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Information Aggregation in Dynamic Markets with Adverse Selection*

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Information Aggregation in Dynamic Markets with Adverse Selection

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Abstract

How effectively does a decentralized marketplace aggregate information that is dispersed throughout the economy? We study this question in a dynamic setting where sellers have private information that is correlated with an unobservable aggregate state. We first characterize equilibria with an arbitrary finite number of informed traders. A common feature is that each seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. We then ask whether the state is revealed as the number of informed traders goes to infinity. Perhaps surprisingly, the answer is no; we provide generic conditions under which information aggregation necessarily fails. In another region of the parameter space, aggregating and non-aggregating equilibria can coexist. We then explore the implications for policies meant to enhance information dissemination in markets. We argue that reporting lags ensure information aggregation while a partially revealing information policy can increase trading surplus.

JEL: G14, G18, D47, D53, D82, D83.

Keywords: Information Aggregation, Decentralized Markets, Adverse Selection, Information Design.

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1 Introduction

Since the seminal work of Hayek (1945), the question of whether markets effectively aggregate dispersed information has been a central one in economics. Formal investigations of this question are typically conducted in a setting with a single (perhaps divisible) asset about which traders have dispersed information. Whether information is aggregated then usually boils down to whether the equilibrium price reveals the value of the asset conditional on the union of traders' information.[1] This broad class of models is natural for many applications from static common-value auctions to dynamic trading in financial markets. For other applications (e.g., real estate, OTC markets), information dispersion arises due to dispersion in ownership, and one is interested in the extent to which aggregate trading behavior across many different assets reveals information about the underlying state of the economy. In this paper, we explore such a setting.

More specifically, we investigate the question of information aggregation in a dynamic setting with many assets, whose values are independently and identically drawn from a distribution that depends on an underlying aggregate state. The value of each asset is privately observed by its seller, who receives offers each period from competitive buyers. We ask whether the history of all transactions reveals the aggregate state as the number of informed sellers in the economy (denoted by N) grows large.

To answer this question, we begin by characterizing the set of equilibria for arbitrary but finitely many N. Due to a complementarity between the amount of information collectively revealed by others and the optimal strategy of an individual seller, multiple equilibria can exist. A feature common to all equilibria is that each individual seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. Intuitively, one might expect that, by the law of large numbers, the state would be revealed as the number of sellers tends to infinity.

Our first main result shows that this intuition is incorrect. We provide necessary and sufficient conditions under which there does not exist a sequence of equilibria that reveal the state as $N \to \infty$. The reason why aggregation fails is that the information content of each individual seller's behavior tends to zero at a rate of $1/N$, just fast enough to offset the additional number of observations. As a result, some information is revealed by the limiting trading behavior, but not enough to precisely determine the underlying state. Roughly speaking, the conditions for non-aggregation require that the correlation of asset values is sufficiently high and that agents

[1] Seminal works on this topic include Grossman (1976), Wilson (1977) and Milgrom (1979). More recent progress on this question has been made by Pesendorfer and Swinkels (1997), Kremer (2002), Albagli et al. (2015), Lauermann and Wolinsky (2013), Mihm and Siga (2017), Bodoh-Creed (2013), and Axelson and Makarov (2017).

are sufficiently patient. Intuitively, these conditions guarantee that if the aggregate state were to be revealed with certainty tomorrow, then the option value of delaying trade today is relatively high.

When these conditions are not satisfied, there exists a sequence of equilibria such that information about the state is aggregated as $N \to \infty$. However, even in this case, information aggregation is not guaranteed. Our second main result shows that there exists a region of the parameter space in which there is coexistence of equilibria that reveal the state with equilibria that do not. The key difference across the two types of equilibria is the rate at which trade declines as the number of informed sellers grows. In the non-aggregating equilibria, trade declines at rate $1/N$ whereas in aggregating equilibria, the rate of trade declines slower than $1/N$. We are not aware of analogous coexistence results in the literature.

Though we do not model them explicitly in this paper, there are a variety of reasons for why information aggregation is a desirable property. For instance, such information may be useful for informing firms' investment decisions (Fishman and Hagerty, 1992; Leland, 1992; Dow and Gorton, 1997), government interventions (Bond et al., 2009; Bond and Goldstein, 2015; Boleslavsky et al., 2017), and monetary policy (Bernanke and Woodford, 1997). Markets that convey more information can also be more useful for providing better incentives to managers (Baumol, 1965; Fishman and Hagerty, 1989) and mitigating the winner's curse in common-value auctions (Milgrom and Weber, 1982).[2]

Given the desirability of markets which aggregate information, our main results give rise to natural questions about market regulation and design. For instance, how should a regulator disclose trading activity to market participants in order to ensure that aggregation obtains? Under what circumstances is concealing information desirable? Is there a trade-off between maximizing trading surplus and information aggregation? We argue that introducing reporting "lags" is a simple mechanism that can be used to ensure that information is aggregated. Creating a delay between when a trade happens and when it is publicly disclosed to market participants prevents the rate of trade from converging to zero, which ensures that the information content of each individual trade remains non-trivial. Clearly then as $N \to \infty$, information is aggregated.

While this simple instrument ensures information is (eventually) aggregated, it uniformly delays revelation and thereby limits the scope for mitigating the adverse selection problem. A social planner can ensure aggregation while increasing trading surplus by revealing some information without delay and some information with delay. One way to accomplish such a revelation policy is to arrange market participants on segmented trading platforms, each with

[2] See Bond et al. (2012) for a survey of both the theoretical and empirical literature on the real effects of information conveyed through markets.

a finite number of traders. Within a platform, traders can observe all trading activity in real time, but across platforms trades are disclosed only with a lag. This structure balances a trade-off between providing the market with information to overcome adverse selection, while not revealing so much that individual trades become uninformative to the point where aggregation fails.

Reporting lags and segmented trading platforms are simple and seemingly empirically relevant.[3] More generally, one could consider a richer class of information revelation policies and characterize the policy that reveals as much information as early as possible while simultaneously maximizing gains from trade. In solving this problem, a novel feedback effect arises. Namely, that the revelation policy itself influences how market participants behave (and therefore the information content of their trading behavior), which in turn influences the information content of whatever is in fact revealed. Though a complete analysis of such a problem is left for future work, we are able to provide sufficient conditions under which the Pareto optimal mechanism involves concealing some information.

Recently, there has been a strong regulatory push towards making financial markets more transparent (i.e., disclosing more information about trading activity to market participants). For example, one of the stated goals of the Dodd-Frank Act of 2010 is to increase transparency and information dissemination in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Our results highlight a potential trade-off for such policies and provide a justification for limiting the amount of information available to market participants (or at least, delaying its disclosure).

The introduction of benchmarks that reveal some aggregate trading information has also received recent attention by policy makers and academics. Duffie et al. (2017) analyze the role of benchmarks (e.g., LIBOR) in revealing information about fundamentals and suggest that the introduction of benchmarks is welfare enhancing. Our analysis highlights an important consideration that is absent in their setting. Namely, that the informational content of the benchmark may change once it is published due to endogenous responses by market participants.

1.1 Related Literature

Kyle (1985) studies a dynamic insider trading model and shows that the insider fully reveals his information as time approaches the end of the trading interval. Foster and Viswanathan (1996) and Back et al. (2000) extend this finding to a model with multiple strategic insiders with

[3] For instance, TRACE delays disseminating information about transactions for certain types of securities. See Hendershott and Madhavan (2015) for a discussion of the various venues on which trading financial securities are traded.

different information. Ostrovsky (2012) further generalizes these findings to a broader class of securities and information structures. He considers a dynamic trading model with finitely many partially informed traders and provides necessary and sufficient conditions on security payoffs for information aggregation to obtain. Our paper differs from these works in that we study a setting with heterogeneous but correlated assets owned by privately informed sellers. We ask whether information aggregates as the number of sellers becomes arbitrarily large. Despite the fact that we look at the limit as $N \to \infty$, the strategic considerations do not vanish in our model since there is an idiosyncratic component to the value of each asset.

Golosov et al. (2014) consider an environment in which a fraction of agents has private information about an asset while the other fraction are uninformed. Agents trade in a decentralized anonymous market through bilateral matches, i.e., signaling with trading histories is not possible. They find that information aggregation obtains in the long run. In contrast, in our setting observing trading histories plays a crucial role: signaling through delay diminishes the amount of trade, thus reducing the information content of the market, leading to the possibility that information aggregation fails.

Lauermann and Wolinsky (2016) study information aggregation in a search market, in which an informed buyer sequentially solicits offers from sellers who have noisy information about the buyer's value. They provide conditions under which information aggregation fails, and they trace this failure to a strong form of winner's curse that arises in a search environment. Although our model is quite different, we share the common feature that the fear of adverse selection hinders trade and thus reduces information generation in markets.

Babus and Kondor (2016) explore how the network structure affects information diffusion in a static OTC model with a single divisible asset. They show that strategic considerations do not influence the degree of information diffusion. However, the network structure combined with a private value component leads to an informational externality that constrains the informativeness of prices and hence the informational efficiency of the economy.

Finally, our paper is related to a growing literature that studies dynamic markets with adverse selection (e.g., Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2012), Fuchs et al. (2016), Daley and Green (2012, 2016)). Our innovation is the introduction of asset correlation, which allows us to study the information aggregation properties of these markets. This paper builds upon our previous work, Asriyan et al. (2017), which demonstrates that multiple equilibria can exist in a setting with two informed agents. Whereas, in this paper we focus on the information aggregation properties in a setting with many informed agents and explore policies related to information design.

2 The Model

There are $N+1$ sellers indexed by $i \in \{1, ..., N+1\}$, with $N \geq 1$. Each seller is endowed with an indivisible asset and is privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$. Seller i has a value c_{θ_i} for her asset, where $c_L < c_H$. The value of a type-θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$. One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and the buyer respectively.

We start by considering a model in which there are two trading periods: $t \in \{1, 2\}$. We generalize our results to an infinite-horizon model in Section 4.[4] In each period, multiple competing buyers make offers to each seller. A buyer whose offer is rejected gets a payoff of zero and exits the game.[5] The payoff to a buyer who purchases an asset of type θ at price p is $v_\theta - p$. Sellers discount future payoffs by a factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ, who agrees to trade at a price p in period t is

$$\left(1 - \delta^{t-1}\right) c_\theta + \delta^{t-1} p. \tag{1}$$

If the seller does not trade at either date, his payoff is c_θ. All players are risk neutral.

The key feature of the model is that asset values are correlated with an unobservable underlying state, S, that takes values in $\{l, h\}$. The unconditional distribution of θ_i is $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$. Assets are mutually independent conditional on the state, but their conditional distributions are given by $\mathbb{P}(\theta_i = L | S = l) = \lambda \in (1 - \pi, 1)$. To allow for arbitrarily high level of correlation, we set $\mathbb{P}(S = h) = \pi$.

Importantly, our correlation structure introduces the possibility that a trade of one asset contains relevant information about the aggregate state and therefore the value of other assets. To capture this possibility, we assume that all transactions are observable. Therefore, prior making offers in the second period, buyers observe the set of assets that traded in the first period. For convenience, we assume that offers are made privately (i.e., the level of rejected offers is not observed by other buyers).

Notice that by virtue of knowing her asset quality, each seller has a private and conditionally independent signal about the aggregate state of nature. Thus, if each seller were to report her information truthfully to a central planner, then the planner would learn the aggregate state with probability one as $N \to \infty$. Our interest is to explore under what conditions the same information can be gleaned from the transaction data of a decentralized market. To ensure

[4] The two-period model facilitates a more precise characterization of the set of equilibria and thus a sharper intuition for our main results.

[5] That buyers are "short-lived" (i.e., make offers in only one period) is a fairly standard assumption in this literature (e.g., Swinkels, 1999; Kremer and Skrzypacz, 2007; Hörner and Vieille, 2009). Our results can be extended to a setting with long-lived buyers if the offers are publicly observable.

that strategic interactions remain relevant, we focus on primitives which satisfy the following assumptions.

Assumption 1. $\pi v_H + (1-\pi)v_L < c_H$.

Assumption 2. $v_L < (1-\delta)c_L + \delta c_H$.

The first assumption, which we refer to as the "lemons" condition, asserts that the adverse selection problem is severe enough to rule out the efficient equilibrium in which all sellers trade immediately. In this equilibrium, trade is uninformative about the underlying state (regardless of N). The second assumption implies a lower bound on the discount factor and ensures that dynamic considerations remain relevant. Our main results do not rely on this assumption but it simplifies exposition and rules out fully separating equilibria, which are also independent of N.

2.1 Strategies and Equilibrium Concept

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer's information set is empty. In the second period, buyers know whether each asset traded in the first period. If asset i trades in the first period, then it is efficiently allocated and it is without loss to assume that buyers do not make offers for it in the second period (Milgrom and Stokey, 1982). The strategy of each seller is a mapping from her information set to a probability of acceptance. Seller i's information includes her type, the set of previous and current offers as well as the information set of buyers.

We use Perfect Bayesian Equilibria (PBE) as our solution concept. This has three implications. First, each seller's acceptance rule must maximize her expected payoff at every information set taking buyers' strategies and the other sellers' acceptance rules as given (*Seller Optimality*). Second, any offer in the support of the buyer's strategy must maximize his expected payoff given his beliefs, other buyers' strategy and the sellers' strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to Bayes' rule whenever possible (*Belief Consistency*).

