Trust in Signals and the Origins of Disagreement

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Distrust in experts and the origins of disagreement

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Abstract

Why do individuals interpret the same information differently? We propose that individuals form beliefs following Bayes' Rule with one exception: when assessing the credibility of experts, they double-dip the data and use already-updated beliefs instead of their priors. This "pre-screening" mechanism explains why individuals jointly disagree about states of the world and the credibility of experts, why the ordering of signals and experts affects final beliefs, and when individuals over- or underreact to new information. In a trading game, pre-screening generates excessive speculation, bubbles, and crashes. Our theory provides a micro-foundation for why individuals disagree about how to interpret the same data.

JEL classification: D91; D83; G41

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1. Introduction

Disagreement is everywhere, over topics ranging from the causes of climate change to the consequences of stimulus spending. A core feature of many disagreements is that individuals disagree not just about the substance of their positions ("Do humans affect climate change?") but also about the credibility of information sources such as experts that inform those positions ("How credible are scientists and their data?"). In debates about economics ("What is the value of

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stimulus spending?"), medicine ("Are vaccinations safe for children?"), and politics ("Was there interference in elections?"), one side typically expresses supreme confidence in their preferred sources while dismissing the other side's sources. Bayesian learning about an unknown state of the world using signals from an information source with uncertain credibility struggles to explain this type of disagreement when all agents observe the same signals and have common priors.

This paper proposes a departure from Bayesian learning called "pre-screening." The central idea of pre-screening is that an agent recognizes that credibility is uncertain but mistakenly treats credibility as an "ancillary parameter," or a parameter necessary to learn with some precision before applying Bayes' Rule to form final beliefs. A pre-screener first forms an updated first-stage belief about credibility. She then forms posterior beliefs by weighing the data using the updated belief about credibility instead of her prior and overlooks that this step "double-dips" the data. In contrast, a Bayesian weighs the data using only her prior beliefs because she carefully separates her priors from the likelihood of the data.

Consider the following example. An individual who is reasonably sure that he weighs 200 pounds steps on a scale that he believes is likely accurate, and the scale reads 300 pounds. Surprised by the reading, a Bayesian's posterior is that the scale is likely inaccurate but that there is some chance he weighs 300 pounds, as he carefully combines the likelihood of the data with his prior belief that the scale was accurate. In contrast, a pre-screener first infers that the scale is inaccurate upon seeing the reading and then combines the likelihood of the data with this updated belief as though he knew the scale was inaccurate all along. This process leads the pre-screener to discount the possibility that he might be 300 pounds too much, on the premise that the scale is inaccurate. Intuitively, the pre-screener thinks: "I now think the scale is not credible, and my beliefs should reflect that I saw non-credible signals."

We motivate pre-screening by observing that "using the data twice" is an error that recurs in practice, particularly when the initial use is for a seemingly legitimate reason—in this case, reducing uncertainty about credibility when the principal object of interest is the state. For example, criticisms of statistical methods such as empirical Bayes and posterior Bayes Factor methods often center around the potential for double-dipping the data (Lindley, 1991; Aitkin, 1991). Similar criticisms exist of applied work in fields as varied as finance and neuroscience (Lo and MacKinlay, 1990; Vul et al., 2009).

Overlooking the use of updated data is plausible as evidence of hindsight bias suggests that individuals who have seen data tend to behave as if they "knew it all along" (Fischhoff, 1975; Hawkins and Hastie, 1990). A pre-screener overlooks her erroneous substitution of updated first-stage beliefs for priors, while a Bayesian carefully distinguishes her prior from subsequent data. Relatedly, evidence on the curse of knowledge (Camerer et al., 1989) suggests that individuals have difficulty conceptualizing what it was like to be uninformed in the past.

Section 2 introduces pre-screening. We assume that signal sources are data-generating processes that produce signals about an uncertain state and abstract from strategic motives to isolate the effects of biased learning. In the base model, we model a source's credibility as its accuracy in discerning the true state of the world (e.g., do economists accurately understand the economy).

Section 3 characterizes three implications of pre-screening for disagreement. First, pre-screening endogenously generates correlated disagreement. Suppose two agents with common priors observe signals with the same objective information content. If the agents are Bayesian, they never disagree. If the agents are pre-screeners, they disagree about the state if and only if they disagree about credibility and credibility objectively matters. Specifically, pre-screener X thinks the objectively-favored state is more likely than pre-screener Y does if and only if X also thinks the source is more credible than Y does.

Second, disagreement about credibility between two pre-screeners occurs if they see signals in different orders, even if they share common priors and see the same objective information content. The reason is that pre-screening, through the repeated substitution of updated beliefs about credibility for priors, leads to path-dependent beliefs whereby early signals have an outsized influence on the interpretation of later signals and thus final posterior beliefs. Bayesian beliefs, in contrast, do not depend on the order of signals.

Third, the model provides conditions for when pre-screeners endogenously over- and under-react to new signals that depend on how the signals affect first-stage beliefs about source credibility. Pre-screening thus provides a unified explanation for seemingly contradictory deviations from Bayes' Rule. For example, the literature suggests that agents exhibit confirmation bias and under-react to disconfirming news that contradicts their beliefs about the state (Rabin and Schrag, 1999). On the other hand, agents also over-react to news that is so disconfirming that it causes agents to re-evaluate their worldview or paradigm (Ortoleva, 2012; Galperti, 2019). Distinct from confirmation bias, pre-screeners can over-react or under-react to confirming and disconfirming news.

Section 4 characterizes pre-screeners' beliefs when there are multiple sources. Pre-screeners can disagree about sources' credibilities and the state even when they observe all sources' signals, as long as they encounter sources in different orders. The reason is that beliefs about an early source's credibility color the interpretation of later sources' signals. Thus, our theory explains why individuals disagree about states of the world and the credibilities of multiple sources of information.

Section 5 considers three extensions. First, we show that the possibility that sources may "slant" signals toward a given state can create further scope for error by pre-screeners. While the base model considers the case where agents do not know a source's accuracy, in reality agents may also be uncertain of whether sources have slant (e.g., whether scientists have an agenda). Second, we extend pre-screening to the case where a source may send multiple signals in one period and show how signal order can be important for persuasion. Third, we show that key results hold when we allow for fading memory.

Section 6 illustrates the implications of pre-screening for prices and trade. In a trading game similar to Harris and Raviv (1993), we compare outcomes when all traders are pre-screeners with outcomes when all traders are Bayesians. The game with pre-screeners features weakly more trade than the game with Bayesians since agents speculate against each other's beliefs in the sense of Harrison and Kreps (1978). Specifically, a pre-screener may buy an asset even if the price is greater than what she thinks it is worth to resell it later to other agents. This speculation is akin to traders "riding the bubble" (Abreu and Brunnermeier, 2003; Brunnermeier and Nagel, 2005). Sharp swings in prices akin to bubbles and crashes, as defined by Barberis (2018), can occur beyond what can be easily explained by a model with Bayesian agents with heterogeneous priors. The application to the trading game illustrates how pre-screening can generate several empirically-relevant dynamics within a unified framework.

Our main contribution is to offer a parsimonious micro-foundation for why individuals disagree about the interpretation of the same data. Our approach suggests that erroneous learning about credibility may play a central role in explaining the joint disagreement over substance and credibility in a wide range of settings. Section 7 summarizes pre-screening's key implications and how they differ from other approaches in the literature, including heterogeneous priors, inattention, correlation neglect, and confirmation bias. Concerning heterogeneous priors (Morris, 1995), our model provides a theory for such priors' origins. We conclude with a discussion of avenues for future research.

2. Model

2.1. Information environment

An agent learns about an unknown state $\theta \in \{A, B\}$ by observing binary signals $s_t \in \{a, b\}$ in each period t from an expert. Experts in our model are data-generating processes (e.g., analysts, advisors, scientists) and are not strategic, and we refer to experts as information sources from now on. A source's credibility is a type c that describes the informativeness of its signals. Nature draws true source credibility independently from the true state. Conditional on state and credibility, signals are independently and identically distributed.

In the main model, we focus on the case where credibility summarizes the accuracy of the source in determining the state. Specifically, a source has credibility $c \in \{L(ow), H(igh)\}$. A high-credibility source has a higher probability of correctly reporting the state than a low-credibility source and is more informative: $P(s_t = a|c, A) = P(s_t = b|c, B) = q_c$ where $q_L < q_H$. We assume $q_c \in [1/2, 1)$: the least accurate possible signal is noise, while even the most accurate possible signal is not perfectly correlated with the true state.

The agent is uncertain whether a source's credibility c is high or low. Uncertain credibility is realistic in many real-world areas of disagreement, such as economics, climate science, medicine, and politics. In economics, few lay individuals have the expertise or training to evaluate primary evidence on these issues. Yet, from the individual's perspective, the economist's ability is uncertain. Sapienza and Zingales (2013) show that American households have sharply different views than economists on questions ranging from the stock market to free trade.

Similarly, few individuals have the expertise to evaluate the extent to which humans affect climate change, yet many people have very strong opinions about the topic. Disagreement between climate "deniers" and supporters of the proposition is largely about the credibility of the majority of the scientific community (Druckman and McGrath, 2019). In medicine, despite consensus on the safety of childhood vaccinations, the anti-vaccine movement has gained traction by casting doubt on the evidence and motives of research. In politics, the proliferation of "fake news" over social media during the recent U.S. presidential election highlights the importance of uncertain expert credibility (The Economist Magazine, 2016).

We focus on a stark environment with no direct signals of source credibility. This is a simplification but reflects the reality that evaluating credibility by comparing predictions to outcomes in a controlled environment is difficult in the real world. Furthermore, credentials (PhDs, Nobel Prizes, and so on) are often of uncertain informativeness themselves and may not be very informative about the credibility of a source's opinions for a specific state. DellaVigna and Pope (2018) run a large experiment in which economists forecast the effectiveness of different incentive treatments on subjects, and find that objective measures of expertise are unrelated to forecast accuracy.

2.2. Learning

Suppose the agent has the prior that the state and credibility are independent with marginal probabilities ω_0^θ for the state and ω_0^c for each credibility type c. Let the agent observe a sequence of n signals, denoted $\mathbf{s}_n = (s_1, s_2, \ldots s_n)$, where one signal is observed each period.

A Bayesian's posterior belief $P(c, \theta|\mathbf{s}_n)$ equals:

$$P(c, \theta|\mathbf{s}_n) = \frac{\left(\prod_{t=1}^n P(s_t|c, \theta)\right) \omega_0^\theta \omega_0^c}{\sum_c \sum_\theta \left(\prod_{t=1}^n P(s_t|c, \theta)\right) \omega_0^\theta \omega_0^c}. \tag{1}$$

When forming her posterior, the Bayesian uses her prior belief ω_0^c about source credibility to weight the likelihood of signals $\left(\prod_{t=1}^n P(s_t|c,\theta)\right)$ in a single step. Her posterior beliefs depend only on her prior and the *information content* of signals, defined as:

Definition 1 *(Information content).* The **information content** of any signal path $\mathbf{s}_n$ is given by the number of "a" signals n_a and the number of "b" signals n_b.

We propose that individuals make a mistake that we call *pre-screening* when faced with the problem of determining how much weight to apply to a source's signals. A pre-screener mistakenly uses updated beliefs about credibility when weighting signals instead of using priors. She updates in two steps. First, she forms an updated first-stage belief about credibility, denoted $\kappa_c(\mathbf{s}_n)$, using Bayes' Rule. Second, she uses this updated belief $\kappa_c(\mathbf{s}_n)$ to weight the signals $\mathbf{s}_n$ in forming her joint posterior of state and credibility, denoted $P^s(c,\theta|\mathbf{s}_n)$. The key mistake is that she uses $\kappa_c(\mathbf{s}_n)$ to evaluate all signals, whereas a Bayesian uses her prior ω_0^c.

To illustrate, suppose a pre-screener observes two signals, one in each period. After observing the first signal (s_1), the first-stage updated belief about credibility, $\kappa_c(\{s_1\})$, is:

$$\kappa_c(\{s_1\}) = \frac{\omega_0^c \sum_\theta P(s_1|c,\theta)\omega_0^\theta}{\sum_c \sum_\theta P(s_1|c,\theta)\omega_0^\theta \omega_0^c}.$$

Using $\kappa_c(\{s_1\})$ to form the joint posterior belief on the state and credibility, $P^s(c,\theta|\{s_1\})$, yields the pre-screener's posterior beliefs after the first signal:

$$P^s(c,\theta|\{s_1\}) = \frac{P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta}{\sum_c \sum_\theta P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta}.$$

After observing the second signal (s_2), the pre-screener's first-stage updated belief about credibility, $\kappa_c(\{s_1,s_2\})$, is:

$$\kappa_c(\{s_1,s_2\}) = \frac{\sum_\theta P(s_2|c,\theta)P^s(c,\theta|\{s_1\})}{\sum_c \sum_\theta P(s_2|c,\theta)P^s(c,\theta|\{s_1\})}.$$

The pre-screener then uses $\kappa_c(\{s_1,s_2\})$ to form her joint posterior belief on the state and credibility by re-weighting all the information from the source. The posterior, $P^s(c,\theta|\{s_1,s_2\})$, equals:

$$P^s(c,\theta|\{s_1,s_2\}) = \frac{P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta}{\sum_c \sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta}.$$

Iterating on the pre-screener's process of repeatedly substituting newly-updated beliefs about credibility for priors allows us to characterize her posterior beliefs.

Definition 2 *(Pre-screener's beliefs).* After observing a signal path $\mathbf{s}_n$ from a source, the **pre-screener's first-stage updated belief** about source credibility, $\kappa_c(\mathbf{s}_n)$, is given by:

$$\kappa_c(\mathbf{s}_n) = \frac{\kappa_c(\mathbf{s}_{n-1})\sum_\theta \left(\prod_{t=1}^n P(s_t|c,\theta)\omega_0^\theta\right)}{\sum_c \kappa_c(\mathbf{s}_{n-1})\sum_\theta \left(\prod_{t=1}^n P(s_t|c,\theta)\omega_0^\theta\right)}, \tag{2}$$

where $\kappa_c(\emptyset) = \omega_0^c$. The **pre-screener's joint posterior** on credibility and the state, $P^s(c,\theta|\mathbf{s}_n)$, is given by:

$$P^s(c,\theta|\mathbf{s}_n) = \frac{\left(\prod_{t=1}^n P(s_t|c,\theta)\right)\kappa_c(\mathbf{s}_n)\omega_0^\theta}{\sum_c \sum_\theta \left(\prod_{t=1}^n P(s_t|c,\theta)\right)\kappa_c(\mathbf{s}_n)\omega_0^\theta}. \tag{3}$$

2.3. Motivation and conceptual foundations

The central idea of pre-screening is that an agent may recognize that credibility is uncertain but mistakenly treats credibility as an "ancillary parameter" when forming her final beliefs, or a parameter that is beneficial or necessary to learn with some precision before weighing the data using Bayes' Rule.[1] Intuitively, a pre-screener forms data-dependent priors for the seemingly-legitimate purpose of estimating an ancillary parameter (the first step) but overlooks double-dipping the data in the final analysis when she uses her data-dependent prior (the second step).

Specifically, the first step of pre-screening, the first-stage update, occurs because the agent faces a complicated problem with uncertainty over two parameters, one of which (credibility) is essential to support the learning process but is not the agent's principal object of study (hence the term "ancillary"). The agent approaches the problem by first reducing uncertainty over the ancillary parameter, a seemingly legitimate reason to use the data. She thus forms updated belief $\kappa_c(\mathbf{s}_n)$ about credibility.[2]

In the second step of pre-screening, the agent feeds the updated belief about credibility as a "data-dependent prior" into her analysis to form her final posterior. For example, if the agent thinks the source is not credible based on the data, she then thinks to herself, "I now think the source is likely not credible, and my beliefs should reflect that I likely saw non-credible signals." She thus replaces ω_0^c with $\kappa_c(\mathbf{s}_n)$.[3] However, she crucially overlooks that using her data-dependent prior in this way "uses the data twice" and double-dips the data. A Bayesian always carefully separates her priors from the likelihood of the data.

We motivate pre-screening in two ways: First, from the observation that "using the data twice" is an error that recurs in practice, and second, from psychological reasons underlying why individuals may double-dip the data.

Two statistical techniques illustrate the possibility of error in practice. Empirical Bayes (EB) methods often initially use the data to reduce parameter uncertainty by calibrating data-dependent priors over nuisance parameters, a seemingly-legitimate reason to use the data (Maritz and Lwin, 1989). For example, one common application first estimates nuisance parameters and plugs them into a second analysis about the primary parameter of interest. However, a researcher may inadvertently double-dip the data if she is not careful. Carlin and Louis (2000) write that early EB authors' consistent "use of Bayesian tools ... while using the data twice (first to help determine the prior, then again in the usual Bayesian way when computing the posterior) was not highly regarded in the Bayesian community at the time."

Posterior Bayes Factor (PBF) methods (Aitkin, 1991) are a variation of Bayes Factor methods that conduct model comparison. The method seeks to reduce uncertainty by first calculating posterior parameter distributions before feeding these posteriors into the second step calculation of Bayes Factors. However, the method plainly uses the data twice by using posterior instead of

[1] We choose the word "ancillary" in line with its dictionary definition: "*ancillary* (adj.): A.1. Subservient, subordinate, ministering (to). ... 3. Designating activities and services that provide essential support to the functioning of a central service ... " (Oxford English Dictionary, 1989).

[2] We view credibility as the ancillary parameter and the state as the principal object of study rather than the other way around. A model where the agent updates on the state first makes predictions that are qualitatively very similar to confirmation bias - for example, there would be no significant distinction between observing conflicting signals from different sources versus a single source.

[3] Definition 3 assumes ex-ante independence of states and credibility. We maintain this assumption both for simplicity and because it isolates the effect of pre-screening on joint beliefs about the state and credibility without assuming any correlation ex-ante. We provide a generalized definition in the Internet Appendix.

prior parameter distributions in the second step, and this double-dip has drawn intense criticism. For example, O'Hagan (1991) writes that the method "suggests that we obtain the posterior distribution using the whole data vector, and then reuse all these data to effect model comparisons. Such a procedure is quite evidently non-coherent."

Pre-screeners make errors analogous to those in these two statistical methods. A pre-screener's first step of narrowing uncertainty over an ancillary parameter is analogous to estimating a nuisance parameter in EB methods and calculating posterior parameter distributions in the PBF method. The second step of overlooking the re-use of data in pre-screening is analogous to plugging estimated nuisance parameters into Bayes' Rule in EB methods and the substitution of the posterior for the prior in PBF methods. These analogies are imperfect since, for example, pre-screeners specifically update about credibility first, and also form a joint distribution over both credibility and the state in the second step. However, they illustrate that mistakenly using the data twice recurs particularly when the first use is to narrow uncertainty about objects that are not the primary object of study.

Individuals have also made the error of first using the data for some purpose before overlooking a second use of the data in several applied settings. Lo and MacKinlay (1990) criticize empirical tests in finance by noting that several statistical tests are invalid where "the construction of test statistics is influenced by empirical relations derived from the very same data used in the test." In neuroscience, Vul et al. (2009) show that published correlations between measures of brain activation and personality are biased because studies use the same data to both calculate the correlations and select which data points to correlate.[4] In both examples, the use of data for a seemingly legitimate purpose (test construction and sample selection) led researchers to double-dip the data when conducting their primary analyses.[5]

Several related psychological reasons underlie why individuals may erroneously double-dip data. Lord et al. (1979) conjectured that individuals make this mistake when noting that their experimental subjects tended to use "evidence already processed ... to bolster the very theory that initially 'justified' the processing bias." As Rabin and Schrag (1999) discuss, the mistake is analogous to a teacher who first assigns a student a low grade because she unfavorably interprets an unclear answer from the student as consistent with priors about low ability, but then goes on to erroneously use the low grade as further or additional evidence of low ability.

Evidence from cognitive psychology about hindsight bias and the curse of knowledge suggest that individuals who have seen data tend to behave as if they "knew it all along" and thus have trouble ignoring information when the context requires using only prior beliefs. Whereas a Bayesian carefully distinguishes her priors from subsequent data, there is a tendency for individuals with "outcome knowledge to overestimate what they would have known without outcome knowledge" (Fischhoff, 1975; Fischhoff and Beyth, 1975; Hawkins and Hastie, 1990). A core tenet of leading cognitive models for hindsight bias (Hoffrage et al., 2000; Hertwig et al., 2003) is that "If knowledge is constantly updated" then "inferences based on updated knowledge may

[4] Kriegeskorte et al. (2009, 2010) dub this "circular analysis" and point out the problem in several fMRI studies.

[5] As another example of why using updated beliefs might seem plausible, Subramanyam (1996) notes that, when the error precision of a normally-distributed signal about a normally-distributed unobserved random variable is unknown, a Bayesian can calculate the posterior mean by applying the updated signal-gain in the linear updating equation for the mean, due to the Law of Iterated Expectations. This is due to the linear relationship between the posterior mean and the realized signal in a Gaussian environment, and the procedure does not recover the joint posterior of the mean and error precision. The non-monotone reaction to surprises in that paper occurs due to how the likelihood combines with a Bayesian's prior beliefs when signal precision is uncertain.

be different from those based on past knowledge" (Hoffrage and Hertwig, 1999).[6] Relatedly, individuals also suffer from the curse of knowledge, or the failure to accurately anticipate the judgments of less-informed individuals (Camerer et al., 1989), as they are unable to ignore their own additional information and imagine what it was like to be uninformed in the past. Madarász (2012) suggests that people overestimate how much others know what they know.

Relatedly, the literature has documented that individuals double-count data due to correlation neglect (DeMarzo et al., 2003; Eyster and Rabin, 2010, 2014; Ortoleva and Snowberg, 2015; Eyster et al., 2018; Enke and Zimmermann, 2019). Individuals who exhibit correlation neglect have difficulty recognizing the double-counting problem inherent in correlated signals and thus tend to double-count data that contains information redundancies. Pre-screeners understand correlation but engage in a different form of double-counting originating from their attempt to resolve uncertainty about credibility more than they objectively should. The specific form of double-counting in pre-screening distinguishes its predictions from those of correlation neglect.

Though the two steps of pre-screening—forming a first-stage belief and then feeding it into final beliefs through a data-dependent prior—each deviate from Bayes Rule, one can consider alternative models that adopt each step individually. We consider such models in Section 7 and highlight how they differ from pre-screening in their mechanisms and predictions.

2.4. An example

An individual who is reasonably sure that he weighs 200 pounds steps on a scale with unknown credibility, and the scale reads 300 pounds. A second reading also shows 300 pounds. What would a Bayesian and pre-screener infer after each signal?

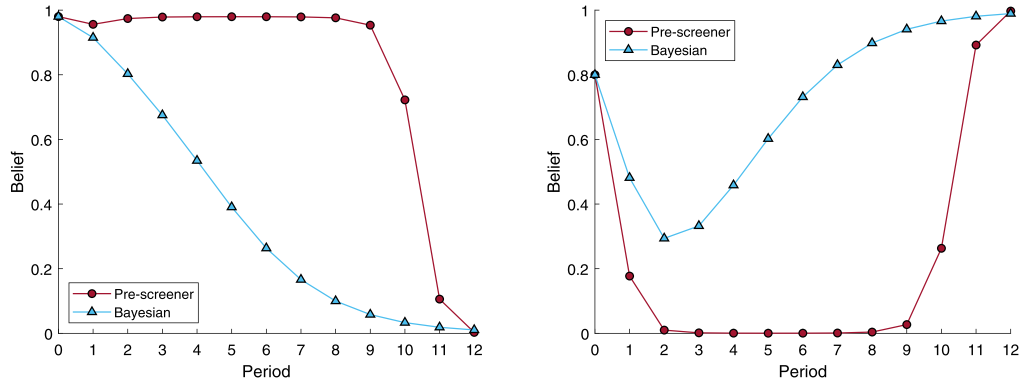
In the following example, the Bayesian's belief about weight moves progressively towards 300 pounds after each signal of 300 pounds, even though he also believes the scale might not be accurate. In contrast, a pre-screener's belief about his weight moves very little towards 300 pounds, and will move *back towards* 200 pounds after the second signal, as he concludes the scale is almost certainly inaccurate. The stark difference in this illustration occurs because the pre-screener erroneously acts as if he had updated beliefs about credibility all along.[7]

Let the individual's weight $\theta \in \{200, 300\}$ pounds be the unknown state of the world, and suppose the scale can read either 200 or 300 pounds. Both the Bayesian and pre-screener are uncertain about the scale's accuracy, which can be accurate ($q_H = 0.9$) or inaccurate ($q_L = 0.5$). They share the same priors that their weight is probably 200 pounds ($\omega_0^{200} = 0.98$) and that the scale is probably accurate ($\omega_0^H = 0.80$). Fig. 1 reports posterior beliefs.

Given prior beliefs, the first reading of $\{s_1\} = \{300\}$ is quite a surprise. The Bayesian's marginal posterior beliefs equal $P(\theta = 200|\{300\}) = 0.91$ and $P(c = H|\{300\}) = 0.48$. Even though the Bayesian's posterior belief is that the scale is likely inaccurate, he is careful to reach his joint posterior beliefs by combining the likelihood of the data with his *prior* belief that the scale is accurate, $\omega_0^H = 0.80$, following Equation (1).

[6] For example, according to the RAFT cognitive process model (Hoffrage et al., 2000), hindsight bias is generated "if individuals are unable to directly retrieve their initial judgment but try to reconstruct it by repeating the original judgment process, this time, however, on the basis of the updated knowledge base" (Blank and Nestler, 2007). An implication of an individual's belief that she "knew it all along" in the past is that she also currently thinks that she "knew it all along" and behaves accordingly. For example, Biais and Weber (2009) show that individuals exhibit more hindsight bias when not explicitly reminded of their prior beliefs, and that hindsight bias is correlated with lower performance among bankers.

[7] The Appendix provides the detailed equations for the pre-screener's and Bayesian's beliefs each period.



(a) Probability of $\theta = 300$ (b) Probability of $c = H$

Fig. 1. **The scale example.** Parameter values equal $(q_H, q_L, \omega_0^{200}, \omega_0^H) = (0.9, 0.5, 0.98, 0.8)$. Signals are $s_t = 300$ each period.

A pre-screener's marginal posteriors equal $P^s(\theta = 200|\{300\}) = 0.96$ and $P^s(c = H|\{300\}) = 0.18$. He reaches his joint posterior beliefs by erroneously combining the likelihood of the data with an *updated* belief that the scale is likely *in*accurate, $\kappa_H(\{300\}) = 0.48$ (Equations (2) and (3), not shown in the figure). This "double-dipping" leads him to update insufficiently towards the belief that he weighs 200 pounds and too much in the direction that the scale is inaccurate.

After a second reading, the Bayesian's marginal posterior beliefs equal $P(\theta = 200|\{300, 300\}) = 0.80$ and $P(c = H|\{300, 300\}) = 0.29$. Notice that the Bayesian's belief about his weight progressively moves away from 200 pounds after each signal (from 0.98 to 0.91 to 0.80), even though his trust in the scale progressively drops.

In contrast, the pre-screener's beliefs equal $P^s(\theta = 200|\{300, 300\}) = 0.97$ and $P^s(c = H|\{300, 300\}) = 0.01$. His posterior probability that he weighs 200 pounds increases (from 0.96 to 0.97), which is the wrong way relative to the Bayesian. This is because he erroneously thinks that his updated belief that the scale is likely inaccurate, $\kappa_H(\{300, 300\}) = 0.09$, should apply to all of the scale's readings, leaving him fairly confident that he is 200 pounds, close to what he initially believed. After all, if the scale is probably inaccurate, then he thinks that all of its readings are more questionable than he originally thought. Put yet another way, the pre-screener thinks: I now think the scale is not credible, and my beliefs should reflect that any concern I had about being overweight stemmed from not-credible readings. Therefore, he is now (erroneously) less concerned about being overweight.

It takes the Bayesian only three signals to begin inferring that the scale is likely accurate. His belief about weight progressively moves away from 200 pounds with each signal. It takes the pre-screener six readings to begin believing that the scale might be accurate, and during this time, his belief about weight is moving towards 200 pounds—the wrong way—before reversing afterwards. Upon the seventh reading, he begins to recognize that the scale may be credible and re-evaluates the evidence he has received in light of more favorable updated first-stage beliefs. He thinks: Perhaps the scale is credible and the readings of 300 pounds that I saw were accurate.

Upon the eleventh reading, the pre-screener thinks he is probably 300 pounds and that the scale is very credible, so he revises his beliefs sharply toward 300 pounds.[8]

3. Pre-screening and disagreement

3.1. Disagreement about the state θ

Our first result, provided in Proposition 1, shows that any disagreement about θ between a Bayesian and pre-screener or two pre-screeners is fully characterized by disagreement about credibility c. We first establish Lemma 1 which provides the key intuition and relates the pre-screener's belief about θ to a Bayesian's belief. We assume all agents share common priors $\left(\omega_0^A, \omega_0^H\right) \in (0,1) \times (0,1)$.

Lemma 1 *(Beliefs about θ). Suppose a pre-screener and Bayesian observe signal path $\mathbf{s}_n$. Pre-screeners and Bayesians share the same posterior conditional probability of the state:*

$$P^s(\theta \mid c; \mathbf{s}_n) = P(\theta \mid c; \mathbf{s}_n), \tag{4}$$

and therefore the pre-screener's marginal posterior belief about θ equals:

$$P^s(\theta \mid \mathbf{s}_n) = P(\theta \mid c = L; \mathbf{s}_n) + [P(\theta \mid c = H; \mathbf{s}_n) - P(\theta \mid c = L; \mathbf{s}_n)] \times P^s(c = H \mid \mathbf{s}_n). \tag{5}$$

Lemma 1 establishes that, after observing $\mathbf{s}_n$, any posterior disagreement about the state between a pre-screener and Bayesian is fundamentally about the credibility of the source's signals. Given a source type c, the two agree about the posterior conditional probability of the θ (Equation (4)). Thus the pre-screener's belief about θ can only differ from a Bayesian's belief if her belief about c differs (Equation (5)). The Lemma leads to Proposition 1.

