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Risk seekers: trade, noise, and the rationalizing effect of market impact on convex preferences

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Risk seekers: trade, noise, and the rationalizing effect of market impact on convex preferences*

Efstathios Avdis

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Abstract

Long-held intuition dictates that information-based trade is impossible without exogenous noise. Risk seekers can resolve this conundrum. Even though such agents have negative risk aversion, they act as utility maximizers because they fully internalize their impact on prices. If their love of risk increases, information decreases in the aggregate, making prices noisier and returns more volatile. If public information becomes more precise, risk sharing decreases but welfare increases, contradicting the Hirshleifer effect. If private information becomes cheaper, liquidity always increases, rendering economies with risk seekers empirically distinct from economies with noise traders or random endowments.

Keywords: Rationality, inefficient markets, information acquisition, liquidity.

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1 Introduction

According to long-held intuition, it is impossible for rational agents to trade with each other based on their information (Milgrom and Stokey, 1982; Tirole, 1982). This conundrum poses a challenge to financial economics in general, and to explaining why markets are inefficient in particular. For markets to be inefficient, we must have an equilibrium in which prices contain noise unrelated to fundamentals. And yet, as the no-trade conundrum prescribes, this noise must come from outside the agents' knowledge—even if we assume their knowledge to be noisy to begin with—because only then can the agents trade on their information, and only then can markets be partially efficient (Grossman and Stiglitz, 1980, and others). It is thus widely believed that inefficient markets are inconsistent with information-based trade, and that to study them we must employ devices such as liquidity trading, random endowments, and behavioral effects.

It is, however, possible for rational agents to trade without us appealing to such devices. As I show in this article, we can generate information-based trade within an otherwise standard economy, if we relax our assumptions about how people think of risk: as opposed to assuming that people avoid risk, we can instead assume that they seek it. As I explain below, such risk seekers are fully rational, and their presence is necessary because, as the literature has shown in various settings, we cannot have equilibrium if everyone is risk neutral or risk averse.

Risk seeking thus enables information-based trade with noisy prices, allowing us to think of prices as imperfect aggregators of diverse information. What is more; it gives us a model where all agents have incentives to acquire costly information, overturning Grossman (1976)'s intuition on the irrelevance of private signals. Finally, as all agents in the economy are shown to be rational, it also helps us distinguish risk-seeking economies from canonical economies with noise traders or random endowments, further implying that risk seeking is not merely a restatement of stochastic mechanisms. In short, to describe inefficient markets we need neither redefine rationality nor look beyond it. We need only expand what we currently categorize as suitable risk attitudes.

It may, of course, appear that risk seeking is a troublesome ingredient of any rational foundation of inefficient markets. After all, intuition from price-taking models dictates that with negative risk aversion the agents' demand-choice problem is not well posed. Nevertheless, for the following reasons, risk seeking is not only rational but also economically salient.

First, the risk seekers' demand choice is well posed indeed. Assuming, as in Kyle (1989), that agents have exponential utility, their second-order condition is the sum of their market impact and of a risk-attitude term, so that—if market impact is large enough—the agents' optimization problem is concave in equilibrium even with negative risk aversion. In fact, the more these risk seekers like risk, the more aggressively they trade, and the more market impact they have. Overpowering the convexity of their preferences, their market impact therefore acts as a self-sustaining force both of their own rationality and of the equilibrium overall. A good analogy is that of a streetcar operator who, while wanting to drive fast to meet his schedule, is also able to pull the breaks when his wagon is about to come off its tracks.

Second, we can motivate risk-seeking behavior on several fronts. As I discuss in the following section, that some people like risk is a common finding in empirical and experimental work. We can thus appeal to risk seeking as an innate preference. Alternatively, we can think of negative risk aversion in three different ways: as a reduced-form model for institutional traders whose contracts reward taking on risk (Brown, Harlow, and Starks, 1996; Chevalier and Ellison, 1997; Panageas and Westerfield, 2009), as an abstraction for utility which stays within the tractable exponential class while speaking to evidence that people are prudent and intemperate (Deck and Schlesinger, 2010), or even as a stylized version of prospect theory (Kahneman and Tversky, 1979) with an exaggerated loss domain.