2.2 Updating

Let σ_i^θ denote the probability that seller i trades in the first period if her asset is type θ. There are two ways in which the prior about seller i is updated between the first and second periods. First, conditional on rejecting the offer in the first period, buyers' interim belief is given by

$$\pi_{\sigma_i} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi(1-\sigma_i^H)}{\pi(1-\sigma_i^H) + (1-\pi)\,(1-\sigma_i^L)} \tag{2}$$

Second, before making offers in the second period, buyers learn about any other trades that took place in the first period. How this information is incorporated into the posterior depends on the trading strategy of the other sellers (i.e., σ_j^θ, $j \neq i$). Let $z^j \in \{0,1\}$ denote the indicator for whether seller j trades in the first period, and let $\mathbf{z} = (z^j)_{j=1}^{N+1}$ and $\mathbf{z}_{-i} = (z^j)_{j \neq i}$. Denote the probability of $\mathbf{z}_{-i}$ conditional on seller i being of type θ by $\rho_\theta^i(\mathbf{z}_{-i})$, which can be written as

$$\rho_\theta^i(\mathbf{z}_{-i}) \equiv \sum_{s \in \{l,h\}} \mathbb{P}\left(S = s | \theta_i = \theta\right) \cdot \prod_{j \neq i} \mathbb{P}\left(z^j | S = s\right), \tag{3}$$

where $\mathbb{P}\left(z^j = 1 | S = s\right) = \sum_{\theta \in \{L,H\}} \sigma_j^\theta \cdot \mathbb{P}\left(\theta_j = \theta | S = s\right)$ is the probability that seller j traded in state s. Provided there is positive probability that i rejects the bid at $t = 1$ and $\mathbf{z}_{-i}$ is realized, we can use equations (2) and (3) to express the posterior probability of seller i being high type conditional on these two events:

$$\pi_i(\mathbf{z}_{-i}; \sigma_i, \sigma_{-i}) \equiv \mathbb{P}(\theta_i = H | z^i = 0, \mathbf{z}_{-i}) = \frac{\pi_{\sigma_i} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right)}{\pi_{\sigma_i} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right) + (1 - \pi_{\sigma_i}) \cdot \rho_L^i(\mathbf{z}_{-i})}. \tag{4}$$

To conserve on notation, we often suppress arguments of π_i.

2.3 Equilibrium Properties

Asriyan et al. (2017) analyze the model with two sellers (i.e., $N = 1$), and they establish several properties that must hold in any equilibrium. These properties extend to the model studied here with an arbitrary number of sellers.

In order to introduce them, we will use the following definitions and notation. We refer to the *bid* for asset i at time t as the maximal offer made across all buyers for asset i at time t. Let $V(\tilde{\pi}) \equiv \tilde{\pi} v_H + (1 - \tilde{\pi}) v_L$ denote buyers' expected value for an asset given an arbitrary belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi, 1)$ be such that $V(\bar{\pi}) = c_H$, and recall that π_i denotes the probability that buyers assign to $\theta_i = H$ prior to making offers in the second period.

Property 1 (Second period) *If seller i does not trade in the first period, then in the second period:*

(i) If $\pi_i > \bar{\pi}$ then the bid is $V(\pi_i)$, which the seller accepts w.p.1.

(ii) If $\pi_i < \bar{\pi}$ then the bid is v_L, which the high type rejects and the low type accepts w.p.1.

(iii) If $\pi_i = \bar{\pi}$, then the bid is $c_H = V(\pi_i)$ with some probability $\phi_i \in [0,1]$ and v_L otherwise.

Note that a high type will only accept a bid higher than c_H. When the expected value of the asset is below c_H (as in (i)), buyers cannot attract both types without making a loss. Thus, only the low type will trade and competition pushes the bid to v_L. When the expected value of the asset is above c_H (as in (ii)), competition forces the equilibrium offer to be the expected value. Finally, when the expected value of the asset is exactly c_H (as in (iii)), buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

Property 2 (First period) *In the first period, the bid for each asset is v_L. The high-type rejects the first period bid with probability 1. The low-type seller accepts with probability $\sigma_i \in [0,1)$.*

By the skimming property, any offer that is acceptable to a high type in the first period is accepted by the low type w.p.1. Assumption 1 implies that any such offer yields negative profits for the buyers. Hence, in equilibrium only low types trade in the first period and competition pushes the bid to v_L. Finally, if $\sigma_i = 1$, then the bid in the second period must be v_H (Property 1). But then the low-type seller i would strictly prefer to delay trade to the second period (Assumption 2), a contradiction.

Notice that Property 1 implies a second period payoff to a type-θ seller i as a function of (π_i, ϕ_i), which we denote by $F_\theta(\pi_i, \phi_i)$, where

$$F_H(\pi_i, \phi_i) \equiv \max\{c_H, V(\pi_i)\}, \tag{5}$$

and

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1-\phi_i) v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases} \tag{6}$$

Properties 1 and 2 also imply that an equilibrium can be fully characterized by $\{\sigma_i, \phi_i\}_{i=1}^{N+1}$.

From seller i's perspective, the strategy of seller $j \neq i$ in the first period is relevant because it influences the distribution of news $\mathbf{z}_{-i}$ and therefore the distribution of π_i. In particular, the (expected) continuation value of a seller from rejecting an offer in the first period can be written as

$$Q_\theta^i(\sigma_i, \sigma_{-i}, \phi_i) \equiv (1-\delta)c_\theta + \delta \sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) F_\theta(\pi_i(\mathbf{z}_{-i}), \phi_i). \tag{7}$$

where σ_{-i} denotes the vector of $\{\sigma_j\}_{j\neq i}$. It is worthwhile to note that because the posterior conditional on rejection is increasing in σ_i, so does Q_L^i. These observations naturally lead to a third useful property that equilibria must satisfy.

Property 3 (Symmetry) *In any equilibrium, $\sigma_i = \sigma > 0$ for all i. If buyer mixing is part of the equilibrium then $\phi_i = \phi$ for all i.*

The key step to prove symmetry is to show that if $\sigma_i > \sigma_j \geq 0$, then $Q_L^i > Q_L^j$. This follows from the fact that, due to imperfect correlation, π_i (and therefore Q_L^i) is more sensitive to i's own trading probability than it is to that of the other players. Note that if $Q_L^i > Q_L^j$, then the low-type seller i strictly prefers to wait, which contradicts $\sigma_i > 0$ being consistent with an equilibrium. That there must be strictly positive probability of trade then follows: if $\sigma_i = 0$ for all i, then no news arrives and buyers in the second period would have the same beliefs as buyer's in the first period. This would imply that the second period bid is v_L but in that case the low-type sellers would be strictly better off by accepting v_L in the first period, which contradicts $\sigma_i = 0$.

2.4 Equilibria

Given Properties 1–3, we can drop the subscripts and denote a candidate equilibrium by the pair (σ, ϕ). Because all equilibria are symmetric, any information about seller i that is contained in news $\mathbf{z}_{-i}$ does not depend on the identity of those who sold but only on the number (or fraction) of other sellers that traded. For example, suppose that $\mathbf{z}_{-i} = \mathbf{z}(K)$ where $\mathbf{z}(K)$ is such that $\sum_{j \neq i} z^j = K \leq N$. Then

$$\rho_\theta^i(\mathbf{z}(K)) = \sum_{s \in \{l,h\}} p_s^K \cdot (1 - p_s)^{N-K} \cdot \mathbb{P}(S = s | \theta_i = \theta),$$

where $p_s \equiv \sigma \cdot \mathbb{P}(\theta_i = L | S = s)$ is the probability that any given seller trades in state s. Naturally, the probability of observing K trades among sellers $j \neq i$ is $\binom{N}{K} \cdot \rho_\theta^i(\mathbf{z}(K))$.

Furthermore, since any equilibrium involves $\sigma \in (0, 1)$, a low-type seller must be indifferent between accepting v_L in the first period and waiting until the second period. The set of equilibria can thus be characterized by the solutions to

$$Q_L(\sigma, \sigma, \phi) = v_L. \tag{8}$$

As we show in the next proposition, there can be multiple solutions to (8) and hence multiple equilibria.

Proposition 1 (Existence and Multiplicity) *An equilibrium always exists. If λ and δ are sufficiently large, there exist multiple equilibria.*

Intuitively, a higher σ has two opposing effects on the seller's continuation value. On the one hand, the posterior beliefs and thus prices in the second period are increasing in σ, which increases the expected continuation value Q_L. On the other hand, as other low types trade more aggressively, the distribution over buyers' posteriors shifts towards lower posteriors, thus decreasing Q_L. The latter force generates complementarities in sellers' trading strategies, which results in multiple equilibria when the correlation between assets is high and traders care sufficiently about the future.

We now turn to our main question, specifically, whether information about the underlying state is aggregated as the number of informed participants grows large. To understand the essence of this question, first notice that the trading behavior of each seller provides an informative signal about the aggregate state. If the seller trades in the first period, than she reveals her asset's type is L, which is more likely when the aggregate state is l than when it is h. Conversely, if the seller does not trade in the first period, then buyers update their beliefs about the asset toward H and their belief about the aggregate state toward h. Moreover, the amount of information revealed in the first period is increasing in the low-type's trading probability, which we now denote by σ_N (in order to explicitly indicate its dependence on the number of other informed participants).

If the information content of each individual trade were to converge to some positive level (i.e., $\lim_{N\to\infty}\sigma_N = \bar{\sigma} > 0$), then information about the state would aggregate. The reason is that by the law of large numbers the fraction of assets traded would concentrate around its population mean $\bar{\sigma}\cdot\mathbb{P}(\theta_i = L|S = s)$, which is strictly greater when the aggregate state is l than when it is h. If, on the other hand, σ_N decreases to zero at a rate weakly faster than $1/N$ (i.e., $\lim_{N\to\infty} N\cdot\sigma_N < \infty$), then information would not aggregate. In this case, despite having arbitrarily many signals about the state, the informativeness of each signal goes to zero fast enough that the overall amount of information does not reveal the true state.

Of course, the equilibrium trading behavior of each individual seller is determined endogenously. Therefore, in order to establish information aggregation properties of equilibria, we need to understand how the set of equilibrium values of σ_N changes with N. Moreover, since different equilibria have different σ_N, the limiting information aggregation properties could be different for different sequences of equilibria. As we will see in the next section, neither of the two cases mentioned in the previous paragraph is pathological.

3 Information Aggregation

We begin by studying the information aggregation properties of equilibria in the first period. Consider a sequence of economies indexed by N (standing for $N+1$ assets), and let σ_N denote an equilibrium trading probability in the first period and π_N^{State} be the buyers' posterior belief that the aggregate state is h, conditional on having observed the outcome of trade in the first period. That is, given a first period trading history $\mathbf{z} = (z^j)_{j=1}^{N+1}$, $\pi_N^{State}(\mathbf{z}) \equiv \mathbb{P}(S = h|\mathbf{z})$. We say that:

Definition 1 *There is* ***information aggregation*** *along a given sequence of equilibria if* $\pi_N^{State} \rightarrow^p 1_{\{S=h\}}$ *as* $N \rightarrow \infty$.

Our notion of information aggregation requires that, upon observing the trading history, buyers (or the econometrician, who observes only whether and when an asset trades) learn all the information available in the market that is relevant to infer the aggregate state. Asymptotically, this is equivalent to asking whether agents' beliefs about the aggregate state become degenerate at the truth.[6]

3.1 A 'Fictitious' Economy

Before presenting our main results, it will be useful to consider a 'fictitious' economy in which buyers observe the true state S via an exogenous signal before making second period offers. This benchmark economy is useful because it approximates the information revealed in the true economy if there is information aggregation. We proceed by deriving a necessary and sufficient condition under which the fictitious economy supports an equilibrium with trade in the first period (Lemma 1). We then show that the same condition is necessary, though not sufficient, for information aggregation (Theorem 1). Intuitively, information aggregation requires trade. But if the fictitious economy does not support an equilibrium with trade, then (by continuity) there cannot exist a sequence of equilibria along which information aggregates.

First, note that Properties 1 and 2 trivially extend to the fictitious economy. Second, observe that conditional on knowing the true state, the information revealed by other sellers is irrelevant for buyers when forming beliefs about seller i. That is, buyers' posterior belief about seller i following a rejection in the first period and observing the true state is s is given by

$$\pi_i^{fict}(s) = \frac{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_i = H|S = s)}{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_i = H|S = s) + (1 - \pi_{\sigma_i}) \cdot \mathbb{P}(\theta_i = L|S = s)}.$$

This implies that seller $i's$ continuation value in the fictitious economy, which we denote by $Q_L^{i,fict}(\sigma_i, \phi_i)$ is independent of the trading strategies of the other sellers. Analogous to (7), the continuation value is given by

$$Q_L^{i,fict}(\sigma_i, \phi_i) = (1 - \delta)c_L + \delta \left(\lambda F_L(\pi_i^{fict}(l), \phi_i) + (1 - \lambda) F_L(\pi_i^{fict}(h), \phi_i) \right)$$

[6]That our definition involves convergence in probability is standard in the literature (see e.g., Kremer (2002)).

Since there are no complementarities, the fictitious economy has a unique equilibrium, which must be symmetric. As in Daley and Green (2012), due to the exogenous arrival of information, it is possible that the equilibrium of the fictitious economy will involve zero probability of trade in the first period.