Proposition 1 *(Disagreement about θ). Let signal path $\mathbf{s}_n$ be given.*

1. *If a pre-screener and Bayesian both observe $\mathbf{s}_n$, the difference between their marginal posterior probabilities over θ equals:*

$$\begin{aligned} P^s(\theta \mid \mathbf{s}_n) - P(\theta \mid \mathbf{s}_n) = &[P(\theta \mid c = H; \mathbf{s}_n) - P(\theta \mid c = L; \mathbf{s}_n)] \\ &\times \left[P^s(c = H \mid \mathbf{s}_n) - P(c = H \mid \mathbf{s}_n)\right]. \end{aligned} \tag{6}$$

   This difference equals zero if and only if $n_a = n_b$ or $\prod_{m=1}^{n} \frac{P(c=H|\mathbf{s}_m)(1-\omega_0^H)}{P(c=L|\mathbf{s}_m)(\omega_0^H)} = 1$, where $\mathbf{s}_m$ is the sequence of the first m signals of $\mathbf{s}_n$.

2. *Let $\Sigma(\mathbf{s}_n)$ be the set of permutations of $\mathbf{s}_n$. Let pre-screeners X and Y observe $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$, respectively, where $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$. The difference between the two pre-screeners' marginal posterior probabilities over θ equals:*

[8] If individuals are uncertain of whether the scale is biased towards one state instead of its accuracy, qualitative insights are similar. Section 5 discusses such "slant" further.

$$P^s\left(\theta \mid \mathbf{s}_n^X\right) - P^s\left(\theta \mid \mathbf{s}_n^Y\right) = \left[P\left(\theta \mid c = H; \mathbf{s}_n\right) - P\left(\theta \mid c = L; \mathbf{s}_n\right)\right] \times \left[P^s\left(c = H \mid \mathbf{s}_n^X\right) - P^s\left(c = H \mid \mathbf{s}_n^Y\right)\right]. \tag{7}$$

This difference equals zero if $n_a = n_b$ *or* $\prod_{m=1}^n \frac{P(c=H|\mathbf{s}_m^X)}{P(c=L|\mathbf{s}_m^X)} = \prod_{m=1}^n \frac{P(c=H|\mathbf{s}_m^Y)}{P(c=L|\mathbf{s}_m^Y)}$.

Proposition 1 says that disagreement about the state occurs (the left-hand side of Equations (6) and (7) is not zero) if and only if credibility objectively matters (the first term on the right-hand side is not zero) and there is disagreement about credibility (the second term is not zero). There is no disagreement about the state if credibility objectively does not matter ($n_a = n_b$) or there is no disagreement about credibility. We discuss the condition for when there is no disagreement about credibility in the next subsection.

A key implication of Proposition 1 is *correlated disagreement*: A pre-screener thinks the state objectively favored by $\mathbf{s}_n$ is more likely than what other agents think if and only if she thinks $c = H$ is more likely. For example, suppose the signals objectively favor state A ($n_a > n_b$). Then $P(\theta = A \mid c = H; \mathbf{s}_n) - P(\theta = A \mid c = L; \mathbf{s}_n) > 0$, and the objective posterior probability of state A is objectively higher when the source is more credible. As a result, differences in beliefs about $\theta = A$ on the left-hand side of Equations (6) and (7) have the same sign as differences in beliefs about $c = H$ on the right-hand side. Identical arguments for B apply when $n_b > n_a$.

To simplify the exposition going forward, we define the following:

Definition 3 *(Over- and under-trust).* Given any signal path $\mathbf{s}_n$, a pre-screener **overtrusts** the source if $P^s(c = H|\mathbf{s}_n) > P(c = H|\mathbf{s}_n)$ and **under-trusts** if $P^s(c = H|\mathbf{s}_n) < P(c = H|\mathbf{s}_n)$.

Part 1 thus says there is correlated disagreement between a pre-screener and a Bayesian in that the pre-screener thinks the objectively-favored state is more likely than a Bayesian thinks if and only if she overtrusts the source. Conversely, she thinks that state is less likely than a Bayesian thinks if and only if she under-trusts the source.

Part 2 says that there is also correlated disagreement between two pre-screeners in that pre-screener X thinks the objectively-favored state is more likely than pre-screener Y thinks if and only if she also thinks the source is more credible. The next section characterizes disagreement about credibility. A key result is that X and Y can disagree about credibility due to different ordering of signals even if $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$ have identical information content.

3.2. Disagreement about credibility c

The reason disagreement about credibility occurs is that a pre-screener's final posteriors erroneously depend on the order of signals. To see why, expand the recursion of $\kappa_c(\mathbf{s}_n)$ in the pre-screener's posterior belief in Equation (3) to obtain:

$$P^s\left(c, \theta \mid \mathbf{s}_n\right) = \frac{\beta_c\left(\mathbf{s}_n\right) \prod_{t=1}^n P\left(s_t \mid c, \theta\right) \omega_0^\theta \omega_0^c}{\sum_c \beta_c\left(\mathbf{s}_n\right) \sum_\theta \prod_{t=1}^n P\left(s_t \mid c, \theta\right) \omega_0^\theta \omega_0^c}, \tag{8}$$

where $\beta_c(\mathbf{s}_n)$ summarizes the cumulative effect of pre-screening through time on beliefs:

$$\beta_c(\mathbf{s}_n) \equiv \left(\sum_\theta P(s_1|c,\theta)\omega_0^\theta\right) \times \left(\sum_\theta P(s_1|c,\theta)P(s_2|c,\theta)\omega_0^\theta\right) \times \ldots$$

$$\times\left(\sum_{\theta} P(s_1|c,\theta)P(s_2|c,\theta)\ldots P(s_n|c,\theta)\omega_0^{\theta}\right)$$
$$=\prod_{m=1}^{n}\left(\sum_{\theta}\left(\prod_{t=1}^{m} P(s_t|c,\theta)\right)\omega_0^{\theta}\right). \tag{9}$$

A pre-screener's posterior beliefs depend on signal order because $\beta_c(\mathbf{s}_n)$ depends on signal order. Early signals appear more often in $\beta_c(\mathbf{s}_n)$ because agents substitute first-stage updated beliefs about credibility for time-0 priors every time they update. When early signals indicate high credibility, β_H is high and β_L is low. This can occur when early signals are consistent with each other (e.g., $\{s_1, s_2\} = \{a, a\}$ when $\omega_0^A = 1/2$) or priors (e.g., $\{s_1\} = \{a\}$ when $\omega_0^A > 1/2$). Conversely, when early signals indicate low credibility, β_H is low and β_L is high.

Pre-screening thus leads to a *first impression bias* where early signals of credibility have an outsized influence on beliefs through their effect on β_c. Proposition 2 characterizes this effect through $\beta_H(\mathbf{s}_n)/\beta_L(\mathbf{s}_n)$, which we shorten to $\beta_H/\beta_L(\mathbf{s}_n)$.

Proposition 2 *(Disagreement about c). Let* $\mathbf{s}_n$ *be given.*

1. $P^s(c = H \mid \mathbf{s}_n) \geq P(c = H \mid \mathbf{s}_n)$ *if and only if* $\beta_H/\beta_L(\mathbf{s}_n) \geq 1$, *with equality holding if and only if* $\beta_H/\beta_L(\mathbf{s}_n) = 1$.
2. *For any* $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$, $P^s\left(c = H \mid \mathbf{s}_n^X\right) \geq P^s\left(c = H \mid \mathbf{s}_n^Y\right)$ *if and only if* $\beta_H/\beta_L(\mathbf{s}_n^X) \geq \beta_H/\beta_L(\mathbf{s}_n^Y)$, *with equality holding if and only if* $\beta_H/\beta_L(\mathbf{s}_n^X) = \beta_H/\beta_L(\mathbf{s}_n^Y)$.

Finally, note that $\beta_H/\beta_L(\mathbf{s}_n) = \prod_{m=1}^{n} \frac{P(c=H|\mathbf{s}_m)(1-\omega_0^H)}{P(c=L|\mathbf{s}_m)(\omega_0^H)}$ *in terms of objective parameters.*

Part 1 says that a pre-screener over-trusts the source if and only if $\beta_H/\beta_L(\mathbf{s}_n) > 1$ and under-trusts the source if and only if $\beta_H/\beta_L(\mathbf{s}_n) < 1$. By Proposition 1, overtrust (under-trust) directly leads the pre-screener to think the objectively-favored state is more (less) likely than a Bayesian thinks. The condition for no disagreement about credibility in Proposition 1 Part 1 is identical to $\beta_H/\beta_L(\mathbf{s}_n) = 1$.

Part 2 makes an analogous statement about disagreement over credibility between two pre-screeners X and Y who observe $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$ that have identical information content but signals in different orders. If the signal order in $\mathbf{s}_n^X$ generates higher $\beta_H/\beta_L(\mathbf{s}_n)$ than $\mathbf{s}_n^Y$ generates, then pre-screener X thinks the source is more credible than pre-screener Y thinks. By Proposition 1, X also thinks the objectively-favored state is more likely than Y thinks.

The Proposition also shows how to interpret the key term $\beta_H/\beta_L(\mathbf{s}_n)$ in terms of objective parameters, $\beta_H/\beta_L(\mathbf{s}_n) = \prod_{m=1}^{n} \frac{P(c=H|\mathbf{s}_m)(1-\omega_0^H)}{P(c=L|\mathbf{s}_m)(\omega_0^H)}$. This formulation of $\beta_H/\beta_L(\mathbf{s}_n)$ demonstrates that a pre-screener's over- or under-trust depends on the cumulative effect of the substitution of first-stage updated beliefs about credibility on subsequent beliefs about credibility. Each mth term of $\beta_H/\beta_L(\mathbf{s}_n)$ is the objective posterior odds that the source is high credibility given subsequence $\mathbf{s}_m$ divided by the prior odds. Because any $\mathbf{s}_m$ necessarily includes the signals previously observed in $\mathbf{s}_{m-1}$, $\mathbf{s}_{m-2}$, and so on, early signals and their initial effects on objective credibility have an outsized influence on $\beta_H/\beta_L(\mathbf{s}_n)$. Thus, a pre-screener's final beliefs are influenced by first impressions about credibility. In contrast, a Bayesian's beliefs are independent of signal order.

Corollary 1 fleshes out the above discussion by considering how changing signal order affects $\beta_H/\beta_L(\mathbf{s}_n)$. Let $\mathbf{s}_n$ contain at least one a and b ($0 < n_a < n$), and consider two permutations that differ only in how they order a single (a, b): Let $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$ with $(s_j^X, s_{j+1}^X) = (a, b)$ and $(s_j^Y, s_{j+1}^Y) = (b, a)$ for some $j < n$. (For example, if $\mathbf{s}_n = (a, a, a, b)$, consider $\mathbf{s}_n^X = \mathbf{s}_n$ and $\mathbf{s}_n^Y = (a, a, b, a)$ with $j = 3$.) Let $n_{a,j-1}$ and $n_{b,j-1}$ be the information content of $\mathbf{s}_{j-1}$ (i.e., the number of a and b signals through $j-1$, respectively), which is common to both $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$. Whether $\beta_H/\beta_L(\mathbf{s}_n^X)$ is greater or less than $\beta_H/\beta_L(\mathbf{s}_n^Y)$ depends on the information content of $\mathbf{s}_{j-1}$ and prior beliefs.

If the information content of $\mathbf{s}_{j-1}$ sufficiently indicates state A, the additional b in $\mathbf{s}_j^Y$ compared to $\mathbf{s}_j^X$ results in $\beta_H/\beta_L(\mathbf{s}_n^Y) < \beta_H/\beta_L(\mathbf{s}_n^X)$ irrespective of prior beliefs (Part 1). The reason is that the additional b in $\mathbf{s}_j^Y$ leads to lower objective beliefs about credibility than $\mathbf{s}_j^X$ since it is inconsistent with the strong evidence favoring A: $\frac{P\left(c=H|\mathbf{s}_j^Y\right)}{P\left(c=L|\mathbf{s}_j^Y\right)} < \frac{P\left(c=H|\mathbf{s}_j^X\right)}{P\left(c=L|\mathbf{s}_j^X\right)}$. This lower belief about credibility at j influences the interpretation of subsequent signals and culminates in $\beta_H/\beta_L(\mathbf{s}_n^Y) < \beta_H/\beta_L(\mathbf{s}_n^X)$.

If the information content of $\mathbf{s}_{j-1}$ only weakly indicates state A, then it is also possible that $\beta_H/\beta_L(\mathbf{s}_n^Y) > \beta_H/\beta_L(\mathbf{s}_n^X)$ when priors strongly favor state B. The reason is that the additional b in $\mathbf{s}_j^Y$ is consistent with priors about the state even though it is inconsistent with the previous information. Thus, $\mathbf{s}_j^Y$ can generate either lower or higher objective beliefs about credibility compared to $\mathbf{s}_j^X$. Because these beliefs color the interpretation of subsequent signals, the ultimate relationship between $\beta_H/\beta_L(\mathbf{s}_n^Y)$ and $\beta_H/\beta_L(\mathbf{s}_n^X)$ is ambiguous. Analogous reasoning applies if the information content of $\mathbf{s}_{j-1}$ favors B (Part 2 of the Corollary).

Corollary 1. *Let $\mathbf{s}_n$ with $0 < n_a < n$ be given, and consider $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$, which are identical except for $(s_j^X, s_{j+1}^X) = (a, b)$ and $(s_j^Y, s_{j+1}^Y) = (b, a)$ for some $j < n$. Let $d_{j-1} = n_{a,j-1} - n_{b,j-1}$.*

1. *Case 1: $d_{j-1} > 0$. There exists a $d_{j-1}^* \geq 0$ such that $\frac{P\left(c=H|\mathbf{s}_{j-1}\right)(1-\omega_0^H)}{P\left(c=L|\mathbf{s}_{j-1}\right)(\omega_0^H)}$ is increasing in d_{j-1} for all $d_{j-1} > d_{j-1}^*$. By implication, if $d_{j-1} > d_{j-1}^*$, then $\beta_H/\beta_L(\mathbf{s}_n^Y) < \beta_H/\beta_L(\mathbf{s}_n^X)$. Otherwise, the effect is ambiguous.*
2. *Case 2: $d_{j-1} < 0$. There exists a $d_{j-1}^{**} \leq 0$ such that $\frac{P\left(c=H|\mathbf{s}_{j-1}\right)(1-\omega_0^H)}{P\left(c=L|\mathbf{s}_{j-1}\right)(\omega_0^H)}$ is decreasing in d_{j-1} for all $d_{j-1} < d_{j-1}^{**}$. By implication, if $d_{j-1} < d_{j-1}^{**}$, then $\beta_H/\beta_L(\mathbf{s}_n^Y) > \beta_H/\beta_L(\mathbf{s}_n^X)$. Otherwise, the effect is ambiguous.*

More broadly, Corollary 1 suggests that pre-screeners tend to interpret signal sequences with early mixed messages about the state as less credible than other signal sequences that have the same objective information content. We show in the Appendix that, for the special case of $\omega_0^A = 1/2$, re-ordering signals so that alternating the signals first generates the least trust in the source, while re-ordering so that the longest consistent streak appears first generates the most trust.[9]

[9] For $\omega_0^A = 1/2$, posterior beliefs about credibility do not depend on whether the information content of $\mathbf{s}_n$ is consistent with prior beliefs about the state. They depend only on the likelihood function, which simplifies the intuition preceding the statement of the Corollary.

Despite the first impression bias, Proposition 3 shows that, as $n \to \infty$, pre-screeners learn the truth if and only if Bayesians do. The reason is that signals are weakly positively correlated with the truth, and therefore sufficient signals overcome the first-impression bias.

Proposition 3 *(Asymptotic agreement). Let c' and θ' be the true credibility and state of the world selected by nature. Pre-screeners and Bayesians agree in the limit, with pre-screeners learning the true state if and only if Bayesians do:*

$$\lim_{n\to\infty} P^s\left(c = c' \mid \mathbf{s}_n\right) = \lim_{n\to\infty} P\left(c = c' \mid \mathbf{s}_n\right) = 1$$

$$\lim_{n\to\infty} P^s\left(\theta = \theta' \mid \mathbf{s}_n\right) = \lim_{n\to\infty} P\left(\theta = \theta' \mid \mathbf{s}_n\right) = \begin{cases} 1 & \text{if } c' = H \text{ or } q_L > 1/2 \\ \omega_0^{\theta'} & \text{if } c' = L \text{ and } q_L = 1/2 \end{cases}.$$

Proposition 3 relies on $q_L \geq 1/2$: pre-screeners learn the truth because signals from both credibility types are weakly positively correlated with the true state. Section 5 introduces an extension where signals may be negatively correlated with the state because sources are *slanted* toward one state. In that case, pre-screeners may become certain of the wrong state even if Bayesians do not learn the truth.

3.3. Endogenous over- and underreaction to new information

Suppose a pre-screener has observed signal path $\mathbf{s}_n$ and has posterior ω_n^s. When do the pre-screener's beliefs about the state over- or under-react in response to the next signal s_{n+1}, compared to a Bayesian endowed with prior ω_n^s? We say that the pre-screener has over-reacted (under-reacted) to new signal s_{n+1} if she updates more (less) toward the state indicated by s_{n+1} than the endowed Bayesian does.

Proposition 4 characterizes how a pre-screener's beliefs react to news in terms of the sign of a new statistic, $\eta(\mathbf{s}_{n+1})$. The sign depends on two terms: (a) whether the information content of $\mathbf{s}_{n+1}$ supports A (equivalent to whether $P(A|H; \mathbf{s}_{n+1}) - P(A|L; \mathbf{s}_{n+1}) > 0$), and (b) whether the evidence increases first-stage trust (whether $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n) > 0$).

Proposition 4 *(Over- and under-reaction to new information). Let ω_n^s equal the pre-screener's joint posterior after signal path $\mathbf{s}_n$, and define:*

$$\eta(\mathbf{s}_{n+1}) \equiv \left(P(A|H; \mathbf{s}_{n+1}) - P(A|L; \mathbf{s}_{n+1})\right)\left(\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n)\right).$$

Then:

1. Relative to Endowed Bayesian:

(a) $P^s[\theta = A|\mathbf{s}_{n+1}] > P[\theta = A|prior = \omega_n^s, \{s_{n+1}\}]$ *if* $\eta(\mathbf{s}_{n+1}) > 0$,
(b) $P^s[\theta = A|\mathbf{s}_{n+1}] < P[\theta = A|prior = \omega_n^s, \{s_{n+1}\}]$ *if* $\eta(\mathbf{s}_{n+1}) < 0$,
(c) $P^s[\theta = A|\mathbf{s}_{n+1}] = P[\theta = A|prior = \omega_n^s, \{s_{n+1}\}]$ *if* $\eta(\mathbf{s}_{n+1}) = 0$.

Furthermore, $\operatorname{sgn}(\eta(\mathbf{s}_{n+1}))$ *is fully determined in terms of objective parameters by the information content of* $\mathbf{s}_{n+1}$ *and the following condition:* $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n) \geq 0$ *if and only if* $P(c = H|\mathbf{s}_{n+1}) \geq \omega_0^H$*, with equality if and only if* $P(c = H|\mathbf{s}_{n+1}) = \omega_0^H$.

2. $P^s(c,\theta|\mathbf{s}_{n+1}) = P^s(c,\theta|prior = \omega_n^s, \{s_{n+1}\})$ *if and only if* $P^s(c|\mathbf{s}_n) = \omega_0^c$.

The key implication of Proposition 4 is that whether agents over- or under-react to a new signal s_{n+1} depends on not only whether the signal confirms or contradicts the existing evidence $\mathbf{s}_n$, but also on the sign of $\eta(\mathbf{s}_{n+1})$. Table 1 maps out the key possibilities. For brevity, we focus our discussion on the case in Panel (a) where the information content of the existing evidence $\mathbf{s}_n$ indicates A and $\eta(\mathbf{s}_{n+1}) \neq 0$. In this case, the sign of $\eta(\mathbf{s}_{n+1})$ is identical to the sign of $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n)$.

For new signals that confirm existing evidence ($s_{n+1} = a$), pre-screeners will over-react if first-stage trust is high (Part 1a) and under-react if first-stage trust is low (Part 1b). The intuition for the under-reaction is that, even though the signal is confirmatory and the pre-screener and endowed Bayesian begin from the same beliefs, $\kappa_H(\mathbf{s}_{n+1})$ may be too low relative to $\kappa_H(\mathbf{s}_n)$ (the effective prior over the informativeness of $\mathbf{s}_{n+1}$ for the endowed Bayesian). The low $\kappa_H(\mathbf{s}_{n+1})$ weighs down the pre-screener's perceived informativeness of the total evidence $\mathbf{s}_{n+1}$, which supports A.

For new signals that contradict existing evidence ($s_{n+1} = b$), pre-screeners will under-react when first-stage trust is high (Part 1a), and over-react when first-stage trust is low (Part 1b). This under-reaction is similar to models of confirmation bias (Rabin and Schrag, 1999), while the over-reaction is the opposite. We label this over-reaction the *undercutting effect*. In this case, contradictory information undercuts the first-stage belief $\kappa_H(\mathbf{s}_{n+1})$ and excessively undermines the history of evidence $\mathbf{s}_{n+1}$ (in particular, the evidence $\mathbf{s}_n$ which supports A) in the second step. In essence, contradictory signals can lead pre-screeners to over-react because they wonder, "Can I trust anything they said?"

The Proposition shows that whether $\kappa_H(\mathbf{s}_{n+1}) \geq \kappa_H(\mathbf{s}_n)$ is equivalent to whether $P(c = H \mid \mathbf{s}_{n+1}) \geq \omega_0^H$. Intuitively, the pre-screener's first-stage belief about high credibility increases when the evidence objectively increases the likelihood that the source is high credibility relative to the

Table 1
Over- and under-reaction to confirming and contradictory news. This table shows when pre-screeners' beliefs over- and underreact to a new signal s_{n+1} for cases where $\eta(\mathbf{s}_{n+1}) \neq 0$. Panel (a) considers the case where the information content of the existing evidence $\mathbf{s}_n$ indicates A while panel (b) considers B. The table omits two cases where the new signal s_{n+1} leads to $\eta(\mathbf{s}_{n+1}) = 0$; that is, when 1) the information content of $\mathbf{s}_{n+1}$ indicates neither state, or 2) $P(c = H \mid \mathbf{s}_{n+1}) = \omega_0^H$. In both of these cases, Part 1(c) of Proposition 4 applies and pre-screeners' and Bayesians' beliefs have the same reaction to s_{n+1}. Finally, note that, from Proposition 4, $\kappa_H(\mathbf{s}_{n+1}) \geq \kappa_H(\mathbf{s}_n)$ if and only if $P(c = H \mid \mathbf{s}_{n+1}) \geq \omega_0^H$, with equality if and only if $P(c = H \mid \mathbf{s}_{n+1}) = \omega_0^H$.

(a) $\mathbf{s}_n$ indicates A	$\kappa_H(\mathbf{s}_{n+1}) > \kappa_H(\mathbf{s}_n)$	$\kappa_H(\mathbf{s}_{n+1}) < \kappa_H(\mathbf{s}_n)$
	$\eta(\mathbf{s}_{n+1}) > 0$	$\eta(\mathbf{s}_{n+1}) < 0$
$s_{n+1} = a$ (confirm)	Overreact	Underreact
$s_{n+1} = b$ (contradict)	Underreact	Overreact
Proposition 4	1(a)	1(b)
(b) $\mathbf{s}_n$ indicates B	$\kappa_H(\mathbf{s}_{n+1}) > \kappa_H(\mathbf{s}_n)$	$\kappa_H(\mathbf{s}_{n+1}) < \kappa_H(\mathbf{s}_n)$
	$\eta(\mathbf{s}_{n+1}) < 0$	$\eta(\mathbf{s}_{n+1}) > 0$
$s_{n+1} = a$ (contradict)	Underreact	Overreact
$s_{n+1} = b$ (confirm)	Overreact	Underreact
Proposition 4	1(b)	1(a)

pre-screener's prior on credibility. This effect occurs when the information content of $\mathbf{s}_{n+1}$ sufficiently favors one state, where stronger information is required when the information contradicts the prior about the state.

The scale example in Section 2.4 combines several of the above intuitions. Suppose state A is that the pre-screener's weight is 300 pounds, state B is that his weight is 200 pounds, and that her prior strongly favors B. The agent initially under-reacts to readings of a but then over-reacts after more readings. According to Proposition 4, the initial under-reaction occurs because $\eta(\mathbf{s}_{n+1}) < 0$. The early signals of a support A: $P(A|H;\mathbf{s}_{n+1}) - P(A|L;\mathbf{s}_{n+1}) > 0$. However, they combine with the strong prior of B to generate low first-stage trust: $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n) < 0$. Thus, the agent initially under-reacts to each successive signal $s_{n+1} = a$ even though they confirm the existing evidence. Eventually, the pre-screener over-reacts to further signals of a because sufficient a's lead to high first-stage trust: $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n) > 0$ and thus $\eta(\mathbf{s}_{n+1}) > 0$.

Part 2 of Proposition 4 shows that the effect of a new signal s_{n+1} on a pre-screener's beliefs cannot be summarized simply by its effect on ω_n^s. This is because the pre-screener re-evaluates all the evidence $\mathbf{s}_{n+1}$ in light of the new first-stage belief $\kappa_H(\mathbf{s}_{n+1})$. In contrast, a Bayesian updates identically irrespective of whether she is endowed with a belief or observes a history of signals consistent with that belief: $P(\theta = A|\mathbf{s}_{n+1}) = P(\theta = A|prior = \omega_n^u, \{s_{n+1}\})$, where ω_n^u equals the Bayesian posterior generated by $\mathbf{s}_n$.

4. Multiple sources

Pre-screener's beliefs are not only influenced by the order of signals, but also by the order of sources. The reason is that a source's signals exert an outsized influence on how the pre-screener interprets signals from subsequent sources.

Suppose the pre-screener receives signals from two sources, with credibilities $c_j \in \{H, L\}$. Source 1 sends signals first, and source 2 sends signals after source 1 is finished. As before, each source $j \in \{1, 2\}$ sends one signal per period t, denoted s_{tj}. Each signal sent by source j is independent of all other signals sent by any source, and nature draws each source's credibility independently of each other. Therefore, $P(s_{tj} \mid c_j, c_k, \theta) = P(s_{tj} \mid c_j, \theta)$ for all t and $j \neq k$. Let $\mathbf{s}_{n_j}$ denote the sequence of n_j signals sent by source j, $\mathbf{s}_{n_1,0}$ denote the sequence when only source 1 has sent its signals, and $\mathbf{s}_{n_1,n_2} = \{\mathbf{s}_{n_1}, \mathbf{s}_{n_2}\}$ denote the entire sequence of signals sent from both sources.

The pre-screener now faces three sources of uncertainty—the two credibility types and the state of the world. We assume agents start with independent priors over all three unknowns $(\omega_0^A, \omega_0^{H_1}, \omega_0^{H_2}) \in (0, 1)^3$ and think of the two sources symmetrically ex-ante with $\omega_0^{H_1} = \omega_0^{H_2}$.

Pre-screening extends naturally from one to multiple sources. The pre-screener first forms a first-stage joint belief about (c_1, c_2) using Bayes' Rule, denoted $\kappa_{c_1c_2}(\cdot)$, and then forms posterior $P^s(c_1, c_2, \theta|\mathbf{s}_{n_1,n_2})$ after substituting $\kappa_{c_1c_2}(\cdot)$ for her prior $\omega_0^{c_1}\omega_0^{c_2}$. For example, upon observing the last signal from source 1, the pre-screener's first-stage updated belief is:

$$\kappa_{c_1c_2}(\mathbf{s}_{n_1,0}) = \frac{\kappa_{c_1c_2}(\mathbf{s}_{n_1-1,0})\left(\sum_\theta \left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^\theta\right)}{\sum_{c_1}\sum_{c_2}\kappa_{c_1c_2}(\mathbf{s}_{n_1-1,0})\left(\sum_\theta \left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^\theta\right)}. \tag{10}$$

The pre-screener's posterior belief is then:

$$P^s(c_1, c_2, \theta|\mathbf{s}_{n_1,0}) = \frac{\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,0})\omega_0^\theta}{\sum_{c_2}\sum_{c_1}\sum_\theta \left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,0})\omega_0^\theta}. \tag{11}$$

Expanding the recursion, we can write her posterior belief as:

$$P^s(c_1, c_2, \theta|\mathbf{s}_{n_1,0}) = \frac{\beta_{c_1}(\mathbf{s}_{n_1})\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{c_1}\omega_0^{c_2}\omega_0^{\theta}}{\sum_{c_2}\sum_{c_1}\beta_{c_1}(\mathbf{s}_{n_1})\sum_{\theta}\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{c_1}\omega_0^{c_2}\omega_0^{\theta}} \tag{12}$$

for $\beta_{c_1}(\mathbf{s}_{n_1}) \equiv \prod_{m=1}^{n_1}\left(\sum_{\theta}\left(\prod_{t=1}^{m} P(s_{t1}|c_1,\theta)\right)\omega_0^{\theta}\right)$ and $\beta_{c_1}(\emptyset) \equiv 1$.