To focus the discussion on the traders, I use a model with one trading period and one risky asset. Opting for simplicity of exposition, I adopt a market structure based on Kyle (1985), albeit without noise traders: there is a representative market maker and many strategic traders who submit orders without observing prices. This type of market, known in the literature as a "market-order" model, allows us to abstract away from effects of learning from prices. This happens without loss of generality for the main message of this paper—as I show by extending the model to include "limit orders," the equilibrium with traders who do observe prices exists if and only if one exists with traders who do not.[1]

I present the main intuition using a benchmark with homogeneous risk seekers. Setting the empirical plausibility of such preferences aside, this model serves two purposes. First, we can elucidate that market impact is directly related to how much the traders like risk—this relationship becomes more opaque once we extend the model to heterogeneous risk attitudes. Second, under information acquisition, we can empirically distinguish the economy with risk seekers from canonical economies with noisy supply, irrespective of whether such supply comes from noise traders or random endowments: when information becomes cheaper, liquidity always increases in the economy with risk seekers, whereas in the ones with noisy supply it may decrease.

This distinction arises because, on the one hand, rational agents fully internalize their impact on the market, scaling back their trading aggression if necessary.[2] Noisy supply, on

[1] This extension also shows that risk seeking generates incentives to use private information, suggesting that a result of Grossman (1976) on the irrelevance of private information is due to a joint assumption of price taking and risk aversion. See Section 3.2 for details.

[2] This effect is also present in Subrahmanyam (1991), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), and others.

the other hand, is exogenous, and does not respond to any changes in the economy. While the rational agents always acquire more information when it becomes cheaper, the way their trading affects the price depends on which economy they are in.

In the risk-seeking economy all agents are informed and rational; here the incentives to compete with others outweigh any concerns to scale back, so that when information becomes cheaper all agents behave more aggressively, increasing liquidity. In the economies with noisy supply, however, even though the informed agents compete more intensely as information becomes cheaper, the rate at which they put information into the price—slowed down by supply noise—may be overwhelmed by their concerns to scale back their aggression. When information becomes cheaper the informed agents may thus behave less aggressively overall, decreasing liquidity.

Beyond the above prediction, risk seeking offers several more takeaways.

We can connect risk-seeking attitudes with the equilibrium amount of risk in a financial asset. More specifically, the inverse of the asset's price informativeness acts as an implicit discount rate, allowing us to think of the price as a discounted noisy dividend. Due to a substitution effect in information acquisition that I discuss below, the price is noisier *in equilibrium* the more the agents like risk. It follows that the more the agents like risk, the less information the price contains, and the riskier the asset returns become.

We can also use risk seeking to sharpen our understanding of the Hirshleifer effect. First stated in Hirshleifer (1971), this effect contends that public information is bad because it destroys risk sharing. Risk seeking overturns this intuition. Using a measure of risk sharing based on a Herfindahl-Hirschman index of equity holdings, I show that risk sharing decreases as public information becomes more precise, but not at the detriment of the risk seekers' welfare. In fact, more precise public information is better for the agents, precisely because it concentrates risk, which they like.

Finally, a question left open by the above is this: what happens if some agents are rational, yet not risk-seeking? Do we still have equilibrium then? To address this point we must use an economy in which risk seeking coexists with other risk attitudes. Nevertheless, by incorporating heterogeneous preferences into standard models we lose quite a lot of tractability. To maintain some of it, I derive two models, both containing just one risk seeker: a market-order model with one risk seeker and one risk averter, and a limit-order model with one risk seeker and finitely many risk neutrals. The resulting economies show that in situations inconsistent with trade—such as with pure risk aversion or pure risk neutrality—injecting even a small amount of risk seeking suffices to generate trade. Internet Appendix B contains the details.

2 Related literature

"One may introduce risk-loving traders."[3] Such is the advice of Tirole (1982), who, together with Milgrom and Stokey (1982), establish foundational results. The takeaway from these studies is stark. To motivate trade, and therefore also equilibrium with noisy prices, we must appeal to one of the following mechanisms: stochastic supply, such as liquidity shocks and noise trading; more narrowly defined trading needs, such as random endowments and hedging concerns; and relaxed rationality, which may vary from differences of opinion to specific cognitive frictions. The theory literature develops largely along these three tracks, with a small but growing strand discussing preferences, and Bond and Eraslan (2010) pointing out that the no-trade results unravel with production.