Lemma 1 *The unique equilibrium of the fictitious economy involves zero probability of trade in the first period (i.e., $\sigma^{fict} = 0$) if and only if*

$$Q_L^{i,fict}(0,0) \geq v_L. \tag{$\star$}$$

Furthermore, ($\star$) holds if and only if λ and δ satisfy the following:

$$\lambda \geq \bar{\lambda} \equiv 1 - \frac{\pi(1-\bar{\pi})}{1-\pi}$$

and

$$\delta \geq \bar{\delta}_\lambda \equiv \frac{v_L - c_L}{\lambda v_L + (1-\lambda)V\left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) - c_L}.$$

This result is intuitive. The equilibrium of the fictitious economy features no trade whenever the low type's option value from delaying trade to the second period is high. This occurs when both the information revealed in the second period is sufficiently informative about the seller's type (i.e., $\lambda \geq \bar{\lambda}$) and for a given correlation the future is sufficiently important (i.e., $\delta \geq \bar{\delta}_\lambda$).

3.2 Main Results

We now establish our first main result, which shows that ($\star$) is also the crucial determinate of the information aggregation properties of equilibria.

Theorem 1 (Aggregation Properties)

(i) If ($\star$) holds with strict inequality, then information aggregation fails along any sequence of equilibria.

(ii) If ($\star$) does not hold, then there exists a sequence of equilibria along which information aggregates.

The proof of the first statement uses the observation that if information were to aggregate, then for N large enough the continuation payoffs of the sellers are close to the continuation payoffs in the fictitious economy. Thus, when ($\star$) holds strictly, delay is also uniquely optimal when there are a large but finite number of assets. But this contradicts Property 3, which states that $\sigma_N \in \{0, 1\}$ cannot be part of an equilibrium for any finite N. In fact, when $(\star)$ holds strictly, the trading probability σ_N is positive but must go to zero at a rate proportional to $1/N$, which is fast enough to prevent information from aggregating. The rate is also slow enough to ensure that the market does not become completely uninformative in the limit. In that case, the bid for any asset in the second period would be v_L with probability arbitrarily close to one; hence, the low types would strictly prefer to trade in the first period (implying $\sigma_N = 1$), which would contradict Property 3.[7]

On the other hand, when the fictitious economy has an equilibrium with positive trade in the first period (i.e., if $(\star)$ does not hold), we can explicitly construct a sequence of equilibria in which the trading probability σ_N is bounded away from zero. Clearly, information is aggregated along such a sequence. Nevertheless, even when aggregating equilibria exist, it is not the case that information will necessarily aggregate along *every* sequence of equilibria.

Theorem 2 (Coexistence) *There exists a $\widehat{\delta} < 1$ such that whenever $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ and λ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails. If either $\lambda < \bar{\lambda}$ or δ is sufficiently small, then information aggregates along any sequence of equilibria.*

To prove the first statement, we first note that for a given $\delta < 1$, if λ is sufficiently large, then we must have $\delta < \bar{\delta}_\lambda$ and thus by Theorem 1 aggregating equilibria must exist. We then show that if we fix δ above a certain threshold, then for a sufficiently large λ, also non-aggregating equilibria must exist. In particular, we explicitly construct a sequence of equilibria in which the second period bid is v_L for all histories except the one in which no seller has traded in the first period. In these equilibria, the probability of the event that *no seller has traded in the first period* remains bounded away from zero, in *both* states of nature. Thus, even as $N \to \infty$, the uncertainty about the state of nature does not vanish.

The second part of Theorem 2 provides sufficient conditions under which information necessarily aggregates. While this result is not particularly surprising, it is instructive to observe that the possibility of aggregation failure requires the two key ingredients of the model: (1) sufficient correlation across assets ($\lambda > \bar{\lambda}$) and that strategic delay is relevant (i.e. δ large enough).

Figure 1 summarizes our main results by illustrating the regions of the parameter space for which aggregation holds and fails as well as the region of coexistence. In the top-right (darkly shaded) region, $(\star)$ holds and hence there do not exist sequences of equilibria that aggregate information. Otherwise, aggregating equilbria exist (Theorem 1). In the bottom-left

[7] Indeed, we can show that whenever information aggregation fails, the distribution of trades converges to a Poisson distribution with mean $N \times \sigma_N \times \mathbb{P}(\theta_i = L | S = s)$.

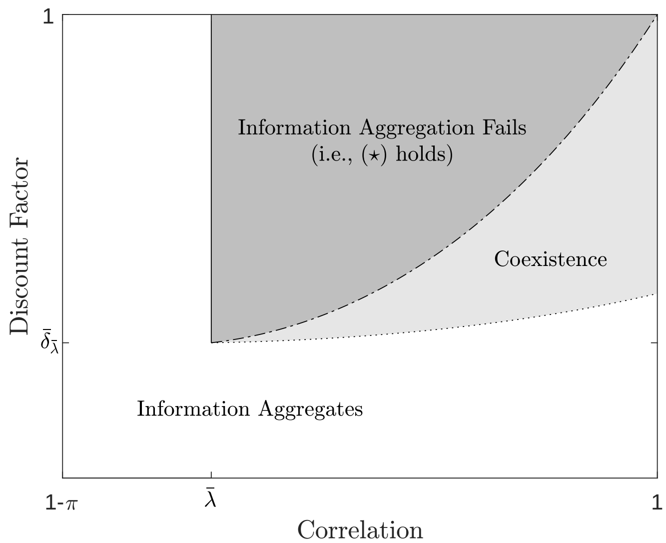

Figure 1: **When does Information Aggregate?** This figure illustrates the regions of the parameter space over which information aggregation obtains or fails.

(unshaded) region, all sequences of equilibria aggregate information and in the middle-right (lightly shaded) region, sequences in which information aggregates coexist with sequences in which information aggregation fails (Theorem 2).

Thus far, we considered the information aggregation properties of equilibria conditional on the trading history in the first period. To consider aggregation in the second period, one can simply extend Definition 1 by requiring that the convergence of buyers' beliefs be conditional on the history of trade over two periods (rather than the first period). Clearly, if information were already aggregated by the first period, it would also be aggregated in the second period. But what if information does not aggregate in the first period? Will trading behavior in the second period provide the additional information necessary to identify the true state?

The answer is that such an outcome is indeed possible. That is, there can exist sequences of equilibria in which information is not aggregated based on first period behavior, but is successfully aggregated from both first and second period behavior.[8]

[8] To illustrate this possibility in more detail, suppose that there exists a sequence of equilibria that achieves the lower bound (i.e., $\pi_i(\mathbf{z}(0)) = \bar{\pi}$) for all N. Clearly, $\pi_i(\mathbf{z}(K)) < \bar{\pi}$ for all $K > 0$. Hence, if at least one seller trades in the first period, then the second-period bid will be v_L for all sellers, and all the remaining low-type sellers will reveal their type by accepting, which aggregates information (since all and only low types trade over the two periods). Now suppose that no sellers trade in the first period (i.e., $\mathbf{z}_{-i} = \mathbf{z}(0)$). Then, in the second period, buyers will mix between a pooling bid with probability ϕ_N and a separating one with probability $(1 - \phi_N)$. If the buyer mixing is independent across sellers, the fraction of sellers who trade in the second period will converge to $\phi_N + (1 - \phi_N)\mathbb{P}(\theta = L|S)$, which also reveals the state provided ϕ_N is bounded away from 1.

However, as we argue in the next section, such aggregating equilibria are merely an artifact of there being no further opportunities to trade after the second period. More generally, in any finite horizon model, it is an equilibrium for a low-type seller to accept an offer of v_L with probability one in the last period, thus revealing her type and information about the aggregate state. If there are always additional trading opportunities, then a "last period" does not exist and such behavior cannot be part of an equilibrium. We formalize this argument in Theorem 3, which demonstrates that our main results regarding the information aggregation properties of equilibria are nearly identical in an infinite horizon model.

4 Infinite Horizon Model

In this section, we extend our main results to a setting with an infinite number of trading opportunities $t \in \{1, 2, ...\}$. Intuitively, one might expect that with more trading periods there are more opportunities to learn from trading behavior and hence more information will be revealed. However, there is a countervailing force; there are more opportunities for (strategic) sellers to signal through delay. It turns out that two factors essentially cancel each other out.

Besides allowing for an infinite number of trading opportunities, the model and the information structure is identical to the one presented in Section 2. The only additional notation we will require is the public history at (the end of) date t, which we denote by $\mathbf{z}^t = \{\mathbf{z}_1, ..., \mathbf{z}_t\}$, consists of the history of all the trades that have taken place at dates prior to and including t. Note that $\mathbf{z}^t$ also corresponds to buyers' information set prior to making offers in date $t + 1$.

Characterizing the set of all possible equilibria in the infinite horizon model is more difficult because the space of relevant histories is a complex object. In principle, the path of play can depend on sellers' beliefs about the quality of other sellers' assets, the distribution of buyers' beliefs about the quality of each seller's asset, the buyers' and the sellers' beliefs about the aggregate state, as well as the number of assets remaining on the market. Nevertheless, we are able to obtain sharp predictions regarding the information aggregation properties of the set of equilibria.

In order to illustrate these findings, we must generalize our notion of information aggregation. Let $\pi_{t,N}^{State}$ denote the buyers' posterior belief that the state is high, conditional on having observed the trading history, $\mathbf{z}^t$, in an economy with $N + 1$ sellers.

Definition 2 *There is* ***information aggregation at date t*** *along a given sequence of equilibria if* $\pi_{t,N}^{State} \to^p 1_{\{S=h\}}$ *as* $N \to \infty$.

We say that *information aggregates* along a given sequence if there exists a $t < \infty$ such that information aggregates at date t. Otherwise, we say that *information aggregation fails*.

In the previous section we argued that, because the second period was also the last trading opportunity, information could aggregate in the second period regardless of whether ($\star$) holds. The following theorem shows that, with an infinite trading horizon, ($\star$) is indeed necessary and sufficient to rule out aggregating equilibria.

Theorem 3 *Consider the infinite horizon model.*

(i) If ($\star$) holds with strict inequality, then information aggregation fails along any sequence of equilibria.

(ii) If ($\star$) does not hold, then there exists a sequence of equilibria along which information aggregates.

(iii) There exists a $\widehat{\delta} < 1$ such that whenever $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ and λ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails.

The proof hinges on arguments similar to those used in the two-period economy. For (i), we show that the earliest date in which information about the state is supposed to aggregate is similar to the first period in a two-period economy. That is, suppose that information aggregates at some date τ but not before. Because ($\star$) holds, the option value of waiting for the state to be revealed is sufficiently high to make sellers strictly prefer to delay trade at date τ. But if sellers do not trade in date τ, then no information is revealed, which means that τ cannot possibly be the earliest date of aggregation.

In order to establish (ii) and (iii), we construct a class of equilibria that essentially share the information aggregation properties of the two-period economy. A feature of this class is that once the belief about the seller weakly exceeds $\bar{\pi}$, all future bids are pooling. When ($\star$) does not hold (i.e., $\delta < \bar{\delta}_\lambda$ or $\lambda < \bar{\lambda}$), we show that such equilibria exist and that there is an equilibrium sequence within this class along which information aggregates. Then, following arguments similar to those for the proof of Theorem 2, we show that under the conditions stated in (iii), there also exists another sequence of equilibria (still within the class) in which aggregation fails.

5 Policy Implications and Market Design

In this section we explore the implications of the model for market design. To do so, we first consider a social planner who wishes to ensure that information aggregates and can decide what information about trading activity should be revealed to whom and when. As mentioned in

Section 1, we leave the motivation for the social planner's objective unspecified though there are a numerous reasons that have been proposed and documented for why information aggregation is a desirable feature of an economy.[9] We then ask whether there is a trade-off between achieving information aggregation and maximizing trading surplus.

5.1 Reporting Lags

One simple way that a planner can ensure aggregation is to introduce a reporting lag. That is, reveal all information about trading activity to all participants in the economy, but only *after* trading has taken place in the second period.[10] By doing so, a low-type seller will trade *as if* there are no other sellers in the market (i.e., as if $N = 0$) and therefore with a strictly positive probability in the first period (by Property 2) that is independent of N. Clearly then, as $N \to \infty$ information is aggregated at the end of the second period when participants observe the fraction of sellers that traded in the first period.[11]

Proposition 2 *If the planner introduces a reporting lag for all trades then there exists a unique equilibrium for any N. Moreover as $N \to \infty$, information is aggregated.*

One potential downside to this approach is that by delaying all information revelation, there are no sources of information to mitigate the adverse selection problem. This leads to a high probability of costly delays and market failures, which reduces the overall trading surplus realized relative to an economy without a reporting lag. Thus, there is a potential trade-off between information aggregation and trading surplus.[12]

Another potential downside of this approach is that it uniformly delays the revelation of information to all market participants. To the extent that the timing of information aggregation is important (e.g., in order to make unmodeled investment decisions), a uniform reporting lag for all trading activity is likely too blunt of an instrument.

5.2 Segmented Trading Platforms

A less drastic alternative to a uniform reporting lag for all trades is to reveal some information without a lag and some information with a lag. One way to accomplish this information

[9]See Bond et al. (2012) for a summary of both theoretical and empirical work on this topic.

[10]Therefore, a buyer interested in purchasing seller i's asset will not observe trading activity of other sellers $j \neq i$ until after the second period.

[11]A similar policy works with a longer trading horizon though it may require a reporting lag of more than one period.

[12]In any equilibrium, the expected payoff to a low-type seller is v_L and buyers make zero profit. Therefore, total welfare can be measured by a high-type seller's equilibrium payoff. Moreover, any increase in total welfare corresponds to a Pareto improvement.