After observing the last signal from source 2, the pre-screener's first-stage updated belief equals:

$$\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2}) = \frac{\left(\sum_{\theta}\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{\theta}\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2-1})}{\sum_{c_2}\sum_{c_1}\left(\sum_{\theta}\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{\theta}\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2-1})}, \tag{13}$$

and her final posterior belief equals:

$$P^s(c_1, c_2, \theta|\mathbf{s}_{n_1,n_2}) = \frac{\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{\theta}}{\sum_{c_2}\sum_{c_1}\sum_{\theta}\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{\theta}}. \tag{14}$$

Expanding the recursion, the pre-screener's final posterior belief equals:

$$P^s(c_1, c_2, \theta|\mathbf{s}_{n_1,n_2}) = \frac{\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{c_1}\omega_0^{c_2}\omega_0^{\theta}\beta_{c_1}(\mathbf{s}_{n_1})\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})}{\sum_{c_2}\sum_{c_1}\sum_{\theta}\left(\prod_{t=n_1+1}^{n_1+n_2} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{c_1}\omega_0^{c_2}\omega_0^{\theta}\beta_{c_1}(\mathbf{s}_{n_1})\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})}, \tag{15}$$

where $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2}) \equiv \prod_{m=n_1+1}^{n_1+n_2}\left(\sum_{\theta}\left(\prod_{t=n_1+1}^{m} P(s_{t2}|c_2,\theta)\right)\left(\prod_{t=1}^{n_1} P(s_{t1}|c_1,\theta)\right)\omega_0^{\theta}\right)$.

Proposition 5 characterizes how perceptions of source 1's credibility exert an outsized influence on how the pre-screener interprets source 2's signals. The Proposition considers how final beliefs change when we re-arrange source 1's signals and thus change $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$. Recall that $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ captures how pre-screening distorts the agent's beliefs about source 1's credibility, with higher values associated with beliefs about higher credibility. The Proposition provides necessary and sufficient conditions under which increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ makes the agent believe source 2 is more credible and that state A is more likely. As a benchmark, a Bayesian's beliefs are always invariant to signal order.

Proposition 5 *(Effects of credibility of source 1 on beliefs). Let* $\mathbf{s}_{n_1,n_2}$ *be given. Re-arranging source 1's signals to increase* $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$*:*

1. *Increases* $P^s(H_1|\mathbf{s}_{n_1,n_2})$.
2. *Increases* $P^s(H_2|\mathbf{s}_{n_1,n_2})$ *if and only if* $\frac{P^s(H_2|H_1;\mathbf{s}_{n_1,n_2})}{P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})} > 1$, *and decreases if and only if strictly less than* ($<$).
3. *Increases* $P^s(A|\mathbf{s}_{n_1,n_2})$ *if and only if* $\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A|L_1;\mathbf{s}_{n_1,n_2})} > 1$, *and decreases if and only if strictly less than* ($<$).

In the proof, we show that $\frac{P^s(H_2|H_1;\mathbf{s}_{n_1,n_2})}{P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})}$ *and* $\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A|L_1;\mathbf{s}_{n_1,n_2})}$ *are fully determined in terms of objective parameters by the information content of* $\mathbf{s}_{n_1,n_2}$.

Part 1 says that re-arranging source 1's signals to increase $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ increases the pre-screener's perception of source 1's credibility, even after source 2 has spoken. Part 2 says that doing so also affects the pre-screener's final belief about source 2's credibility. The intuition is that increasing the pre-screener's perception of source 1's credibility increases (decreases) her perception of source 2's credibility if source 2's early signals are sufficiently consistent (inconsistent) with source 1's information content. Such consistency (inconsistency) leads the pre-screener to think that source 2 is more (less) likely to be high credibility when source 1 is also the high credibility type than when source 1 is the low credibility type. In this way, a pre-screener's "first impression" of source 1's credibility casts a shadow over how the pre-screener ultimately views source 2.

Table 2 illustrates how Part 2 works when we re-arrange signals to increase source 1's credibility. In the examples, source 1's information content indicates state A, and source 2's information content indicates state B. Going from Row 1 to Row 2, rearranging source 1's a signals so that they appear earlier increases $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ and her credibility. This change decreases source 2's credibility since source 2's early signals are inconsistent with source 1's information. The same re-arrangement of source 1's signals in Rows 3 and 4 increases source 2's credibility since source 2's early signals are consistent with source 1's information.

Part 3 summarizes how re-arranging source 1's signals affects beliefs about the state. Increasing the credibility of source 1 by increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ moves beliefs about state through two effects. First, there is a direct effect of increasing source 1's credibility that moves beliefs toward the state suggested by the source 1's information content. Second, there is an indirect effect: increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ changes source 2's credibility per Part 2. If increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ decreases source 2's credibility, then the indirect effect moves the agents' beliefs away from the state suggested by source 2's information content. For example, if source 2's information content suggests B, then the indirect effect moves beliefs away from B. If $\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})$ increases source 2's credibility, the indirect effect moves beliefs toward the state suggested by source 2's information content.

Intriguingly, it is possible for the indirect effect to overturn the direct effect when they conflict, so that increasing source 1's credibility moves beliefs away from the state suggested by source 1. Corollary 2 first shows that a necessary condition is that source 2's information must be objectively stronger than source 1's information. It next shows that the direct effect always dominates the indirect effect when source 2's information is weaker than source 1's information. In the proof of Proposition 5, we provide further examples of each case.

Table 2
Pre-screener's and Bayesian's beliefs with multiple sources. Parameter values equal $(q_H, q_L, \omega_0^A, \omega_0^{H_1}, \omega_0^{H_2}) = (0.7, 0.55, 0.5, 0.5, 0.5)$. In all rows, the Bayesian's posterior beliefs are $P(H_1|\mathbf{s}_{n_1,n_2}) = 0.441$, $P(H_2|\mathbf{s}_{n_1,n_2}) = 0.400$, and $P(A|\mathbf{s}_{n_1,n_2}) = 0.506$.

Row	$\mathbf{s}_{n_1}$	$\mathbf{s}_{n_2}$	$\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$	$\frac{P^s(H_2|H_1;\mathbf{s}_{n_1,n_2})}{P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})}$	$\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A|L_1;\mathbf{s}_{n_1,n_2})}$	$P^s(H_1|\mathbf{s}_{n_1,n_2})$	$P^s(H_2|\mathbf{s}_{n_1,n_2})$	$P^s(A|\mathbf{s}_{n_1,n_2})$
1	$\{a,b,a\}$	$\{b,b,b,a,a\}$	0.720	0.522	1.423	0.131	0.368	0.463
2	$\{a,a,b\}$	$\{b,b,b,a,a\}$	0.974	0.522	1.423	0.169	0.361	0.470
3	$\{a,b,a\}$	$\{a,a,b,b,b\}$	0.720	1.055	1.337	0.224	0.279	0.492
4	$\{a,a,b\}$	$\{a,a,b,b,b\}$	0.974	1.055	1.337	0.283	0.280	0.500

Corollary 2. *Without loss of generality, suppose $n_{a1} > n_{b1}$. A necessary condition for increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ to decrease $P^s(A|\mathbf{s}_{n_1,n_2})$ is $|n_{a2} - n_{b2}| > n_{a1} - n_{b1}$. A sufficient condition for increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ to increase $P^s(A|\mathbf{s}_{n_1,n_2})$ is $|n_{a2} - n_{b2}| \leq n_{a1} - n_{b1}$.*

5. Extensions

In the Internet Appendix, we consider three extensions. First, we consider a setting where agents are uncertain about whether the source may "slant" signals toward a given state, in that the source probabilistically flips a signal towards a certain state before reporting it to the agent. For example, an A-slanted source may report a when her true signal was b with some fixed probability. Sources are non-strategic as before. We show that the presence of slant can lead the pre-screener's beliefs to be sufficiently wrong that she becomes certain of the state even when the evidence should not change beliefs about the state from priors. The possibility of slant thus creates further scope for error by pre-screeners.

Second, we extend pre-screening to the case where a source may send multiple signals in one period. We show that a source can countervail other sources more effectively by delivering signals in a simultaneous "blast" rather than sequentially. This effect further illustrates how the timing of signals can be important for persuasion.

Third, we show that key results hold when we allow for fading memory. Pre-screening assumes that agents have memory over previous signals in that they evaluate the likelihood of previous signals in light of first-stage updated beliefs $\kappa_c(\mathbf{s}_n)$ every period. We allow for fading memory in the manner of Mangel (1990) and Nagel and Xu (2019) by assuming that signals farther in the past receive less weight in the likelihood function. The pre-screening process is otherwise identical. We show that Propositions 1, 2, and 4 hold when comparing the pre-screener with fading memory to the Bayesian with fading memory.

6. Application: speculative trade, bubbles, and crashes

The goal of this section is to illustrate the implications of pre-screening for prices and trade. We adopt a simple trading game comparable to the model of Harris and Raviv (1993), which features Bayesian agents who exogenously "agree to disagree" about source credibility. We show that a similar game featuring pre-screeners can generate rapid changes in prices and positions that are similar to speculative trade, bubbles, and crashes.

We first show that trading volume between pre-screeners weakly exceeds trading volume between Bayesians due to the speculative motive defined by Harrison and Kreps (1978). We then show that pre-screening can lead to bubbles and crashes following the definition outlined by Barberis (2018, p. 88). Prices can rise sharply before crashing endogenously, with some traders "riding the bubble" and generating abnormal volume beyond what a Bayesian benchmark produces.

6.1. Pricing and speculative trade

6.1.1. Trading environment

Harris and Raviv (1993) features two groups of risk-neutral Bayesian agents who trade because they have different (exogenously fixed) beliefs over signal credibility. We adopt an analogous trading environment but with pre-screeners. Two groups of risk-neutral traders, X and Y, trade shares of a risky asset at dates $t = 1, 2, \ldots, T$. The asset makes a single random payment

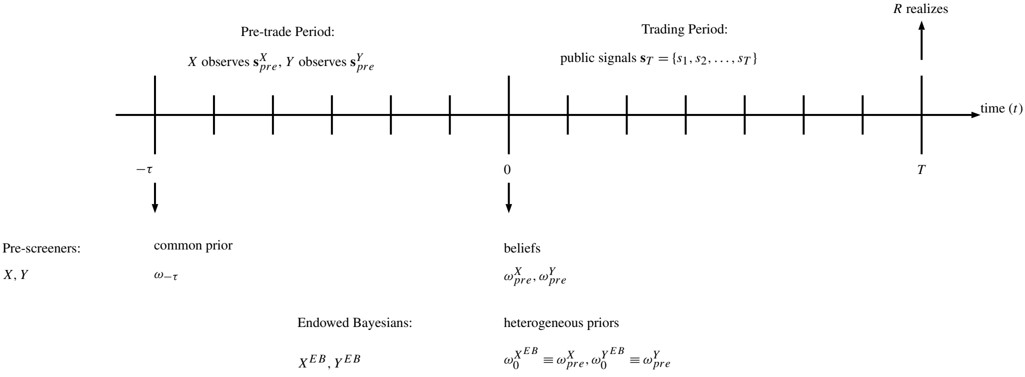


Fig. 2. Timeline of trading game.

of R immediately after the end of date T. If the state θ is A, then the payoff is $R = 1$. If the state θ is B, then the payoff is $R = 0$. There are a fixed number of shares available normalized to 1 with no short sales. There is a risk-free asset whose return is zero.

At each date $t = 1, 2, \ldots T$, both groups X and Y first observe a common public signal $s_t \in \{a, b\}$, after which they update their beliefs and can trade at price p_t determined in equilibrium. Each signal s_t is independent and identically distributed conditional on the true payoff, and comes from a single unslanted source with unknown accuracy q. Let the public signal path as of period t be denoted $\mathbf{s}_t = \{s_1, \ldots, s_t\}$ where $n_{a,t}$ and $n_{b,t}$ are the number of a's and b's in $\mathbf{s}_t$, respectively.

How do prices and trade in the game between pre-screeners compare with prices and trade when agents are Bayesians? If all traders had common priors just before trade opens at $t = 1$, all agents would have identical beliefs during the trading period, regardless of whether they are Bayesians or pre-screeners, because they observe the same signals in the same order.

To make things interesting, we assume that pre-screeners observe signals during τ pre-trade "burn-in" periods starting in period $(-\tau + 1) < 0$ and ending at the end of period 0. X and Y share common priors $\omega_{-\tau}$ before they see any signals. As before, we assume that the beliefs about the state and credibility are independent in the prior: $\omega_{-\tau} = \omega^\theta_{-\tau}\omega^c_{-\tau}$. During the pre-trade periods, X and Y separately observe one signal per period cumulating in signal path $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$, which generates beliefs ω^X_{pre} and ω^Y_{pre} at the end of period 0. Fig. 2 illustrates.

We assume that the two pre-trade signal paths $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$ have the same information content but that X and Y observe this information in different orders. For example, X might observe $\mathbf{s}^X_{pre} = \{a, a, b, b\}$ while Y observes $\mathbf{s}^Y_{pre} = \{a, b, a, b\}$. This assumption ensures that any trade between pre-screeners is due to differently-experienced first impressions of credibility early in their life prior to time 1, not differences in objective information or priors. Specifically, we assume that $n^j_{a,pre} = n^j_{b,pre}$ for $j \in \{X, Y\}$, where $n^j_{a,pre}$ is the number of a signals observed in $\mathbf{s}^j_{pre}$. By Proposition 1, this implies $P^s(H|\mathbf{s}^X_{pre}) \neq P^s(H|\mathbf{s}^Y_{pre})$, but $P^s(A|\mathbf{s}^X_{pre}) = P^s(A|\mathbf{s}^Y_{pre}) = \omega^\theta_{-\tau}$.

We adopt the market structure of Harris and Raviv (1993) where Y has sufficient market power each period to make a take-it-or-leave-it (TIOLI) offer to X. We also assume that agents "agree to disagree": X does not revise her beliefs irrespective of any offers from Y or even knowing Y's beliefs, and vice versa. We elaborate on higher-order beliefs below.

To make apples-to-apples comparisons, we compare outcomes in two games. First, we consider a benchmark trading game between X^{EB} and Y^{EB}, two risk-neutral "Endowed Bayesians" who start time 1 with heterogeneous priors equal to ω^X_{pre} and ω^Y_{pre}, the beliefs of the two pre-screeners at the end of the pre-trade period. We then move to the trading game where X and Y are pre-screeners.

6.1.2. Bayesian benchmark

Endowed Bayesians $\{X^{EB}, Y^{EB}\}$ begin $t = 1$ with priors $\omega_0^{X^{EB}} = \omega^X_{pre}$ and $\omega_0^{Y^{EB}} = \omega^Y_{pre}$, respectively. By construction of ω^X_{pre} and ω^Y_{pre}, such priors are heterogeneous in that agents ex-ante agree about the cash flow but disagree about source credibility: $P_0^{X^{EB}}(H) \neq P_0^{Y^{EB}}(H)$ but $P_0^{X^{EB}}(A) = P_0^{Y^{EB}}(A) = \omega^A_{-\tau}$. Although agents learn about credibility, equilibrium outcomes are similar to Harris and Raviv (1993). We sketch the analysis below.

Group X^{EB}'s reservation price each period, and thus the equilibrium price p_t^{EB}, equals X^{EB}'s expectation of the final cash flow, $E_t^{X^{EB}}(R)$. (To de-clutter notation, we drop the EB superscript in the rest of this paragraph and the next.) The reason is that X believes the expected value of future trade with Y is zero. Y makes TIOLI offers, so X anticipates that Y will offer Y's perception of X's reservation price in any subsequent period $s > t$. X knows that Y knows that X is a Bayesian, and X also knows that Y knows X's current beliefs but "agrees to disagree." Working backwards from period T, X's best guess about what Y will offer in future periods $s > t$ is $E_t^X(p_s) = E_t^X E_s^Y E_s^X(R) = E_t^X E_s^X(R) = E_t^X(R)$. This is the same as X's best guess about the final cash flow, so X expects no value from future trade with Y.

Given the equilibrium price, trade occurs whenever beliefs about R cross the threshold where $E_t^Y(R) = E_t^X(R)$. Given common priors about the state, this occurs whenever the number of a and b signals crosses the threshold $n_{a,t} = n_{b,t}$.

6.1.3. Pre-screening

Pre-screeners X and Y also begin $t = 1$ with beliefs ω^X_{pre} and ω^Y_{pre} from having observed $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$, respectively. As in the EB analysis, the equilibrium price will depend on X's belief about what Y will offer X in the future and thus on the fact that agents agree to disagree about the expected value of R. Apart from the EB analysis, whether a pre-screener realizes her own pre-screening and whether she thinks that others also realize any pre-screening is crucial. Our analysis has hitherto not required specification of these higher-order beliefs.

We assume that a pre-screener thinks she forms rational and dynamically consistent beliefs using Bayes' Rule and is unaware of her pre-screening and others' perceptions about her pre-screening. This assumption naturally extends from the premise that agents overlook the first-stage substitution of updated beliefs for priors. It also naturally extends from hindsight bias, as such bias prevents the pre-screener from realizing that her past beliefs are dynamically inconsistent and that any offers in the market may be inconsistent with what she anticipated in the past. Instead, a pre-screener thinks she is a Bayesian and that others think she is a Bayesian. Her perceptions about herself as a Bayesian extend to all higher order beliefs. Thus, for example, X thinks X is a Bayesian, and X thinks Y thinks X is a Bayesian, and so forth.

We also assume that agents, although unaware of their own pre-screening, recognize that other agents pre-screen and that other agents are oblivious to their own pre-screening. This assumption is consistent with experimental evidence in psychology and economics about the "bias blind spot," that one can recognize cognitive or motivational biases more in others than in oneself

(Pronin et al., 2002; Ehrlinger et al., 2005; Fedyk, 2021; West et al., 2012). Thus, for example, X thinks Y is a pre-screener and X thinks that Y thinks that Y is a Bayesian.

The combined effect of these higher-order beliefs with the TIOLI market structure is that X's reservation price each period, and thus the equilibrium price p_t, equals X's pre-screened belief $E_t^X(R)$.[10] As in the EB case, X anticipates that Y will offer what Y believes is X's reservation price in future periods $s > t$ because Y makes TIOLI offers. Since X thinks Y thinks X is Bayesian, X thinks that Y will calculate X's belief in the future by combining X's current belief with the next period's signal using Bayes' Rule.[11] X's expected value of this future belief, calculated using the likelihood of future signals given her current beliefs and working backwards from period T, equals $E_t^X(R)$. As a result, X believes the future value of trade with Y is zero. Y understands X's calculation,[12] and thus offers $p_t = E_t^X(R)$.[13]

Given these assumptions, Proposition 6 shows that pre-screeners $\{X, Y\}$ trade whenever EB's $\{X^{EB}, Y^{EB}\}$ trade, but also trade when EB's do not. Part 1 shows that pre-screeners trade whenever their beliefs about fundamental value cross ($n_{a,t} = n_{b,t}$), which is also when EB's trade.

Part 2 shows that pre-screeners also engage in "speculative behavior" defined by Harrison and Kreps (1978): "an investor may buy the stock now so as to sell it later for more than he thinks it is actually worth, thereby reaping capital gains." Suppose Y believes that the asset's fundamental value is lower than X believes because she thinks the source is less credible than X does. Y may nevertheless buy and hold the asset speculatively: $E_t^Y(p_{t+1}) > E_t^X(R) = p_t$ even though $E_t^Y(R) < E_t^X(R) = p_t$, so long as disagreement between X and Y is not too large and there is enough existing good cash flow news. The reason is that Y correctly believes X will under-react to (disconfirming) bad news and over-react to further good cash flow news (using Proposition 4), and Y is not too skeptical about prospects of such further news, leading Y to expect an upward drift in the price. However, X believes they are Bayesians in the future and fail to anticipate this upward drift. Analogously, X holds the asset speculatively in the symmetric case, when Y believes the source is more credible than X does after the pre-trade signals and bad cash flow news arrives after trade opens.

Overall, Proposition 6 implies that the extent of excess speculative trade due to pre-screening depends on the extent of disagreement about credibility and how disagreement originates. In the game with pre-screeners, initial disagreement about credibility originates from differing first impressions of credibility from signals prior to trade, $\omega_{pre}^X \neq \omega_{pre}^Y$. Even though we endow identical disagreement in the game with Bayesians, $\omega_0^{j^{EB}} = \omega_{pre}^j$ for $j \in \{X, Y\}$, and even though

[10] As Harris and Raviv (1993) note, if one assumes the equilibrium price is competitively set each period, then the price is determined by the beliefs of potentially different groups through time. Scheinkman and Xiong (2003) analyze a market with this added significant complication. Our stated goal is more modest: we seek to analyze whether the belief dynamics of pre-screening can generate interesting implications in the simplest structure with results that are easily comparable to Harris and Raviv (1993).

[11] X thus anticipates wrong offers in the future from Y, and Y's realized offer in periods $s > t$ will differ from X's anticipated possibilities. Given X's hindsight bias, this does not faze X ex-post. In future periods $s > t$, X thinks he arrived at $E_s^X(R)$ rationally and that the offer is consistent with what he rationally anticipated in the past. Similarly, Y knows X's current belief because Y knows X is a pre-screener, even though X thinks Y knows X's current belief was obtained through Bayesian updating. We thank an anonymous referee for clarifying our thinking on this point.

[12] Specifically, Y knows X thinks Y thinks X is a Bayesian.

[13] One can alternatively consider what happens if pre-screeners were "sophisticated" in that they recognize their own pre-screening yet somehow continue to pre-screen each period. In this case, agents would recognize their own dynamic inconsistency, and X may think there is value to future trade with Y. Our assumptions abstract from this significant complication and focus on a more modest goal.

all traders see a common signal path when trade is open, outcomes differ across the two games because of how new signals interact with pre-screeners' first impressions of credibility.

Proposition 6 *(Speculative trade). Let pre-screeners X and Y observe pre-period signal paths $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$, where $n^j_{a,pre} = n^j_{b,pre} \geq 2$ for signal paths $j \in \{X, Y\}$, and then public signal path $\mathbf{s}_t$. Let Bayesians X^{EB} and Y^{EB} be endowed with priors that equal the pre-screeners' posterior beliefs after the pre-period, $\omega_0^{X^{EB}} = \omega^X_{pre}$ and $\omega_0^{Y^{EB}} = \omega^Y_{pre}$, and observe $\mathbf{s}_t$. Let $\omega^A_{-\tau} = 1/2$.*[14]

The price in the pre-screeners' game is $p^s_t = E^X_t(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})$, and the price in the EB game is $p^{EB}_t = E^{X^{EB}}_t(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})$. Groups X and Y trade weakly more than groups X^{EB} and Y^{EB}:

1. *If $P^s(H|\mathbf{s}^X_{pre}) \neq P^s(H|\mathbf{s}^Y_{pre})$, then X^{EB} and Y^{EB} trade* only *when beliefs cross threshold $n_{a,t} = n_{b,t}$. Pre-screeners X and Y also trade when beliefs cross threshold $n_{a,t} = n_{b,t}$.*
2. *(Speculative trade) Suppose $P^s(H|\mathbf{s}^X_{pre}) > P^s(H|\mathbf{s}^Y_{pre})$. There exists at least one signal path $\mathbf{s}^Y_{pre}$ such that Y holds the asset (buys it from X) if the following conditions are satisfied:*
   - *(a) State A is objectively more likely: $n_{a,t} > n_{b,t}$.*
   - *(b) Group X (weakly) under-reacts to disconfirming news: $P(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t, s_{t+1} = b\}) \geq \omega^H_{-\tau}$.*

X holds the asset in the symmetric case, where all of the above inequalities are reversed.

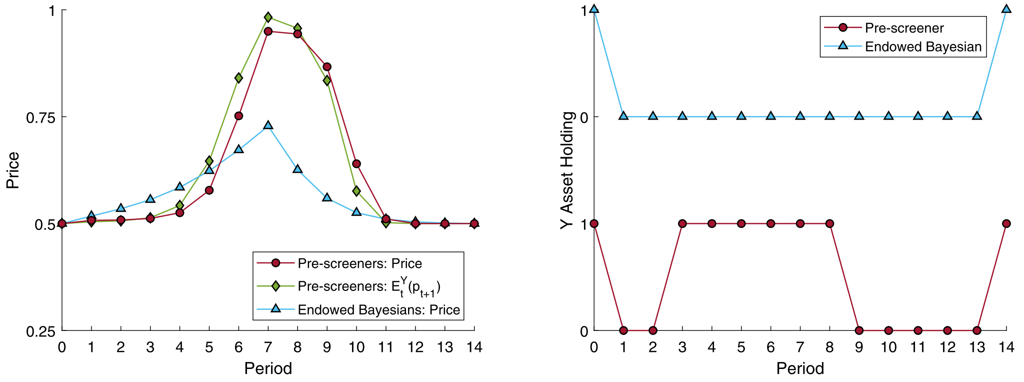
6.2. Bubbles and crashes

Pre-screening can generate rapid changes in prices and positions in line with several features of bubbles, crashes, and speculation described by Barberis (2018, p. 88), beyond what the EB framework can explain. Fig. 3 provides a numerical example, which we discuss in narrative form to build intuition. We consider the case where, at the start of trade, 1) X trusts the source more than Y does and 2) X and Y agree about the state, due to differences in the order of pre-trade signals. After trade opens, a string of good cash flow news arrives, followed by bad cash flow news.

The game begins with Y holding the asset in period 0. In period 1, good cash flow news arrives and X thinks $R = 1$ is more likely than Y does, leading Y to sell the asset to X, in both the pre-screening and EB frameworks. Good cash flow news arrives through period 7, and through this point, prices rise sharply and trade occurs in the pre-screening game, both of which are abnormal relative to the EB game. The trade in period 3 reflects group Y increasing asset exposure: Y speculatively "rides the bubble" and buys the asset from X even though the price is higher than Y's belief about fundamental value (Proposition 6 Part 2). As more good cash flow news arrives, prices rise substantially beyond the price in the EB game. The reason is that agents develop too much trust in the source and become too optimistic about cash flows.

Bad cash flow news begins to arrive in period 8. Prices in the pre-screening game initially remain high. This is because X thinks the history of good cash flow news provided by the source is credible and under-reacts to the bad cash flow news, behavior that is akin to confirmation

[14] For clarity of exposition, the Proposition supposes that $\omega^A_{-\tau} = 1/2$. As the proof details, this assumption is not required for Part 2 and can be partially relaxed in Part 1. Specifically, the proof provides sufficient conditions for regions of $(\omega^A_{-\tau}, q_L, q_H)$ where Part 1 applies.



(a) Prices **(b)** Asset holdings

Fig. 3. **Trading game.** This figure plots outcomes from a trading game between two groups of traders, X and Y, described in Section 6, when they are either both pre-screeners or both endowed Bayesians. Prices are in Panel (a) and the asset holdings of group Y are in Panel (b). Realized signals in periods 1-7 are good cash flow news ('a'), while periods 8-14 have bad cash flow news ('b'). Trader beliefs at period 0 are equal to the beliefs that pre-screeners would have after observing $\{a, a, b, b\}$ for X and $\{a, b, a, b\}$ for Y, starting from common priors. Parameters are $(q_H, q_L, \omega^A_{-\tau}, \omega^H_{-\tau}, T, \tau) = (0.8, 0.5, 0.5, 0.3, 14, 4)$.

bias (Proposition 4 Part 1a). As more bad cash flow news comes in, X begins to doubt whether they believe anything the source reported before, due to the contradiction with the previously-reported good cash flow news (the "undercutting effect" of Proposition 4 Part 1b). Anticipating this possibility, Y sells the asset to X in period 9. Bad news continues to arrive, and prices steeply decline as X's belief about source credibility collapses. In period 14, X and Y share the same beliefs about the state, and Y buys the asset back from X.

In sum, the paths depicted in Fig. 3 in the pre-screening game reasonably constitute a bubble and crash. Prices rise steeply and speculative trade occurs, both in excess of what happens in the EB game. Prices then crash endogenously due to revisions in beliefs in response to bad news. This endogenous crash distinguishes our model from those that assume a crash due to the exogenous realization of cash flows.[15] Thus, pre-screening generates rapid changes in prices and positions that are hard to generate in a Bayesian framework.

Proposition 7 provides the formal result beyond the specific signals and parameters considered in Fig. 3. It shows when and how pre-screening game can generate bubbles, crashes, and volume. Part 1 shows that over-valuation (under-valuation) occurs when consistent (inconsistent) good news about cash flows leads the pre-screener to trust (distrust) the source so much that she over-reacts (under-reacts) to additional good news. Part 2 shows that this can accelerate into bubbles and crashes: when overvaluation occurs, rises and subsequent falls in prices are steeper when traders are pre-screeners rather than Bayesian. As in Fig. 3, prices initially under-react to bad cash flow news before beliefs over-react and prices crash. Part 3 shows there exists at least one pre-path $\mathbf{s}^Y_{pre}$ such that speculative trade occurs during the bubble. Agents "ride the bubble"

[15] Hong and Stein (2003) model an endogenous crash caused by the revelation of hidden information from pessimistic investors. In our model, there is no hidden information since all investors see the same information. The Internet Appendix shows that confirmation bias does not produce such sudden price declines, because agents always under-react to contradictory information.

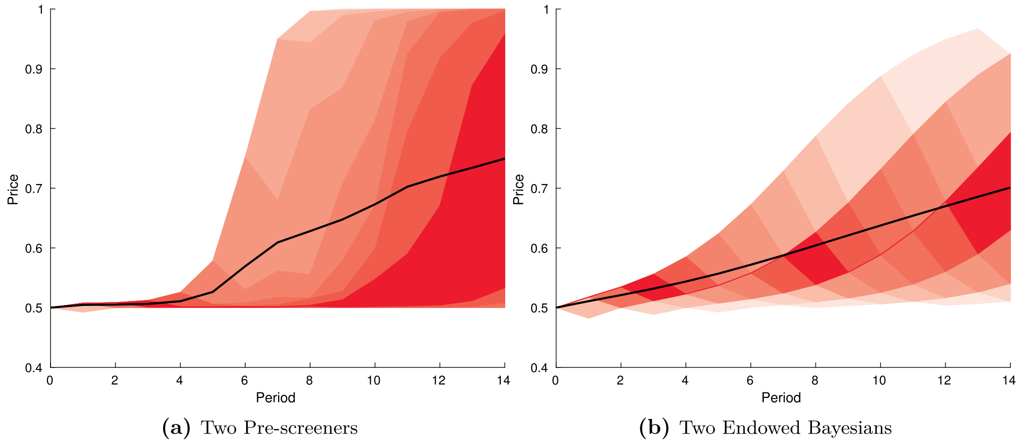


Fig. 4. **Trading game price distributions.** This figure plots a fan chart of the distribution of prices in 10,000 simulated trading games with identical parameters as Fig. 3. Darker regions indicate a greater frequency of prices. The solid line indicates the average price each period. The simulations assume $c = H$ and $R = 1$.

because they anticipate that others will over-react to future good news. These features provide a theory of bubbles and crashes that match key features described by Barberis (2018, p. 88).