The earliest study of noisy prices assumes that the supply of the traded asset varies exogenously (Grossman, 1976). Grossman and Stiglitz (1980) and Hellwig (1980) develop this idea

[3] Even though this advice appears explicitly at the bottom of p. 1167 in Tirole (1982), risk seekers are not mentioned again therein.

further, establishing workhorse models in the information economics of financial markets. As Dow and Gorton (2008) discuss, there are two economic interpretations of stochastic supply: one which portrays variation in supply as liquidity shocks, and another, which argues that some investors trade because of irrational—and therefore "noisy"—motives.

A number of papers propose other theories. Diamond and Verrecchia (1981) and Verrecchia (1982) introduce random endowments of shares to both generate trade and avoid full revelation of the asset's fundamental value. Endowments are also known to generate trade in markets which are competitive and incomplete (Blume et al., 2006; Gottardi and Rahi, 2013). Related papers—such as Wang (1994), Dow and Gorton (1997), Albuquerque and Miao (2014), and others—replace endowment shocks with hedging concerns, and discuss a variety of topics that require trade.

Another approach assumes that rationality is bounded, typically due to heterogeneous beliefs, imperfect learning, or overconfidence. Using heterogeneous beliefs, Harrison and Kreps (1978) study speculation, while Morris (1994) analyzes the implications of such beliefs for trade.[4] Constraining investors' learning capacity à la Sims (2003), Peng and Xiong (2006) discuss how prices are affected by inattention. A different model of imperfect learning is explored in Vives and Yang (2018), where investors process price information subject to receivers' noise as in Myatt and Wallace (2012). Using overconfidence as yet another type of bounded rationality, Scheinkman and Xiong (2003) discuss how it can generate bubbles, while Kyle, Obizhaeva, and Wang (2018) use it to study markets with traders who agree to disagree about the precision of each other's information.

There are also papers that relax specific assumptions of rational expectations. One such

[4] As models with heterogeneous beliefs typically assume that traders do not condition on prices, their heterogeneity is often referred to as "disagreement," a term paraphrasing Aumann (1976), or as "difference-of-opinion," a term appearing in Harris and Raviv (1993). The more recent literature explores how heterogeneous belief affects the empirical attributes of prices (David, 2008; Banerjee, 2011; Heyerdahl-Larsen and Illeditsch, 2019, and others).

example is Banerjee and Green (2015), where uninformed investors are uncertain about whether the individuals they trade against are informed traders or noise traders. Other examples use investors with private, i.e. different, valuations for the same asset. Biais and Bossaerts (1998) use this assumption to formally establish Keynes's intuition for financial markets as beauty contests, Vives (2011) uses it to discuss competition in supply schedules, and Rostek and Weretka (2012) use it to study how market size affects information aggregation.

A final strand of the theory literature explores how preferences may elicit trade. It is possible to generate trade with preferences which are non-state-additive (Dow, Werlang, and Madrigal, 1990), non-consequential (Halevy, 2004), or heterogeneous and time-inseparable (Xiao, 2019). The preferences I use do not have such features; the only departure from the usual homogeneous Constant Absolute Risk Aversion (CARA) preferences is that they are convex rather than concave.

Outside the scope of theory work, that some people like risk is a common finding, both empirically and experimentally. Using simple gambles, Coombs and Pruitt (1960) carry out experiments with 99 American subjects, finding that about one third of their subjects prefer higher variance. Assuming a representative utility-maximizing agent with objective knowledge of probabilities, and using more than $20,000$ horse races in New York, Ali (1977) estimates that the representative race bettor has convex utility.

More recently, Kachelmeier and Shehata (1992) elicit certainty equivalents for lotteries with 80 Chinese subjects under significant monetary incentives. Their evidence suggests that a fraction of subjects likes risk. Also under significant monetary incentives, Holt and Laury (2002) measure relative risk-aversion coefficients in experiments with 175 American subjects, producing negative estimates for a fraction of them. Finally, while the aforementioned studies use small populations, evidence of risk seeking is consistently found in studies with populations in the order of thousands, both in Germany (Dohmen et al., 2011; Crosetto and Filippin, 2013) and in the Netherlands (von Gaudecker et al., 2011; Noussair et al., 2014).

Taken together, this evidence suggests that risk-seeking preferences are natural. Under price-taking assumptions, we would either exclude risk seekers from participating in markets, or we would be left with a puzzle. The model I present next would explain that puzzle away.