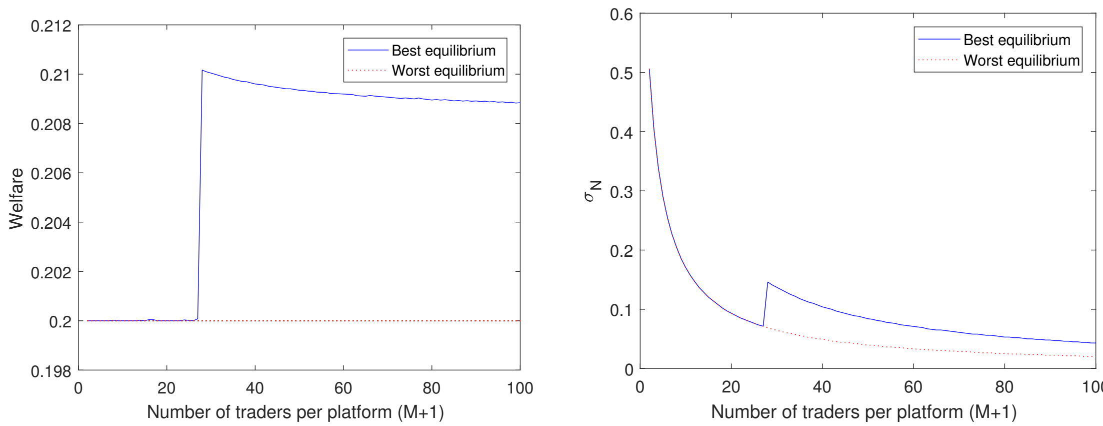

Figure 2: The left panel illustrates how the welfare per trader depends on the number of traders per platform. The right panel shows the corresponding strategy of the seller in the first period. The parameters are such that $(\star)$ holds and, hence, absent intervention aggregating equilibria do not exist.

revelation policy is to arrange traders on different *platforms*. Buyers and sellers can observe trading behavior of others on the same platform in real time (i.e., immediately after it occurs and before the second trading period), but across platforms trading behavior is only revealed with a lag. Suppose that there are a total of $M + 1$ sellers assigned to each platform. Then, within the platform, each seller will behave *as if* $N = M$. Again by Property 2, with only a finite number of other traders, each low-type seller i will trade with strictly positive probability in the first-period and therefore information will aggregate when the trading activity across all platforms is revealed after the second period.[13]

The advantage of this approach is twofold. First, traders will have access to at least some information prior to the second period. And perhaps more importantly, the planner can increase total trading surplus by appropriately choosing the number of traders on each platform. Figure 2 illustrates an example in which the optimal number of traders on each platform is finite. The solid blue line in the left panel illustrates the total surplus in the "best" equilibrium (i.e., the equilibrium with the highest trading surplus).[14] Notice that a new equilibrium with discretely higher trading surplus emerges when $M = 27$. In this example, it is also the case that information aggregation fails absent some form of intervention (i.e., $(\star)$ holds). Therefore, by arranging market participants on segmented trading platforms and introducing a reporting lag across platforms, the social planner can *both* improve welfare and achieve information aggregation (albeit with delay).

[13] An alternative way to achieve the same information revelation policy is to publicly reveal the trading behavior of a finite subsample of the population.

[14] Figures 2 and 3 can alternatively be interpreted as illustrating the comparative static on the number of traders in the economy without any policy intervention (where $N = M$).

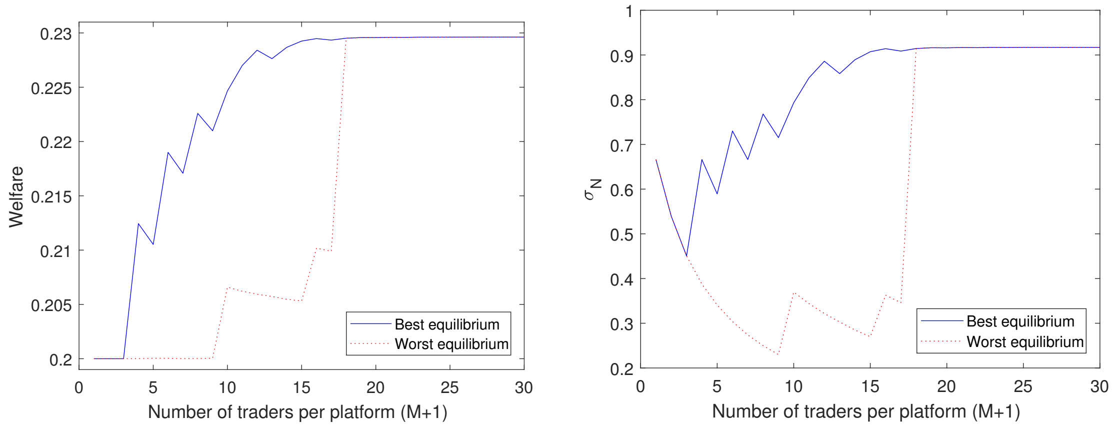

Figure 3: The left panel illustrates how the welfare per trader depends on the number of traders per platform. The right panel shows the corresponding strategy of a low-type seller in the first period. The parameters are such that absent intervention only aggregating equilibria exist.

That there is an interior optimal number of traders per platform for maximizing welfare is driven by the fact that more information revelation leads to an endogenous response by sellers. Namely, for large enough M the seller trades less aggressively (see the right panel of Figure 2), which has a negative effect on welfare.

On the other hand, Figure 3 shows that the effect is not always negative. More specifically, it illustrates that, for a different set of parameters, increasing the number of traders still leads to a response by sellers, but the effect is positive: as M increases, sellers trade more aggressively, which leads to higher welfare. In this example, the parameters are such that, information aggregates (in the first period) along all sequences of equilibria absent intervention. This example suggests that when information aggregates in the first period without intervention, there is no need to delay aggregation in order to maximize trading surplus. However, as we will show in the next section, this conclusion is not true once we allow for more general revelation policies.

5.3 Information Design

In the previous two subsections, we considered several specific types of information revelation policies. More generally, one might be interested in considering a broader class of information revelation policies and find the policy that maximizes welfare subject to revealing as much information as early as possible. While a complete analysis of such a problem is beyond the scope of this paper, we are able to provide sufficient conditions under which maximizing welfare necessitates concealing some information.

Before doing so, it is useful to compare this problem to the literature on Bayesian persuasion

(Kamenica and Gentzkow, 2011; Rayo and Segal, 2010) and "information design" problems more generally (Bergemann and Morris, 2013, 2016).[15] On one hand, the problems are quite similar. Both involve designing an information revelation policy to induce other players to take certain desired actions. On the other hand, the planner's problem in our setting must take into account a novel feedback effect. Namely, the policy influences the information content of trading behavior, and therefore the information content of whatever is revealed. In short, the statistical properties of the information the planner can reveal, which is typically exogenous in a Bayesian persuasion setting, depends on the policy itself.

Proposition 3 *Suppose that* $(\star)$ *is violated (so that aggregating equilibria exist) and* δ *is sufficiently large. Then, when* N *is large enough, the social planner can (generically) increase the trading surplus in the best equilibrium with a partially revealing information policy.*

We establish the result in two steps. First, we show that if the social planner knew the state, then it would be sub-optimal for her to fully reveal it, i.e., she would prefer to reveal a noisy signal of the true state. A simple policy that the planner can use to increase surplus is as follows. When the true state is low, the planner makes a report of 0. When the true state is high, the planner makes a report of 1 with probability $1 - \gamma$ and a report of 0 with probability γ. In the proof of Proposition 3, we show that there exists a $\gamma > 0$ such that this policy leads to strictly higher trading surplus. Denote this policy by Γ. Intuitively, the reason why suppressing information can increase surplus is similar to the intuition for the example in Figure 2. By doing so, the planner reduces the low-type's payoff from delaying trade, which thereby induces the seller to trade with a higher probability in the first period and reduces costly delays.

Second, we show that there exists an information policy and a sequence of equilibria such that the posterior beliefs of traders converge to the ones under Γ. This policy involves a threshold such that when the total fraction of sellers who trade in the first period is above the threshold, the planner makes a report of 0, and when the total fraction of sellers who trade is below the threshold, the planner makes a report of 1 with probability $1 - \gamma$ and makes a report of 0 with probability γ. An interesting feature of this revelation policy is that information is in fact aggregated given the planner's information even though it does not aggregate based on what is publicly revealed.

[15] Bergemann and Morris (2017) provide a more general treatment of information design problems drawing a distinction between whether the designer has an informational advantage (as in Bayesian persuasion) or not (as in communication games). In our model, the planner has no informational advantage ex-ante but has a technology for acquiring one in the interim. Another important distinction of our setting is that the planner has only limited means by which she can elicit information.

6 Concluding Remarks

We study the information aggregation properties of decentralized dynamic markets in which traders have private information about the value of their asset, which is correlated with some underlying 'aggregate' state of nature. We provide necessary and sufficient conditions under which information aggregation necessarily fails. Further, we show that when these conditions are violated, there can be a coexistence of non-trivial equilibria in which information about the state aggregates with equilibria in which aggregation fails. Our findings suggest there are important differences in the aggregation properties of multi-asset decentralized markets (as studied here) and single-asset centralized markets as typically explored in the literature.

We argue that our theory has implications for policies meant to enhance information dissemination in asset markets. In particular, a social planner seeking information aggregation (when it otherwise would fail) can do so by introducing a reporting lag. Reporting lags come at the cost of reducing trading activity and therefore trading surplus. A realistic and preferable alternative is to reveal some information in real time and some information with a lag, which can be accomplished by organizing traders on segmented trading platforms. More generally, we show that there is indeed a trade-off between ensuring information aggregation and maximizing total welfare: a planner can increase total surplus through an information policy that partially obscures the true aggregate state.

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A Proofs for Sections 2 and 3

Proof of Property 1. See Lemma 1 in Asriyan et al. (2017). ■

Skimming Property. Since $c_H > c_L$ and $F_H \geq F_L$, the continuation value of the low type seller from rejecting the bid v_L in the first period satisfies:

$$\begin{aligned} Q_L^i &= (1-\delta)\cdot c_L + \delta \cdot \mathbb{E}_L\{F_L(\pi_i,\phi_i)\} \\ &< (1-\delta)\cdot c_H + \delta \cdot \mathbb{E}_L\{F_L(\pi_i,\phi_i)\} \\ &\leq (1-\delta)\cdot c_H + \delta \cdot \mathbb{E}_L\{F_H(\pi_i,\phi_i)\}. \end{aligned}$$

Therefore, in order to prove that $Q_H^i > Q_L^i$, it is sufficient to show that $\mathbb{E}_H\{F_H(\pi_i,\phi_i)\} \geq \mathbb{E}_L\{F_H(\pi_i,\phi_i)\}$. Recall that F_H is increasing in π_i and independent of ϕ_i. Hence, the desired inequality is implied by proving that conditional on $\theta_i = H$, the random variable π_i (weakly) first-order stochastically dominates π_i conditional on $\theta_i = L$.

Note that the distribution of π_i in the second period is a function of the trading probabilities of the seller i and of the realization of news from sellers $j \neq i$, $z_i^j \in \{0,1\}$. Fix the interim belief π_{σ_i}, and consider news $\mathbf{z}'_{-i}$ and $\mathbf{z}''_{-i}$ (which occur with positive probability) such that the posterior π_i satisfies $\pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i})$, i.e., $\mathbf{z}'_{-i}$ is "better news" for seller i than $\mathbf{z}''_{-i}$. But note that:

$$\frac{\pi_{\sigma_i}\cdot\rho_H^i(\mathbf{z}'_{-i})}{\pi_{\sigma_i}\cdot\rho_H^i(\mathbf{z}'_{-i}) + (1-\pi_{\sigma_i})\cdot\rho_L^i(\mathbf{z}'_{-i})} = \pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i}) = \frac{\pi_{\sigma_i}\cdot\rho_H^i(\mathbf{z}''_{-i})}{\pi_{\sigma_i}\cdot\rho_H^i(\mathbf{z}''_{-i}) + (1-\pi_{\sigma_i})\cdot\rho_L^i(\mathbf{z}''_{-i})},$$

which implies that $\frac{\rho_H(\mathbf{z}'_{-i})}{\rho_L(\mathbf{z}'_{-i})} \geq \frac{\rho_H(\mathbf{z}''_{-i})}{\rho_L(\mathbf{z}''_{-i})}$, i.e. the ratio of distributions $\frac{\rho_H(\cdot)}{\rho_L(\cdot)}$ satisfies the monotone likelihood ratio property. This in turn implies that $\rho_H(\cdot)$ first-order stochastically dominates $\rho_H(\cdot)$, which establishes the result. ■

Proof of Property 2. See Lemma 3 in Asriyan et al. (2017). ■

Proof of Property 3. The proof that all equilibria involve strictly positive probability of trade in the first period is in the text. We show here that all equilibria must be symmetric. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$ for some $A, B \in \{1,...,N\}$. We establish the result by first showing that the beliefs for seller A are more favorable than for seller B, following all news realizations; then we show that good news about seller A are more likely to arrive than good news about seller B.