Proposition 7 *(Bubbles and crashes). Let pre-screeners X and Y observe pre-period signal paths* $\mathbf{s}^X_{pre}$ *and* $\mathbf{s}^Y_{pre}$*, where* $n^j_{a,pre} = n^j_{b,pre} \geq 2$ *for signal paths* $j \in \{X, Y\}$*, and then public signal path* $\mathbf{s}_t$*. Let Bayesians* X^{EB} *and* Y^{EB} *be endowed with priors that equal the pre-screeners' posterior beliefs after the pre-period,* $\omega_0^{X^{EB}} = \omega^X_{pre}$ *and* $\omega_0^{Y^{EB}} = \omega^Y_{pre}$*, and observe* $\mathbf{s}_t$*. Let* $\omega^A_{-\tau} \in (0, 1)$*.*

1. *(Over- and under-valuation) Wlog, suppose* $n_{a,t} > n_{b,t}$*. Under-valuation (*$p^s_t < p^{EB}_t$*) occurs if and only if* $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) < P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$*. Over-valuation (*$p^s_t > p^{EB}_t$*) occurs if and only if* $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$*.*
2. *(Bubbles and crashes) Consider a path* $\mathbf{s}_T$ *such that* $n_{a,t} > n_{b,t}$ *for* $t \in (0, T)$ *with* $n_{a,T} = n_{b,T}$*. If there exists* $\hat{t} \in (0, T)$ *such that* $p^s_{\hat{t}} > p^{EB}_{\hat{t}} > \omega^A_{-\tau}$ *where* $p^k_{\hat{t}} \equiv \max p^k_t$ *for* $k \in \{s, EB\}$*, the average price change of* p^s_t *must be strictly greater than the average price change of* p^{EB}_t *for* $t \in [0, \hat{t}]$ *(bubbles) and* $t \in [\hat{t}, T]$ *(crashes). Moreover, the pre-screeners exhibit initial under-reaction relative to the endowed Bayesians after the peak:* $|p^s_{\hat{t}+1} - p^s_{\hat{t}}| < |p^{EB}_{\hat{t}+1} - p^{EB}_{\hat{t}}|$*.*
3. *("Riding the bubble") Given any signal path* $\mathbf{s}_T$ *such that* $p^s_{\hat{t}} > p^{EB}_{\hat{t}} > \omega^A_{-\tau}$*, there exists at least one signal path* $\mathbf{s}^Y_{pre}$ *such that speculative trade between pre-screeners occurs (e.g., Y holds the asset at* $t = \hat{t}$*, at least).*

Figs. 4 and 5 give a broader sense of the possible outcomes suggested by Propositions 6 and 7 by plotting the distribution of prices and trade from 10,000 simulated trading games. We assume the same parameters that underlie the example in Fig. 3, that $c = H$, and that the true asset payoff is $R = 1$. Fig. 4 illustrates that the average price path in the game with two pre-screeners is higher than in the game with two EB agents, with a greater propensity for rapid run-ups in price. Fig. 5

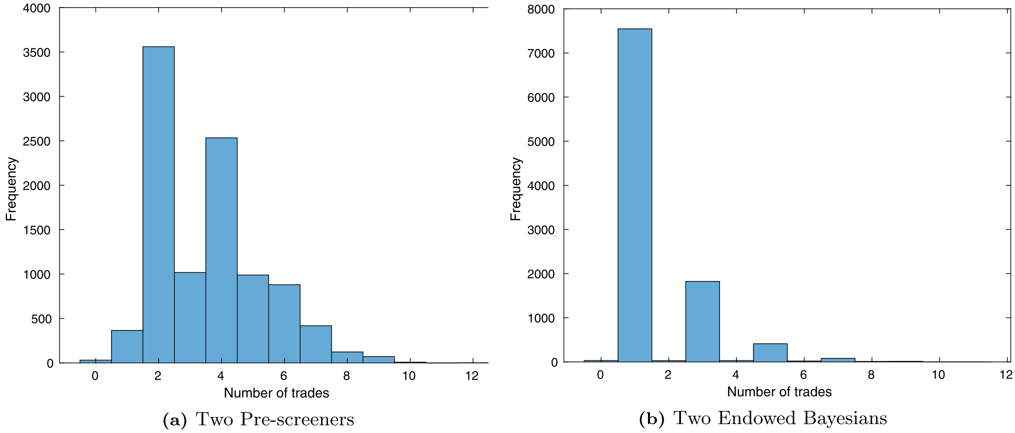


Fig. 5. **Histogram of number of trades.** This figure plots a histogram of the total number of trades in 10,000 simulated trading games with identical parameters as Fig. 3. The simulations assume $c = H$ and $R = 1$.

shows that there is more trade: the average (median) number of trades in the game with two pre-screeners is 3.6 (4) compared to 1.6 (1) in the game with two EB agents.

7. Discussion and literature

7.1. Two alternative models

We consider two models that separately adopt each step of pre-screening to highlight how they differ in their mechanisms and predictions. Recall that a pre-screener first forms an updated belief about credibility. Second, she forms final posterior beliefs by following Bayes' Rule using updated beliefs instead of priors.[16]

In "Alternative Model 1" (AM1), an agent follows the same first step as pre-screening but a different second step that assumes agents separately update on credibility *and* the state. Specifically, agents form a marginal belief about credibility but also form a separate marginal belief about the state before combining them in a final posterior that equals the product of the two marginals. This model of "pure separate updating" effectively assumes that agents do not understand joint distributions or correlation in beliefs (Koçak, 2018, considers a similar model). If, for example, an AM1 agent saw signals $\{a, a\}$, she would think $P(c = H \mid \theta = A) = P(c = H \mid \theta = B)$ even though $\{a, a\}$ rationally suggests that H is more likely if the state is A than if the state is B.

In "Alternative Model 2" (AM2), an agent follows the same second step as pre-screening but a different first step that assumes agents first form *joint* beliefs. Specifically, in the first step, an AM2 agent forms an updated belief over (c, θ) using Bayes' Rule before forming her final belief in the second step by applying Bayes' Rule again using the updated joint belief as her prior. Agents in this model of "pure double dipping" are equivalent to agents who use Bayes' Rule but who have seen too many copies of early signals. For example, if an AM2 agent saw signals

16 We thank an anonymous referee for motivating the following discussion. The Internet Appendix contains formal details.

$\{a, a, b, b, b\}$, we show she has the same beliefs as a Bayesian who saw eleven a and nine b signals.

These two models differ from pre-screening in mechanisms, foundations, and empirical predictions. In terms of mechanisms, pre-screeners understand that beliefs about one unknown parameter are related to beliefs about other parameters through correlation, unlike AM1 agents. Pre-screeners only substitute an updated belief about credibility in the second step, unlike AM2 agents who substitute an updated belief about the entire joint distribution as their prior.

In terms of conceptual and psychological foundations, pre-screening begins from the premise that agents may form data-dependent priors for the seemingly-legitimate purpose of estimating an ancillary parameter (the first step) but overlook double-dipping the data in the final analysis (the second step) as outlined in Section 2.3. In contrast, the premise of AM1 is that agents do not understand how unknown parameters are related through correlation. The premise of AM2 is that agents double-dip the data but do not treat one parameter as ancillary.

Pre-screening, AM1, and AM2 are each plausible in different ways; for example, AM1 seems plausible since joint distributions may be difficult quantities for individuals to process. Ultimately, which model (among these or others below) more accurately describes learning is an open empirical question. We next discuss different testable empirical predictions that future research can use to further evaluate the plausibility of alternative models.

7.2. *Empirical predictions and other approaches*

The first empirical prediction of pre-screening is *correlated disagreement* (Lemma 1 and Proposition 1). Correlated disagreement is a testable prediction that agents' opinions about which side of an issue is correct should be positively correlated with their beliefs about the credibility of information sources on the same side of the issue. For example, in individual-level survey data, individuals' opinions on whether climate change is real should be positively correlated with their opinions on the credibility of scientists who think climate change is real.

Other models of agents' beliefs about both credibility and an unknown state roughly fall into three categories: 1) They do not predict correlated disagreement, 2) They do predict correlated disagreement since agents have fixed beliefs (i.e., do not learn) about credibility, or 3) They do predict correlated disagreement due to agents' heterogeneous priors. In category 1 are models such as AM1 and AM2. As the Internet Appendix elaborates, these models do not generate correlated disagreement since AM1 agents ignore correlation and AM2 agents double-dip the data through joint beliefs rather than through beliefs about credibility alone. In category 2 are models such as Harrison and Kreps (1978), Scheinkman and Xiong (2003), Kandel and Pearson (1995), and Harris and Raviv (1993). In category 3 are Bayesian models with heterogeneous beliefs such as Acemoglu et al. (2016), Sethi and Yildiz (2016) and Suen (2004).[17] Models in categories 2 and 3 exogenously assume the positive correlation noted above and hence make no prediction about how the correlation comes about. Since the correlation is endogenous in our

[17] Glaeser and Sunstein (2014) consider how polarization from common information can occur when consumers have different priors about senders' motives. Gentzkow and Shapiro (2006) and Mullainathan and Shleifer (2005) show that different media sources may generate disagreement by slanting news to build a reputation or to cater to consumers' preferences for beliefs. Without heterogeneous priors in these models, media slant generates biased beliefs, but not disagreement. Our results suggest that erroneous learning about credibility leads to demand distortions that complement these strategic supply distortions. Morris (1995) reviews the literature on heterogeneous priors.

model, additional tests below can isolate whether pre-screening is the underlying mechanism relative to these models.

The second empirical prediction of pre-screening relates to the first-impression bias inherent in a pre-screener's beliefs. The psychology literature has accumulated substantial evidence of such a bias (e.g., Asch, 1946; Anderson, 1965; Hogarth and Einhorn, 1992; Uleman and Kressel, 2013). A specific empirical test of pre-screening would ideally study two groups of individuals who receive the same objective information, where one group is exogenously treated with early signals suggesting a credible source and the other with early signals suggesting a less credible source. Pre-screening predicts that the first group believes the source is more credible relative to the second group (Proposition 2 and Corollary 1) *and* that the state is more likely (Proposition 1). Such a joint test would distinguish pre-screening relative to models featuring path-dependent beliefs about only the state (e.g., confirmation bias in Rabin and Schrag, 1999). An experimental setting where signal order is within the control of the researcher would be particularly suited to test this prediction.

The third empirical prediction of pre-screening relates to when agents under- and over-react to new information (Proposition 4). Empirical evidence in the literature suggests that individuals tend to exhibit confirmation bias, or under-reaction to disconfirming news about the state (Lord et al., 1979; Griffin and Tversky, 1992; Rabin and Schrag, 1999; Fryer et al., 2019; Gentzkow et al., 2018). However, individuals also overreact to disconfirming news if it causes them to re-evaluate their worldview or paradigm (Ortoleva, 2012; Galperti, 2019). De Filippis et al. (forthcoming) find evidence that individuals over-react to contradictory signals, citing a similar mechanism, and Grether (1992) and Holt and Smith (2009) experimentally find that rare events cause larger deviations from Bayes' Rule. Other evidence suggests that agents over-react to signals due to over-confidence (e.g., Hirshleifer, 2015).

Pre-screening predicts that whether agents over- or under-react to news depends on how the source's evidence interacts with the effect of news on credibility (Proposition 4). In contrast with confirmation bias, pre-screeners can over-react to news that disconfirms existing evidence if such news is also sufficiently bad news about credibility. Empirically, whether or not agents underreact to news that disconfirms existing evidence should depend on an interaction term with credibility.[18]

The fourth empirical prediction of pre-screening relates to multiple signal sources. Pre-screening predicts that agents can disagree about the credibility of multiple sources even when they observe all signals from all sources, as long as they encounter sources in different orders (Proposition 5). This prediction distinguishes our theory from the growing literature on rational attention (Sims, 2003, 2006; Gabaix et al., 2006) and selective attention (Schwartzstein, 2014; Nimark and Sundaresan, 2019; Kominers et al., 2019). Broadly speaking, theories of attention suggest that "putting all the signals on the table" and forcing agents to see all signals should help resolve disagreement.

In contrast, pre-screeners disagree about the credibility of signals *that they all see*. Empirical evidence suggests that the differential interpretation of commonly-observed information is an important feature of real-world disagreement. Kandel and Zilberfarb (1999), Lahiri and Sheng (2008) and Patton and Timmermann (2010) provide evidence that differences in information sets

[18] To cleanly contrast confirmation bias with pre-screening, empirical tests would experimentally study settings where agents begin with neutral priors on the state. The reason is that confirmation bias relates to how agents react to news that disconfirm beliefs while pre-screening relates to how agents react to news that disconfirms existing evidence. In a setting with neutral priors on the state, news that disconfirms the existing evidence is equivalent to news that disconfirms beliefs.

do not explain disagreement among macroeconomic forecasters and emphasize the importance of differences in how forecasters interpret information. Cookson and Niessner (2020) provide evidence that differing signal interpretation is an important component of disagreement about firm stock prices.

Tests of this prediction could thus examine whether agents disagree because they have not seen other sources' information. Pre-screening predicts that both sides are well aware of each side's sources but ultimately discount them as non-credible. For example, pre-screening predicts that disagreement about climate change occurs even when all agents have seen the same information, whereas theories of attention broadly predict that disagreement occurs because one side has not seen the other side's sources. Tests of this prediction could also study how varying the order in which agents encounter sources affects beliefs, analogous to the tests of signal order.[19]

8. Conclusion

The key predictions of pre-screening are that: 1) Differing first impressions about credibility generate disagreement, 2) Disagreement about states of the world and credibility are endogenously correlated, and 3) Pre-screeners can over- and under-react depending on how signals interact with beliefs about credibility. New information sources may not resolve disagreement when they should, and agents can become certain of incorrect states if sources are slanted. In a trading game, pre-screening can generate price bubbles and crashes along with speculative trades, with traders "riding the bubble" along the way, even in an environment where Bayesians with heterogeneous priors would not do so.

Future research can extend our theory and apply it to several settings. For example, extending the theory beyond the two-state model may yield additional insights. The two-step process of pre-screening would be nearly identical, and we speculate that qualitative insights such as the occurrence of disagreement and the existence of order effects in beliefs would be similar. However, such an extended model may yield even richer predictions about the dynamics of beliefs and over- and under-reaction to news. Developing context-specific models or empirically testing the predictions of Section 7 may help illuminate disagreement in contexts as varied as climate change, medicine, and politics. Overall, exploring the endogenous reasons for why individuals jointly disagree about subject matter and the credibility of sources is a fruitful area for future research.

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[19] As a final note, relative to other behavioral models in the literature, one concern is that pre-screeners can employ many computations. A distinguished literature suggests that boundedly rational agents follow heuristics due to cognitive limitations (Simon, 1957; Gigerenzer and Selten, 2002; Selten, 2002). In contrast, our approach is in the spirit of work that models systematic conceptual deviations from rationality, even though agents may make more computations. For example, Enke (2020) emphasizes the importance of conceptual rather than computational errors in inference. Agents in Brunnermeier and Parker (2005) calculate optimal subjective beliefs. Agents in Bénabou and Tirole (2002) forget bad news, but also account for their self-serving beliefs. Agents in Kominers et al. (2019) process the decision value of signals before updating. Agents in Fryer et al. (2019) optimally consider how to interpret ambiguous signals, trading off short-run and long-run informational value.

Laibson, Jon Lewellen, Juhani Linnainmaa, Ted O'Donoghue, Martin Oehmke, Piotr Orlowski, Carol Osler, John Patty, Davide Pettenuzzo, Uday Rajan, Tanya Rosenblat, Jesse Shapiro, Kelly Shue, Kathy Spier, Phillip Stocken, Courtney Stoddard, Savitar Sundaresan, Dustin Tingley, Wei Xiong, Leeat Yariv, Muhamet Yildiz, seminar participants at Brandeis University, Cornell University, Harvard University, London School of Economics, MIT Sloan, Princeton University, The Ohio State University, University of California at San Diego (Rady), University of Chicago, and conference participants at the Behavioral Economics Annual Meeting, CSWEP CeMENT conference, Duke Behavioral Models of Politics Conference, HEC/McGill Winter Finance Conference, North American Summer Meeting of the Econometric Society, Stanford Institute for Theoretical Economics, University of British Columbia Winter Finance Conference, and the Western Finance Association Annual Meetings for comments. Part of this research was conducted while Cheng was at the Tuck School of Business, Dartmouth College, whom Cheng thanks for support.

Appendix A. Proofs of main propositions

A.1. Equations for beliefs of pre-screener and Bayesian

To illustrate the mechanics that drive the divergence between the beliefs of a pre-screener and a Bayesian, as in Section 2.4, we expand the recursion of $\kappa_c(\mathbf{s}_n)$ in the pre-screener's posterior beliefs. For brevity, we show the evolution of beliefs for three signals.

After the first signal s_1, the pre-screener's beliefs are:

$$\begin{aligned}
\kappa_c(\{s_1\}) &= \frac{\sum_\theta P(s_1|c,\theta)\omega_0^\theta\omega_0^c}{\sum_c\sum_\theta P(s_1|c,\theta)\omega_0^\theta\omega_0^c} \\
P^s(c,\theta|\{s_1\}) &= \frac{P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta}{\sum_c\sum_\theta P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta} \\
&= \frac{[\sum_\theta P(s_1|c,\theta)\omega_0^\theta]P(s_1|c,\theta)\omega_0^\theta\omega_0^c}{\sum_c[\sum_\theta P(s_1|c,\theta)\omega_0^\theta]\sum_\theta P(s_1|c,\theta)\omega_0^\theta\omega_0^c}.
\end{aligned}$$

In contrast, the Bayesian's posterior beliefs after the first signal are:

$$P(c,\theta|\{s_1\}) = \frac{P(s_1|c,\theta)\omega_0^c\omega_0^\theta}{\sum_c\sum_\theta P(s_1|c,\theta)\omega_0^c\omega_0^\theta}.$$

After the second signal s_2, the pre-screener's beliefs are:

$$\begin{aligned}
\kappa_c(\{s_1,s_2\}) &= \frac{\sum_\theta P(s_2|c,\theta)P^s(c,\theta|\{s_1\})}{\sum_c\sum_\theta P(s_2|c,\theta)P^s(c,\theta|\{s_1\})} \\
&= \frac{\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta}{\sum_c\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1\})\omega_0^\theta} \\
&= \frac{[\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]\omega_0^c}{\sum_c[\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]\omega_0^c} \\
P^s(c,\theta|\{s_1,s_2\}) &= \frac{P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta}{\sum_c\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta} \\
&= \frac{[\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}{\sum_c\sum_\theta[\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}.
\end{aligned}$$

In contrast, the Bayesian's posterior beliefs after the second signal are:

$$P(c,\theta|\{s_1,s_2\}) = \frac{P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}{\sum_c\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}.$$

After the third signal s_3, the pre-screener's beliefs are:

$$\begin{aligned}\kappa_c(\{s_1,s_2,s_3\}) &= \frac{\sum_\theta P(s_3|c,\theta)P^s(c,\theta|\{s_1,s_2\})}{\sum_c\sum_\theta P(s_3|c,\theta)P^s(c,\theta|\{s_1,s_2\})}\\ &= \frac{\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta}{\sum_c\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2\})\omega_0^\theta}\\ &= \frac{[\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]\omega_0^c}{\sum_c[\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]\omega_0^c}\end{aligned}$$

$$P^s(c,\theta|\{s_1,s_2,s_3\}) = \frac{P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2,s_3\})\omega_0^\theta}{\sum_c\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\kappa_c(\{s_1,s_2,s_3\})\omega_0^\theta}$$

$$= \frac{[\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}{\sum_c\sum_\theta[\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^\theta][\sum_\theta P(s_1|c,\theta)\omega_0^\theta]P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c.\omega_0^\theta}$$

In contrast, the Bayesian's posterior beliefs after the third signal are:

$$P(c,\theta|\{s_1,s_2,s_3\}) = \frac{P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}{\sum_c\sum_\theta P(s_3|c,\theta)P(s_2|c,\theta)P(s_1|c,\theta)\omega_0^c\omega_0^\theta}.$$

A.2. *Proof of Lemma 1*

Proof. Without loss of generality, let $(c,\theta)=(L,B)$. The posterior odds ratio of (L,B) versus (L,A) equals:

$$\begin{aligned}\frac{P^s(L,B\mid \mathbf{s}_n)}{P^s(L,A\mid \mathbf{s}_n)} &= \frac{\beta_L(\mathbf{s}_n)\prod_{t=1}^n P(s_t\mid L,B)\,\omega_0^B\omega_0^L}{\beta_L(\mathbf{s}_n)\prod_{t=1}^n P(s_t\mid L,A)\,\omega_0^A\omega_0^L}\\ &= \frac{\prod_{t=1}^n P(s_t\mid L,B)\,\omega_0^B\omega_0^L}{\prod_{t=1}^n P(s_t\mid L,A)\,\omega_0^A\omega_0^L}\\ &= \frac{P(L,B\mid \mathbf{s}_n)}{P(L,A\mid \mathbf{s}_n)}.\end{aligned}$$

Since $\frac{x}{y}=\frac{x'}{y'}$ implies $\frac{x}{x+y}=\frac{x'}{x'+y'}$ and $P^s(\theta=B\mid L;\mathbf{s}_n)=\frac{P^s(L,B|\mathbf{s}_n)}{P^s(L,B|\mathbf{s}_n)+P^s(L,A|\mathbf{s}_n)}$, the conclusion follows. □

A.3. *Proof of Proposition 1*

1. Since $P^s(\theta\mid c;\mathbf{s}_n)=P(\theta\mid c;\mathbf{s}_n)$, we have:

$$\begin{aligned}P^s(\theta\mid \mathbf{s}_n)-P(\theta\mid \mathbf{s}_n) =& P(\theta\mid H;\mathbf{s}_n)\times\left[P^s(H\mid \mathbf{s}_n)-P(H\mid \mathbf{s}_n)\right]\\ &+P(\theta\mid L;\mathbf{s}_n)\times\left[P^s(L\mid \mathbf{s}_n)-P(L\mid \mathbf{s}_n)\right]\\ =& P(\theta\mid H;\mathbf{s}_n)\times\left[P^s(H\mid \mathbf{s}_n)-P(H\mid \mathbf{s}_n)\right]\\ &+P(\theta\mid L;\mathbf{s}_n)\times\left[P(H\mid \mathbf{s}_n)-P^s(H\mid \mathbf{s}_n)\right]\\ =& \left[P(\theta\mid H;\mathbf{s}_n)-P(\theta\mid L;\mathbf{s}_n)\right]\times\left[P^s(H\mid \mathbf{s}_n)-P(H\mid \mathbf{s}_n)\right].\end{aligned}$$

Without loss of generality, consider $\theta = B$:

$$\begin{aligned}
P(B \mid H; \mathbf{s}_n) &= P(B \mid L; \mathbf{s}_n) \\
\frac{P(H, B \mid \mathbf{s}_n)}{P(H, B \mid \mathbf{s}_n) + P(H, A \mid \mathbf{s}_n)} &= \frac{P(L, B \mid \mathbf{s}_n)}{P(L, B \mid \mathbf{s}_n) + P(L, A \mid \mathbf{s}_n)} \\
\frac{P(H, B \mid \mathbf{s}_n)}{P(H, A \mid \mathbf{s}_n)} &= \frac{P(L, B \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} \\
\left(\frac{1-q_H}{q_H}\right)^{n_a-n_b} \frac{(1-\omega_0^A)\omega_0^H}{\omega_0^A \omega_0^H} &= \left(\frac{1-q_L}{q_L}\right)^{n_a-n_b} \frac{(1-\omega_0^A)\omega_0^L}{\omega_0^A \omega_0^L}.
\end{aligned}$$

Therefore, $P(\theta \mid H; \mathbf{s}_n) - P(\theta \mid L; \mathbf{s}_n) = 0$ if and only if $n_a = n_b$. Moreover, $P^s(H \mid \mathbf{s}_n) = P(H \mid \mathbf{s}_n)$ if and only if $\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} = 1$:

$$\begin{aligned}
P^s(H \mid \mathbf{s}_n) &= P(H \mid \mathbf{s}_n) \\
\frac{P^s(H \mid \mathbf{s}_n)}{P^s(L \mid \mathbf{s}_n)} &= \frac{P(H \mid \mathbf{s}_n)}{P(L \mid \mathbf{s}_n)} \\
\frac{\beta_H(\mathbf{s}_n) \sum_\theta \prod_{t=1}^n P(s_t \mid H, \theta)\, \omega_0^\theta \omega_0^c}{\beta_L(\mathbf{s}_n) \sum_\theta \prod_{t=1}^n P(s_t \mid L, \theta)\, \omega_0^\theta \omega_0^c} &= \frac{\sum_\theta \prod_{t=1}^n P(s_t \mid H, \theta)\, \omega_0^\theta \omega_0^c}{\sum_\theta \prod_{t=1}^n P(s_t \mid L, \theta)\, \omega_0^\theta \omega_0^c} \\
\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} &= 1.
\end{aligned}$$

From the definition of $\beta_c(\mathbf{s}_n)$,

$$\begin{aligned}
\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} &= \frac{\prod_{m=1}^n \sum_\theta \left(\prod_{t=1}^m P(s_t \mid H, \theta)\, \omega_0^\theta\right)}{\prod_{m=1}^n \sum_\theta \left(\prod_{t=1}^m P(s_t \mid L, \theta)\, \omega_0^\theta\right)} \\
&= \prod_{m=1}^n \frac{\omega_0^L \sum_\theta \left(P(\mathbf{s}_m \mid H, \theta)\, \omega_0^\theta \omega_0^H\right)}{\omega_0^H \sum_\theta \left(P(\mathbf{s}_m \mid L, \theta)\, \omega_0^\theta (1-\omega_0^H)\right)} \\
&= \prod_{m=1}^n \frac{P(c = H \mid \mathbf{s}_m)(1-\omega_0^H)}{P(c = L \mid \mathbf{s}_m)\, \omega_0^H}.
\end{aligned} \tag{A.1}$$

Therefore, $P^s(c = H \mid \mathbf{s}_n) - P(c = H \mid \mathbf{s}_n) = 0$ if and only if $\prod_{m=1}^n \frac{P(c=H|\mathbf{s}_m)(1-\omega_0^H)}{P(c=L|\mathbf{s}_m)\omega_0^H} = 1$.

2. This follows from Lemma 1 and the proof of Part 1 of Proposition 1.

A.4. *Proof of Proposition 2*

Shown in the proof of Part 1 of Proposition 1.

