) becomes

$$E\Pi_A(\tau_\eta, \tau_\xi)\big|_{\tau_u=0} = \frac{m^2}{9} + \frac{9\tau_\theta + 4\tau_\eta}{36\tau_\theta\left(\tau_\theta + \tau_\eta\right)} + \frac{\tau_\xi}{9\tau_\theta\left(\tau_\theta + \tau_\xi\right)}.$$

Taking derivatives shows $\frac{\partial E\Pi_A\left(\tau_\eta,\tau_\xi\right)\big|_{\tau_u=0}}{\partial\tau_\eta} < 0$. Thus, no disclosure is a dominant strategy, which implies that the nondisclosure equilibrium serves as the unique equilibrium (i.e., $\tau_\eta^* = \tau_\xi^* = 0$).

The multiplicity result follows immediately from Part (a) of Theorem 3.

Using the expression of $\tau_\eta^* = \tau_\xi^* = \frac{\gamma^2}{5\tau_u} + o(1)$ and the profit expression in equation (15), I can show that $E\Pi_A\left(\frac{\gamma^2}{5\tau_u}, \frac{\gamma^2}{5\tau_u}\right) - E\Pi_A(0,0)$ has the same sign as $\Delta\Pi(\tau_u) = -2000\tau_\theta^4\tau_u^3 - 1000\gamma^2\tau_\theta^2\tau_\delta\tau_u^2 + 5\gamma^4\left(32\tau_\theta^2 - 25\tau_\delta^2 - 40\tau_\theta\tau_\delta\right)\tau_u + 4\gamma^6\left(4\tau_\theta - 5\tau_\delta\right)$. Thus, when τ_u is sufficiently small, $\Delta\Pi(\tau_u) > 0$ provided $4\tau_\theta > 5\tau_\delta$.

Proof of Part (b)

By $\tau_\eta^* = \tau_\xi^* = \frac{\gamma^2}{5\tau_u} + o(1)$, it is clear that τ_η^* decreases with τ_u and diverges to ∞ as $\tau_u \to 0$.

By the expression of τ_s and a_f in Proposition 2, direct computations show that τ_s^* and a_f increase with τ_η, and that $\tau_s^* \to \infty$ and $a_f \to 1$, as $\tau_\eta^* \to \infty$.

Proof of Theorem 4

In the main text, I have already proved $ER_L^* \geq ER_L^\varnothing$ and $CE_S^* \leq CE_S^\varnothing$. Now I prove $E\Pi_A^* > E\Pi_A^\varnothing$ and $CS^* > CS^\varnothing$.

Proof of $E\Pi_A^* > E\Pi_A^\varnothing$

Let $\tau_s(\tau_\eta, \tau_\xi)$ denote the expression of τ_s in Proposition 2, i.e., $\tau_s(\tau_\eta, \tau_\xi) = \frac{16\tau_u(\tau_\theta+\tau_\eta)^2\left(\tau_\theta+\tau_\xi\right)^2}{\gamma^2\left(2\tau_\theta+\tau_\xi+\tau_\eta\right)^2}$. By profit expression (15), firm A's expected profit in the setting with learning from prices is:

$$E\Pi_A^* = \max_{\tau_\eta}\left[\frac{m^2}{9} + \frac{9\tau_\theta + 4\tau_\eta}{36\tau_\theta(\tau_\theta+\tau_\eta)} + \frac{\tau_\xi^*}{9\left(\tau_\theta+\tau_\xi^*\right)\tau_\theta} + \frac{\tau_s\left(\tau_\eta,\tau_\xi^*\right)}{9\tau_\delta\left(\tau_s\left(\tau_\eta,\tau_\xi^*\right)+\tau_\delta\right)}\right].$$

Similarly, firm A's expected profit in the benchmark setting without learning from prices is:

$$E\Pi_A^\varnothing = \max_{\tau_\eta}\left[\frac{m^2}{9} + \frac{9\tau_\theta + 4\tau_\eta}{36\tau_\theta(\tau_\theta+\tau_\eta)}\right] = \frac{m^2}{9} + \frac{1}{4\tau_\theta}.$$

Hence,

$$\begin{aligned}
E\Pi_A^* &= \max_{\tau_\eta}\left[\frac{m^2}{9}+\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}+\frac{\tau_s\left(\tau_\eta,\tau_\xi^*\right)}{9\tau_\delta\left(\tau_s\left(\tau_\eta,\tau_\xi^*\right)+\tau_\delta\right)}\right]+\frac{\tau_\xi^*}{9\left(\tau_\theta+\tau_\xi^*\right)\tau_\theta} \\
&> \max_{\tau_\eta}\left[\frac{m^2}{9}+\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}+\frac{\tau_s\left(\tau_\eta,\tau_\xi^*\right)}{9\tau_\delta\left(\tau_s\left(\tau_\eta,\tau_\xi^*\right)+\tau_\delta\right)}\right] \\
&> \max_{\tau_\eta}\left[\frac{m^2}{9}+\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}\right] \\
&\geq \frac{m^2}{9}+\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)} \text{ for any } \tau_\eta \Rightarrow \\
E\Pi_A^* &> \max_{\tau_\eta}\left[\frac{m^2}{9}+\frac{9\tau_\theta+4\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}\right]=E\Pi_A^{\varnothing}.
\end{aligned}$$