Consider the posterior belief about seller $i \in \{A,B\}$ following some news $\mathbf{z}_{-i} = \left(z_i^j\right)_{j\neq i}$:

$$\pi_i\left(\mathbf{z}_{-i}\right) = \frac{\pi_{\sigma_i}\cdot\rho_H^i\left(\mathbf{z}_{-i}\right)}{\pi_{\sigma_i}\cdot\rho_H^i\left(\mathbf{z}_{-i}\right) + (1-\pi_{\sigma_i})\cdot\rho_L^i\left(\mathbf{z}_{-i}\right)}$$

where we can express $\rho_\theta^i(\mathbf{z}_{-i})$ as:

$$\rho_\theta^i(\mathbf{z}_{-i}) = \sum_{s\in\{l,h\}} \mathbb{P}(S=s|\theta_i=\theta)\cdot\mathbb{P}\left((z_i^j)_{j\neq i,i'}|S=s\right)\cdot\mathbb{P}(z_i^{i'}|S=s)$$

for $i,i'\in\{A,B\}$ and $i'\neq i$. Note that $\rho_\theta^i(\mathbf{z}_{-i})$ depends on $\sigma_{i'}$ only through the term $\mathbb{P}\left(z_i^{i'}|S\right)$. We now show that $\sigma_A>\sigma_B$ implies that:

$$\frac{1-\pi_{\sigma_A}}{\pi_{\sigma_A}}\cdot\frac{\rho_L^A(\mathbf{z}_{-i})}{\rho_H^A(\mathbf{z}_{-i})} < \frac{1-\pi_{\sigma_B}}{\pi_{\sigma_B}}\cdot\frac{\rho_L^B(\mathbf{z}_{-i})}{\rho_H^B(\mathbf{z}_{-i})}, \tag{9}$$

which will establish that $\pi_A(\mathbf{z}_{-i})>\pi_B(\mathbf{z}_{-i})$ for all news $\mathbf{z}_{-i}$. There are two cases to consider, depending on whether $z_i^{i'}=0$ or $z_i^{i'}=1$.

If $z_i^{i'}=1$, then $\mathbb{P}\left(z_i^{i'}=1|S=s\right)=\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$ and the likelihood ratio $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in σ_i but is independent of $\sigma_{i'}$. Intuitively, if seller i' traded, her type is revealed to be low, and the intensity with which she trades is irrelevant for updating. But then inequality (9) follows because π_{σ_i} is increasing in σ_i.

If $z_i^{i'}=0$, then $\mathbb{P}\left(z_i^{i'}=0|S=s\right)=1-\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$, and now the likelihood ratio $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in both σ_i and $\sigma_{i'}$. However, given that both i and i' did not trade (both are good news for i), inequality (9) follows because the assets i and i' are imperfectly correlated and $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ is more sensitive to trading probability σ_i than to $\sigma_{i'}$.

Finally, note that $\sigma_A>\sigma_B$ also implies that the probability that seller B trades and releases bad news about seller A is lower than the probability that seller A trades and releases bad news about seller B. Since the posteriors following good news are higher than following bad news, this establishes the result. ■

Proof of Proposition 1. To prove existence of an equilibrium, it suffices to show there exists a $(\sigma,\phi)\in[0,1]^2$ such that equation (8) holds, i.e., $Q_L(\sigma,\sigma,\phi)=v_L$ where the second argument states that all other sellers also trade with intensity σ. Note that by varying σ from 0 to 1, Q_L ranges from $[(1-\delta)c_L+\delta v_L,(1-\delta)c_L+\delta v_H]$. By continuity of Q_L and Assumption 2, the intermediate value theorem gives the result.

Consider the following two candidate equilibria. We will refer to an equilibrium in which the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(0))=\bar{\pi}$ as a *low trade* equilibrium, and when the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(N))=\bar{\pi}$ as a *high trade* equilibrium. Although there can be other equilibria as well, we do not focus on them. We will now show that the *high trade* and the *low trade* equilibria coexist when λ and δ are large enough.

1. <u>Low trade.</u> Note that there is at most one low trade equilibrium since the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(\mathbf{z}(0))=\bar{\pi}$. Let x be

the value of σ such that $\pi_i(\mathbf{z}(0); x, x) = \bar{\pi}$ (x denotes the trading probability of all $N+1$ sellers). As ϕ varies from 0 and 1, $Q_L(x, x, \phi)$ varies continuously from $(1-\delta)c_L + \delta v_L$ to $(1-\delta)c_L + \delta(\rho_L^i(\mathbf{z}(0))v_L + (1-\rho_L^i(\mathbf{z}(0)))c_H)$ where $\rho_L^i(\mathbf{z}(0)) > 0$. Hence, there exists a $\bar{\delta}_\lambda < 1$, such that $Q_L(x, x, 1) = v_L$. Clearly, a low trade equilibrium exists if $\delta > \bar{\delta}_\lambda$. Moreover, it is straightforward to show that $\sup_\lambda \rho_L^i(\mathbf{z}(0)) < 1$. Hence, this equilibrium exists if δ is larger than $\bar{\delta} \equiv \sup_{\lambda \in (1-\pi, 1)} \widehat{\delta}_\lambda < 1$.

2. High trade. Note that there is at most one high trade equilibrium since the trading intensity σ is fully pinned down by the requirement that $\pi_i(\mathbf{z}(N)) = \bar{\pi}$. Let y be the value of σ such that $\pi_i(\mathbf{z}(N); y, y) = \bar{\pi}$. As ϕ varies from 0 to 1, Q_L varies continuously from

$$(1-\delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))v_L + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i}) V(\pi_i(\mathbf{z}_{-i}; y, y)) \right)$$

to

$$(1-\delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))c_H + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i}) V(\pi_i(\mathbf{z}_{-i}; y, y)) \right).$$

Hence, we have $\lim_{\lambda \to 1} \rho_L^i(\mathbf{z}(N)) = 1$, and it follows that the range of Q_L converges to the interval $((1-\delta)c_L + \delta v_L, (1-\delta)c_L + \delta c_H]$ as λ goes to 1. By Assumption 2, v_L is in this interval. This establishes the existence of the threshold $\bar{\lambda}_\delta$ such that the high trade equilibrium exists whenever $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$.

Thus, we conclude that multiple equilibria exist when $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$. ■

Proof of Lemma 1. Uniqueness of equilibrium follows from the fact that $Q_L^{i,fict} = (1-\delta)c_L + \delta v_H > v_L$ when $\sigma_i = 1$, and because $Q_L^{i,fict}$ is monotonically increasing in σ_i, and in ϕ_i when buyer mixing is part of an equilibrium. Hence, the unique equilibrium must feature no trade if $Q_L^{i,fict}(0,0) \geq v_L$. Finally, it is straightforward to check that $Q_L^{i,fict}(0,0) \geq v_L$ holds if and only if $\lambda \geq \bar{\lambda}$ and $\delta \geq \bar{\delta}_\lambda$. ■

For the proof of Theorem 1, it will be useful to reference the following lemma, which is straightforward to verify so the proof is omitted. Let $\pi_i(s; \sigma)$ denote the buyers' posterior belief about seller i following a rejection, conditional on observing that the state is s. Then, for $s \in \{l, h\}$, we have:

$$\pi_i(s; \sigma) = \frac{\pi_\sigma \cdot \mathbb{P}(S = s | \theta_i = H)}{\pi_\sigma \cdot \mathbb{P}(S = s | \theta_i = H) + (1 - \pi_\sigma) \cdot \mathbb{P}(S = s | \theta_i = L)},$$

where as before π_σ is the interim belief.

Lemma A.1 *Given a sequence $\{\sigma_N\}_{N=1}^{\infty}$ of trading probabilities along which information aggregates, we also have convergence of posteriors: $\pi_i(\mathbf{z}_{-i};\sigma_N) \to^p \pi_i(S;\sigma_N)$ as $N \to \infty$.*

Proof of Theorem 1. Part (i). Suppose to the contrary that $(\star)$ holds with strict inequality, but that information aggregation obtains. Recall that in equilibrium, for any N, we must have:

$$v_L = Q_L^i(\sigma_N, \phi_i) = (1-\delta)\, c_L + \delta \sum_{\mathbf{z}_{-i}} \rho_L^i\left(\mathbf{z}_{-i}\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right),$$

where

$$\begin{aligned}\sum_{\mathbf{z}_{-i}} \rho_L^i\left(\mathbf{z}_{-i}\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) &= \sum_{s=l,h} \mathbb{P}\left(S=s|\theta_i=L\right) \sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=s\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) \\ &> \lambda \cdot v_L + (1-\lambda) \cdot \sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=h\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right).\end{aligned}$$

Since by Lemma A.1, $\pi_i\left(\mathbf{z}_{-i};\sigma_N\right) \to^p \pi_i\left(h;\sigma_N\right)$ when the state is h, and because $(\star)$ holding strictly implies that $\pi_i\left(h;\sigma_N\right) > \pi_i\left(h;0\right) > \bar{\pi}$, we have that for a given $\epsilon > 0$, if N is large enough, then:

$$\sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=h\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) > V\left(\pi_i\left(h;\sigma_N\right)\right) - \epsilon.$$

Therefore, we conclude that for sufficiently large N:

$$\begin{aligned}v_L = Q_L^i\left(\sigma_N, \phi_i\right) &> (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;\sigma_N\right)\right)\right) - \delta \cdot (1-\lambda) \cdot \epsilon \\ &> (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;0\right)\right)\right) - \delta \cdot (1-\lambda) \cdot \epsilon.\end{aligned}$$

Since ϵ was arbitrary, it must be that:

$$v_L \geq (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;0\right)\right)\right),$$

which violates $(\star)$ holding with strict inequality, a contradiction.

Part (ii). If $(\star)$ does not hold, then in the fictitious economy, the unique equilibrium trading probability in the first period must satisfy $\sigma^* > 0$. We next construct an equilibrium sequence $\{\sigma_N\}$ of the actual economy such that the sequence is uniformly bounded away from zero, which then implies that information aggregates along this sequence. First, consider a sequence $\{\widehat{\sigma}_N\}$, not necessarily an equilibrium one, such that $\widehat{\sigma}_N = \widehat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but strictly below σ^*. Along such a sequence, information clearly aggregates and, by Lemma A.1, $\pi_i\left(\mathbf{z}_{-i}, \widehat{\sigma}_N\right) \to^p \pi_i\left(S, \widehat{\sigma}_N\right)$. Therefore, combined with the fact that $\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N) = \pi_i(\mathbf{z}_{-i}, \widehat{\sigma}) < \pi_i(\mathbf{z}_{-i}, \sigma^*)$, there exists an N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N), \phi_i)\} < \mathbb{E}_L^{fict} \{F_L(\pi_i(S, \sigma^*))\} = \frac{v_L - (1-\delta) \cdot c_L}{\delta},$$

where the last equality holds since $\sigma^* > 0$ implies that, in the fictitious economy, the low type must be indifferent to trading at $t = 1$ and delaying trade to $t = 2$. The correspondence $\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}, \sigma), \cdot)\}$ is upper hemicontinuous in σ for each N, and has a maximal value of v_H that is strictly greater than $\mathbb{E}_L^{fict} \{F_L(\pi_i(S, \sigma^*))\}$. Hence, for each $N > N^*$, we can find a σ_N such that $\sigma_N \geq \widehat{\sigma}_N > 0$ and $\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}, \sigma_N), \phi_i)\} = \frac{v_L - (1-\delta) \cdot c_L}{\delta}$. This delivers the desired equilibrium sequence $\{\sigma_N\}$ along which information aggregates. ■

Proof of Theorem 2. We establish the conditions for the coexistence of aggregating and non-aggregating equilibria. To do so, we first show that if $\lambda > \bar{\lambda}$, there exists a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist if $\delta > \delta_2(\lambda)$. Second, we show that for λ large enough $\delta_2(\lambda) < \bar{\delta}_\lambda$. Therefore, both non-aggregating and aggregating equilibria exist if $\delta \in (\delta_2(\lambda), \bar{\delta}_\lambda)$, since ($\star$) is violated (see Theorem 1).

Consider a candidate sequence of equilibria with trading probabilities $\{\sigma_N\}$, such that $\sigma_N = \kappa_N \cdot N^{-1}$ and:

$$\pi_i\left(\mathbf{z}(0); \kappa_N \cdot N^{-1}\right) = \bar{\pi} \tag{10}$$

Solving (10) for κ_N and taking the limit as $N \to \infty$ gives $\kappa_N \to \kappa$ where

$$\kappa \equiv \frac{1}{\lambda - \frac{(1-\lambda)(1-\pi)}{\pi}} \cdot \log\left(\frac{\lambda - \left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \frac{(1-\lambda)(1-\pi)}{\pi}}{\left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) - (1-\lambda)}\right) \in (0, \infty). \tag{11}$$

Seller i expects to receive an offer of v_L in all events other than $\mathbf{z}(0)$ and an expected offer $\phi_i c_H + (1-\phi_i) v_L$ for some $\phi_i \in [0,1]$ in the event $\mathbf{z}(0)$. Therefore, the sequence of trading probabilities defined above constitutes an equilibrium if δ is sufficiently high and the probability of the event $\mathbf{z}(0)$ conditional on the seller's type being low is bounded away from zero. To establish the latter, note that:

$$\begin{aligned}
\mathbb{P}(\mathbf{z}(0)|\theta_i = L) &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot (1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N \\
&= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot \left(1 - \kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N \\
&\to \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L|S = s)} > 0,
\end{aligned}$$

where the last limit follows from Lemma A.2. In these equilibria, information fails to aggregate because as a result $\mathbb{P}(\mathbf{z}(0)|S=s)$ is bounded away from zero in both states of nature (see Lemma A.4). Thus, for each $\lambda > \bar{\lambda}$, we have established the existence of a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist whenever $\delta > \delta_2(\lambda)$. Finally, from (11) we have that:

$$\lim_{\lambda\to 1}\sum_{s=l,h}\mathbb{P}\left(S=s|\theta_i=L\right)\cdot e^{-\kappa\cdot\mathbb{P}(\theta_i=L|S=s)} = \frac{1-\bar{\pi}}{\bar{\pi}}\cdot\frac{\pi}{1-\pi}\in(0,1),$$

and hence $\lim_{\lambda\to 1}\delta_2(\lambda) < 1$. Letting $\widehat{\delta} = \lim_{\lambda\to 1}\delta_2(\lambda)$ and noting that $\lim_{\lambda\to 1}\bar{\delta}_\lambda = 1$ implies the result.