A.5. *Proof of Corollary 1*

Equation (A.1) shows how a pre-screener's over- or under-trust depends on the cumulative effect of the signals on their beliefs about credibility. Each individual mth term of $\beta_H/\beta_L(\mathbf{s}_n)$ is the objective odds that the source of high credibility relative to priors, given the information content of subsequence $\mathbf{s}_m$. Each mth term of Equation (A.1) is strictly greater than 1 if and only if

$$\left(\sum_{\theta}\left(\prod_{t=1}^{m}P(s_t|H,\theta)\right)\omega_0^{\theta}\right)-\left(\sum_{\theta}\left(\prod_{t=1}^{m}P(s_t|L,\theta)\right)\omega_0^{\theta}\right)\geq 0. \tag{A.2}$$

Without loss of generality, consider the $m=n$th term of Equation (A.1) and suppose $n_a \geq n_b$ for $\mathbf{s}_n$. Let $d \equiv n_a - n_b$. Expanding Equation (A.2), the $m=n$th term of Equation (A.1) is strictly greater than 1 if and only if

$$\begin{aligned}&\omega_0^A q_H^{n_a}(1-q_H)^{n_b}+(1-\omega_0^A)(1-q_H)^{n_a}q_H^{n_b}\\ &\quad > \omega_0^A q_L^{n_a}(1-q_L)^{n_b}+(1-\omega_0^A)(1-q_L)^{n_a}q_L^{n_b}\end{aligned} \tag{A.3}$$

We can re-write Equation (A.3) as

$$\left(\frac{q_H(1-q_H)}{q_L(1-q_L)}\right)^{n_b}\left(\frac{\omega_0^A q_H^d+(1-\omega_0^A)(1-q_H)^d}{\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d}\right)>1. \tag{A.4}$$

Note that Equation (A.4) is strictly decreasing in n_b. We show that $G(d)\equiv \frac{\omega_0^A q_H^d+(1-\omega_0^A)(1-q_H)^d}{\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d}$ is increasing in d when $d \geq 0$ is sufficiently large:

$$\begin{aligned}\frac{\partial G}{\partial d}=&\frac{\left(\omega_0^A q_H^d\ln(q_H)+(1-\omega_0^A)(1-q_H)^d\ln(1-q_H)\right)\left(\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d\right)}{\left(\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d\right)^2}\\ &-\frac{\left(\omega_0^A q_H^d+(1-\omega_0^A)(1-q_H)^d\right)\left(\omega_0^A q_L^d\ln(q_L)+(1-\omega_0^A)(1-q_L)^d\ln(1-q_L)\right)}{\left(\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d\right)^2}.\end{aligned} \tag{A.5}$$

The numerator of Equation (A.5) is positive if and only if

$$\begin{aligned}&\frac{\omega_0^A q_H^d\ln(q_H)+(1-\omega_0^A)(1-q_H)^d\ln(1-q_H)}{\omega_0^A q_H^d+(1-\omega_0^A)(1-q_H)^d}\\ &\quad>\frac{\omega_0^A q_L^d\ln(q_L)+(1-\omega_0^A)(1-q_L)^d\ln(1-q_L)}{\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d}.\end{aligned}$$

Thus the numerator of Equation (A.5) is positive if the following holds:

$$\frac{\partial}{\partial q}\left(\frac{\omega_0^A q^d\ln(q)+(1-\omega_0^A)(1-q)^d\ln(1-q)}{\omega_0^A q^d+(1-\omega_0^A)(1-q)^d}\right)>0,$$

which we re-write as

$$\frac{\partial}{\partial q}\left(\frac{\ln(q)+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d\ln(1-q)}{1+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d}\right)>0. \tag{A.6}$$

The numerator of Equation (A.6) is

$$\begin{aligned}&\left(\frac{1}{q}+(\frac{1-\omega_0^A}{\omega_0^A})d(\frac{1-q}{q})^{d-1}(\frac{-q-(1-q)}{q^2})-(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1}{1-q})\right)\\ &\times\left(1+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d\right)\end{aligned}$$

$$
\begin{aligned}
&-\left(\ln(q)+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d\ln(1-q)\right)\left((\frac{1-\omega_0^A}{\omega_0^A})d(\frac{1-q}{q})^{d-1}(\frac{-q-(1-q)}{q^2})\right)\\
&=\frac{1}{q}+(\frac{1-\omega_0^A}{\omega_0^A})d(\frac{1-q}{q})^{d-1}(\frac{2q-1}{q^2})\ln(1-q)-(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d(\frac{1}{1-q})\\
&+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d(\frac{1}{q})\\
&+(\frac{1-\omega_0^A}{\omega_0^A})^2d(\frac{1-q}{q})^{2d-1}(\frac{2q-1}{q^2})\ln(1-q)-(\frac{1-\omega_0^A}{\omega_0^A})^2(\frac{1-q}{q})^{2d}(\frac{1}{1-q})\\
&-(\frac{1-\omega_0^A}{\omega_0^A})d(\frac{1-q}{q})^{d-1}(\frac{2q-1}{q^2})\ln(q)-(\frac{1-\omega_0^A}{\omega_0^A})^2d(\frac{1-q}{q})^{2d-1}(\frac{2q-1}{q^2})\ln(1-q)\\
&=\frac{1}{q}+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d(\frac{2q-1}{q(1-q)})(d\ln(\frac{1-q}{q})-1)-(\frac{1-\omega_0^A}{\omega_0^A})^2(\frac{1-q}{q})^{2d}(\frac{1}{1-q}).
\end{aligned}
\tag{A.7}
$$

The first term of Equation (A.7) is positive and the second and third terms are negative. The third term is clearly increasing in d and $\lim_{d\to\infty}-(\frac{1-\omega_0^A}{\omega_0^A})^2(\frac{1-q}{q})^{2d}(\frac{1}{1-q})=0$. The second term is also increasing in d:

$$
\begin{aligned}
&\frac{\partial}{\partial d}\left((\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d(\frac{2q-1}{q(1-q)})(d\ln(\frac{1-q}{q})-1)\right)\\
&\quad=(\frac{1-\omega_0^A}{\omega_0^A})(\frac{2q-1}{q(1-q)})(\frac{1-q}{q})^d d\left(\ln(\frac{1-q}{q})\right)^2>0,
\end{aligned}
$$

and its limit as $d\to\infty$ is zero:

$$
\begin{aligned}
&\lim_{d\to\infty}(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q}{q})^d(\frac{2q-1}{q(1-q)})(d\ln(\frac{1-q}{q})-1)\\
&\quad=(\frac{1-\omega_0^A}{\omega_0^A})(\frac{2q-1}{q(1-q)})\lim_{d\to\infty}\left(\ln(\frac{1-q}{q})\frac{d}{(\frac{q}{1-q})^d}-(\frac{1-q}{q})^d\right)\\
&\quad=(\frac{1-\omega_0^A}{\omega_0^A})(\frac{2q-1}{q(1-q)})\lim_{d\to\infty}\left(\ln(\frac{1-q}{q})\frac{1}{d(\frac{q}{1-q})^{d-1}}-(\frac{1-q}{q})^d\right)\\
&\quad=0.
\end{aligned}
$$

This implies that there exists some $\overline{d}\geq 0$ such that Equation (A.7) is positive for all $d>\overline{d}$ and negative for all $d<\overline{d}$. Thus, there exists some $\overline{d}\geq 0$ such that $G(d)$ is increasing in d for all $d>\overline{d}$. Moreover, since Equation (A.7) is clearly increasing in ω_0^A, then $\overline{d}$ is decreasing in ω_0^A.

Finally, we know that $\lim_{d\to\infty}G(d)=\infty$:

$$
\lim_{d\to\infty}\frac{\omega_0^A q_H^d+(1-\omega_0^A)(1-q_H)^d}{\omega_0^A q_L^d+(1-\omega_0^A)(1-q_L)^d}=\lim_{d\to\infty}\frac{(\frac{q_H}{q_L})^d+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q_H}{q_L})^d}{1+(\frac{1-\omega_0^A}{\omega_0^A})(\frac{1-q_L}{q_L})^d}=\frac{\infty+0}{1+0}=\infty.
$$

Since $G(d)$ is increasing in d for all $d > \overline{d}$, $G(0) = 1$, and $\lim_{d\to\infty} G(d) = \infty$, then for any $n_b \geq 0$, there exists some $d^* \geq 0$ such that Equation (A.4) equals 1 for $d = d^*$, Equation (A.4) is less than 1 for $d < d^*$, and Equation (A.4) is greater than 1 and increasing in d for $d > d^*$. Since the right-hand side of Equation (A.4) decreases in n_b, then d^* increases in n_b. Since $\overline{d}$ is decreasing in ω_0^A, then d^* is decreasing in ω_0^A.

Analogous results hold if $n_b \geq n_a$ so that $d \leq 0$, since Equation (A.3) would instead become

$$\left(\frac{q_H(1-q_H)}{q_L(1-q_L)}\right)^{n_a}\left(\frac{\omega_0^A(1-q_H)^d + (1-\omega_0^A)q_H^d}{\omega_0^A(1-q_L)^d + (1-\omega_0^A)q_L^d}\right) > 1. \tag{A.8}$$

Analogously, Equation (A.8) is strictly decreasing in n_a and there exists some $d^{**} \leq 0$ such that Equation (A.8) equals 1 for $d = d^{**}$, Equation (A.8) is less than 1 for $d > d^{**}$, and Equation (A.8) is greater than 1 and decreasing in d for $d < d^{**}$. Likewise, d^{**} is decreasing in ω_0^A.

Let $n_{a,m}$ be the number of a's in subsequence $\mathbf{s}_m$, $n_{b,m}$ be the number of b's in subsequence $\mathbf{s}_m$, and let $d_m = n_{a,m} - n_{b,m}$. We have shown from Equation (A.4) that if $n_{a,m} \geq n_{b,m}$ so $d_m \geq 0$, then an mth term of $\beta_H/\beta_L(\mathbf{s}_n)$ is decreasing in $n_{b,m}$. We have also show that an mth term of $\beta_H/\beta_L(\mathbf{s}_n)$ is increasing in d when $d_m > d_m^*$. Note that mechanically, if $n_{b,m+1} = n_{b,m} + 1$ then $d_{m+1} = d_m - 1$.

Likewise, if $n_{b,m} \geq n_{a,m}$ so $d_m \leq 0$, then an mth term of $\beta_H/\beta_L(\mathbf{s}_n)$ is decreasing in $n_{a,m}$, and it is decreasing in d_m when $d_m < d_m^{**}$. Mechanically, if $n_{a,m+1} = n_{a,m} + 1$ then $d_{m+1} = d_m + 1$.

This implies the following: Consider $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$ where $n_{b,j}^X = n_{b,j}^Y - 1$ and $n_{b,m}^X = n_{b,m}^Y$ for all $m \neq j$.

1. If $n_{a,j-1} \geq n_{b,j-1}$, then for any $n_{b,j-1} \geq 0$ there exists some $d_{j-1}^* \geq 0$ such that $\frac{P(c=H|\mathbf{s}_{j-1})(1-\omega_0^H)}{P(c=L|\mathbf{s}_{j-1})(\omega_0^H)}$ is increasing in d_{j-1} for all $d_{j-1} > d_{j-1}^*$. For any j such that $n_{a,j-1} \geq n_{b,j-1}$ and $d_{j-1} > d_{j-1}^*$, then $\beta_H/\beta_L(\mathbf{s}_n^X) > \beta_H/\beta_L(\mathbf{s}_n^Y)$.
   Let $n_{a,j-1} \geq n_{b,j-1}$. Sequences $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$ are ordered identically except that $(s_j^X, s_{j+1}^X) = (a,b)$ while $(s_j^Y, s_{j+1}^Y) = (b,a)$. From Equation (A.1), it follows that $\beta_H/\beta_L(\mathbf{s}_n^X) > \beta_H/\beta_L(\mathbf{s}_n^Y)$ if and only if
   $$\frac{P(c=H|\mathbf{s}_j^X)}{P(c=L|\mathbf{s}_j^X)} > \frac{P(c=H|\mathbf{s}_j^Y)}{P(c=L|\mathbf{s}_j^Y)}. \tag{A.9}$$
   Note that $n_{b,j}^Y = n_{b,j}^X + 1$ so $d_j^X = d_{j-1}^X + 1$ and $d_j^Y = d_{j-1}^X - 1$. Since $d_{j-1} > d_{j-1}^*$, then $d_j^X > d_j^Y \geq d_{j-1}^*$. Thus Equation (A.9) holds.
2. If $n_{b,j-1} \geq n_{a,j-1}$, then for any $n_{a,j-1} \geq 0$, there exists some $d_{j-1}^{**} \leq 0$ such that $\frac{P(c=H|\mathbf{s}_{j-1})(1-\omega_0^H)}{P(c=L|\mathbf{s}_{j-1})(\omega_0^H)}$ is decreasing in d_{j-1} for all $d_{j-1} < d_{j-1}^{**}$. For any j such that $n_{b,j-1} \geq n_{a,j-1}$ and $d_{j-1} < d_{j-1}^{**}$, then $\beta_H/\beta_L(\mathbf{s}_n^X) < \beta_H/\beta_L(\mathbf{s}_n^Y)$.
   Analogous argument as above.

A.6. Special Case of Corollary 1

Corollary 3 shows the implications of Corollary 1 in the special case where the prior on the state does not color the pre-screener's interpretation of signals ($\omega_0^A = 1/2$). It illustrates the intuition for how signal order generates first impression bias by showing that pre-screeners erroneously believe that the timing of signal reversals is itself informative, in that a pattern of few (more) initial reversals inflates (deflates) their beliefs about source credibility. Consider the sequences $\{a,a,b\}$ and $\{a,b,a\}$, which have identical information content. Under $\{a,a,b\}$, the first two signals $\{a,a\}$ are consistent and objectively indicate high credibility. Under $\{a,b,a\}$, the first two signals $\{a,b\}$ are inconsistent and objectively indicate low credibility. In both cases, the pre-screener overinfers credibility from the first two signals, and this first impression of credibility colors the pre-screener's interpretation of the third signal. As a result, $P^s(c=H|\{a,a,b\}) > P^s(c=H\{(a,b,a\})$. Corollary 3 broadens this example and shows that re-ordering the signals so that the longest consistent streak appears first generates the most trust in the source, while alternating the signals first generates the least trust. In contrast, a Bayesian's final beliefs are independent of signal order.

Corollary 3. *Let $\mathbf{s}_n$ be a signal path with $n_a \geq n_b$ be given. Let $\mathbf{s}_n^X \equiv \{a,a,\ldots,b,b,\ldots\}$ and $\mathbf{s}_n^Y \equiv \{a,b,a,b,\ldots,a,a,\ldots\}$ where $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$. If $(\omega_0^A, \omega_0^H) = (1/2, \hat{\omega})$ where $\hat{\omega} \in (0,1)$, then $\mathbf{s}_n^X = \operatorname{argmax}_{\mathbf{s}\in\Sigma(\mathbf{s}_n)} \beta_H/\beta_L(\mathbf{s})$ and $\mathbf{s}_n^Y = \arg\min_{\mathbf{s}\in\Sigma(\mathbf{s}_n)} \beta_H/\beta_L(\mathbf{s})$.*

Proof. Let $\mathbf{s}_n$ a set of signals with $n_a \geq n_b$, and $\Sigma(\mathbf{s}_n)$ be the permutations of $\mathbf{s}_n$. Consider any pair of sequences $\mathbf{s}_n^X, \mathbf{s}_n^Y \in \Sigma(\mathbf{s}_n)$.

By Proposition 2, a necessary and sufficient condition for $P^s(c=H|\mathbf{s}_n^X) > P^s(c=H|\mathbf{s}_n^Y)$ is $\beta_H/\beta_L(\mathbf{s}_n^X) > \beta_H/\beta_L(\mathbf{s}_n^Y)$. Consider $\mathbf{s}_n^X$ and $\mathbf{s}_n^Y$, where the first j signals are ordered identically and $n-2 \geq j \geq 1$, the two sequences differ in the $j+1$ and $j+2$ signals, and then all subsequent signals are identical (i.e., terms $j+3$ through n). Suppose the first j terms contain k a's and $j-k$ b's, where $k \geq j-k$. Let $s_{j+1}^X = a$, $s_{j+2}^X = b$, $s_{j+1}^Y = b$, and $s_{j+2}^Y = a$. (For example, $\mathbf{s}_n^X$ could be $\{a,a,b,a,b,a\}$ and $\mathbf{s}_n^Y$ could be $\{a,a,b,b,a,a\}$, so $j=3$, $k=2$.) Then $\beta_H/\beta_L(\mathbf{s}_n^X) > \beta_H/\beta_L(\mathbf{s}_n^Y)$ whenever $k > j-k$ and $\beta_H/\beta_L(\mathbf{s}_n^X) = \beta_H/\beta_L(\mathbf{s}_n^Y)$ whenever $k = j-k$. To see this, note that, given Equation (9), all of the terms are identical for $\beta_c(\mathbf{s}_n^X)$ and $\beta_c(\mathbf{s}_n^Y)$ except term $j+1$. When $\omega_0^A = 1/2$, then $\beta_H/\beta_L(\mathbf{s}_n^X) \geq \beta_H/\beta_L(\mathbf{s}_n^Y)$ if and only if

$$
\begin{aligned}
&\left(q_H^{k+1}(1-q_H)^{j-k} + (1-q_H)^{k+1} q_H^{j-k}\right)\left(q_L^k(1-q_L)^{j-k+1} + (1-q_L)^k q_L^{j-k+1}\right)\\
&- \left(q_L^{k+1}(1-q_L)^{j-k} + (1-q_L)^{k+1} q_L^{j-k}\right)\left(q_H^k(1-q_H)^{j-k+1} + (1-q_H)^k q_H^{j-k+1}\right) \geq 0\\
&(q_H - q_L)\left((q_H q_L)^{2k-j} - ((1-q_H)(1-q_L))^{2k-j}\right)\\
&+ (q_H + q_L - 1)\left((q_H(1-q_L))^{2k-j} - (q_L(1-q_H))^{2k-j}\right) \geq 0.
\end{aligned}
$$

We can verify that both terms on the left-hand side are positive when $k > j-k$ and zero when $k = j-k$. Thus, $P^s(c=H|\mathbf{s}_n^X) > P^s(c=H|\mathbf{s}_n^Y)$ when $k > j-k$. Using this result, we can iteratively apply it to order sequences in $\Sigma(\mathbf{s}_n)$ by decreasing trust, by starting with the sequence with the least reversals (all a's followed by all b's), and iteratively switching the first b and last a to generate sequences in which the first b moves forward. For example, $P^s(c=H|\{a,a,a,a,b,b\}) > P^s(c=H|\{a,a,a,b,a,b\}) > P^s(c=H|\{a,a,b,a,a,b\}) > P^s(c=H|\{a,b,a,a,a,b\})$. Then, $P^s(c=H|\{a,a,a,b,b,a\}) > P^s(c=H|\{a,a,b,a,b,a\}) > P^s(c=H|\{a,b,a,a,b,a\})$, where $P^s(c=H|\{a,a,a,b,a,b\}) > P^s(c=H|\{a,a,a,b,b,a\})$ and $P^s(c=H|\{a,b,a,a,a,b\}) > P^s(c=H|\{a,b,a,a,b,a\})$. We continue this procedure (and applying the result that $P^s(c=H|\mathbf{s}_n^X) > P^s(c=H|\mathbf{s}_n^Y)$ when $k > j-k$) to establish that $\{a,a,a,a,b,b\}$ generates the most trust and $\{a,b,a,b,a,a\}$ generates the least trust.

Thus, if $(\omega_0^A, \omega_0^H) = (1/2, \hat{\omega})$ where $\hat{\omega} \in (0, 1)$, then $\mathbf{s}_n^X = \operatorname{argmax}_{\mathbf{s} \in \Sigma(\mathbf{s}_n)} \beta_H/\beta_L(\mathbf{s})$ and $\mathbf{s}_n^Y = \arg\min_{\mathbf{s} \in \Sigma(\mathbf{s}_n)} \beta_H/\beta_L(\mathbf{s})$, where $\mathbf{s}_n^X \equiv \{a, a, \ldots, b, b, \ldots\}$ and $\mathbf{s}_n^Y \equiv \{a, b, a, b, \ldots, a, a, \ldots\}$. □

A.7. Proof of Proposition 3

Proof. Suppose $\left(c', \theta'\right) = (L, A)$. The proof shows that the posterior odds ratio of any other (c, θ) versus (L, A) converges to zero when $q_L > 1/2$, which we assume unless otherwise noted. If $q_L = 1/2$, the posterior odds ratios of (L, B) versus (L, A) converge to ω_0^B/ω_0^A. Arguments for other $\left(c', \theta'\right)$ are similar. We consider convergence almost surely.

Bayesian. Suppose $\left(c', \theta'\right) = (L, A)$. We show the asymptotic belief for every other (c, θ) is zero through standard arguments.

Let $(c, \theta) = (L, B)$. The posterior odds ratio of (L, B) versus (L, A) equals:

$$\frac{P(L, B \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} = \frac{q_L^{n_b}(1-q_L)^{n_a}\omega_0^B\omega_0^L}{q_L^{n_a}(1-q_L)^{n_b}\omega_0^A\omega_0^L} = \left(\frac{1-q_L}{q_L}\right)^{n_a-n_b}\frac{(1-\omega_0^A)}{\omega_0^A} \to 0,$$

since $q_L > 1/2$ implies $(n_a - n_b) \to \infty$ and $\frac{1-q_L}{q_L} < 1$.[20] If $q_L = 1/2$, the posterior odds ratio equals ω_0^B/ω_0^A as $(1-q_L)/q_L = 1$.

Now let $(c, \theta) = (H, A)$. The posterior odds ratio of (H, A) versus (L, A) equals:

$$\frac{P(H, A \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} = \frac{q_H^{n_a}(1-q_H)^{n_b}\omega_0^A\omega_0^H}{q_L^{n_a}(1-q_L)^{n_b}\omega_0^A\omega_0^L} = \left(\frac{q_H^{n_a/n}(1-q_H)^{n_b/n}}{q_L^{n_a/n}(1-q_L)^{n_b/n}}\right)^n \frac{\omega_0^H}{\omega_0^L} \to 0.$$

The formal limit proof of the last line is as follows. Let $\varepsilon > 0$ be given. Since $x^{n_a/n}(1-x)^{n_b/n}$ converges with limit $x^{q_L}(1-x)^{1-q_L}$ for $x \in \{q_L, q_H\}$, that means for any $\nu > 0$, there exists an N_1 large enough so that, for any $n > N_1$:

$$\left|\frac{q_H^{n_a/n}(1-q_H)^{n_b/n}}{q_L^{n_a/n}(1-q_L)^{n_b/n}} - \frac{q_H^{q_L}(1-q_H)^{1-q_L}}{q_L^{q_L}(1-q_L)^{1-q_L}}\right| < \nu.$$

Note that $\eta \equiv \frac{q_H^{q_L}(1-q_H)^{1-q_L}}{q_L^{q_L}(1-q_L)^{1-q_L}} < 1$ since $f(x) = x^{q_L}(1-x)^{1-q_L}$ is a strictly decreasing function for $x \in (q_L, 1)$.[21] This implies that we can also choose ν small enough so that there exists an N_1 such that $\frac{q_H^{n_a/n}(1-q_H)^{n_b/n}}{q_L^{n_a/n}(1-q_L)^{n_b/n}} < 1 - \left(\frac{1-\eta}{2}\right)$ and in particular is bounded away from 1 for any $n > N_1$. Therefore, there exists an $N_2 > N_1$ such that $\left(\frac{q_H^{n_a/n}(1-q_H)^{n_b/n}}{q_L^{n_a/n}(1-q_L)^{n_b/n}}\right)^n < \varepsilon$ for $n > N_2$. If $q_L = 1/2$, the posterior odds ratio also equals zero, and a similar argument applies for (H, B) versus (L, B). Intuitively, conditional on the state, the agent learns the true type (L) for sure based on the long-run frequency.

[20] Note that $n_a - n_b = 2n\left(\frac{n_a}{n} - \frac{1}{2}\right)$, which diverges to $+\infty$ when $q_L > 1/2$ since $n_a/n \to q_L$.

[21] Let $f(x) = x^{q_L}(1-x)^{1-q_L}$. Note that $f(x)$ is decreasing if and only if $g(x) = q_L \log x + (1-q_L)\log(1-x)$ is decreasing. Taking derivatives:

$$g'(x) = \frac{q_L}{x} - \frac{1-q_L}{1-x}.$$

Therefore, we have $g'(x) < 0 \Leftrightarrow \frac{q_L}{1-q_L} < \frac{x}{1-x} \Leftrightarrow q_L < x$.

Now let $(c, \theta) = (H, B)$. The posterior odds ratio of (H, B) versus (H, A) equals:

$$\frac{P(H, B \mid \mathbf{s}_n)}{P(H, A \mid \mathbf{s}_n)} = \frac{q_H^{n_b}(1-q_H)^{n_a}}{q_H^{n_a}(1-q_H)^{n_b}} \frac{\omega_0^B \omega_0^H}{\omega_0^A \omega_0^H} = \left(\frac{1-q_H}{q_H}\right)^{n_a - n_b} \frac{(1-\omega_0^A)}{\omega_0^A} \to 0,$$

for $q_L > 1/2$.

Thus:

$$\begin{aligned}\lim_{n\to\infty} \frac{P(H, B \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} &= \lim_{n\to\infty}\left[\frac{P(H, B \mid \mathbf{s}_n)}{P(H, A \mid \mathbf{s}_n)} \frac{P(H, A \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)}\right] \\ &= \lim_{n\to\infty} \frac{P(H, B \mid \mathbf{s}_n)}{P(H, A \mid \mathbf{s}_n)} \lim_{n\to\infty} \frac{P(H, A \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} = 0,\end{aligned}$$

for $q_L > 1/2$.

If $q_L = 1/2$, then:

$$\lim_{n\to\infty} \frac{P(H, B \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} = \frac{q_H^{n_b}(1-q_H)^{n_a}}{q_L^{n_a}(1-q_L)^{n_b}} \frac{\omega_0^B \omega_0^H}{\omega_0^A \omega_0^L} = 2^n q_H^{n_b}(1-q_H)^{n_a} \frac{\omega_0^B \omega_0^H}{\omega_0^A \omega_0^L}.$$

But:

$$2^n q_H^{n_b}(1-q_H)^{n_a} = \left(2 q_H^{\frac{n_b}{n}}(1-q_H)^{\frac{n_a}{n}}\right)^n.$$

The inside term on the right-hand side converges to $2q_H^{\frac{1}{2}}(1-q_H)^{\frac{1}{2}}$. Observe $q_H \in (1/2, 1)$ implies $q_H(1-q_H) < 1/4$ and thus $2q_H^{\frac{1}{2}}(1-q_H)^{\frac{1}{2}} < 1$. This means we can choose N sufficiently large so that $2q_H^{\frac{n_b}{n}}(1-q_H)^{\frac{n_a}{n}}$ is less than and bounded away from 1 for all $n > N$. This in turn implies $\left(2q_H^{\frac{n_b}{n}}(1-q_H)^{\frac{n_a}{n}}\right)^n \to 0$.

Pre-screener. The pre-screener's belief equals:

$$\begin{aligned}P^s(c, \theta \mid \mathbf{s}_n) &= \frac{\beta_c(\mathbf{s}_n)\prod_{t=1}^n P(s_t \mid c, \theta)\,\omega_0^\theta \omega_0^c}{\sum_c \beta_c(\mathbf{s}_n)\sum_\theta \prod_{t=1}^n P(s_t \mid c, \theta)\,\omega_0^\theta \omega_0^c} \\ \beta_c(\mathbf{s}_n) &= \prod_{m=1}^n \sum_\theta \left(\prod_{t=1}^m P(s_t \mid c, \theta)\,\omega_0^\theta\right).\end{aligned}$$

Let $(c, \theta) = (L, B)$. The posterior odds ratio of (L, B) versus (L, A) equals:

$$\frac{P^s(L, B \mid \mathbf{s}_n)}{P^s(L, A \mid \mathbf{s}_n)} = \frac{\beta_L(\mathbf{s}_n)\prod_{t=1}^n P(s_t \mid L, B)\,\omega_0^B \omega_0^L}{\beta_L(\mathbf{s}_n)\prod_{t=1}^n P(s_t \mid L, A)\,\omega_0^A \omega_0^L} = \frac{\prod_{t=1}^n P(s_t \mid L, B)(1-\omega_0^A)}{\prod_{t=1}^n P(s_t \mid L, A)\,\omega_0^A} \to 0,$$

by the same arguments as the Bayesian. If $q_L = 1/2$, the posterior odds ratio evidently equals ω_0^B/ω_0^A as it does for a Bayesian.

Let $(c, \theta) = (H, A)$. The posterior odds ratio of (H, A) versus (L, A) equals:

$$\frac{P^s(H, A \mid \mathbf{s}_n)}{P^s(L, A \mid \mathbf{s}_n)} = \frac{\beta_H(\mathbf{s}_n)\prod_{t=1}^n P(s_t \mid H, A)\,\omega_0^A \omega_0^H}{\beta_L(\mathbf{s}_n)\prod_{t=1}^n P(s_t \mid L, A)\,\omega_0^A \omega_0^L}$$

From Equation (A.1), $\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} = \prod_{m=1}^n \frac{P(c=H|\mathbf{s}_m)(1-\omega_0^H)}{P(c=L|\mathbf{s}_m)\omega_0^H}$. Because the Bayesian learns the truth, we have $\frac{P(c=H|\mathbf{s}_n)\omega_0^L}{P(c=L|\mathbf{s}_n)\omega_0^H} \to 0$. In particular, there exists an N such that for all $n > N$, $\frac{P(c=H|\mathbf{s}_n)\omega_0^L}{P(c=L|\mathbf{s}_n)\omega_0^H} <$

1. This implies that for any $\varepsilon > 0$, there exists an $N_1 > N$ such that $\prod_{m=1}^{n} \frac{P(c=H|\mathbf{s}_m)\omega_0^L}{P(c=L|\mathbf{s}_m)\omega_0^H} < \varepsilon$ for all $n > N_1$. Therefore, $\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} \to 0$. But then:

$$\frac{P^s(H, A \mid \mathbf{s}_n)}{P^s(L, A \mid \mathbf{s}_n)} = \frac{\beta_H(\mathbf{s}_n) \prod_{t=1}^{n} P(s_t \mid H, A)\, \omega_0^A \omega_0^L}{\beta_L(\mathbf{s}_n) \prod_{t=1}^{n} P(s_t \mid L, A)\, \omega_0^A \omega_0^H} = \frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} \frac{P(H, A \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} \to 0.$$

If $q_L = 1/2$, the posterior odds ratio also equals zero. A similar argument applies for (H, B) versus (L, B).