3 Risk-seeking traders

The economy unfolds in one trading period, comprising N utility-maximizing traders—there are no noise traders, endowment shocks, or hedging concerns. There is one risky asset with dividend $D \sim \mathcal{N}\left(0, \tau_D^{-1}\right)$ and price P, and one riskless asset whose return is normalized to zero. Each trader $n = 1, \ldots, N$ observes the signal

$$s_n = D + \varepsilon_n \tag{1}$$

about the dividend, where ε_n, $n = 1, \ldots, N$ are independent random variables, with distribution $\mathcal{N}\left(0, \tau_n^{-1}\right)$, independent of D. Trader n's information set is $\mathcal{F}_n$—as I discuss shortly, this may be either the σ-algebra of s_n or that of (s_n, P).

Given his information set, trader n has mean-variance preferences with risk-preference parameter δ, so that his utility is

$$u\left(\pi_n; \mathcal{F}_n\right) = \mathbb{E}\left[\pi_n \middle| \mathcal{F}_n\right] - \frac{1}{2}\delta \mathrm{Var}\left(\pi_n \middle| \mathcal{F}_n\right), \tag{2}$$

where $\pi_n = X_n(D - P)$ is his profit.[5]

[5]Given that state variables are Normal, the utility in (2) is equivalent to expected CARA utility over profit with coefficient δ, conditional on $\mathcal{F}_n$. The first-order and second-order conditions of either formulation are identical.

I use two information structures: one in which traders observe only the signals in (1), and another, in which they also observe the price. For each structure I derive two types of equilibria: one with pure exchange, and one with information acquisition, with the latter type nesting the former under the assumption that precision costs are

$$c(\tau_n) = \frac{\tau_n^2}{4\psi}. \tag{3}$$

The parameter $\psi > 0$, the inverse of the marginal cost of precision, measures "how easy" it is to acquire information.

Definition 1 *A* trading equilibrium *is a price function together with non-zero demand functions that satisfy the traders' first-order conditions, taking the traders' effect on the price into account and holding their signal precisions fixed, where either*

(i) the traders do not observe the price, and the price is set by the break-even condition of a representative market maker, or

(ii) the traders observe the price, and the price is set by market clearing among the traders.

The equilibrium in (i) is one with market orders*, while the equilibrium in (ii) is one with* limit orders*. An equilibrium is* rational *if the second-order conditions of all traders are satisfied. Finally, an* information equilibrium *is a trading equilibrium together with signal precisions that maximize the traders' ex-ante utility, taking the trading equilibrium as given.*

We thus have four equilibria altogether, with limit orders or with market orders, and with or without information acquisition. I begin with the simplest one, the trading equilibrium with market orders, bringing in the other ones where necessary.

3.1 A benchmark model of risk seeking with market orders

There is a representative market maker who sets the price equal to his conditional expectation of the dividend given the aggregate order flow. Following standard conjectures, the demand strategy of trader n is linear in his signal,

$$X_n = \beta_n s_n, \tag{4a}$$

and the price is linear in aggregate order flow,

$$P = \lambda \left(\sum_{n=1}^{N} X_n \right). \tag{4b}$$

It is well known that, under standard assumptions, no trading equilibrium exists without exogenous noise, as long as traders are (weakly) risk averse (Milgrom and Stokey, 1982; Tirole, 1982). This result, however, is overturned if we allow risk seeking. All proofs are in the Appendix.

Theorem 2 *If $\delta \geq 0$, no rational trading equilibrium exists. If $\delta < 0$, a unique symmetric rational trading equilibrium with the structure of* (4) *exists, which has the form*

$$\beta_n = \frac{\tau_n}{2\rho + \delta}, \tag{5a}$$

where ρ is a constant that satisfies

$$\frac{1}{\lambda} = \frac{\tau_D}{\rho} + \sum_{i=1}^{N} \beta_i. \tag{5b}$$

The equilibrium values of ρ and λ are

$$\rho = -\delta, \tag{6a}$$

and

$$\lambda = \frac{-\delta}{\tau_D + \sum_{i=1}^{N} \tau_i}. \tag{6b}$$

In addition, a unique information equilibrium exists that supports this trading equilibrium with endogenously homogeneous precisions.

Given the provisions of the trade literature, it may appear counterintuitive that an equilibrium exists. It is important, however, to point out that risk seeking falls outside the constraints of the traditional no-trade results. In fact, as the above theorem suggests, traders who are willing to take risk are also willing to trade.

Several questions now arise. Why is there an equilibrium at all? Standard intuition from price-taking models dictates that risk seekers trade very aggressively, and may thus attempt to hold infinite positions in the asset, thereby destroying equilibrium.[6] What is more; are risk seekers rational? Can we think of them, that is, as utility-maximizing agents whose second-order conditions are not violated?