Next, we establish that when $\lambda < \bar{\lambda}$ or δ is sufficiently small, then only aggregating equilibria exist. First, suppose that $\lambda < \bar{\lambda}$ and assume to the contrary that information aggregation fails along some sequence of equilibria, and pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞ (See Lemma A.3 for the existence of such a subsequence). But note that for each N, we have $\pi_i(\mathbf{z}_{-i},\sigma_N) \leq \pi_i(h,\sigma_N)$, i.e., the posterior beliefs must be weakly lower than if the state were revealed to be high. Since $\pi_i(h,\sigma_N)$ is continuous in σ_N, and since $\lambda < \bar{\lambda}$ implies that $\pi_i(h,0) < \bar{\pi}$, it follows that for N large enough all posterior beliefs are strictly below $\bar{\pi}$. But then for N large, $Q_L^i < v_L$ and therefore $\sigma_N = 1$, contradicting Property 2.

Second, consider $\hat{\delta}$ defined by $v_L = (1-\hat{\delta})c_L + \hat{\delta}V(\pi)$, and assume that $\delta < \hat{\delta}$ (Note that Assumption 2 can still be satisfied since $V(\pi) < c_H$). Suppose to the contrary that information aggregation fails along a sequence of equilibria, and again pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞. By continuity, we must also have that $\pi_{\sigma_N} \to \pi$ along this subsequence. But, note that for each N along this subsequence, it must be that:

$$\begin{aligned} v_L = Q_L^i(\sigma_N,\phi_i) &= (1-\delta)c_L + \delta\mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i},\sigma_N),\phi_i)\} \\ &\leq (1-\delta)c_L + \delta\mathbb{E}_L\{V(\pi_i(\mathbf{z}_{-i},\sigma_N))\} \\ &\leq (1-\delta)c_L + \delta V(\pi_{\sigma_N}). \end{aligned}$$

where the first inequality follows immediately from (6) and the second from the fact that V is linear function and $\pi_i(\mathbf{z}_{-i},\sigma_N)$ is a supermartingale conditional on $\theta_i = L$. Because $\delta < \hat{\delta}$ and $V(\pi_{\sigma_N}) \to V(\pi)$, the last expression is strictly lower than v_L for N large enough, a contradiction.
■

Lemma A.2 *Let $\{\alpha_x\}$ be any non-negative sequence of real numbers such that $\alpha_x \to \alpha$ as $x \to \infty$ where $\alpha \in (0,1)$. Then $\left(\frac{x-\alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$.*

Proof. Assume that for any $\gamma \in (0,1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Then, given $\epsilon > 0$ so that $\epsilon < \alpha < 1-\epsilon$, if x is large enough then $|\alpha_x - \alpha| < \epsilon$, $\left(\frac{x-\alpha-\epsilon}{x}\right)^x \geq e^{-\alpha-\epsilon} - \epsilon$, and

$\left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon}+\epsilon$. This in turn implies that:

$$e^{-\alpha-\epsilon}-\epsilon \leq \left(\frac{x-\alpha-\epsilon}{x}\right)^x \leq \left(\frac{x-\alpha_x}{x}\right)^x \leq \left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon}+\epsilon.$$

Since ϵ is arbitrary, we conclude that $\left(\frac{x-\alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$. Next, we prove the supposition that for any $\gamma \in (0,1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Note that $\left(\frac{x-\gamma}{x}\right)^x = e^{x\cdot\log\left(\frac{x-\gamma}{x}\right)}$ and by L'Hospital's rule:

$$\lim_{x\to\infty} x \cdot \log\left(\frac{x-\gamma}{x}\right) = \lim_{x\to\infty}\frac{\log\left(\frac{x-\gamma}{x}\right)}{x^{-1}} = -\lim_{x\to\infty}\frac{\gamma\cdot x}{x-\gamma} = -\gamma.$$

By continuity, $\lim_{x\to\infty} e^{x\cdot\log\left(\frac{x-\gamma}{x}\right)} = e^{-\gamma}$. ■

Lemma A.3 *Suppose that there is a sequence of equilibria $\{\sigma_N\}$ along which information aggregation fails. Then there exist a subsequence of equilibria with trading probabilities $\{\sigma_{N_m}\}$ such that for some $0<\underline{\kappa}<\bar{\kappa}<\infty$, we have $\underline{\kappa}<\sigma_{N_m}N_m<\bar{\kappa}$ for all m.*

Proof. Suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have $\lim_{m\to\infty}\sigma_{N_m}N_m=\infty$. Let X_i denote the indicator that takes value of 1 if seller i has traded in the first period. Define $Y_{N_m}=N_m^{-1}\cdot\sum_{i=1}^{N_m}X_i$ be the fraction of sellers who have traded in the first period, and note that conditional on the state being s, Y_{N_m} has a mean p_{s,N_m} and variance $N_m^{-1}\cdot p_{s,N_m}\cdot(1-p_{s,N_m})$, where recall that $p_{s,N_m}=\sigma_{N_m}\cdot\mathbb{P}(\theta_i=L|S=s)$. Since $p_{l,N_m}>p_{h,N_m}$,

$$\begin{aligned}\mathbb{P}\left(Y_{N_m}\geq\frac{p_{h,N_m}+p_{l,N_m}}{2}|S=h\right) &= \mathbb{P}\left(Y_{N_m}-p_{h,N_m}\geq\frac{p_{l,N_m}-p_{h,N_m}}{2}|S=h\right)\\ &\leq \mathbb{P}\left((Y_{N_m}-p_{h,N_m})^2\geq\left(\frac{p_{l,N_m}-p_{h,N_m}}{2}\right)^2|S=h\right)\end{aligned}$$

And by Markov's inequality:

$$\begin{aligned}\mathbb{P}\left((Y_{N_m}-p_{h,N_m})^2\geq\left(\frac{p_{l,N_m}-p_{h,N_m}}{2}\right)^2|S=h\right) &\leq \frac{\mathbb{E}\left\{(Y_{N_m}-p_{h,N_m})^2|S=h\right\}}{\left(\frac{p_{l,N_m}-p_{h,N_m}}{2}\right)^2}\\ &= \frac{N_m^{-1}\cdot p_{h,N_m}\cdot(1-p_{h,N_m})}{\left(\frac{p_{l,N_m}-p_{h,N_m}}{2}\right)^2}\\ &= 4\cdot\frac{\sigma_{N_m}\cdot\mathbb{P}(\theta_i=L|S=h)-\sigma_{N_m}^2\cdot\mathbb{P}(\theta_i=L|S=h)^2}{N_m\cdot\sigma_{N_m}^2\cdot(\mathbb{P}(\theta_i=L|S=l)-\mathbb{P}(\theta_i=L|S=h))^2}\end{aligned}$$

which by our assumption tends to 0 as $m \to \infty$. By a similar reasoning, we have that:

$$\begin{aligned}
\mathbb{P}\left(Y_{N_m} < \frac{p_{h,N_m} + p_{l,N_m}}{2} | S = l\right) &= \mathbb{P}\left(p_{l,N_m} - Y_{N_m} > \frac{p_{l,N_m} - p_{h,N_m}}{2} | S = l\right) \\
&\leq \mathbb{P}\left((p_{l,N_m} - Y_{N_m})^2 > \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = l\right) \\
&\leq \frac{\mathbb{E}\left\{(Y_{N_m} - p_{l,N_m})^2 | S = l\right\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= \frac{N_m^{-1} \cdot p_{l,N_m} \cdot (1 - p_{l,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = l\right) - \sigma_{N_m}^2 \cdot \mathbb{P}\left(\theta_i = L | S = l\right)^2}{N_m \cdot \sigma_{N_m}^2 \cdot \left(\mathbb{P}\left(\theta_i = L | S = l\right) - \mathbb{P}\left(\theta_i = L | S = h\right)\right)^2}
\end{aligned}$$

which again tends to 0 as $m \to \infty$. Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction.

Next, suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have that $\lim_{m\to\infty} \sigma_{N_m} N_m = 0$. Then, given any $\epsilon > 0$ and m large enough, we have:

$$(1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right))^{N_m} = \left(\frac{N_m - \sigma_{N_m} \cdot N_m \cdot \mathbb{P}\left(\theta_i = L | S = s\right)}{N_m}\right)^{N_m} \geq \left(\frac{N_m - \epsilon}{N_m}\right)^{N_m}$$

for $s \in \{l, h\}$, where the last expression converges to $e^{-\epsilon}$ by Lemma A.2. Since ϵ is arbitrary, $(1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right))^{N_m}$ goes to 1 as $m \to \infty$. Hence, we have that for $\theta \in \{L, H\}$:

$$\mathbb{P}\left(Y_{N_m} = 0 | \theta_i = \theta\right) = \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = \theta\right) \cdot (1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right))^{N_m} \to 1.$$

Now, consider the posterior belief about the seller conditional on event that no seller has traded. For any m, since the low type must expect offers above v_L with positive probability and since $\mathbf{z}(0)$ is the best possible news, it must be that:

$$\begin{aligned}
&\pi_i(\mathbf{z}(0), \sigma_{N_m}) \geq \bar{\pi} \\
&\qquad \iff \frac{\pi_{\sigma_{N_m}} \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = H)}{\pi_{\sigma_{N_m}} \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = H) + (1 - \pi_{\sigma_{N_m}}) \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = L)} \geq \bar{\pi}.
\end{aligned}$$

But note that, since $\sigma_{N_m} \to 0$ and $\pi_{\sigma_{N_m}}$ is continuous, the left-hand side converges to $\pi < \bar{\pi}$, a contradiction. ■

Lemma A.4 *Consider a sequence of equilibria with trading probabilities* $\{\sigma_N\}$ *such that* $\sigma_N N <$

$\bar{\kappa}$ *for some* $\bar{\kappa} < \infty$. *Then* $\mathbb{P}(Y_N = 0|S = s)$ *is bounded away from zero, uniformly over* N, *for* $s \in \{l, h\}$.

Proof. We have that $\mathbb{P}(Y_N = 0|S = s) = (1 - p_{s,N})^N$ for $s \in \{l, h\}$. By assumption, $p_{s,N} \leq N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)$. Therefore,

$$\mathbb{P}(Y_N = 0|S = s) \geq \left(1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N$$

and by Lemma A.2, $\lim_{N\to\infty} (1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s))^N = e^{-\bar{\kappa}\cdot\mathbb{P}(\theta_i = L|S=s)} > 0$. ■

B Proof of Theorem 3

We establish parts (i)-(iii) of Theorem 3 separately.

Proof of Theorem 3, part (i). We proceed by contradiction and suppose to the contrary that there is some finite date t at which information aggregates. In particular, suppose that information has not aggregated before t, but it aggregates at t. Consider seller i who trades with probability in $(0, 1)$ at t. We know that the number of such sellers must grow to ∞ with N, since otherwise there would be insufficient information learned at t. Without loss of generality assume that all sellers trade with probability in $(0, 1)$ at t. By the skimming property, the bid for this seller's asset must be v_L, which the high type rejects whereas the low type accepts with some probability $\sigma_{i,N} \in (0, 1)$.

Let $Q^{i,N}_{L,t}$ denote the low type seller i's continuation value from rejecting a bid v_L at time t. Define

$$\bar{Q}^N_t \equiv (1 - \delta) \cdot c_L + \delta \cdot (\lambda_{L,t} \cdot v_L + (1 - \lambda_{L,t}) \cdot V(\pi_i(h; 0))), \tag{12}$$

where (i) $\lambda_{L,t} = \mathbb{P}_t(S = l|\theta_i = L)$ is the posterior belief that the state is l conditional on trading history up to period t and the seller's type being L, and (ii) $\pi_i(h; 0)$ is the posterior belief about the seller i conditional on the state being h. In Lemma B.1, we show that:

$$\lim_{N\to\infty} \mathbb{P}_t\left(Q^{i,N}_{L,t} \geq \bar{Q}^N_t\right) = 1, \tag{13}$$

i.e., $\bar{Q}^N_t$ provides a lower bound on the low type's continuation value. Next, we use this result to show that with probability bounded away from zero in both states of nature, if N is large enough, then $Q^{i,N}_{L,t} > v_L$. This immediately implies that the low types strictly prefer to delay trade at t, contradicting aggregation and thus establishing our result.

Since ($\star$) holds, $\bar{Q}^N_1 > v_L$. Thus, information aggregation must fail in the first period. In Lemma B.2, we show that failure of information aggregation at t implies that the probability of the event that no seller trades in that period must be bounded away from zero, uniformly over

N, in both states of nature. Because this event is 'good' news about the state, then following it in the first period, we have $\lambda_{L,2} < \lambda_{L,1}$ and, thus, $\bar{Q}_2^N > v_L$. But then again information aggregation must fail in the second period and, therefore, the probability that no seller trades in the second period must remain bounded away from zero in both states of nature. Repeating this argument until period t, we can construct a history that occurs with probability bounded away from zero in both states of nature, in which $\bar{Q}_t^N > v_L$, as was stated above. ■

Proof of Theorem 3, part (ii). The proof is by construction. Consider a candidate equilibrium in which for any period t, the following properties hold:

(i) If $\pi_{i,t} < \bar{\pi}$, then the bid is v_L, which the low type accepts w.p. $\sigma_t \in [0, 1)$ whereas the high type rejects w.p.1.

(ii) If $\pi_{i,t} > \bar{\pi}$, then the bid is $V(\pi_{i,t})$ and both types accept it w.p.1.

(iii) If $\pi_{i,t} = \bar{\pi}$, then the bid is $V(\pi_{i,t})$ w.p. ϕ_t (and both types accept it w.p.1) and is v_L w.p. $1 - \phi_t$ (and both types reject it).