Let $(c, \theta) = (H, B)$. The posterior odds ratio of (H, B) versus (L, A) equals:

$$\frac{P^s(H, B \mid \mathbf{s}_n)}{P^s(L, A \mid \mathbf{s}_n)} = \frac{\beta_H(\mathbf{s}_n) \prod_{t=1}^{n} P(s_t \mid H, B)\, \omega_0^B \omega_0^L}{\beta_L(\mathbf{s}_n) \prod_{t=1}^{n} P(s_t \mid L, A)\, \omega_0^A \omega_0^H} = \frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} \frac{P(H, B \mid \mathbf{s}_n)}{P(L, A \mid \mathbf{s}_n)} \to 0.$$

since $\frac{\beta_H(\mathbf{s}_n)}{\beta_L(\mathbf{s}_n)} \to 0$ from the arguments above and $\frac{P(H,B|\mathbf{s}_n)}{P(L,A|\mathbf{s}_n)} \to 0$ from the arguments for the Bayesian. □

A.8. *Proof of Proposition 4*

1. Let ω_n^s equal the pre-screener's joint posterior after the sequence $\mathbf{s}_n$.
   First, note that each joint belief on the state and credibility for the prior ω_n^s, denoted $\omega_n^{c\theta}$, is given by

$$\omega_n^{c\theta} \equiv P^s(c, \theta|\mathbf{s}_n) = \frac{\left(\prod_{t=1}^{n} P(s_t|c, \theta)\right) \omega_0^\theta \omega_0^c \beta_c(\mathbf{s}_n)}{\sum_\theta \sum_c \left(\prod_{t=1}^{n} P(s_t|c, \theta)\right) \omega_0^\theta \omega_0^c \beta_c(\mathbf{s}_n)}. \tag{A.10}$$

   Thus, the Bayesian's posterior belief given the biased prior is

$$P(\theta = A|\text{prior} = \omega_n^s, \{s_{n+1}\}) = \frac{\omega_0^A \sum_c \left(\prod_{t=1}^{n+1} P(s_t|c, A)\right) \omega_0^c \beta_c(\mathbf{s}_n)}{\sum_\theta \omega_0^\theta \sum_c \left(\prod_{t=1}^{n+1} P(s_t|c, \theta)\right) \omega_0^c \beta_c(\mathbf{s}_n)}.$$

   In contrast, the pre-screener's posterior belief after observing $\mathbf{s}_{n+1}$ is

$$P^s(\theta = A|\mathbf{s}_{n+1}) = \frac{\omega_0^A \sum_c \left(\prod_{t=1}^{n+1} P(s_t|q, A)\right) \omega_0^c \beta_c(\mathbf{s}_{n+1})}{\sum_\theta \omega_0^\theta \sum_c \left(\prod_{t=1}^{n+1} P(s_t|c, \theta)\right) \omega_0^c \beta_c(\mathbf{s}_{n+1})},$$

   where

$$\beta_c(\mathbf{s}_{n+1}) = \prod_{m=1}^{n+1} \left(\sum_\theta \left(\prod_{t=1}^{m} P(s_t|c, \theta) \right) \omega_0^\theta \right) = \beta_c(\mathbf{s}_n) \left(\sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|c, \theta) \right) \omega_0^\theta \right). \tag{A.11}$$

   Substituting all of the preceding information into $P^s(\theta = A|\mathbf{s}_{n+1}) > P(\theta = A|prior = \omega_n^s, \{s_{n+1}\})$, the inequality is only satisfied if

$$\omega_0^A (1 - \omega_0^A) \omega_0^H (1 - \omega_0^H) \beta_L(\mathbf{s}_n) \beta_H(\mathbf{s}_n)$$
$$\times \underbrace{\left(\left(\sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|H, \theta) \right) \omega_0^\theta \right) - \left(\sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|L, \theta) \right) \omega_0^\theta \right) \right)}_{X}$$

$$\times \underbrace{\left(\left(\prod_{t=1}^{n+1} P(s_t|H,A)\right)\left(\prod_{t=1}^{n+1} P(s_t|L,B)\right) - \left(\prod_{t=1}^{n+1} P(s_t|H,B)\right)\left(\prod_{t=1}^{n+1} P(s_t|L,A)\right)\right)}_{Y}$$

$$> 0. \tag{A.12}$$

By direct calculation, $Y > 0$ if and only if $P(A|H;\mathbf{s}_{n+1}) > P(A|L;\mathbf{s}_{n+1})$, $Y = 0$ if and only if $P(A|H;\mathbf{s}_{n+1}) - P(A|L;\mathbf{s}_{n+1}) = 0$, and $Y < 0$ if and only if $P(A|H;\mathbf{s}_{n+1}) < P(A|L;\mathbf{s}_{n+1})$. Moreover, note that $\kappa_c(\mathbf{s}_n) \equiv \frac{\beta_c(\mathbf{s}_n)\omega_0^c}{\sum_c \beta_c(\mathbf{s}_n)\omega_0^c}$. Then Equation (A.11) implies that $\kappa_H(\mathbf{s}_{n+1}) > \kappa_H(\mathbf{s}_n)$ if and only if

$$\omega_0^H(1-\omega_0^H)\beta_H(\mathbf{s}_n)\beta_L(\mathbf{s}_n) \times \left(\left(\sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|H,\theta)\right)\omega_0^\theta\right) - \left(\sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|L,\theta)\right)\omega_0^\theta\right)\right) > 0,$$

which is the requirement that $X > 0$. Thus, $X > 0$ if and only if $\kappa_H(\mathbf{s}_{n+1}) > \kappa_H(\mathbf{s}_n)$, $X = 0$ if and only if $\kappa_H(\mathbf{s}_{n+1}) = \kappa_H(\mathbf{s}_n)$, and $X < 0$ if and only if $\kappa_H(\mathbf{s}_{n+1}) < \kappa_H(\mathbf{s}_n)$.

Since the sign of Equation (A.12) is determined by the sign of XY, this yields the proposition.

Finally, note that $P(H|\mathbf{s}_{n+1}) = \frac{\omega_0^H \sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta}{\sum_c \omega_0^c \sum_\theta \left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta}$. By direct calculation, $P(H|\mathbf{s}_{n+1}) > \omega_0^H$ if and only if $X > 0$. Thus, $\kappa_H(\mathbf{s}_{n+1}) > \kappa_H(\mathbf{s}_n)$ if and only if $P(\theta = H|\mathbf{s}_{n+1}) > \omega_0^H$, $\kappa_H(\mathbf{s}_{n+1}) = \kappa_H(\mathbf{s}_n)$ if and only if $P(c = H|\mathbf{s}_{n+1}) = \omega_0^H$, and $\kappa_H(\mathbf{s}_{n+1}) < \kappa_H(\mathbf{s}_n)$ if and only if $P(c = H|\mathbf{s}_{n+1}) < \omega_0^H$.

Below, we relate $\eta(\mathbf{s}_{n+1})$ to properties of the signal sources.

By direct calculation, $P(A|H;\mathbf{s}_{n+1}) - P(A|L;\mathbf{s}_{n+1}) \geq 0$ if and only if the number of a's is greater than or equal to the number of b's contained in $\mathbf{s}_{n+1}$.

Above we have shown that $\kappa_H(\mathbf{s}_{n+1}) - \kappa_H(\mathbf{s}_n) \geq 0$ if and only if $P(\theta = H|\mathbf{s}_{n+1}) \geq \omega_0^H$, which is true if and only if $X \geq 0$.

Let m_a be the number of a's in $\mathbf{s}_{n+1}$ and m_b be the number of b's in $\mathbf{s}_{n+1}$. Expanding X yields:

$$\omega_0^A q_H^{m_a}(1-q_H)^{m_b} + (1-\omega_0^A)(1-q_H)^{m_a} q_H^{m_b} \geq \omega_0^A q_L^{m_a}(1-q_L)^{m_b} + (1-\omega_0^A)(1-q_L)^{m_a} q_L^{m_b}. \tag{A.13}$$

Without loss of generality, suppose $m_a \geq m_b$. Then defining $d \equiv m_a - m_b$ where $d \geq 0$, we can re-write Equation (A.13) as

$$\left(\frac{q_H(1-q_H)}{q_L(1-q_L)}\right)^{m_b}\left(\frac{\omega_0^A q_H^d + (1-\omega_0^A)(1-q_H)^d}{\omega_0^A q_L^d + (1-\omega_0^A)(1-q_L)^d}\right) \geq 1, \tag{A.14}$$

which is identical to Equation (A.4). As we have shown in the proof of Proposition 2, for any $m_b > 0$, there exists some $d^* \geq 0$ such that Equation (A.14) equals 1 for $d = d^*$, Equation (A.14) is less than 1 for $d < d^*$, and Equation (A.14) is greater than 1 for $d > d^*$. Further, d^* increases in m_b and decreases in ω_0^A. The analogous result applies to $m_b \geq m_a$. Thus $P(c = \mathbf{s}_{n+1}) \geq \omega_0^H$ when the information content of $\mathbf{s}_{n+1}$ sufficiently favors one state, where stronger information is required when it is contrary to the prior on the state.

2. First, note that $P^s(c, \theta|\mathbf{s}_{n+1})$ is equal to

$$P^s(c,\theta|\mathbf{s}_{n+1}) = \frac{\beta_c(\mathbf{s}_{n+1})\left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c}{\sum_c \beta_c(\mathbf{s}_{n+1})\sum_\theta\left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c} \tag{A.15}$$

where $\beta_c(\mathbf{s}_{n+1})$ is described by Equation (A.11). Second, applying the generalized prescreening described in the Internet Appendix, $P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\})$ is equal to

$$P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\}) = \frac{\beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)P(s_{t+1}|c,\theta)\omega_n^{c\theta}}{\sum_c\sum_\theta \beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)P(s_{t+1}|c,\theta)\omega_n^{c\theta}},$$

where $\beta_{c\theta}(s_{n+1}) = \sum_\theta P(s_{n+1}|c,\theta)\omega_n^{c\theta}$ and $\omega_n^{c\theta}$ is described by Equation (A.10) and $\beta_c(\mathbf{s}_n)$ is described by Equation (9). Substituting this into $P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\})$ yields:

$$\begin{aligned}
&P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\}) \\
&= \frac{\beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)P(s_{t+1}|c,\theta)\beta_c(\mathbf{s}_n)\left(\prod_{t=1}^{n} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c}{\sum_c\sum_\theta \beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)P(s_{t+1}|c,\theta)\beta_c(\mathbf{s}_n)\left(\prod_{t=1}^{n} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c} \\
&= \frac{\beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)\beta_c(\mathbf{s}_n)\left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c}{\sum_c\sum_\theta \beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right)\beta_c(\mathbf{s}_n)\left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta\omega_0^c},
\end{aligned} \tag{A.16}$$

where

$$\beta_{c\theta}(s_{n+1})\left(\frac{1}{\sum_\theta \omega_n^{c\theta}}\right) = \beta_c(\mathbf{s}_n)\left(\sum_\theta\left(\prod_{t=1}^{n+1} P(s_t|c,\theta)\right)\omega_0^\theta\right)\left(\frac{\omega_0^c}{\sum_\theta \omega_n^{c\theta}}\right).$$

Equation (A.11) implies that Equation (A.16) equals Equation (A.15) if and only if $\omega_0^c = \sum_\theta \omega_n^{c\theta}$. Since $\omega_n^{c\theta} \equiv P^s(c,\theta|\mathbf{s}_n)$, then $P^s(c,\theta|\mathbf{s}_{n+1}) \neq P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\})$ if $P^s(c|\mathbf{s}_n) \neq \omega_0^c$ and $P^s(c,\theta|\mathbf{s}_{n+1}) = P^s(c,\theta|\text{prior} = \omega_n^s, \{s_{n+1}\})$ if $P^s(c|\mathbf{s}_n) = \omega_0^c$.

A.9. Proof of Proposition 5

Proof. Analogous to the one-source case, for the two-source case we can relate $\kappa_{c_2c_1}(\cdot)$ to $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{c_1}(\mathbf{s}_{\mathbf{n_1}})$ in the following way:

$$\kappa_{c_1c_2}(\mathbf{s}_{n_1,n_2}) = \frac{\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{c_1}(\mathbf{s}_{n_1})\omega_0^{c_1}\omega_0^{c_2}}{\sum_{c_1}\sum_{c_2}\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{c_1}(\mathbf{s}_{n_1})\omega_0^{c_1}\omega_0^{c_2}}.$$

Note that we can also write $\beta_{c_1}(\mathbf{s}_{n_1})$ and $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})$ as

$$\begin{aligned}
\beta_{c_1}(\mathbf{s}_n) &= \prod_{t_1=1}^{n_1}\frac{1}{\omega_0^{c_1}\omega_0^{c_2}}P(c_1,c_2|\mathbf{s}_{t_1}) \\
\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2}) &= \prod_{t_2=n_1+1}^{n_1+n_2}\frac{1}{\omega_0^{c_1}\omega_0^{c_2}}P(c_1,c_2|\mathbf{s}_{n_1,t_2}).
\end{aligned} \tag{A.17}$$

This means that $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{c_1}(\mathbf{s}_n)$ separates the key components of the pre-screener's credibility weight into two pieces. The term $\beta_{c_1}(\mathbf{s}_n)$ clearly depends on the order in which the pre-screener observes source 1's signals, and governs the first impressions of source 1's credibility (after source 1 has sent all its signals and before source 2 has sent any signals). The term $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})$ is the effect on the pre-screener's credibility weight from evaluating source 2's signals against source 1's signals, *holding aside the first impressions of source 1's credibility*. That is, this is how the pre-screener evaluates the signals from source 2 given source 1's entire body of signals, so $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2})$ does not depend on the order of source 1's signals.

For ease of exposition, let $\pi_{c_1c_2\theta} = \left(\prod_{t=n_1}^{n_1+n_2} P(s_{t2}|c_1, c_2, \theta)\right)\left(\prod_{t=n_1}^{n_1} P(s_{t1}|c_1, c_2, \theta)\right)$ where $c_1 \in \{H, L\}$, $c_2 \in \{H, L\}$ and $\theta \in \{A, B\}$. let $c_1 = \overline{c}_1$ and $c_2 = \overline{c}_2$.

1. We can write the pre-screener's posterior belief that source 1 is type H as the following:

$$
\begin{aligned}
&P^s(H_1|\mathbf{s}_{n_1,n_2}) \\
&= \frac{\kappa_{H_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1H_2\theta}\omega_0^\theta\right) + \kappa_{H_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1L_2\theta}\omega_0^\theta\right)}{\kappa_{H_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1H_2\theta}\omega_0^\theta\right) + \kappa_{H_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1L_2\theta}\omega_0^\theta\right) + \kappa_{L_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{L_1H_2\theta}\omega_0^\theta\right) + \kappa_{L_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{L_1L_2\theta}\omega_0^\theta\right)} \\
&= \frac{\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}\left(\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1H_2\theta}\omega_0^\theta\right) + \beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1L_2\theta}\omega_0^\theta\right)\right)}{\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}\left(\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1H_2\theta}\omega_0^\theta\right) + \beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{H_1L_2\theta}\omega_0^\theta\right)\right) + \left(\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{L_1H_2\theta}\omega_0^\theta\right) + \beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{L_1L_2\theta}\omega_0^\theta\right)\right)}.
\end{aligned}
\tag{A.18}
$$

   Note that only the term $\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$ depends on the order of source 1's signals. By direct differentiation of Equation (A.18), it is straightforward to show that $\frac{\partial P^s(H_1|\mathbf{s}_{n_1,n_2})}{\partial\left(\beta_{H_1}/\beta_{L_1}(\mathbf{s}_{n_1})\right)} > 0$.

2.

Lemma A.1. *$P^s(c_2|c_1;\mathbf{s}_{n_1,n_2})$ and $P^s(\theta|c_1;\mathbf{s}_{n_1,n_2})$ do not depend on the order of source 1's signals.*

Proof.

$$
\begin{aligned}
P^s(\overline{c}_2|\overline{c}_1;\mathbf{s}_{n_1,n_2}) &= \frac{\kappa_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{\overline{c}_1\overline{c}_2\theta}\omega_0^\theta\right)}{\sum_{c_2}\kappa_{\overline{c}_1c_2}(\mathbf{s}_{n_1,n_2})\left(\sum_\theta \pi_{\overline{c}_1c_2\theta}\omega_0^\theta\right)} \\
&= \frac{\beta_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\beta_{\overline{c}_1}(\mathbf{s}_{n_1})\omega_0^{\overline{c}_1}\omega_0^{\overline{c}_2}\left(\sum_\theta \pi_{\overline{c}_1\overline{c}_2\theta}\omega_0^\theta\right)}{\sum_{c_2}\beta_{\overline{c}_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{\overline{c}_1}(\mathbf{s}_{n_1})\omega_0^{\overline{c}_1}\omega_0^{c_2}\left(\sum_\theta \pi_{\overline{c}_1c_2\theta}\omega_0^\theta\right)} \\
&= \frac{\beta_{\overline{c}_1}(\mathbf{s}_{n_1})\omega_0^{\overline{c}_1}\beta_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\omega_0^{\overline{c}_2}\left(\sum_\theta \pi_{\overline{c}_1\overline{c}_2\theta}\omega_0^\theta\right)}{\beta_{\overline{c}_1}(\mathbf{s}_{n_1})\omega_0^{\overline{c}_1}\sum_{c_2}\beta_{\overline{c}_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{c_2}\left(\sum_\theta \pi_{\overline{c}_1c_2\theta}\omega_0^\theta\right)} \\
&= \frac{\beta_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\omega_0^{\overline{c}_2}\left(\sum_\theta \pi_{\overline{c}_1\overline{c}_2\theta}\omega_0^\theta\right)}{\sum_{c_2}\beta_{\overline{c}_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{c_2}\left(\sum_\theta \pi_{\overline{c}_1c_2\theta}\omega_0^\theta\right)}.
\end{aligned}
\tag{A.19}
$$

None of the terms in Equation (A.19) depend on the order of source 1's signals. Analogously, let $\theta = \overline{\theta}$. We can conduct the same exercise to obtain:

$$
P^s(\overline{\theta}|\overline{c}_1;\mathbf{s}_{n_1,n_2}) = \frac{\omega_0^{\overline{\theta}}\left(\sum_{c_2}\kappa_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\pi_{\overline{c}_1c_2\overline{\theta}}\right)}{\sum_\theta \omega_0^\theta\left(\sum_{c_2}\kappa_{\overline{c}_1\overline{c}_2}(\mathbf{s}_{n_1,n_2})\pi_{\overline{c}_1c_2\theta}\right)}
$$

$$= \frac{\omega_0^{\overline{\theta}} \left(\sum_{c_2} \pi_{\overline{c}_1 c_2 \overline{\theta}} \beta_{\overline{c}_1 c_2}(\mathbf{s}_{n_1,n_2}) \omega_0^{c_2} \right)}{\sum_{c_2} \beta_{\overline{c}_1 c_2}(\mathbf{s}_{n_1,n_2}) \omega_0^{c_2} \left(\sum_\theta \pi_{\overline{c}_1 c_2 \theta} \omega_0^{\theta} \right)}. \tag{A.20}$$

None of the terms in Equation (A.20) depend on the order of source 1's signals. □

Note that

$$\begin{aligned}
&P^s(H_2|\mathbf{s}_{n_1,n_2}) \\
&= P^s(H_2|H_1;\mathbf{s}_{n_1,n_2})P^s(H_1|\mathbf{s}_{n_1,n_2}) + P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})P^s(L_1|\mathbf{s}_{n_1,n_2}) \\
&= P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) + P^s(H_1|\mathbf{s}_{n_1,n_2})[P^s(H_2|H_1;\mathbf{s}_{n_1,n_2}) - P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})]
\end{aligned}$$

where

$$\begin{aligned}
P^s(H_2|H_1;\mathbf{s}_{n_1,n_2}) &= \frac{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)\kappa_{H_1H_2}(\mathbf{s}_{n_1,n_2})}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)\kappa_{H_1H_2}(\mathbf{s}_{n_1,n_2}) + P(\mathbf{s}_{n_1,n_2}|H_1,L_2)\kappa_{H_1L_2}(\mathbf{s}_{n_1,n_2})} \\
P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) &= \frac{P(\mathbf{s}_{n_1,n_2}|L_1,H_2)\kappa_{L_1H_2}(\mathbf{s}_{n_1,n_2})}{P(\mathbf{s}_{n_1,n_2}|L_1,H_2)\kappa_{L_1H_2}(\mathbf{s}_{n_1,n_2}) + P(\mathbf{s}_{n_1,n_2}|L_1,L_2)\kappa_{L_1L_2}(\mathbf{s}_{n_1,n_2})}.
\end{aligned}$$

Holding source 1's information content fixed, we have shown that $P^s(H_1|\mathbf{s}_{n_1,n_2})$ increases in $\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$. By Lemma A.1, $P^s(H_2|H_1;\mathbf{s}_{n_1,n_2})$ and $P^s(H_2|L_1;\mathbf{s}_{n_1,n_2})$ do not vary with $\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$. Expanding this out, we have $P^s(H_2|H_1;\mathbf{s}_{n_1,n_2}) - P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) > 0$ if and only if

$$\begin{aligned}
\frac{\kappa_{H_1H_2}(\mathbf{s}_{n_1,n_2})\kappa_{L_1L_2}(\mathbf{s}_{n_1,n_2})}{\kappa_{H_1L_2}(\mathbf{s}_{n_1,n_2})\kappa_{L_1H_2}(\mathbf{s}_{n_1,n_2})} &> \frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)} \\
\frac{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\beta_{H_1}(\mathbf{s}_1)\beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1}(\mathbf{s}_1)}{\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\beta_{H_1}(\mathbf{s}_1)\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1}(\mathbf{s}_1)} &> \frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)} \\
\frac{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})}{\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})} &> \frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)}.
\end{aligned}$$

Using Equation (A.17), we can say that $P^s(H_2|H_1;\mathbf{s}_{n_1,n_2}) - P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) > 0$ if and only if

$$\begin{aligned}
&\frac{\prod_{t_2=n_1+1}^{n_1+n_2} \frac{1}{(\omega_0^H)^2(1-\omega_0^H)^2} P(H_1,H_2|\mathbf{s}_{n_1,t_2})P(L_1,L_2|\mathbf{s}_{n_1,t_2})}{\prod_{t_2=n_1+1}^{n_1+n_2} \frac{1}{(\omega_0^H)^2(1-\omega_0^H)^2} P(H_1,L_2|\mathbf{s}_{n_1,t_2})P(L_1,H_2|\mathbf{s}_{n_1,t_2})} \\
&\quad > \frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)} \\
&\frac{\prod_{t_2=n_1+1}^{n_1+n_2} P(H_1,H_2|\mathbf{s}_{n_1,t_2})P(L_1,L_2|\mathbf{s}_{n_1,m_2})}{\prod_{t_2=n_1+1}^{n_1+n_2} P(H_1,L_2|\mathbf{s}_{n_1,t_2})P(L_1,H_2|\mathbf{s}_{n_1,t_2})} > \frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)}.
\end{aligned} \tag{A.21}$$

However, we can actually write the right-hand side of Equation (A.21) in the following way:

$$\begin{aligned}
&\frac{P(\mathbf{s}_{n_1,n_2}|H_1,L_2)P(\mathbf{s}_{n_1,n_2}|L_1,H_2)}{P(\mathbf{s}_{n_1,n_2}|H_1,H_2)P(\mathbf{s}_{n_1,n_2}|L_1,L_2)} \\
&\quad = \frac{\sum_\theta \left(P\left(s_{n_1,n_2} \mid H_1, L_2, \theta\right) \omega_0^\theta \right) \sum_\theta \left(P\left(s_{n_1,n_2} \mid L_1, H_2, \theta\right) \omega_0^\theta \right)}{\sum_\theta \left(P\left(s_{n_1,n_2} \mid H_1, H_2, \theta\right) \omega_0^\theta \right) \sum_\theta \left(P\left(s_{n_1,n_2} \mid L_1, L_2, \theta\right) \omega_0^\theta \right)}
\end{aligned}$$

$$= \frac{\frac{\sum_\theta P(s_{n_1,n_2}|H_1,L_2)\omega_0^\theta\omega_0^H\omega_0^L}{P(s_{n_1,n_2})}\frac{\sum_\theta P(s_{n_1,n_2}|L_1,H_2)\omega_0^\theta\omega_0^H\omega_0^L}{P(s_{n_1,n_2})}}{\frac{\sum_\theta P(s_{n_1,n_2}|H_1,H_2)\omega_0^H\omega_0^H\omega_0^\theta}{P(s_{n_1,n_2})}\frac{\sum_\theta P(s_{n_1,n_2}|L_1,L_2)\omega_0^\theta\omega_0^L\omega_0^L}{P(s_{n_1,n_2})}}$$

$$= \frac{P\left(H_1, L_2 \mid s_{n_1,n_2}\right) P\left(L_1, H_2 \mid s_{n_1,n_2}\right)}{P\left(H_1, H_2 \mid s_{n_1,n_2}\right) P\left(L_1, L_2 \mid s_{n_1,n_2}\right)}.$$

Thus Equation (A.21) can also be written as

$$\left(\frac{\prod_{t_2=n_1+1}^{n_1+n_2} P(H_1,H_2|\mathbf{s}_{n_1,t_2})P(L_1,L_2|\mathbf{s}_{n_1,t_2})}{\prod_{t_2=n_1+1}^{n_1+n_2} P(H_1,L_2|\mathbf{s}_{n_1,t_2})P(L_1,H_2|\mathbf{s}_{n_1,t_2})}\right) \times \left(\frac{P\left(H_1, H_2 \mid s_{n_1,n_2}\right) P\left(L_1, L_2 \mid s_{n_1,n_2}\right)}{P\left(H_1, L_2 \mid s_{n_1,n_2}\right) P\left(L_1, H_2 \mid s_{n_1,n_2}\right)}\right) > 1.$$

3. We will perform an analogous decomposition on the pre-screener's posterior belief about A given $\mathbf{s}_{n_1,n_2}$.

$$\begin{aligned}
P^s(A|\mathbf{s}_{n_1,n_2}) &= P(A|H_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_1,H_2|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|H_1,L_2;\mathbf{s}_{n_1,n_2})P^s(H_1,L_2|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,H_2;\mathbf{s}_{n_1,n_2})P^s(L_1,H_2|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_1,L_2|\mathbf{s}_{n_1,n_2}) \\
&= P(A|H_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|H_1,\mathbf{s}_{n_1,n_2})P^s(H_1|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|H_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|H_1,\mathbf{s}_{n_1,n_2})P^s(H_1|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|L_1,\mathbf{s}_{n_1,n_2})P^s(L_1|\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|L_1,\mathbf{s}_{n_1,n_2})P^s(L_1|\mathbf{s}_{n_1,n_2}) \\
&= P(A|L_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|L_1,\mathbf{s}_{n_1,n_2}) \\
&+ P^s(H_1|\mathbf{s}_{n_1,n_2})\big(P(A|H_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|H_1;\mathbf{s}_{n_1,n_2}) \\
&+ P(A|H_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|H_1;\mathbf{s}_{n_1,n_2}) \\
&- P(A|L_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) \\
&- P(A|L_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|L_1;\mathbf{s}_{n_1,n_2})\big) \\
P^s(A|\mathbf{s}_{n_1,n_2}) &= P(A|L_1,H_2;\mathbf{s}_{n_1,n_2})P^s(H_2|L_1;\mathbf{s}_{n_1,n_2}) \\
&+ P(A|L_1,L_2;\mathbf{s}_{n_1,n_2})P^s(L_2|L_1,\mathbf{s}_{n_1,n_2}) \\
&+ P^s(H_1|\mathbf{s}_{n_1,n_2})\big(P^s(A|H_1;\mathbf{s}_{n_1,n_2}) - P^s(A|L_1;\mathbf{s}_{n_1,n_2})\big). \qquad \text{(A.22)}
\end{aligned}$$

By Lemma A.1, the only part of Equation (A.22) that changes with $\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$ is $P^s(H_1|\mathbf{s}_{n_1,n_2})$. Thus, $P^s(A|\mathbf{s}_{n_1,n_2})$ increases with $\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$ if and only if $\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A|L_1;\mathbf{s}_{n_1,n_2})} > 1$, and decreases if and only if $<$.

Note that

$$P^s(A|H_1;\mathbf{s}_{n_1,n_2}) = \frac{\sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^A \kappa_{H_1c_2}}{\sum_\theta \sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^\theta \kappa_{H_1c_2}}$$
$$= \frac{\sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^A \beta_{H_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{H_1}(\mathbf{s}_{n_1})\omega_0^{c_2}\omega_0^{H_1}}{\sum_\theta \sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^\theta \beta_{H_1c_2}(\mathbf{s}_{n_1,n_2})\beta_{H_1}(\mathbf{s}_{n_1})\omega_0^{c_2}\omega_0^{H_1}}$$
$$= \frac{\sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^A \beta_{H_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{c_2}\omega_0^{H_1}}{\sum_\theta \sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^\theta \beta_{H_1c_2}(\mathbf{s}_{n_1,n_2})\omega_0^{c_2}\omega_0^{H_1}}. \tag{A.23}$$

The numerator of Equation (A.23) is expanded out as

$$\sum_{c_2} P(\mathbf{s}_{n_1,n_2}|c_2,H_1,A)\omega_0^A \beta_{c_2H_1}(\mathbf{s}_{n_1,n_2})\omega_0^{c_2}\omega_0^{H_1}$$
$$= \beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\omega_0^A\omega_0^H\omega_0^H P(\mathbf{s}_{n_1,n_2}|H_2,H_1,A)$$
$$+ \beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\omega_0^A\omega_0^H\omega_0^L P(\mathbf{s}_{n_1,n_2}|L_2,H_1,A),$$

and we can analogously expand the denominator. Multiplying the numerator and denominator of Equation (A.23) by $\frac{1}{P^s(\mathbf{s}_{n_1,n_2})}$, we can write this as

$$P^s(A|H_1;\mathbf{s}_{n_1,n_2})$$
$$= \frac{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})P(H_1,H_2,A|\mathbf{s}_{n_1,n_2}) + \beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})P(H_1,L_2,A|\mathbf{s}_{n_1,n_2})}{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})P(H_1,H_2|\mathbf{s}_{n_1,n_2}) + \beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})P(H_1,L_2|\mathbf{s}_{n_1,n_2})}.$$

Likewise,

$$P^s(A|L_1;\mathbf{s}_{n_1,n_2})$$
$$= \frac{\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})P(L_1,H_2,A|\mathbf{s}_{n_1,n_2}) + \beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})P(L_1,L_2,A|\mathbf{s}_{n_1,n_2})}{\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})P(L_1,H_2|\mathbf{s}_{n_1,n_2}) + \beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})P(L_1,L_2|\mathbf{s}_{n_1,n_2})}.$$

Thus, we have

$$\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A|L_1;\mathbf{s}_{n_1,n_2})} = \frac{\frac{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})P(H_1,H_2,A|\mathbf{s}_{n_1,n_2})+\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})P(H_1,L_2,A|\mathbf{s}_{n_1,n_2})}{\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})P(H_1,H_2|\mathbf{s}_{n_1,n_2})+\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})P(H_1,L_2|\mathbf{s}_{n_1,n_2})}}{\frac{\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})P(L_1,H_2,A|\mathbf{s}_{n_1,n_2})+\beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})P(L_1,L_2,A|\mathbf{s}_{n_1,n_2})}{\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})P(L_1,H_2|\mathbf{s}_{n_1,n_2})+\beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})P(L_1,L_2|\mathbf{s}_{n_1,n_2})}},$$

where recall that $\beta_{c_1c_2}(\mathbf{s}_{n_1,n_2}) = \prod_{t_2=n_1+1}^{n_1+n_2} P(c_1,c_2|\mathbf{s}_{n_1,t_2})$.