These questions are connected. Examining the traders' optimal trading intensity in (5a), we can see that their demands are finite and well-defined, while examining the traders' optimal utility,

$$u(\pi_n; s_n) = \frac{1}{2}\beta_n^2 s_n^2 \left[2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)\right], \tag{7}$$

where P_{-n} is the price excluding the impact of trader n, we can see that if trading intensity

[6] The first-order condition of a price-taking trader with CARA utility, risk-aversion coefficient δ, information set $\mathcal{F}$, and optimal demand X is $X = \mathbb{E}\left[D - P \middle| \mathcal{F}\right] / \left\{\delta \text{Var}\left(D - P \middle| \mathcal{F}\right)\right\}$. The second-order condition is $-\delta \text{Var}\left(D - P \middle| \mathcal{F}\right)$, which is violated for negative δ. Thus, under price-taking assumptions, negative risk aversion implies that the demand does not correspond to a maximum.

and market impact are finite, then so is utility. Crucially, however, both for rationality and for the willingness to trade, we can see also that the square bracket in (7) must be positive.

We can recognize this bracket as the negative of the traders' second-order condition. Similarly to Kyle (1989), it consists of two parts: market impact, and risk aversion multiplied by conditional variance; the second-order condition is thus satisfied if the sum of the two parts is overall positive. Negative risk aversion is therefore admissible, as long as market impact offsets it.

In fact, as Theorem 2 tells us, there is a quantity which captures exactly how negative the traders' risk aversion is: the quantity ρ. As it equals $-\delta$ in equilibrium, we can call it "risk appetite." But more importantly for understanding the economic mechanism that brings about equilibrium, ρ appears both in the denominator of trading intensity for individuals, and in the relation that ties market impact together with trading intensity in the aggregate.

More specifically, Equation (5a) shows that, holding risk appetite fixed, how aggressively a risk seeker trades depends on his love of risk, with his trading intensity increasing as δ becomes more negative. As the trader is strategic, he realizes that he is not acting alone in the economy, and that other traders also trade aggressively. The extent to which his trading aggression depends on other traders is measured by risk appetite; as ρ increases, his trading intensity decreases.

As Equation (5b) shows next, ρ is an aggregate quantity, being related both to market impact and aggregate trading intensity:

$$\rho = \tau_D \left(\frac{1}{\lambda} - \sum_{i=1}^{N} \beta_i \right)^{-1}. \tag{8}$$

Holding aggregate trading intensity fixed, ρ increases as traders gain market impact, and similarly, holding market impact fixed, ρ increases as all traders taken together trade more aggressively.

This relation implies that if other traders start trading more aggressively, any given trader in isolation will react by trading *less* aggressively, as if he was pulling the breaks on a speeding wagon about to come off its tracks. A similar reaction takes place if other traders increase their impact on prices, an effect which is also present in models without risk seeking (Kyle, 1985; Subrahmanyam, 1991; Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996).

Overall, Theorem 2 reveals that risk appetite performs a dual role; it works as market impact, and in equilibrium it turns out to also equal the traders' love for risk. This property is intuitive, because the more each trader likes risk, the more aggressively he trades, amplifying his market impact. Going back to the second-order condition contained in (7), in equilibrium the traders' love for risk amplifies their market impact so much that it overwhelms the negative risk-aversion term.

Risk seeking thus enables equilibrium, and in a way that is not possible under standard assumptions. In price-taking models risk seekers want to trade too much, destroying equilibrium; in strategic models risk averters do not want to trade enough, annuling equilibrium. Making traders strategic yet risk seeking gets the balance right.

3.2 Risk seeking with limit orders

Here I extend the above results. Traders now maximize their utility by observing the price P in addition to their private signals. There is no explicit market maker, and I assume—in contrast to Kyle (1989), Diamond and Verrecchia (1981) and others—that the net aggregate demand of the utility maximizers is zero almost surely.

More specifically, trader n conditions his demand on his signal and the price. Following standard methodology, I continue to assume that prices and demands are linear. Letting the demand function of trader n be

$$X_n = \beta_n s_n - \gamma_n P, \tag{9a}$$

the price that clears the market is

$$P = \lambda \left(D \sum_{n=1}^{N} \beta_n + \sum_{k=1}^{N} \beta_k \varepsilon_k \right) \tag{9b}$$

where $\lambda = \left(\sum_{n=1}^{N} \gamma_n \right)^{-1}$. This price function is similar to that in Grossman (1976), but with the difference that traders are neither price takers nor risk averse.