The only off-equilibrium path event in a candidate satisfying (i)-(iii) is a rejection when $\pi_{i,t} > \bar{\pi}$, in which case the interim belief as given by Bayes rule is not well defined. For such cases, we specify $\pi_{\sigma_i,t} = \pi_{i,t}$ (i.e., unexpected rejections are attributed to random trembles).[16]

We will now verify that an equilibrium satisfying (i)-(iii) exists (with off-path beliefs as specified immediately above). To do so, consider any history and let N_t ($\mathcal{N}_t$) denote the number (set) of sellers who have not yet traded at the beginning of period t. Notice that the seller's value function under the proposed equilibrium is the same as in (5) and (6), where (π_i, ϕ_i) is replaced by $(\pi_{i,t}, \phi_t)$. By symmetry of the candidate, $\pi_{i,t} = \pi_{j,t} = \pi_t$ for all $j \neq i \in \mathcal{N}_t$. We now show that there exists (σ_t, ϕ_{t+1}) such that profitable deviations do not exist:

(i) Suppose that $\pi_t < \bar{\pi}$. Since continuation values in period $t + 1$ are the same as in the two-period model, we know by Proposition 1, that for any N_t, there exists at least one (σ_t, ϕ_{t+1}) pair such that the low types' continuation value is exactly v_L. Hence, a low-type seller is willing to mix. Clearly, a high-type seller strictly prefers to reject.

(ii) Suppose that $\pi_t > \bar{\pi}$ and seller i rejects, Since all other seller accept w.p.1. there is no information revealed by other sellers and therefore (given the off-path specification above) $\pi_{i,t+1} = \pi_t$. Therefore, rejecting the offer leads to a payoff of $(1 - \delta)c_\theta + \delta F_\theta(\pi_t) < V(\pi_t)$.

[16] The precise specification of off-path beliefs is not crucial for the construction, any $\pi_{\sigma_i,t} \leq \min\{g(\pi_{i,t}), 1\}$ will suffice, where $g(\pi_{i,t}) > \pi_{i,t}$ is such that $(1 - \delta)c_H + \delta V(g(\pi_{i,t})) = V(\pi_{i,t})$ and ensures that a high type cannot profitably deviate from rejecting.

(iii) Suppose that $\pi_t = \bar{\pi}$. If trade does not occur, buyers in period $t+1$ attribute all rejections to a low offer made by buyers in period t. Hence, if the seller rejects, $\pi_{i,t+1} = \pi_{i,t}$ and by the same argument as in (ii), such a deviation is not profitable for the seller.

That buyers do not have a profitable deviation from the candidate follows a similar reasoning to the argument for Property 1 in the two-period model. Belief consistency is by construction. Thus, there exists an equilibrium of the candidate form.

Notice that, by construction, the second period payoff to a type-θ seller is the same as in the two-period model. Therefore, if ($\star$) does not hold then following the same argument as in the proof of Theorem 1, we can construct a sequence of first-period trading probabilities $\{\sigma_{N,1}\}$ that are uniformly bounded away from zero, which ensures information aggregates in the first period. ■

Proof of Theorem 3, part (iii). From Theorem 3, part (ii), when ($\star$) does not hold, there is a sequence of equilibria along which information aggregates. In the class of equilibria constructed in the proof of Theorem 3 (ii), equilibrium play in the first play coincides with the equilibrium play of all equilibria in the two period economy of Section 3. As a result, under the same conditions as in Theorem 2 (namely, that $\delta \in (\hat{\delta}, \bar{\delta}_\lambda)$), there exists a sequence of equilibria along which information aggregation fails in the first period. Furthermore, by construction of this sequence of equilibria (see proof of Theorem 2), (i) the probability of the event that no seller trades in the first period remains bounded away from zero in both states of nature and (ii) the posterior belief about the seller in the second period following this event is equal to $\bar{\pi}$. But then, following this event, by construction no additional information about the state is revealed through trade. ■

In what follows, we prove the two lemmas used in the proof of Theorem 3, part (i).

Lemma B.1 *Suppose that ($\star$) holds, and information aggregates in period t but not before. Then* $\lim_{N\to\infty} \mathbb{P}_t \left(Q_{L,t}^{i,N} \geq \bar{Q}_t^N \right) = 1$.

Proof. The low type's continuation value from rejecting bid v_L at date t is:

$$\begin{aligned} Q_{L,t}^{i,N} &= (1-\delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = l \right\} + (1-\lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right) \\ &> (1-\delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot v_L + (1-\lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right), \end{aligned}$$

where $\mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = s \right\}$ denotes the low type's expected payoff conditional on history up to t and the state being s. For the inequality, we used the fact that the payoffs at $t+1$ must be strictly above v_L with positive probability, since otherwise no seller would be willing to delay trade to $t+1$. We next show that, for any $\varepsilon > 0$,

$$\lim_{N\to\infty} \mathbb{P}_t \left(F_{L,t+1}^{i,N} \geq V\left(\pi_i(h;0)\right) - \varepsilon | S = h \right) = 1, \tag{14}$$

which, since ε is arbitrary, will establish the result.

Suppose that the state is h and let T be the smallest number such that:

$$(1-\delta^T) \cdot c_H + \delta^T \cdot v_H < V(\pi_i(h;0)),$$

which is finite since $(\star)$ implies $V\left(\pi_i\left(h;0\right)\right) > c_H$. Since information aggregates at t (by hypothesis), we can choose N large enough so that (w.p. close to 1) the agents' belief that the state is h is close to 1 in the periods $t+1$ through $t+1+T$. Let us consider histories in which this is the case. If we show that (w.p. going to 1 as N goes to ∞) the bid at $t+1$ is pooling and both seller types accept the bid, then we are done.

Suppose to the contrary that for any N, there is strictly positive probability (bounded away from zero) that the bid is not pooling at $t+1$. There are two cases to consider at t. First, it could be that, with probability bounded away from zero, the buyers make a bid that is rejected by both types. Second, it could be that, with probability bounded away from zero, the bid is v_L and the low types accept it with positive probability.

The first case is straightforward to rule out, since otherwise the buyers could profitably deviate and attract both seller types to trade at date t. For the second case, note that also at $t+2$, with probability bounded away from zero, the bid v_L must be made and accepted by the low type with some probability. Otherwise, if the pooling bid were made instead (w.p. close to 1), the low type would not be willing to trade at $t+1$ (Assumption 2). We can repeat this argument until and including period T and construct sub-histories that occur with probability bounded away from zero, in which the buyers make a bid v_L which is accepted with positive probability by the low types in periods $t+1$ through $t+1+T$.

Let Ω_τ denote the set of sub-histories at $\tau \in \{t+1, t+1+T\}$ in which the bid is v_L in periods $t+1$ through τ, and let ω_τ denote an element of Ω_τ. For $\tau' > \tau$, let $\Omega_{\tau'}|\omega_\tau$ denote the sub-histories in $\Omega_{\tau'}$ that have ω_τ as a predecessor. Now, for any $\omega_\tau \in \Omega_\tau$, in order for buyers not to be able to attract the high type at τ, it must be that:

$$V\left(\pi_{i,\tau}(\omega_\tau)\right) \leq Q_{H,\tau}^{i,N}(\omega_\tau), \tag{15}$$

i.e., the high type would weakly prefer to reject a pooling offer and get his continuation value.

The high type's continuation value in turn satisfies:

$$Q_{H,\tau}^{i,N}(\omega_\tau) \leq (1-\delta)\cdot c_H + \delta \cdot \max\left\{\mathbb{E}_H\{Q_{H,\tau+1}^{i,N}|\{\Omega_{\tau+1}|\omega_\tau\}\}, \mathbb{E}_H\{V(\pi_{i,\tau+1})|\{\Omega_{\tau+1}|\omega_\tau\}^c\}\right\}.$$

Since the beliefs that the state is h are arbitrarily close to 1 in all periods $\tau \in \{t+1, t+1+T\}$, the posterior beliefs about the seller are arbitrarily close to each other in any such period τ. Hence, combining with (15), for any $\varepsilon > 0$, we can choose N large enough so that:

$$Q_{H,\tau}^{i,N}(\omega_\tau) \leq (1-\delta)\cdot c_H + \delta \cdot \mathbb{E}_H\{Q_{H,\tau+1}^{i,N}|\{\Omega_{\tau+1}|\omega_\tau\}\} + \varepsilon$$

for all $\tau \in \{t+1, t+1+T\}$, which implies that:

$$Q_{H,t+1}^{i,N}(\omega_{t+1}) \leq \left(1-\delta^T\right)\cdot c_H + \delta^T \cdot \mathbb{E}_H\{Q_{H,t+1+T}^{i,N}|\{\Omega_{t+1+T}|\omega_{\tau+1}\}\} + \hat{\varepsilon},$$

where $\hat{\varepsilon}$ can be made small by choosing ε small. Since the value to the seller in any period cannot exceed v_H, then with probability approaching 1 as N goes to ∞,

$$V(\pi_i(h;0)) - \hat{\varepsilon} \leq V\left(\pi_{i,t+1}(\omega_{t+1})\right) \leq \left(1-\delta^T\right)\cdot c_H + \delta^T \cdot v_H + \hat{\varepsilon},$$

which, since $\hat{\varepsilon}$ is arbitrary, contradicts our choice of T. ■

Lemma B.2 *Suppose that information aggregation fails at t along a sequence of equilibria with time-t trading probabilities $\{\sigma_{i,N}\}$. Then there is a subsequence of equilibria along which the probability that no seller trades at t remains bounded away from zero, in both states of nature.*

Proof. Assume that there is a subsequence of equilibria with trading probabilities $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m}\sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m. Note that $1-\sigma_{i,N_m}\cdot P(\theta_i = L|S=s) \geq e^{-\sigma_{i,N_m}\cdot K}$ for any K satisfying $1 - P(\theta_i = L|S=l) \geq e^{-K}$. But for any such K, we have:

$$\begin{aligned}
P(\text{no seller trades at } t|S=s) &= \Pi_{i=1}^{N_m}\left(1-\sigma_{i,N_m}\cdot P(\theta_i = L|S=s)\right) \\
&\geq \Pi_{i=1}^{N_m} e^{-\sigma_{i,N_m}\cdot K} \\
&= e^{-K\cdot\sum_{i=1}^{N_m}\sigma_{i,N_m}} \\
&\geq e^{-K\cdot\bar{\kappa}} > 0,
\end{aligned}$$

which establishes the result.

We are left to prove the assertion that there is a subsequence $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m}\sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m. Suppose to the contrary that for all subsequences $\lim_{m\to\infty}\sum_{i=1}^{N_m}\sigma_{i,N_m} = \infty$. Let $X_i \in \{0,1\}$ denote the indicator that seller i has traded and $Y_{N_m} = N_m^{-1} \sum_{i=1}^{N_m} X_i$ denote the fraction of sellers who have traded. Let $p_{i,N_m}(s) = \sigma_{i,N_m} \cdot \mathbb{P}(\theta_i = L|S = s)$ and note that:

$$\mu_{N_m}(s) \equiv \mathbb{E}\{Y_{N_m}|S = s\} = N_m^{-1} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s),$$

and

$$\nu_{N_m}(s) \equiv \mathbb{E}\left\{(Y_{N_m} - \mu_{N_m}(s))^2 |S = s\right\} = N_m^{-2} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s) \cdot (1 - p_{i,N_m}(s)).$$

Since $\mu_{N_m}(l) > \mu_{N_m}(h)$,

$$\begin{aligned} P\left(Y_{N_m} \geq \frac{\mu_{N_m}(h) + \mu_N(l)}{2}|S = h\right) &= P\left(Y_{N_m} - \mu_{N_m}(h) \geq \frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}|S = h\right) \\ &\leq P\left((Y_{N_m} - \mu_{N_m}(h))^2 \geq \left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2 |S = h\right). \end{aligned}$$

And, by Markov's inequality:

$$\begin{aligned} P\left((Y_{N_m} - \mu_{N_m}(h))^2 \geq \left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2 |S = h\right) &\leq \frac{\nu_{N_m}(h)}{\left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2} \\ &= \frac{N_m^{-2} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(h) \cdot (1 - p_{i,N_m}(h))}{\left(N_m^{-1} \cdot \sum_{i=1}^{N_m} \frac{p_{i,N_m}(l) - p_{i,N_m}(h)}{2}\right)^2} \\ &= 4 \cdot \frac{\sum_{i=1}^{N_m} \sigma_{i,N_m} \cdot P(\theta_i = L|S = h) - \sum_{i=1}^{N_m} \sigma_{i,N_m}^2 \cdot P(\theta_i = L|S = h)^2}{\left(\sum_{i=1}^{N_m} \sigma_{i,N_m}\right)^2 \cdot (P(\theta_i = L|S = l) - P(\theta_i = L|S = h))^2}, \end{aligned}$$

which by our assumption tends to 0 as m goes to ∞. By a similar reasoning, we have that:

$$\mathbb{P}\left(Y_{N_m} < N_m^{-1} \cdot \sum_{i=1}^{N_m} \frac{p_{i,N_m}(l) + p_{i,N_m}(h)}{2}|S = l\right) \to 0,$$

as m goes to ∞. Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction. ■

C Proofs for Section 5

Proof of Proposition 2. If trades are only reported at the end of the second period, then the offers made at the beginning of the second period cannot be conditioned on the trading activity of any other seller. Hence, the equilibrium trading strategies (σ, ϕ) are the same as in a model with $N = 1$, and given by the solution to:

$$v_L = Q_L^i(\sigma_i, \phi_i) = (1-\delta)c_L + \delta F_L(\pi_{\sigma_i}, \phi_i), \tag{16}$$

where we omit the argument σ_i from the continuation value since Q_L^i does not depend on it. Notice, because there is no information received before offers are made in the second period, the posterior belief in the second period is simply given by the interim belief π_{σ_i} that is given in equation (2). Since Q_L^i is monotonic in σ_i, and in ϕ_i when buyer mixing is part of equilibrium, the equilibrium must be unique, with $\sigma_i = \sigma > 0$ by Property 3. Because each low-type seller trades with strictly positive probability independent of N, information is (trivially) aggregated when trades are reported at the end of the second period. ■

Proof of Proposition 3. From Theorem 1, we know that if the fictitious economy features a strictly positive probability of trade in the first period, then in the true economy, sequences of equilibria along which information aggregates exist. Furthermore, it is straightforward to show that all aggregating sequences of equilibria (if there are multiple) yield the same asymptotic welfare, and that the welfare in such equilibria is higher than welfare in any non-aggregating equilibria (if the latter also exist). Thus, we need to achieve welfare improvement over equilibria which aggregate information.