To illustrate possible cases of Part 3: In Table A.1, both sources' information content indicate state A. From Row 1 to Row 2, and from Row 3 to Row 4, changing the order of source 1's signals so that a's come earlier increases source 1's credibility. In Rows 1 and 2, source 2's early signals are consistent with source 1's information content, so the direct and indirect effects both move the pre-screener's final beliefs toward A. In Rows 3 and 4, source 2's early signals are inconsistent with source 1's information content, so the indirect effect is that source 2's credibility *decreases* even though source 2 objectively agrees with source 1. Moreover, the indirect effect on source 2's credibility is stronger the direct effect on source 1's credibility, so the pre-screener's final beliefs move *away from* A (note that $|n_{a2} - n_{b2}| > n_{a1} - n_{b1}$ here). □

Table A.1

Pre-screener's and Bayesian's beliefs with multiple sources. Parameter values equal $(q_H, q_L, \omega_0^A, \omega_0^{H_1}, \omega_0^{H_2}) = (0.7, 0.55, 0.5, 0$

beliefs are $P(H_1|\mathbf{s}_{n_1,n_2}) = 0.51442$, $P(H_2|\mathbf{s}_{n_1,n_2}) = 0.72946$, and $P(A|\mathbf{s}_{n_1,n_2}) = 0.96971$.

Row	$\mathbf{s}_{n_1}$	$\mathbf{s}_{n_2}$	$\frac{\beta_{H_1}(\mathbf{s}_{n_1})}{\beta_{L_1}(\mathbf{s}_{n_1})}$	$\frac{P^s(H_2\|H_1;\mathbf{s}_{n_1,n_2})}{P^s(H_2\|L_1;\mathbf{s}_{n_1,n_2})}$	$\frac{P^s(A\|H_1;\mathbf{s}_{n_1,n_2})}{P^s(A\|L_1;\mathbf{s}_{n_1,n_2})}$	$P^s(H$
1	$\{a, b, a\}$	$\{a, a, a, a, a, b, b, b, b, b, a, a, a, a, a, a, a, a\}$	0.71993	1.085837	1.01246	0.5194
2	$\{a, a, b\}$	$\{a, a, a, a, a, b, b, b, b, b, a, a, a, a, a, a, a, a\}$	0.97450	1.085837	1.01246	0.5940
3	$\{a, b, a\}$	$\{b, b, b, b, b, a, a, a, a, a, a, a, a, a, a, a, a, a\}$	0.71993	0.78062	0.99856	0.0210
4	$\{a, a, b\}$	$\{b, b, b, b, b, a, a, a, a, a, a, a, a, a, a, a, a, a\}$	0.97450	0.78062	0.99856	0.0283

A.10. Proof of Corollary 2

Proof. From Proposition 5 Part 3, a necessary and sufficient condition for $P^s(A|\mathbf{s}_{n_1,n_2})$ increasing in $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ is $\frac{P^s(A|H_1;\mathbf{s}_{n_1,n_2})}{P^b(A|L_1;\mathbf{s}_{n_1,n_2})} > 1$. For ease of exposition, let

$$\pi_{c_1c_2\theta} = \left(\prod_{t=n_1}^{n_1+n_2} P(s_{t2}|c_1, c_2, \theta)\right)\left(\prod_{t=n_1}^{n_1} P(s_{t1}|c_1, c_2, \theta)\right)$$

where $c_1, c_2 \in \{H, L\}$ and $\theta \in \{A, B\}$.

Expanding out $P^s(A|H_1; \mathbf{s}_{n_1,n_2}) - P^s(A|L_1; \mathbf{s}_{n_1,n_2})$, we obtain:

$$\begin{aligned}
&\text{sgn}\left(P^s(A|H_1; \mathbf{s}_{n_1,n_2}) - P^s(A|L_1; \mathbf{s}_{n_1,n_2})\right) \\
&\quad = (\omega_0^H)^2 \beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})\left(\pi_{H_1H_2A}\pi_{L_1H_2B} - \pi_{L_1H_2A}\pi_{H_1H_2B}\right) \\
&\quad + \omega_0^H(1-\omega_0^H)\beta_{H_1H_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})\left(\pi_{H_1H_2A}\pi_{L_1L_2B} - \pi_{L_1L_2A}\pi_{H_1H_2B}\right) \\
&\quad + \omega_0^H(1-\omega_0^H)\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\beta_{L_1H_2}(\mathbf{s}_{n_1,n_2})\left(\pi_{H_1L_2A}\pi_{L_1H_2B} - \pi_{L_1H_2A}\pi_{H_1L_2B}\right) \\
&\quad + (1-\omega_0^H)^2 \beta_{L_1L_2}(\mathbf{s}_{n_1,n_2})\beta_{H_1L_2}(\mathbf{s}_{n_1,n_2})\left(\pi_{H_1L_2A}\pi_{L_1L_2B} - \pi_{L_1L_2A}\pi_{H_1L_2B}\right).
\end{aligned} \tag{A.24}$$

The first and fourth terms of Equation (A.24) are strictly positive since $n_{a1} > n_{b1}$:

$$\begin{aligned}
&\pi_{H_1H_2A}\pi_{L_1H_2B} - \pi_{L_1H_2A}\pi_{H_1H_2B} \\
&\quad = [q_H(1-q_H)]^{n_{a2}+n_{b2}}[q_H(1-q_H)q_L(1-q_L)]^{n_{b1}} \\
&\qquad \times [(q_H(1-q_L))^{n_{a1}-n_{b1}} - (q_L(1-q_H))^{n_{a1}-n_{b1}}] > 0 \\
&\pi_{H_1L_2A}\pi_{L_1L_2B} - \pi_{L_1L_2A}\pi_{H_1L_2B} \\
&\quad = [q_L(1-q_L)]^{n_{a2}+n_{b2}}[q_H(1-q_H)q_L(1-q_L)]^{n_{b1}} \\
&\qquad \times [(q_H(1-q_L))^{n_{a1}-n_{b1}} - (q_L(1-q_H))^{n_{a1}-n_{b1}}] > 0.
\end{aligned}$$

Suppose that $n_{b2} - na2 \geq 0$. Then the third term is also positive since $n_{b2} \geq n_{a2}$ and $n_{a1} > n_{b1}$:

$$\begin{aligned}
&\pi_{H_1L_2A}\pi_{L_1H_2B} - \pi_{L_1H_2A}\pi_{H_1L_2B} \\
&\quad = [q_H(1-q_H)q_L(1-q_L)]^{n_{b1}+n_{a2}} \\
&\qquad \times [(q_H(1-q_L))^{n_{b2}-n_{a2}+n_{a1}-n_{b1}} - (q_L(1-q_H))^{n_{b2}-n_{a2}+n_{a1}-n_{b1}}] > 0.
\end{aligned}$$

The second term is non-negative if $n_{b2} - n_{a2} \leq n_{a1} - n_{b1}$, but negative if $n_{b2} - n_{a2} > n_{a1} - n_{b1}$:

$$\begin{aligned}
&\pi_{H_1H_2A}\pi_{L_1L_2B} - \pi_{L_1L_2A}\pi_{H_1H_2B} \\
&\quad = [q_H(1-q_H)q_L(1-q_L)]^{n_{b1}+n_{a2}}[(q_H(1-q_L))^{n_{a1}-n_{b1}}(q_L(1-q_H))^{n_{b2}-n_{a2}} \\
&\qquad - (q_H(1-q_L))^{n_{b2}-n_{a2}}(q_L(1-q_H))^{n_{a1}-n_{b1}}].
\end{aligned}$$

Thus if $n_{b2} - n_{a2} \geq 0$, then a necessary condition for $P^s(A|\mathbf{s}_{n_1,n_2})$ to be decreasing in $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ is $n_{b2} - n_{a2} > n_{a1} - n_{b1}$. If $n_{b2} - n_{a2} \leq n_{a1} - n_{b1}$, then $P^s(A|\mathbf{s}_{n_1,n_2})$ is decreasing in $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$.

Suppose that $n_{a2} - nb2 \geq 0$. Then the second term is also positive since $n_{a2} \geq n_{b2}$ and $n_{a1} > n_{b1}$:

$$
\begin{aligned}
&\pi_{H_1H_2A}\pi_{L_1L_2B} - \pi_{L_1L_2A}\pi_{H_1H_2B} \\
&\quad = [q_H(1-q_H)q_L(1-q_L)]^{n_{b1}+n_{b2}} \\
&\qquad \times [(q_H(1-q_L))^{n_{a2}-n_{b2}+n_{a1}-n_{b1}} - (q_L(1-q_H))^{n_{a2}-n_{b2}+n_{a1}-n_{b1}}] > 0.
\end{aligned}
$$

The third term is non-negative if $n_{a2} - n_{b2} \leq n_{a1} - n_{b1}$, but negative if $n_{a2} - n_{b2} > n_{a1} - n_{b1}$:

$$
\begin{aligned}
&\pi_{H_1L_2A}\pi_{L_1H_2B} - \pi_{L_1H_2A}\pi_{H_1L_2B} \\
&\quad = [q_H(1-q_H)q_L(1-q_L)]^{n_{b1}+n_{b2}}[(q_H(1-q_L))^{n_{a1}-n_{b1}}(q_L(1-q_H))^{n_{a2}-n_{b2}} \\
&\qquad - (q_H(1-q_L))^{n_{a2}-n_{b2}}(q_L(1-q_H))^{n_{a1}-n_{b1}}].
\end{aligned}
$$

Thus if $n_{a2} - n_{b2} \geq 0$, then a necessary condition for $P^s(A|\mathbf{s}_{n_1,n_2})$ to be decreasing in $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ is $n_{b2} - n_{a2} > n_{a1} - n_{b1}$. If $n_{a2} - n_{ab} \leq n_{a1} - n_{b1}$, then $P^s(A|\mathbf{s}_{n_1,n_2})$ is increasing in $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$.

Thus, a necessary condition for increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ to decrease $P^s(A|\mathbf{s}_{n_1,n_2})$ is $|n_{a2} - n_{b2}| > n_{a1} - n_{b1}$. A sufficient condition for increasing $\beta_{H_1}/\beta_{L_1}(\mathbf{s_{n_1}})$ to increase $P^s(A|\mathbf{s}_{n_1,n_2})$ is $|n_{a2} - n_{b2}| \leq n_{a1} - n_{b1}$. □

Appendix B. Application: Speculative trade, bubbles, and crashes

B.1. Proof of Proposition 6 (speculative trade)

Let $(\omega^A_{-\tau}, \omega^H_{-\tau}) = (\hat{\theta}, \hat{\omega})$ for any $\hat{\theta} \in (0, 1)$ and $\hat{\omega} \in (0, 1)$. After observing $n_{a,t} = n_{b,t}$ (in any order), the disagreement about the state is zero by Proposition 1.

Lemma B.1. *If group Y offers the price and group X takes the price, then in any period t the price of the asset is $p^s_t = E^X_t(R)$. If group Y^{EB} offers the price and group X^{EB} takes the price, then in any period t the price of the asset is $p_t = E^{X^{EB}}_t(R)$.*

Proof. We can determine the price in any period $t \in \{1, 2, \dots T\}$ by backwards induction, as in the proof of Lemma 2 of Harris and Raviv (1993). In period T, Y buys from X if and only if $E^Y_T(R) \geq p_T$ where p_T is the time-T price. Since Y has all the bargaining power, she offers to buy at $p_T = E^X_T(R)$, which X accepts (and receives zero expected utility). Conversely, Y sells to X if and only if $E^Y_T(R) < p_T$, and sells at $p_T = E^X_T(R)$.

In period $T-1$, consider what price Y would offer if she wants to trade (buy or sell). X is willing to buy if $E^X_{T-1}(p_T) \geq p_{T-1}$. Since X expects Y to offer $p_T = E^X_T(R)$ in period T, then $E^X_{T-1}(p_T) = E^X_{T-1}(E^Y_T(E^X_T(R))$. Since X thinks Y thinks X is Bayesian, X thinks that Y will calculate X's period-T belief by taking X's current $T-1$ belief and combine it with the likelihood of the new signal s_T using Bayes' Rule. Call this value $E^{X,u}_T(R)$. Then from the perspective of X in period $T-1$, the two possible outcomes in period T for what Y offers are either $E^{X,u}_T(R \mid s_T = a)$ or $E^{X,u}_T(R \mid s_T = b)$. This implies that $E^X_{T-1}(E^Y_T(E^X_T(R)) = E^X_{T-1}(E^{X,u}_T(R))$.

Given $E^X_{T-1}(E^Y_T(E^X_T(R)) = E^X_{T-1}(E^{X,u}_T(R))$, we will now show that $E^X_{T-1}(E^{X,u}_T(R)) = E^X_{T-1}(R)$ even though X is anticipating the wrong offer $E^{X,u}_T(R)$. Suppose X holds beliefs $E^X_{T-1}(R)$, $P^X_{T-1}(s_T = a)$ and $P^X_{T-1}(s_T = b)$, but falsely believes these were derived from Bayes' Rule when in fact they were the result of pre-screening. X will try to calculate:

$$E_{T-1}^{X}\left(E_{T}^{Y}E_{T}^{X}(R)\right)=E_{T-1}^{X}\left(E_{T}^{X,u}(R)\right)=P_{T-1}^{X,b}(s_T=a)\,E_{T}^{X,u}(R\mid s_T=a)$$
$$+P_{T-1}^{X,b}(s_T=b)\,E_{T}^{X,u}(R\mid s_T=b). \qquad \text{(B.1)}$$

Direct calculation shows:

$$P_{T-1}^{X}(s_T=a)=\sum_c\sum_\theta P(s_T=a\mid c,\theta)\,P^s(c,\theta\mid \mathbf{s}_{T-1})$$

$$P^s(c,\theta\mid\mathbf{s}_{T-1})=\frac{\beta_c(\mathbf{s}_{T-1})\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}$$

$$\Rightarrow P_{T-1}^{X}(s_T=a)=\frac{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta P(s_T=a\mid c,\theta)\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}$$

$$E_{T}^{X,u}(R\mid s_T=a)=\frac{\beta_c(\mathbf{s}_{T-1})\prod_{t=1}^{T-1}P(s_T=a\mid c,\theta)\,P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_T=a\mid c,\theta)\,P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}$$

$$\Rightarrow P_{T-1}^{X}(s_T=a)\,E_{T}^{X,u}(R\mid s_T=a)=\frac{\beta_c(\mathbf{s}_{T-1})\prod_{t=1}^{T-1}P(s_T=a\mid c,\theta)\,P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}$$

$$P_{T-1}^{X}(s_T=b)\,E_{T}^{X,u}(R\mid s_T=b)=\frac{\beta_c(\mathbf{s}_{T-1})\prod_{t=1}^{T-1}P(s_T=b\mid c,\theta)\,P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}$$

Since $P(s_T=a\mid c,\theta)+P(s_T=b\mid c,\theta)=1$, then:

$$\begin{aligned}&P_{T-1}^{X}(s_T=a)\,E_{T}^{X,u}(R\mid s_T=a)+P_{T-1}^{X}(s_T=b)\,E_{T}^{X,u}(R\mid s_T=b)\\&=\frac{\beta_c(\mathbf{s}_{T-1})\sum_c\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}{\sum_c\beta_c(\mathbf{s}_{T-1})\sum_\theta\prod_{t=1}^{T-1}P(s_t\mid c,\theta)\,\omega_0^\theta\omega_0^c}\\&=E_{T-1}^{X}(R).\end{aligned}\qquad \text{(B.2)}$$

Thus, $E_{T-1}^{X}\left(E_{T}^{X,u}(R)\right)=E_{T-1}^{X}(R)$.

Combining the insights above:

$$E_{T-1}^{X}(p_T)=E_{T-1}^{X}(E_{T}^{Y}(E_{T}^{X}(R))=E_{T-1}^{X}\left(E_{T}^{X,u}(R)\right)=E_{T-1}^{X}(R).$$

Since Y offers X so that X has zero reservation utility, Y offers $p_{T-1}=E_{T-1}^{X}(p_T)=E_{T-1}^{X}(R)$. Note that Y must know how X does the calculation in Equation (B.2). Therefore, Y must know that X thinks Y thinks X is a Bayesian. Likewise, X is willing to sell if $p_{T-1}\geq E_{T-1}^{X}(R)$ so Y will offer $p_{T-1}=E_{T-1}^{X}(R)$. And so on for all preceding periods. Thus, in any given period t, we have $p_t^s=E_t^X(R)$.

An analogous argument applies for trade between X^{EB} and Y^{EB}. Thus, in any given period $t=1,2,\dots,T$, we have $p_t^{EB}=E_t^{X^{EB}}(R)$. □

Lemma B.2. *In period T, agent Y will hold the asset if and only if $E_T^Y(R)\geq E_T^X(R)$. In period $t=1,2\dots,T-1$, agent Y will hold the asset if and only if $E_t^Y(E_{t+1}^X(R))\geq E_T^X(R)$.*

Likewise, in period T, agent Y^{EB} will hold the asset if and only if $E_T^{Y^{EB}}(R)\geq E_T^{X^{EB}}(R)$. In period $t=1,2\dots,T-1$, agent Y^{EB} will hold the asset if and only if $E_t^{Y^{EB}}(E_{t+1}^{Y^{EB}}(R))\geq E_T^{X^{EB}}(R)$.

Proof. In period T, Y's expected utility from buying is $E_T^Y(R) - p_T$ and her expected utility from not buying is 0. Thus, Y will buy if and only if $E_T^Y(R) \geq p_T$. Since $p_T = E_T^X(R)$ by Lemma B.1, then Y will buy if and only if $E_T^Y(R) \geq E_T^X(R)$. If Y is holding the asset, Y's expected utility from selling is p_T and her expected utility from not selling is $E_T^Y(R)$. Since $p_T = E_T^X(R)$ by Lemma B.1, then in period T, Y will buy if and only if $E_T^Y(R) \leq E_T^X(R)$.

In period $T-1$, Y's expected utility from buying over not buying is $E_{T-1}^Y(p_T) - p_{T-1}$, so Y will buy if and only if $E_{T-1}^Y(p_T) - p_{T-1} \geq 0$. Since $p_T = E_T^X(R)$ and Y knows that X is a pre-screener, then $E_{T-1}^Y(p_T) = E_{T-1}^Y(E_T^X(R))$. Since $p_{T-1} = E_{T-1}^X(R)$, then Y will buy if and only if $E_{T-1}^Y(E_T^X(R)) \geq E_{T-1}^X(R)$. And so on for all preceding periods. Thus, in any period $t = 1, 2 \ldots, T-1$, agent Y will hold the asset if and only if $E_t^Y(E_{t+1}^X(R)) \geq E_T^X(R)$.

An analogous argument applies to show the trading behavior of Y^{EB} when she trades with X^{EB}. □

Lemma B.3. *Suppose that pre-screener X observes signal path $\mathbf{s}_{pre}^X$ in the pre-period. In contrast, suppose pre-screener Y observes signal path $\mathbf{s}_{pre}^Y$ in the pre-period. Then both pre-screeners observe public signal path $Z = \mathbf{s}_t$ in the trading period. Let $n_{a,pre}^X = n_{a,pre}^Y$ and $n_{b,pre}^X = n_{b,pre}^Y$ where $n_{a,pre}^j + n_{b,pre}^j = n_{pre}^j$ for $j \in \{X, Y\}$, so signal paths $\mathbf{s}_{pre}^X$ and $\mathbf{s}_{pre}^Y$ contain the same information content. Let $c \in \{v, w\}$.*

If $P^s(v|\mathbf{s}_{pre}^X) > P^s(v|\mathbf{s}_{pre}^Y)$, then $P^s(v|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}) > P^s(v|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})$.

Proof. From the proof of Proposition 2, we already know that the necessary and sufficient condition for $P^s(v|\mathbf{s}_{pre}^X) > P^s(v|\mathbf{s}_{pre}^Y)$ is that $\beta_v^X \beta_w^Y - \beta_w^X \beta_v^Y > 0$.[22] Suppose that this holds.

Analogously, we can only have $P^s(v|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}) > P^s(v|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})$ if $\beta_v^{\{X,Z\}} \beta_w^{\{Y,Z\}} - \beta_w^{\{X,Z\}} \beta_v^{\{Y,Z\}} > 0$. Note that

$$\beta_c^{\{X,Z\}} = \beta_c^X b_c$$
$$\beta_c^{\{Y,Z\}} = \beta_c^Y b_c,$$

where

$$b_c = \prod_{m=1}^{t} \left(\sum_{\theta} \left(\prod_{i=-\tau+1}^{0} P(s_i|c,\theta) \right) \left(\prod_{i=1}^{m} P(s_i|c,\theta) \right) \omega_{-\tau}^{\theta} \right)$$

because $\prod_{i=-\tau+1}^{0} P(s_i|c,\theta)$ is the same for signal paths $\mathbf{s}_{pre}^X$ and $\mathbf{s}_{pre}^Y$ since they have the same information content. Thus,

$$\begin{aligned} \beta_v^{\{X,Z\}} \beta_w^{\{Y,Z\}} - \beta_w^{\{X,Z\}} \beta_v^{\{Y,Z\}} &= \beta_v^X b_v \beta_w^Y b_w - \beta_w^X b_w \beta_v^Y b_v \\ &= b_v b_w (\beta_v^X \beta_w^Y - \beta_w^X \beta_v^Y) > 0, \end{aligned}$$

since $\beta_v^X \beta_w^Y - \beta_w^X \beta_v^Y > 0$. □

Lemma B.4. *Suppose a pre-screener or Bayesian with prior $\omega_0 = \omega_0^{\theta} \omega_0^{c}$ observes $n_{a,t} = n_{b,t}$ signals. Then $P^s(\theta, c|\mathbf{s}_t) = P^s(\theta|\mathbf{s}_t) P^s(c|\mathbf{s}_t)$ and $P(\theta, c|\mathbf{s}_t) = P(\theta|\mathbf{s}_t) P^s(c|\mathbf{s}_t)\ \forall\, \theta, c$.*

[22] Note that this property does not require any restrictions on the c type-space or on the number a's and b's in the pre-period signals, only that X and Y have the same information content.

Proof. This is easily verified by direct calculation. □

First, suppose without loss of generality that groups X and Y observe $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$ such that $P^s(H|\mathbf{s}^X_{pre}) > P^s(H|\mathbf{s}^Y_{pre})$.

Lemma B.5. *Suppose two Bayesians X^{EB} and Y^{EB} are endowed with priors that coincide with the pre-screeners' posterior beliefs after the pre-period: $\omega_0^{X^{EB}} = \omega^X_{pre}$ and $\omega_0^{Y^{EB}} = \omega^Y_{pre}$. Then the* only *threshold at which X^{EB} and Y^{EB} trade is $n_{a,t} = n_{b,t}$: X^{EB} holds the asset when $n_{a,t} > n_{b,t}$ and Y^{EB} holds the asset when $n_{a,t} < n_{b,t}$.*

Proof. By Lemma B.4, each Bayesian is endowed with independent priors on the state and credibility, where $\omega_0^{j^{EB},A} = \omega^A_{-\tau}$ and $\omega_0^{j^{EB},H} = P^s(c|\mathbf{s}^j_{pre})$ where $j \in \{X, Y\}$. Clearly, the analogous argument and conclusion of Lemma B.3 apply to two endowed Bayesians: If $\omega_0^{X^{EB},H} > \omega_0^{Y^{EB},H}$, then $P(H|prior = \omega_0^{X^{EB}}, \mathbf{s}_t\}) > P(H|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t\})$. Thus, X^{EB} always trusts the source more than Y^{EB} does. By Proposition 1, this means that $P(H|prior = \omega_0^{X^{EB}}, \mathbf{s}_t) > P(H|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t)$ if and only if $P(A|prior = \omega_0^{X^{EB}}, \mathbf{s}_t) > P(A|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t)$ when $n_{a,t} > n_{b,t}$. Likewise, $P(H|prior = \omega_0^{X^{EB}}, \mathbf{s}_t) > P(H|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t)$ if and only if $P(A|prior = \omega_0^{X^{EB}}, \mathbf{s}_t) < P(A|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t)$ when $n_{a,t} < n_{b,t}$. When $n_{a,t} = n_{b,t}$, then by direct calculation $P(A|prior = \omega_0^{X^{EB}}, \mathbf{s}_t) = P(A|prior = \omega_0^{Y^{EB}}, \mathbf{s}_t) = \omega^A_{-\tau}$.

Thus, $E_t^{X^{EB}}(R) > E_t^{Y^{EB}}(R)$ when $n_{a,t} > n_{b,t}$, $E_t^{X^{EB}}(R) < E_t^{Y^{EB}}(R)$ when $n_{a,t} < n_{b,t}$, and $E_t^{X^{EB}}(R) = E_t^{Y^{EB}}(R)$ when $n_{a,t} = n_{b,t}$. Combining this with the law of iterated expectations also implies that $E_t^{X^{EB}}(R) > E_t^{Y^{EB}}[E_{t+1}^{X^{EB}}(R)] > E_t^{Y^{EB}}(R)$ when $n_{a,t} > n_{b,t}$, $E_t^{X^{EB}}(R) < E_t^{Y^{EB}}[E_{t+1}^{X^{EB}}(R)] < E_t^{Y^{EB}}(R)$ when $n_{a,t} < n_{b,t}$, and $E_t^{X^{EB}}(R) = E_t^{Y^{EB}}[E_{t+1}^{X^{EB}}(R)] = E_t^{Y^{EB}}(R)$ when $n_{a,t} = n_{b,t}$. Thus, by Lemma B.2, the only threshold at which X^{EB} and Y^{EB} trade in the trading period is $n_{a,t} = n_{b,t}$ (i.e., when the two sides "switch sides" at $n_{a,t} = n_{b,t}$). □

Consider pre-screeners X and Y. By Lemma B.3, since $P^s(H|\mathbf{s}^X_{pre}) > P^s(H|\mathbf{s}^Y_{pre})$, then $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P^s(H|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$. By Proposition 1, this means that when $n_{a,t} > n_{b,t}$, $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P^s(H|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$ if and only if $P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P^s(A|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$. Likewise, when $n_{a,t} < n_{b,t}$, $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P(H|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$ if and only if $P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) < P^s(A|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$. When $n_{a,t} = n_{b,t}$, then by direct calculation $P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) = P^s(A|\{\mathbf{s}^Y, \mathbf{s}_t\}) = \omega^A_{-\tau}$.

Thus, $E_t^X(R) > E_t^Y(R)$ when $n_{a,t} > n_{b,t}$, $E_t^X(R) < E_t^Y(R)$ when $n_{a,t} < n_{b,t}$, and $E_t^X(R) = E_t^Y(R)$ when $n_{a,t} = n_{b,t}$.

To determine trade between pre-screeners, we need to compare $E_t^Y(E_{t+1}^X(R))$ to $E_t^X(R)$.

First, we will show that if $\omega^A_{-\tau} = 1/2$, then pre-screeners always trade whenever their endowed Bayesian counterparts do.

Lemma B.6. *Suppose two pre-screeners have independent priors $\omega_{-\tau} = \omega^\theta_{-\tau}\omega^c_{-\tau}$ where $\omega^\theta_{-\tau} = 1/2$, and they observe pre-period signal paths $j \in \{X, Y\}$, respectively, where $n^j_{a,pre} = n^j_{b,pre} \geq 2$. Then X and Y always trade at the threshold $n_{a,t} = n_{b,t}$: X holds the asset when $n_{a,t} - n_{b,t} = 1$ and Y holds the asset when $n_{b,t} - n_{a,t} = 1$.*

Proof. Let ω_t^X denote pre-screener X's posterior belief after observing $\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}$. Note that:

$$\begin{aligned}&E_t^X(E_{t+1}^X(R)) - E_t^X(R)\\&= P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X,\mathbf{s}_t\})[P^s(A|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) - P(A|prior=\omega_t^X, s_{t+1}=a)]\\&+[1-P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X,\mathbf{s}_t\})][P^s(A|\{\mathbf{s}_{pre}^X,s_{t+1}=b\}) - P(A|prior=\omega_t^X,s_{t+1}=b)].\end{aligned}$$

Suppose $n_{a,t}=n_{b,t}+1$. By Lemma B.3, $P^s(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t\}) > P^s(H|\{\mathbf{s}_{pre}^Y,\mathbf{s}_t\})$. By Proposition 1, $E_t^X(R) > E_t^Y(R)$. We will show that for any $\omega_0^A \in (0,1)$, $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) < \omega_{-\tau}^H$ is a sufficient condition for X to hold the asset.

Suppose that $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) < \omega_{-\tau}^H$. By Proposition 4, $P^s(A|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) - P(A|prior=\omega_t^X,s_{t+1}=a) < 0$ and $P^s(A|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=b\}) - P(A|prior=\omega_t^X, s_{t+1}=b)=0$. This implies that $E_t^Y(E_{t+1}^X(R)) - E_t^X(R) < 0$. Thus, X holds the asset when $n_{a,t}=n_{b,t}+1$.

Suppose $n_{b,t}=n_{a,t}+1$. By Lemma B.3, $P^s(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t\}) > P^s(H|\{\mathbf{s}_{pre}^Y,\mathbf{s}_t\})$. By Proposition 1, $E_t^X(R) < E_t^Y(R)$. We will show that for any $\omega_0^A \in (0,1)$, $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=b\}) < \omega_{-\tau}^H$ is a sufficient condition for Y to hold the asset. Applying the preceding argument symmetrically, by Proposition 4, $P^s(A|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=b\}) - P(A|prior=\omega_t^X,s_{t+1}=b) > 0$ and $P^s(A|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) - P(A|prior=\omega_t^X,s_{t+1}=a)=0$. This implies that $E_t^Y(E_{t+1}^X(R)) - E_t^X(R) > 0$. Thus, Y holds the asset when $n_{b,t}=n_{a,t}+1$.

We verify that $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}\}) < \omega_{-\tau}^H$ is satisfied for $\omega_{-\tau}^A = 1/2$.