Proof of $CS^* > CS^{\varnothing}$

Using the FOC of consumers' problem, I can compute:

$$CS^* = \frac{E\left[\left(q_A^*+q_B^*\right)^2\right]}{2}.$$

Then using the expressions of q_A^* and q_B^* in Proposition 2, I can obtain

$$CS^* = \frac{2m^2}{9}+\frac{9\tau_\theta+16\tau_\eta}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}+\frac{2\tau_s}{9\tau_\delta\left(\tau_s+\tau_\delta\right)}.$$

Similarly, in the setting without learning from prices,

$$CS^{\varnothing} = \frac{2m^2}{9}+\frac{1}{4\tau_\theta}.$$

Thus,

$$CS^*-CS^{\varnothing} = \frac{7\tau_\eta^*}{36\tau_\theta\left(\tau_\theta+\tau_\eta\right)}+\frac{2\tau_s^*}{9\tau_\delta\left(\tau_s^*+\tau_\delta\right)} > 0 \Rightarrow CS^* > CS^{\varnothing}.$$

References

Amador, M., Weill, P. O., 2010, "Learning from prices: Public communication and welfare," *Journal of Political Economy* 118, 866–907.

Arya, A., Mittendorf, B., Yoon, D-H., 2016, "Public disclosures in the presence of suppliers and competitors," Working Paper.

Bagnoli, M, Watts, S., 2015, "Competitive intelligence and disclosure," *RAND Journal of Economics* 46, 709–729.

Bakke T. E., Whited, T. M., 2010, "Which firms follow the market? An analysis of corporate investment decisions," *Review of Financial Studies* 23, 1941–1980.

Banerjee, S., Davis, J., Gondhi, N., 2017, "When transparency improves, must prices reflect fundamentals better?", Working paper.

Bhattacharya, S., Ritter, J. R., 1983, "Innovation and communication: Signaling with partial disclosure," *Review of Economic Studies* 50, 331–46.

Black, F., 1976, "The pricing of commodity contracts," *Journal of Financial Economics* 3, 167-179.

Bond, P., Edmans, A., Goldstein, I., 2012, "The real effects of financial markets," *Annual Review of Financial Economics* 4, 339–360.

Bond, P., Goldstein, I., 2015, "Government intervention and information aggregation by prices," *Journal of Finance* 70, 2777–2811.

Chen, Q., Goldstein, I., Jiang, W., 2007, "Price informativeness and investment sensitivity to stock price," *Review of Financial Studies* 20, 619–650.

Cheng, I. H., Xiong, W., 2014, "The financialization of commodity markets," *Annual Review of Financial Economics* 6, 419–441.

Darrough, M., 1993, "Disclosure policy and competition: Cournot vs. Bertrand," *The Accounting Review* 68, 534–561.

Easley, D., O'Hara, M., Yang, L., 2016, "Differential access to price information in financial markets," *Journal of Financial and Quantitative Analysis* 51, 1071–1110.

Fama, E. F., Miller, M., 1972, *The Theory of Finance.* Holt, Rinehart & Winston, New York.

Foster, G., 1986, Financial Statement Analysis. Englewood Cliffs, NJ: Prentice-Hall.

Foucault, T., Frésard, L., 2014, "Learning from peers' stock prices and corporate investment," *Journal of Financial Economics* 111, 554–577.

Gal-Or, E., 1985, "Information sharing in oligopoly," *Econometrica* 53, 329–343.

Gal-Or, E., 1986, "Information transmission—Cournot and Bertrand equilibria," *Review of Economic Studies* 53, 85–92.

Gao, P., Liang, P. J., 2013, "Informational feedback, adverse selection, and optimal disclosure policy," *Journal of Accounting Research* 51, 1133–1158.

Goldstein, I., Huang, C., 2017, "The real effects of almost uninformative prices," Working Paper.

Goldstein, I., Yang, L., 2015, "Information diversity and complementarities in trading and information acquisition," *Journal of Finance* 70, 1723–1765.

Goldstein, I., Yang, L., 2017, "Information disclosure in financial markets," *Annual Review of Financial Economics* 9, 101–125.

Goldstein, I., Yang, L., 2018, "Good disclosure, bad disclosure," *Journal of Financial Economics*, Forthcoming.

Grossman, S., Stiglitz, J., 1980, "On the impossibility of informationally efficient markets," *American Economic Review* 70, 393–408.

Han, B., Tang, Y., Yang, L., 2016, "Public information and uninformed trading: Implications for market liquidity and efficiency," *Journal of Economic Theory* 163, 604–643.

Hayek, F., 1945, "The use of knowledge in society," *American Economic Review* 35, 519–530.

Hirshleifer, J. 1971, "The private and social value of information and the reward to inventive activity," *American Economic Review* 61, 561–574.

Kurlat, P., Veldkamp, L., 2015, "Should we regulate financial information?", *Journal of Economic Theory* 158, 697–720.

Luo, Y., 2005, "Do insiders learn from outsiders? Evidence from mergers and acquisitions," *Journal of Finance* 60, 1951–1982.

Raith, M., 1996, "A general model of information sharing in oligopoly," *Journal of Economic Theory* 71, 260–288.

Singh, N., Vives, X., 1984, "Price and quantity competition in a differentiated duopoly," *The RAND Journal of Economics* 15, 546–554.

Vives, X., 1984, "Duopoly information equilibrium: Cournot and Bertrand," *Journal of Economic Theory* 34, 71–94.

Vives, X., 2008, "Information sharing among firms," *The New Palgrave Dictionary of Economics*, ed. by S. N. Durlauf and L. E. Blume, Basingstoke: Palgrave Macmillan.