There are two cases to consider, depending on whether λ is greater or smaller than $\bar{\lambda}$. In both cases, we will choose δ large enough but still ensure that $(\star)$ is violated.

For now, let us assume that the planner knows the state. We will then show this information policy can be implemented in the actual economy.

<u>Case (1)</u>. Suppose that in the unique equilibrium of the fictitious economy there is full pooling in the second period when $s = h$. Then

$$v_L = Q_L^{i,fict} = (1-\delta)\, c_L + \delta \cdot \left(\mathbb{P}(S = l|\theta_i = L) \cdot v_L + (1 - \mathbb{P}(S = l|\theta_i = L)) \cdot V\left(\pi_i\left(h; \sigma_i\right)\right)\right), \tag{17}$$

with $\sigma_i > 0$. For this to be the equilibrium, it must be that $\lambda > \bar{\lambda}$ and $\delta \in (\hat{\delta}, \bar{\delta}_\lambda)$ for some $\hat{\delta} < \bar{\delta}_\lambda$ where $\bar{\delta}_\lambda$. Whereas the low type's welfare is v_L, the high type's welfare is given by:

$$Q_H^{i,fict} = (1-\delta)\, c_H + \delta \cdot \left(\mathbb{P}(S = l|\theta_i = H) \cdot c_H + (1 - \mathbb{P}(S = l|\theta_i = H)) \cdot V\left(\pi_i\left(h; \sigma_i\right)\right)\right). \tag{18}$$

Consider the following information policy at the beginning of the second period, before offers are made. The planner's report is given by a random variable ω^{fict} that takes values in $\{0, 1\}$, and satisfies the following property. When the state is low, then $\omega^{fict} = 0$. When the state is high, then $\omega^{fict} = 1$ w.p. $1-\gamma$ and $\omega^{fict} = 0$ w.p. γ. When $\gamma = 0$, the planner fully reveals the state, whereas when $\gamma = 1$ she fully conceals it.

As in Section 3.1, we can define a fictitious economy in which agents who trade for asset i do not observe whether other assets traded, but they observe the realization of the random variable ω^{fict} after first period trade but before making offers in the second period. From type θ seller's perspective, the probability that $\omega^{fict} = 0$ is:

$$\mathbb{P}(\omega^{fict} = 0|\theta_i = \theta) = \mathbb{P}\left(S = l|\theta_i = \theta\right) + \gamma \cdot \left(1 - \mathbb{P}\left(S = l|\theta_i = \theta\right)\right). \tag{19}$$

If γ is small, it is straightforward to show that the unique equilibrium of this fictitious economy must feature $v_L = Q_L^{i,fict}(\gamma)$, where:

$$Q_L^{i,fict}(\gamma) = (1-\delta)\, c_L + \delta \cdot \left(\mathbb{P}(\omega^{fict} = 0|\theta_i = L) \cdot v_L + \left(1 - \mathbb{P}(\omega^{fict} = 0|\theta_i = L)\right) \cdot V\left(\pi_i\left(1; \hat{\sigma}_i\right)\right)\right), \tag{20}$$

for some $\hat{\sigma}_i^{\gamma} > 0$, where note that posterior belief is conditioned on the event that $\omega^{fict} = 1$. We make the dependence of the continuation value on γ explicit, as it will be useful in what follows. Again, the low type's welfare is v_L, whereas the high type's welfare is:

$$Q_H^{i,fict}(\gamma) = (1-\delta)\, c_H + \delta \cdot \left(\mathbb{P}(\omega^{fict} = 0|\theta_i = H) \cdot c_H + \left(1 - \mathbb{P}(\omega^{fict} = 0|\theta_i = H)\right) \cdot V\left(\pi_i\left(1; \hat{\sigma}_i^{\gamma}\right)\right)\right). \tag{21}$$

Next, we show that in the fictitious economy, it can be Pareto improving to conceal some information; namely, that there exists a $\gamma > 0$ such that $Q_H^{i,fict}(\gamma) > Q_H^{i,fict}(0)$.

Since in the equilibrium of the fictitious economy, the low type's value is v_L for any γ, it must be that in equilibrium $\left(1 - \mathbb{P}(\omega^{fict} = 0|\theta_i = L)\right) \cdot \pi_i\left(1; \hat{\sigma}_i^{\gamma}\right)$ is equal to some $x \in (0,1)$ that is independent of γ, when γ is sufficiently small. We can express the low type's indifference condition in equation (20) as:

$$\left(1 - \left(\lambda + \gamma \cdot (1-\lambda)\right)\right) \cdot \pi_i\left(1; \hat{\sigma}_i^{\gamma}\right) = \frac{(1-\delta) \cdot (v_L - c_L)}{\delta \cdot (v_H - v_L)}, \tag{22}$$

which uniquely pins down $\hat{\sigma}_i^{\gamma}$ and the belief $\pi_i\left(1; \hat{\sigma}_i\right)$ as a function of γ. Furthermore,

$$\frac{d\pi_i\left(1, \hat{\sigma}_i^{\gamma}\right)}{d\gamma}|_{\gamma=0} = \pi_i(1, \hat{\sigma}_i^{\gamma}) > 0. \tag{23}$$

Thus, we have that:

$$\delta^{-1}\left(Q_H^{i,fict}(\gamma)-(1-\delta)\,c_H\right)=\left(\frac{(1-\lambda)\,(1-\pi)}{\pi}+\gamma\cdot\left(1-\frac{(1-\lambda)\,(1-\pi)}{\pi}\right)\right)\cdot c_H$$
$$+(1-\gamma)\cdot\left(1-\frac{(1-\lambda)\,(1-\pi)}{\pi}\right)\cdot V\left(\pi_i\left(1;\hat{\sigma}_i^{\gamma}\right)\right),$$

which is differentiable in γ and, using (23), we have:

$$\frac{dQ_H^{i,fict}(\gamma)}{d\gamma}|_{\gamma=0}=\delta\cdot\left(1-\frac{(1-\lambda)\,(1-\pi)}{\pi}\right)\cdot(c_H-v_L)>0. \tag{24}$$

This establishes that $Q_H^{i,fict}(\gamma)>Q_H^{i,fict}(0)$ for $\gamma\in(0,\bar{\gamma})$ for some small $\bar{\gamma}>0$. Therefore, in the fictitious economy, concealing some information is strictly Pareto improving.

Case (2). Consider next the case in which in the unique equilibrium of the fictitious economy, we have $\pi_i\,(h;\sigma_i)=\bar{\pi}$ and $v_L=Q_L^{i,fict}$, where

$$Q_L^{i,fict}=(1-\delta)\,c_L+\delta\cdot\left(\mathbb{P}(S=l|\theta_i=L)\cdot v_L+(1-\mathbb{P}(S=l|\theta_i=L))\cdot(\phi_i c_H+(1-\phi_i)v_L)\right), \tag{25}$$

for some $\phi_i<1$. For this to be the equilibrium, it must be that $\lambda<\bar{\lambda}$ and $\delta>\frac{v_L-c_L}{\lambda v_L+(1-\lambda)c_H-c_L}$. Thus, the low type's welfare is v_L, whereas the high type's welfare is $Q_H^{i,fict}=c_H$.

Consider the same revelation policy as in Case (1). If γ is large enough, i.e. is above some threshold $\hat{\gamma}$, it is straightforward to show that the unique equilibrium of the fictitious economy must feature $v_L=Q_L^{i,fict}(\gamma)$, where:

$$Q_L^{i,fict}(\gamma)=(1-\delta)\,c_L+$$
$$+\delta\cdot\left(\mathbb{P}(\omega^{fict}=0|\theta_i=L)\cdot(\phi_i c_H+(1-\phi_i)v_L)+\left(1-\mathbb{P}(\omega^{fict}=0|\theta_i=L)\right)\cdot V(\pi_i(1,\hat{\sigma}_i^{\gamma}))\right)$$

for some $\hat{\sigma}_i^{\gamma}>0$ such that $\pi_i(0,\hat{\sigma}_i^{\gamma})=\bar{\pi}$ and $\phi_i\in(0,1)$. But then, since the offer following $\omega^{fict}=1$ is strictly above c_H, the high type's welfare must satisfy $Q_H^{i,fict}(\gamma)>c_H=Q_H^{i,fict}(0)$.

Thus, we have shown that, if the planner knows the state, then she can achieve Pareto improvement through a partially revealing information policy. In what follows, we show how such an information policy can be implemented.

Implementation. In the actual economy, with arbitrarily many but a finite number of assets, the planner does not observe the state S. Instead, she observes the outcome of trade in the first period, i.e., how many or what fraction of sellers have traded. Thus, we need to construct an information policy measurable with respect to trading behavior rather than the state S.

To this end, consider the following information policy. Fix $\gamma\in(0,1)$ and suppose that the planner reports a binary signal $\omega_N \in \{0,1\}$ with the following properties. If the planner observes that the fraction of sellers who traded is more than $\tau^\gamma \equiv \hat{\sigma}_i^\gamma \frac{\mathbb{P}(\theta_i=L|S=h)+\mathbb{P}(\theta_i=L|S=l)}{2}$, then $\omega_N = 0$. Instead, if she observes a fraction weakly less than τ^γ, then $\omega_N = 1$ w.p. $1-\gamma$ and $\omega_N = 0$ w.p. γ. It is again straightforward to show that, under this information policy, when there are $N+1$ assets, all equilibria must still satisfy the Properties 1 through 3, and that the continuation value of type-θ seller from rejecting bid v_L in the first period is:

$$Q^i_{\theta,N}(\sigma_N, \phi_i) = (1-\delta) c_L + \delta \sum_{\omega_N} \mathbb{P}(\omega_N|\theta_i = \theta) F_L (\pi_i(\omega_N; \sigma_N), \phi_i), \tag{26}$$

when σ_N is the trading probability of the low type in the first period. An equilibrium is again characterized by a pair (σ_N, ϕ_i) satisfying $Q^i_{\theta,N}(\sigma_N, \phi_i) = v_L$, just as in equation 8. It exists because $Q^i_{L,N}(0,\cdot) < v_L < Q^i_{L,N}(1,\cdot)$, and $Q^i_{L,N}(x,\cdot)$ is upper hemicontinuous in x.

Take $\varepsilon > 0$ small, and consider constants $\tilde{\sigma}^0 \in \left(\hat{\sigma}_i^\gamma - \varepsilon, \hat{\sigma}_i^\gamma - \frac{\varepsilon}{2}\right)$ and $\tilde{\sigma}^1 \in \left(\hat{\sigma}_i^\gamma + \frac{\varepsilon}{2}, \hat{\sigma}_i^\gamma + \varepsilon\right)$. If these were the equilibrium trading probabilities, then as N grows large the distribution of the report ω_N would converge to that of the fictitious economy: $\mathbb{P}(\omega_N = 0|\theta_i = \theta) \to \mathbb{P}\left(\omega^{fict} = 0|\theta_i = \theta\right)$ and, as a result, the posterior beliefs would converge as well: $\pi_i(\omega_N; \tilde{\sigma}^j) \to^p \pi_i\left(\omega^{fict}; \tilde{\sigma}^j\right)$ for $j \in \{0,1\}$. From (26), it follows that when N is large enough, then:

$$Q^i_{L,N}\left(\tilde{\sigma}^0, \cdot\right) < Q^{i,fict}_L(\gamma) = v_L < Q^i_{L,N}\left(\tilde{\sigma}^1, \cdot\right),$$

where recall that $Q^{i,fict}_L(\gamma)$ is the equilibrium continuation value in the fictitious economy and is therefore equal to v_L. By continuity, there exists a pair $(\tilde{\sigma}_N, \phi_i)$ such that $\tilde{\sigma}_N \in (\tilde{\sigma}^0, \tilde{\sigma}^1)$ and $Q^i_{L,N}(\tilde{\sigma}_N, \phi_i) = v_L$. It follows that we can construct a sequence $\{\tilde{\sigma}_N\}$ of equilibrium trading probabilities in the actual economy such that, for N large, $|\tilde{\sigma}_N - \hat{\sigma}_i^\gamma| < \varepsilon$, i.e., it must be that $\tilde{\sigma}_N \to \hat{\sigma}_i^\gamma$ since ε can be chosen arbitrarily small. Note also that the distribution of the ω^N under this equilibrium sequence also converges to the distribution of report ω^{fict}. But then it follows that $Q^i_{H,N}(\tilde{\sigma}_N, \phi_i) \to Q^{i,fict}_H(\gamma)$, which establishes the result. ∎