Theorem 3 *If $\delta \geq 0$, no rational trading equilibrium exists. If $\delta < 0$, a unique symmetric rational trading equilibrium with the structure of* (9) *exists. It has the form*

$$\beta_n = \frac{\tau_n}{(\nu + 1)\rho + \delta} \tag{10a}$$

for some constants ρ and ν that satisfy

$$\frac{1}{\lambda} = \frac{\tau_D}{\nu \rho} + \sum_{i=1}^{N} \beta_i, \tag{10b}$$

while γ_n is the same for all traders, and it depends on δ, ρ, and ν in a manner shown explicitly in Equation (57) *of the Appendix. The equilibrium values of ρ and ν are*

$$\rho = -\delta, \tag{11a}$$

and

$$\nu = 1 - \frac{1}{N}, \tag{11b}$$

while

$$\lambda = \nu \frac{-\delta}{\tau_D + \sum_{i=1}^{N} \tau_i}. \tag{11c}$$

In addition, a unique information equilibrium exists that supports this trading equilibrium with endogenously homogeneous precisions.

Comparing the limit-order equilibria of Theorem 3 to the market-order equilibria of Theorem 2, we can draw the following economic conclusions.

Juxtaposing the trading intensity and liquidity of the two models, we can see that limit orders behave as a scaled version of market orders, with ν, being one-to-one with the number of traders, controlling this scale. If, however counterfactually, we forced ν to be one, then every quantity in Theorem 3 reduces to its counterpart in Theorem 2.[7] This scaling connects the two models at a formal level—nothing surprising, given that one model generalizes the other. Nonetheless, a deeper economic lesson is that letting traders observe prices does not unravel the intuition we gain from the simpler model.

In fact, our intuition is enhanced. As with market orders, risk seeking with limit orders enables an equilibrium where traders behave as rational optimizers. This equilibrium not only lies outside the scope of Milgrom and Stokey (1982) and Tirole (1982), but it also shows that the intuition in Grossman (1976) rests on price-taking behavior. Whereas Grossman's price takers ignore their private information because it is already in the price, the traders of this setting recognize that the price contains their information only if they trade, thus also recognizing that the price summarizes the information of all other traders, making private information valuable because it can be used in conjunction with the price to learn about other traders' private information.

Still, as Theorem 3 reminds us, going beyond price taking is not enough to get us an

[7] This happens even to the price coefficient; as Equation (57) shows, if we force $\nu = 1$ then $\gamma_n = 0$.

equilibrium—unless we go as far as risk seeking. In that case, as we can see in (10a), trading intensity is again inversely related to risk appetite, reflecting that, being strategic, the traders internalize how they affect prices. As above, realizing that everyone else also trades aggressively, each individual scales back their trading, in a manner that, after all, is not unique to risk seeking. What is unique here is that risk seekers are their own best medicine. Acting as a self-sustaining effect, their strategic reaction to others' aggression ensures that demand remains finite and that market impact remains positive, thereby guaranteeing equilibrium, and a rational one at that.

3.3 The effect of risk seeking on prices

In either of the economies described above, the price behaves as a discounted version of the dividend. Even though there is no explicit discount rate, there is an implicit type of "noise-based discounting" that connects the price to dividend expectations in a manner analogous to well-known relations in asset pricing. This discounting features the signal-to-noise ratio of the price, which I denote by $\mathcal{Q}$, as a measure of price informativeness.

Using trading intensity and liquidity for either economy, we can write

$$P = \frac{1}{1+\mathcal{Q}^{-1}} \left(D + \sum_{k=1}^{N} \frac{\beta_k}{\sum_{n=1}^{N} \beta_n} \varepsilon_k \right) \tag{12a}$$

and

$$\frac{1}{1+\mathcal{Q}^{-1}} = 1 - \frac{\mathrm{Var}(D-P)}{\mathrm{Var}(D)}. \tag{12b}$$

Equation (12a) says that the price is a noisy version of the dividend, discounted at a net rate identical to the inverse of price informativeness (Lemma I.1 contains the proof). The dividend is thus discounted more when the price contains more noise—or, equivalently by

(12b), when the return is riskier—whereas the opposite happens when the price contains more information. As the price is a public signal, this discounting is consistent with the intuition of Easley and O'Hara (2004) that stocks with more public information have a lower risk premium; such stocks contain dividend expectations with less discounting.