Suppose $n_{a,t}=n_{b,t}+1$. By direct calculation, $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) < \omega_{-\tau}^H$ if and only if

$$1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)^{n_b}\left(\frac{(q_L)^2+(1-q_L)^2}{(q_H)^2+(1-q_H)^2}\right),$$

where the first term of the right-hand side is strictly greater than one and strictly increasing in n_b. Thus a sufficient condition for $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=a\}) < \omega_{-\tau}^H$ is $1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)\left(\frac{(q_L)^2+(1-q_L)^2}{(q_H)^2+(1-q_H)^2}\right)$, which is satisfied since $\frac{\partial}{\partial q}(q(1-q)(q^2+(1-q)^2)) = -(2q-1)^3 \leq 0$.

Suppose $n_{b,t}=n_{a,t}+1$. Analogously, by direct calculation, $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=b\}) < \omega_{-\tau}^H$ if and only if

$$1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)^{n_a}\left(\frac{(1-q_L)^2+(q_L)^2}{(1-q_H)^2+(q_H)^2}\right),$$

where the first term of the right-hand side is strictly greater than one and strictly increasing in n_a. Thus a sufficient condition for $P(H|\{\mathbf{s}_{pre}^X,\mathbf{s}_t,s_{t+1}=b\}) < \omega_{-\tau}^H$ is $1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)\left(\frac{(1-q_L)^2+(q_L)^2}{(1-q_H)^2+(q_H)^2}\right)$, which is satisfied. Thus, X holds the asset when $n_{a,t}=n_{b,t}+1$ and Y holds the asset when $n_{b,t}=n_{a,t}+1$. At $n_{a,t}=n_{b,t}$, $E_t^Y(E_{t+1}^X(R)) = E_t^X(R)$ so Y is indifferent about holding or not holding the asset.

Then X and Y always trade at the threshold $n_{a,t}=n_{b,t}$: X holds the asset when $n_{a,t}-n_{b,t}=1$ and Y holds the asset when $n_{b,t}-n_{a,t}=1$. □

To show that speculative trade can occur when agents are pre-screeners, suppose a signal path $\mathbf{s}_t$ with $n_{a,t} > n_{b,t}$. We have shown in Lemma B.5 that endowed Bayesians will never trade in this case, and X will hold the asset. Moreover, we know that $p_t = E_t^X(R) > E_t^Y(R)$ when $n_{a,t} > n_{b,t}$.

Thus, the pre-screeners will only trade in period t if there exists some point at which Y buys the asset from X because $E_t^Y(E_{t+1}^X(R)) > E_t^X(R)$. Note that we can re-write this as:

$$
\begin{aligned}
&E_t^Y(E_{t+1}^X(R)) - E_t^X(R)\\
&= P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})E_{t+1}^X(R|s_{t+1}=a)\\
&+ (1-P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\}))E_{t+1}^X(R|s_{t+1}=b)\\
&- P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\})E_{t+1}(R|prior=\omega_t^X, s_{t+1}=a)\\
&- (1-P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}))E_{t+1}(R|prior=\omega_t^X, s_{t+1}=b)\\
&= P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\})\left(E_{t+1}^X(R|s_{t+1}=a) - E_{t+1}(R|prior=\omega_t^X, s_{t+1}=a)\right)\\
&+ (1-P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}))\left(E_{t+1}^X(R|s_{t+1}=b) - E_{t+1}(R|prior=\omega_t^X, s_{t+1}=b)\right)\\
&+ \left(P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}) - P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})\right)\\
&\times \left(E_{t+1}^X(R|s_{t+1}=b) - E_{t+1}^X(R|s_{t+1}=a)\right).
\end{aligned} \tag{B.3}
$$

By Proposition 4, Equation (B.3) is negative if we have signals such that $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1} = a\}) \le \omega_{-\tau}^H$. Thus, a necessary condition for Equation (B.3) to be positive is that we have signals such that $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=a\}) > \omega_{-\tau}^H$.

To demonstrate existence of the speculative trade, fix the parameters $q_c \in [1/2, 1)$, prior $\omega_{-\tau}$, and signal paths $\mathbf{s}_{pre}^X$ and $\mathbf{s}_t$ such that $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=b\}) \ge \omega_{-\tau}^H$, so the sum of the first two terms of Equation (B.3) is strictly positive. Since $n_a > n_b$ and $E_t^X(R) > E_t^Y(R)$, then the third term of Equation (B.3) is strictly negative because $P^s(s_{t+1} = a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\} > P^s(s_{t+1} = a|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})$ whenever $E_t^X(R) > E_t^Y(R)$. Note that the third term of Equation (B.3) is strictly increasing in $P^s(s_{t+1} = a|\{\mathbf{s}_{pre}^Y, \mathbf{s}_t\})$ and equals zero if $\mathbf{s}_{pre}^Y = \mathbf{s}_{pre}^X$. We will show that there exists some signal path $\bar{\mathbf{s}}_{pre}^Y$ such that Equation (B.3) is satisfied whenever $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1} = b\}) \ge \omega_{-\tau}^H$, so $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=b\}) \ge \omega_{-\tau}^H$ is a sufficient condition for extra trade.

To find the sequence(s) $\mathbf{s}_{pre}^Y$ to satisfy Equation (B.3) when $n_{a,pre}^Y = n_{a,pre}^X$ and $n_{b,pre}^Y = n_{b,pre}^X > 1$: Let $\mathbf{s}_{pre}^Y$ be identical to $\mathbf{s}_{pre}^X$ in all positions except for the last reversal pair (a, b), if it exists, in the sequence $\mathbf{s}_{pre}^X$. Call the positions of this pair $j+1$ and $j+2$ (so this means that $s_{j+1}^X = a$ and $s_{j+2}^X = b$). If $n_{a,pre}^j > n_{b,pre}^j$ (i.e., more a's have been observed than b's in the subsequence $\mathbf{s}_j^X$, which is the first j signals of $\mathbf{s}_{pre}^X$), then replace this (a, b) with (b, a) so that $s_{j+1}^Y = b$ and $s_{j+2}^Y = a$. If $n_{a,pre}^j \le n_{b,pre}^j$ or the pair (a, b) does not exist in $\mathbf{s}_{pre}^X$, then instead find the last reversal pair (b, a) in the sequence $\mathbf{s}_{pre}^X$. Call the positions of this pair $k+1$ and $k+2$ (so this means that $s_{k+1}^X = b$ and $s_{k+2}^X = a$). If $n_{b,pre}^k > n_{a,pre}^k$ (i.e., more b's have been observed than a's in the subsequence $\mathbf{s}_k^X$, which is the first k signals of $\mathbf{s}_{pre}^X$), then replace this (b, a) with (a, b) so that $s_{k+1}^Y = a$ and $s_{k+2}^X = b$. If $n_b^k \le n_a^k$, then continue by finding the second-to-last reversal pair (a, b) and applying this procedure, and so on. By the argument made in the proof of Corollary 3, this constructed sequence generates the greatest degree of trust such that we still have $P^s(H|\mathbf{s}_{pre}^Y) < P^s(H|\mathbf{s}_{pre}^X)$, and therefore by Lemma B.3 and Proposition 1 it generates the greatest belief in A such that we still have $E_t^Y(R|\{\bar{\mathbf{s}}_{pre}^Y, \mathbf{s}_t\})) < E_t^X(R|\{\mathbf{s}^X, \mathbf{s}_t\})$. By Corollary 3, we can construct such a sequence $\mathbf{s}_{pre}^Y$ as long as $\mathbf{s}_{pre}^X$ is not the sequence that generates the minimal degree of trust. This is already satisfied by assumption that $P^s(H|\mathbf{s}^X_{pre}) > P^s(H|\mathbf{s}^Y_{pre})$. We can continue constructing sequences that lead to decreasing degrees of trust by iterating in this procedure for each constructed $\mathbf{s}^Y_{pre}$.

Then there exists some signal path $\bar{\mathbf{s}}^Y_{pre}$ such that:

$$E^X_t(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > E^Y_t(R|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\})),$$

and:

$$E^Y_t(E^X_{t+1}(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}) - E^X_t(R) = 0.$$

This implies that for any signal path $\mathbf{s}^Y_{pre}$ that results in beliefs such that $P^s(s_{t+1} = a|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P^s(s_{t+1} = a|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\}) > P^s(s_{t+1} = a|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\})$, we thus have $E^X_t(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > E^Y_t(R|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}))$ and $E^Y_t(E^X_{t+1}(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}) - E^X_t(R) > 0$. Thus, Y will buy the asset from X. Likewise, for any signal path $\mathbf{s}^Y_{pre}$ that results in beliefs such that $P^s(s_{t+1} = a|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P^s(s_{t+1} = a|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}) > P^s(s_{t+1} = a|\{\mathbf{s}^Y_{pre}, \mathbf{s}_t\})$, we have $E^X_t(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > E^Y_t(R|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}))$ and $E^Y_t(E^X_{t+1}(R|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})|\{\bar{\mathbf{s}}^Y_{pre}, \mathbf{s}_t\}) - E^X_t(R) < 0$. Thus, Y will not buy the asset from X.

Suppose that $n_{a,t} < n_{b,t}$. As we have already shown, $p_t = E^X_t(R) < E^Y_t(R)$ when $n_{a,t} < n_{b,t}$. Thus, $E^X_t(R) < E^Y_t(E^X_{t+1}(R))$ when $n_{a,t} < n_{b,t}$ and no trade occurs because Y always holds the asset. Thus, $n_{a,t} > n_{b,t}$ is a necessary condition for speculative trade between pre-screeners to occur.

Suppose that $\mathbf{s}^X_{pre} = \mathbf{s}^Y_{pre}$. Since Y buys the asset whenever $E^Y_t(E^X_{t+1}(R)) \geq E^X_t(R)$, then Y always buys the asset if $\mathbf{s}^X_{pre} = \mathbf{s}^Y_{pre}$. Therefore, when $\mathbf{s}^X_{pre} = \mathbf{s}^Y_{pre}$, there is no trade between X and Y, nor between X^{EB} and Y^{EB}.

Second, suppose groups X and Y observe $\mathbf{s}^X_{pre}$ and $\mathbf{s}^Y_{pre}$ such that $P^s(H|\mathbf{s}^X_{pre}) < P^s(H|\mathbf{s}^Y_{pre})$. We can apply the exact same analysis as in the preceding case of $P^s(H|\mathbf{s}^X_{pre}) > P^s(H|\mathbf{s}^Y_{pre})$ to show that the mirror image holds when $P^s(H|\mathbf{s}^X_{pre}) < P^s(H|\mathbf{s}^Y_{pre})$. For brevity, we do not repeat it in detail. In particular, it is easy to show that Y will always hold the asset when $n_{a,t} > n_{b,t}$, and X will buy it at $n_{a,t} = n_{b,t}$. The key portion is that extra trade can only occur if $n_{b,t} < n_{a,t}$ and $E^Y_t(E^X_{t+1}(R)) - E^X_t(R) > 0$ as in Equation (B.3). Again by Proposition 4, Equation (B.3) is negative if we have signals such that $P(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t, s_{t+1} = a\}) \geq \omega^H_{-\tau}$. Thus, a necessary condition for Equation (B.3) to be positive is that we have signals such that $P(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t, s_{t+1} = a\}) < \omega^H_{-\tau}$. Likewise, the sufficient condition is that $P(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t, s_{t+1} = b\}) \leq \omega^H_{-\tau}$. When the sufficient condition is satisfied given $\mathbf{s}^X_{pre}$ and $\mathbf{s}_t$, then there exists at least one signal path $\mathbf{s}^Y_{pre}$ such that Y holds the asset.

Trade when $\omega^A_{-\tau} \neq 1/2$

If $\omega^A_{-\tau} \neq 1/2$, then all proofs in Proposition 6 apply with the exceptions of Lemma B.6 and Corollary 3. We discuss the generalizations below.

First, Lemma B.6 can be generalized to Lemma B.7.

Lemma B.7. *If $P^s(H|\mathbf{s}^X_{pre}) \neq P^s(H|\mathbf{s}^Y_{pre})$, then X^{EB} and Y^{EB} trade* only *when beliefs cross at $n_{a,t} = n_{b,t}$. A sufficient condition for pre-screeners X and Y to trade when beliefs cross at $n_{a,t} = n_{b,t}$ is $P(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t, s_{t+1}\}) < \omega^H_{-\tau}$ when $|n_{a,t} - n_{b,t}| = 1$.*

Proof. Let ω_t^X denote pre-screener X's posterior belief after observing $\{\mathbf{s}_{pre}^X, \mathbf{s}_t\}$. Note that:

$$\begin{aligned} E_t^X(E_{t+1}^X(R)) - E_t^X(R) = {} & P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\})[P^s(A|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=a\}) \\ & - P(A|prior=\omega_t^X, s_{t+1}=a)] \\ & + [1-P^s(s_{t+1}=a|\{\mathbf{s}_{pre}^X, \mathbf{s}_t\})][P^s(A|\{\mathbf{s}_{pre}^X, s_{t+1}=b\}) \\ & - P(A|prior=\omega_t^X, s_{t+1}=b)]. \end{aligned}$$

Suppose $n_{a,t}=n_{b,t}+1$. As shown in Lemma B.6 for any $\omega_0^A \in (0,1)$, $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=a\}) < \omega_{-\tau}^H$ is a sufficient condition for X to hold the asset.

Suppose $n_{b,t}=n_{a,t}+1$. As shown in Lemma B.6 for any $\omega_0^A \in (0,1)$, $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=b\}) < \omega_{-\tau}^H$ is a sufficient condition for Y to hold the asset.

At $n_{a,t}=n_{b,t}$, who holds the asset depends on ω_0^A. If $\omega_0^A=1/2$, then $E_t^Y(E_{t+1}^X(R))=E_t^X(R)$ so Y is indifferent about holding the asset. If $\omega_0^A \neq 1/2$, then $E_t^Y(E_{t+1}^X(R)) \neq E_t^X(R)$ so Y either strictly prefers to hold or not hold.

Sufficient conditions to have $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}\}) < \omega_{-\tau}^H$ when $|n_{a,t}-n_{b,t}|=1$ are as follows.

Suppose $n_{a,t}=n_{b,t}+1$. By direct calculation, $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=a\}) < \omega_{-\tau}^H$ if and only if

$$1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)^{n_b} \left(\frac{\omega_0^A(q_L)^2+(1-\omega_0^A)(1-q_L)^2}{\omega_0^A(q_H)^2+(1-\omega_0^A)(1-q_H)^2}\right),$$

where the first term of the right-hand side is strictly greater than one and strictly increasing in n_b. Thus a sufficient condition for $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=a\}) < \omega_{-\tau}^H$ is $1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)\left(\frac{\omega_0^A(q_L)^2+(1-\omega_0^A)(1-q_L)^2}{\omega_0^A(q_H)^2+(1-\omega_0^A)(1-q_H)^2}\right)$, which can also be written as

$$\begin{aligned} & q_L(1-q_L)^3 - q_H(1-q_H)^3 \\ & \quad - \omega_0^A[q_L(1-q_L)^3 - q_H(1-q_H)^3 + q_H^3(1-q_H) - q_L^3(1-q_L)] > 0. \end{aligned} \tag{B.4}$$

Suppose $n_{b,t}=n_{a,t}+1$. Analogously, by direct calculation, $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=b\}) < \omega_{-\tau}^H$ if and only if

$$1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)^{n_a} \left(\frac{\omega_0^A(1-q_L)^2+(1-\omega_0^A)(q_L)^2}{\omega_0^A(1-q_H)^2+(1-\omega_0^A)(q_H)^2}\right),$$

where the first term of the right-hand side is strictly greater than one and is strictly increasing in n_a. Thus a sufficient condition for $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}=b\}) < \omega_{-\tau}^H$ is $1 < \left(\frac{q_L(1-q_L)}{q_H(1-q_H)}\right)\left(\frac{\omega_0^A(1-q_L)^2+(1-\omega_0^A)(q_L)^2}{\omega_0^A(1-q_H)^2+(1-\omega_0^A)(q_H)^2}\right)$, which can also be written as

$$\begin{aligned} & q_H^3(1-q_H) - q_L^3(1-q_L) \\ & \quad - \omega_0^A[q_H^3(1-q_H) - q_L^3(1-q_L) + q_L(1-q_L)^3 - q_H(1-q_H)^3] < 0. \end{aligned} \tag{B.5}$$

Thus, sufficient conditions to have $P(H|\{\mathbf{s}_{pre}^X, \mathbf{s}_t, s_{t+1}\}) < \omega_{-\tau}^H$ when $|n_{a,t}-n_{b,t}|=1$ are that Equations (B.4) and (B.5) are satisfied.

These sufficient conditions do not unduly constrain the set of parameters $(\omega_{-\tau}^A, q_L, q_H)$ such that Lemma B.7 applies. As shown in the proof of Lemma B.6, these conditions are satisfied for any $1/2 \leq q_L < q_H < 1$ when $\omega^A_{-\tau} = 1/2$. Further, Equations (B.4) and (B.5) are satisfied for all $\omega^A_{-\tau} \in (0, 1)$ whenever we have (q_L, q_H) such that $b \equiv q_H^3(1 - q_H) - q_L^3(1 - q_L) \leq 0$. Note that $\frac{\partial b}{\partial q_H} = q_H^2(3 - 4q_H)$, which is strictly positive for $q_H < 3/4$ and strictly negative for $q_H > 3/4$. Combining with the fact that $b(q_H = q_L) = 0$ and $b(q_H = 1) < 0$, this implies that $b \leq 0$ whenever $q_L \geq 3/4$. Also, for any $q_L \in [1/2, 3/4)$, then there exists a unique $\underline{q}_H \in (q_L, 1)$ satisfying $b(\underline{q}_H) = 0$ such that for all $q_H \geq \underline{q}_H$, $b \leq 0$. □

Second, for $\omega^A_{-\tau} \neq 1/2$, one can find the sequences of pre-trade signals ordered by degrees of trust (analogous to Corollary 3), and therefore demonstrate the existence of speculative trade.

B.2. Proof of Proposition 7

Since the pre-screener and endowed Bayesian have the same beliefs at the start of the trading period, $p_0^s = p_0^{EB} = \omega^A_{-\tau}$. As can be seen from the proof of Proposition 1, $p_T^s = p_T^{EB} = \omega^A_{-\tau}$ when $n_{a,T} = n_{b,T}$. Also $n_{a,t} > n_{b,t}$ implies that $p_t^s > \omega^A_{-\tau}$ and $p_t^{EB} > \omega^A_{-\tau}$.

1. First, we demonstrate existence of $p_t^s > p_t^{EB} > \omega^A_{-\tau}$. Note that

$$\begin{aligned} p_t^s &= P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) \\ &= \frac{\omega^A_{-\tau} \sum_c \left(\prod_{i=-\tau+1}^0 P(s_i|q, A)\right)\left(\prod_{i=1}^t P(s_i|q, A)\right) \omega^c_{-\tau} \beta_c(\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})}{\sum_\theta \omega^\theta_{-\tau} \sum_c \left(\prod_{i=-\tau+1}^0 P(s_i|q, A)\right)\left(\prod_{i=1}^t P(s_i|c, \theta)\right) \omega^c_{-\tau} \beta_c(\{\mathbf{s}^X_{pre}, \mathbf{s}_t\})} \\ p_t^{EB} &= P(A|prior = \omega^X_{pre}, \mathbf{s}_t) \\ &= \frac{\omega^A_{-\tau} \sum_c \left(\prod_{i=-\tau+1}^0 P(s_i|q, A)\right)\left(\prod_{i=1}^t P(s_i|q, A)\right) \omega^c_{-\tau} \beta_c(\mathbf{s}^X_{pre})}{\sum_\theta \omega^\theta_{-\tau} \sum_c \left(\prod_{i=-\tau+1}^0 P(s_i|q, A)\right)\left(\prod_{i=1}^t P(s_i|c, \theta)\right) \omega^c_{-\tau} \beta_c(\mathbf{s}^X_{pre})} \end{aligned}$$

where

$$\begin{aligned} \beta_c(\mathbf{s}^X_{pre}) &= \prod_{m=-\tau}^{0} \left(\sum_\theta \left(\prod_i^m P(s_i|c, \theta) \right) \omega^\theta_{-\tau} \right) \\ \beta_c(\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) &= \beta_c(\mathbf{s}^X_{pre}) \prod_{m=1}^{n} \left(\sum_\theta \left(\prod_{i=-\tau+1}^{0} P(s_i|c, \theta) \right) \left(\prod_{i=1}^{m} P(s_i|c, \theta) \right) \omega^\theta_{-\tau} \right). \end{aligned}$$

By direct comparison, $p_t^s > p_t^{EB}$ if and only if $\omega^H_{-\tau}(1 - \omega^H_{-\tau})\beta_H(\mathbf{s}^X_{pre})\beta_L(\mathbf{s}^X_{pre})FG > 0$, where

$$\begin{aligned} F &= \left(\prod_{m=1}^{t} \left(\sum_\theta \left(\prod_{t=-\tau}^{0} P(s_i|H, \theta) \right) \left(\prod_{i=1}^{m} P(s_i|H, \theta) \right) \omega^\theta_{-\tau} \right) \right) \\ &\quad - \left(\prod_{m=1}^{t} \left(\sum_\theta \left(\prod_{t=-\tau}^{0} P(s_i|L, \theta) \right) \left(\prod_{i=1}^{m} P(s_i|L, \theta) \right) \omega^\theta_{-\tau} \right) \right) \\ G &= \left(\prod_{i=-\tau+1}^{0} P(s_i|H, A) \right) \left(\prod_{i=1}^{t} P(s_i|H, A) \right) \left(\prod_{i=-\tau+1}^{0} P(s_i|L, B) \right) \left(\prod_{i=1}^{t} P(s_i|L, B) \right) \end{aligned}$$

$$-\left(\prod_{i=-\tau+1}^{0} P(s_i|H,B)\right)\left(\prod_{i=1}^{t} P(s_i|H,B)\right)\left(\prod_{i=-\tau+1}^{0} P(s_i|L,A)\right)\left(\prod_{i=1}^{n} P(s_i|L,A)\right),$$

where we have already shown in the Proof of Proposition 4 that $G > 0$ when $n_{a,t} > n_{b,t}$, since $n^X_{a,pre} = n^X_{b,pre}$. Thus, $p^s_t > p^{EB}_t$ if and only if $F > 0$. Moreover, we can easily show that $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$ if and only if $F > 0$. Thus, $p^s_t > p^{EB}_t > \omega^A_{-\tau}$ if and only if $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$. By the argument in Corollary 3, there exists at least one path $\mathbf{s}_t$ that generates $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) > P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$. For example, if $\mathbf{s}_t$ contains $n_{a,t} > 1$ signals, we can show that there exists a unique threshold $\overline{n}_{a,t}(n_{b,t})$ such that $F > 0$ if $n_{a,t} > \overline{n}_{a,t}(n_{b,t})$ and $F \leq 0$ if $n_{a,t} \leq \overline{n}_{a,t}(n_{b,t})$. By direct computation, $F < 0$ when $n_{a,t} = n_{b,t}$, $\frac{\partial F}{\partial n_{a,t}} > 0$, and $\lim_{n_{a,t}\to\infty} F = \infty$. Thus, there exists a unique threshold $\overline{n}_{a,t}$ such that $p^s_t > p^{EB}_t$ for all $\overline{n}_{a,t} < n_{a,t} \leq n_{a,T}$ and $P^s_t \leq p^{EB}_t$ for all $0 \leq n_a \leq \overline{n}_a$.

Clearly, we can reverse the inequalities to show $p^{EB}_t > p^s_t > \omega^A_{-\tau}$ if and only if $P^s(H|\{\mathbf{s}^X_{pre}, \mathbf{s}_t\}) < P(H|prior = \omega^X_{pre}, \mathbf{s}_t)$.

2. Suppose we have $\mathbf{s}_T$ such that $p^s_{\hat{t}} > p^{EB}_{\hat{t}} > \omega^A_{-\tau}$. Since $p^s_0 = p^{EB}_0 = p^s_T = p^{EB}_T = \omega^A_{-\tau}$ and $p^s_{\hat{t}} > p^{EB}_{\hat{t}}$, then the average price change of p^s_t must be strictly greater than the average price change of p^{EB}_t for $t \in [0, \hat{t}]$ and $t \in [\hat{t}, T]$.
By Proposition 4, $p^s_1 < p^{EB}_1$ because $P(H|\{\mathbf{s}^X_{pre}, s_1 = a\}) < \omega^H_{-\tau}$ for $n^X_{a,pre} = n^X_{b,pre} \geq 1$. This implies that we can only have $p^b_{\hat{t}} > p^{EB}_{\hat{t}} \geq 0$ if there are sufficiently many a's that the pre-screener over-reacts (under-reacts) to confirming (disconfirming) signals (i.e., more a signals). By Proposition 4, if at least one such t' such that $p^s_{t'} > p^{EB}_{t'}$ exists, then it must be that $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{t'}) > \omega^H_{-\tau}$ for at least one of these t' dates. Suppose there exists some t'' such that $p^s_{t''} > p^{EB}_{t''}$ but $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{t'}) \leq \omega^H_{-\tau}$. Since $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{t'}) \leq \omega^H_{-\tau}$, then there must necessarily be a lower proportion of a's than b's observed at t'' than at t'. Thus it cannot be that $p^s_{t''} = \max p^s_t$ and we must have that $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}) > \omega^H_{-\tau}$ whenever $p^s_{\hat{t}} > p^{EB}_{\hat{t}}$.
Suppose $p^s_{\hat{t}} = p^{EB}_{\hat{t}}$ and $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}) > \omega^H_{-\tau}$ but $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b) < \omega^H_{-\tau}$. Since $p^s_{\hat{t}} \equiv \max P^s_t$, this implies $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, \mathbf{s}_t) < \omega^H_{-\tau}$ for all signal paths $\mathbf{s}_t$ with $t \in (\hat{t}, T]$ because they are below the peak. By Proposition 7, this implies that the pre-screener under-reacts to each a and over-reacts to each b in $t \in (\hat{t}, T]$. But this means that for any t such that $n_{a,t} = n_{b,t}$, $p^s_t < p^{EB}_t$, which cannot be true. Thus, $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b) \geq \omega^H_{-\tau}$ when $p^s_{\hat{t}} = p^{EB}_{\hat{t}}$. If $p^s_{\hat{t}} > p^{EB}_{\hat{t}}$, then the Bayesian posterior belief in accuracy must be even higher than when $p^s_{\hat{t}} = p^{EB}_{\hat{t}}$. Thus $P(H|\mathbf{s}^X, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b) \geq \omega^H_{-\tau}$ when $p^s_{\hat{t}} \geq p^{EB}_{\hat{t}}$.
Since $p_{\hat{t}} \equiv \max p_t$, then $s_{\hat{t}+1} = b$ and $p^{EB}_{\hat{t}+1} - p^{EB}_{\hat{t}} < 0$ and $p^s_{\hat{t}+1} - p^s_{\hat{t}} < 0$. Moreover, because have we have shown that $P(H|\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b) \geq \omega^H_{-\tau}$, then $P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b\}) > P(A|prior = \omega^b_{\hat{t}}, s_{\hat{t}+1} = b)$ by Proposition 7, where each joint belief for the prior $\omega^b_{\hat{t}}$, denoted by $\omega^{c\theta}_{\hat{t}}$, is the pre-screener's belief after observing signal path $\mathbf{s}^X_{pre}$ and the public path $\mathbf{s}_{\hat{t}}$:

$$\begin{aligned}\omega^{c\theta}_{\hat{t}} &\equiv P^s(c,\theta|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}\}) \\ &= \frac{\omega^A_{-\tau}\sum_c\left(\prod_{i=-\tau+1}^{0} P(s_i|q,A)\right)\left(\prod_{i=1}^{\hat{t}} P(s_i|q,A)\right)\omega^c_{-\tau}\beta_c(\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}\})}{\sum_\theta \omega^\theta_{-\tau}\sum_c\left(\prod_{i=-\tau+1}^{0} P(s_i|q,A)\right)\left(\prod_{i=1}^{\hat{t}} P(s_i|c,\theta)\right)\omega^c_{-\tau}\beta_c(\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}\})}.\end{aligned}$$

Since $p^s_{\hat{t}} = P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}\}) = P(A|prior = \omega^b_{\hat{t}})$ and $p^{EB}_{\hat{t}} = P(A|prior = \omega^X_{pre}, \mathbf{s}_{\hat{t}})$ by definition, then $p^s_{\hat{t}} > p^{EB}_{\hat{t}}$ implies $P(A|prior = \omega^b_{\hat{t}}) > P(A|prior = \omega^X_{pre}, \mathbf{s}_{\hat{t}})$. Thus $0 > P(A|prior = \omega^b_{\hat{t}}, s_{\hat{t}+1} = b) - P(A|prior = \omega^b_{\hat{t}}) > P(A|prior = \omega^X_{pre}, \{\mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b\}) - P(A|prior = \omega^X_{pre}, \mathbf{s}_{\hat{t}})$. Combining this with the fact that $P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b\}) > P(A|prior = \omega^b_{\hat{t}}, s_{\hat{t}+1} = b)$ by Proposition 7, then $0 > P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b\}) - P^s(A|\{\mathbf{s}^X_{pre}, \mathbf{s}_{\hat{t}}\}) > P(A|prior = \omega^X_{pre}, \{\mathbf{s}_{\hat{t}}, s_{\hat{t}+1} = b\}) - P(A|prior = \omega^X_{pre}, \mathbf{s}_{\hat{t}})$. Thus, $|p^s_{\hat{t}+1} - p^s_{\hat{t}}| < |p^{EB}_{\hat{t}+1} - p^{EB}_{\hat{t}}|$.

3. If $p^s_{\hat{t}} > p^{EB}_{\hat{t}} > \omega^A_{-\tau}$, then we have already shown in the proof of Proposition 7 that the necessary and sufficient conditions given in Proposition 6 must hold. Thus, there exists at least one signal path $\mathbf{s}^Y_{pre}$ such that speculative trade between pre-screeners occurs (e.g., Y holds the asset at $t = \hat{t}$, at least).

Appendix C. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2021.105401.

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