We can also draw out a connection between price informativeness and liquidity. From Theorem 3 it follows that

$$\mathcal{Q} + 1 = \lambda^{-1} \rho \, \nu \, \tau_D^{-1} \tag{13}$$

for the limit-order equilibria, while the same relation follows from Theorem 2 for the market-order equilibria with ν being one. Using Equation (13) we can say that, holding risk seeking constant, a more liquid asset has a more informative price, while using system (12) we can add that a more liquid asset also has less volatile returns and a lower discount rate.

Varying risk seeking we obtain the following comparative statics.

Lemma 4 *In the trading equilibria, as risk seeking increases trading intensity and liquidity decrease, but price informativeness is unaffected. In the information equilibria, as risk seeking increases trading intensity, liquidity, price informativeness, and precision all decrease.*

Irrespective of whether precisions are endogenous or not, the more everyone likes risk, the more everyone scales back their trading aggression, implying that trading intensity and liquidity decrease in risk seeking. With exogenous precisions, however, price informativeness is unaffected because neither trading intensity nor liquidity affects price noise directly. For that to happen, we need the traders to acquire information, opening up a channel for their preferences to affect their precisions.

As in many models of financial markets, other traders acquiring information is a substitute for an individual trader acquiring information. Holding equilibrium effects fixed, as risk seeking increases other traders want to hold more of the asset, making them trade more aggressively, pushing the price closer to the fundamental. While this may increase price informativeness out of equilibrium, in equilibrium each trader responds strategically, scaling back his trading intensity. Because any information acquired thus becomes less useful, the more risk seeking increases the more each trader substitutes away from acquiring information, and in such a strong fashion that the overall amount of information goes down.

The consequence is that as risk seeking increases prices become noisier. It follows that even though we need some risk seeking to get an equilibrium at all, the more of it we have, the more noise we get. We can thus say that risk seeking does not only generate trade, but it also amplifies noise, going back to an argument of Dow and Gorton (1997) that trade and noise are inseparable.

3.4 A Herfindahl-Hirschman honing of the Hirshleifer effect

One of the most intriguing—yet puzzling—results in the economics of financial markets is the Hirshleifer (1971) effect. Contrary to casual intuition that information is good, in its most abstract form the Hirshleifer effect holds that information is bad because it destroys the market for insurance, diminishing the motives for agents to trade assets for risk-sharing purposes. While Hirshleifer (1971)'s model is stylized, this effect is present in many models with asymmetric information, where we can see that increasing the precision of public information is detrimental to traders' welfare.

Nevertheless, as empirical work in the area is scarce, and as a number of theories disallow full welfare analysis due to noise trading, the Hirshleifer effect remains partially examined.[8] In fact, the intuition behind it is compound: first, information destroys risk sharing, and second, information destroys welfare. As I explain below, we must attach a qualifying statement on the second portion, thereby sharpening our understanding of the economic forces

[8] See Hakansson et al. (1982) for work prior to 1983 and Gottardi and Rahi (2014) for more recent results.

at play.

To explore the effect appropriately, we must be able to measure risk sharing. I thus adopt a well-known measure of concentration from industrial organization, the Herfindahl-Hirschman Index (HHI), using its inverse to measure risk sharing. As trader n's holdings X_n is are in dollar-denominated shares, the proportion of the asset he holds (his "market share" in HHI parlance) is

$$x_n = \frac{X_n}{\sum_{i=1}^{N} X_i}, \tag{14}$$

and thus the realized HHI of asset holdings is

$$H = \sum_{i=1}^{N} x_i^2. \tag{15}$$

I use $\mathcal{H} = \mathbb{E}[H]$ to measure demand concentration—and $\mathcal{H}^{-1}$ to measure risk sharing—with the following note of caution. On the one hand, because aggregate demand is stochastic in the market-order model, the quantity in (15) is an intractable random variable.[9] In the limit-order model, on the other hand, the denominator is deterministic, and what is more, it is zero by assumption. This presents little difficulty, because it is straightforward to introduce positive (yet deterministic) aggregate supply in the limit-order model. I thus confine the analysis to limit orders with positive supply, with the advantage that such a model is not only tractable, but also that its implications are robust to learning from prices.

Finally, as the exercise below is about varying the precision of public information, I assume that the signal of trader n is

$$s_n = D + \eta + \varepsilon_n, \tag{16}$$

[9]The difficulty arises from that (15) is a ratio of two *correlated* chi-squares.

where the distribution of η is $\mathcal{N}\left(0,\tau_\eta^{-1}\right)$, independent of D and of ε_n, $n=1,\ldots,N$. The precisions of the trader-specific noises ε_n are exogenous and, to simplify exposition, they are homogeneous.

Theorem 5 *In the trading equilibrium with limit orders and risk seekers, as the public precision τ_η increases, welfare increases but risk sharing $\mathcal{H}$ decreases.*

The takeaway is that in a risk-seeking economy more precise public information increases welfare, contradicting the second part of Hirshleifer's intuition. The first part is nonetheless still active, because the traders share risk less when they have more information. These two properties are consistent exactly because the traders like risk; having to take on more risk, a byproduct of information destroying insurance, caters to their risk-seeking preferences.

4 Comparison with noise-trading economies

Is it possible to distinguish a risk-seeking economy from existing models with trade, such as those with noise traders, random endowments, or hedging concerns?

One answer to this question is that, as we can see in Theorems 2 and 3, the precisions of each trader can be made endogenous. This is in contrast to models with stochastic supply, as the inherent assumed exogeneity of supply makes it difficult to endogenize noise.[10] Moreover, while random endowments are useful in studying the informational efficiency of prices (Verrecchia, 1982), their randomness is merely a modeling device, meant to represent an economy with diverse asset holdings. Nevertheless, without prior trading, it is unclear how such diversity is borne out.

[10] Exceptions include Admati and Pfleiderer (1988) who use discretionary and nondiscretionary liquidity traders, and Han, Tang, and Yang (2016) who reduce noise to a function of trading benefits.

Another answer comes by appealing to empirically observable characteristics. Whereas quantities such as signals and trades are difficult to measure in reality, quantities such as liquidity and information costs do have empirical counterparts. Deriving how liquidity responds to changes in information costs would thus allow us to compare risk seeking with well-known mechanisms for trade.

To do so, I juxtapose the risk-seeking economies with three widely-used alternatives, modifying or simplifying them where necessary for exposition. As these alternatives vary in their assumptions, I pair them up with risk-seeking economies according to the types of orders they assume, comparing that of Section 3.1 to a market-order economy with noise traders, and that of Section 3.2 to two limit-order economies, one with noise traders and one with random endowments. I leave the comparison with hedging concerns for Section 4.1 below.

The first alternative extends Kyle (1985) to many traders, similarly to Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996). There is a representative market maker, a population of N risk-neutral traders with signals as in (1), and a block of noise traders whose supply of the asset is $\theta \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$, independent of everything else.

The second alternative is a simplified version of Kyle (1989), with all the utility maximizers risk neutral and informed with signals as in (1). The noise traders are the same as in the first alternative.

The third alternative is a limit-order model with random endowments. As with a model employed in Kyle and Lee (2018), this model is Diamond and Verrecchia (1981) with strategic demands instead of price-taking demands, enabling an equitable comparison with risk seeking, which requires market impact. Because this model merges random endowments with Kyle (1989), some of its components warrant a brief mention.

Trader n is endowed with z_n shares of the asset, where z_n, $n = 1, \ldots, N$ are independent and identically distributed (iid) random variables, with distribution $\mathcal{N}(0, \tau_z^{-1})$, independent of D and of ε_n, $n = 1, \ldots, N$. Trader n's information set is the sigma algebra of (s_n, P, z_n). His utility is that of (2), with two differences: share endowments affect the profit via $\pi_n = X_n(D - P) + z_n P$, while traders are risk averse.[11]

Demands are linear, with form

$$X_n = \beta_n s_n - \gamma_n P + \kappa_n z_n, \tag{17}$$

and markets clear when aggregate demand equals aggregate supply,

$$\sum_{n=1}^{N} X_n = \sum_{n=1}^{N} z_n, \tag{18}$$

from which it follows that the price is

$$P = \lambda \left[\sum_{k=1}^{N} \beta_k s_k + \sum_{k=1}^{N} (\kappa_k - 1) z_k \right] \tag{19}$$

with $\lambda = \left(\sum_{k=1}^{N} \gamma_k \right)^{-1}$.

In all economies, the rational traders acquire information ex-ante subject to the information costs in (3). As the three above alternatives are variations of established models, I relegate their derivations to the Internet Appendix (Sections A.1, A.2, and A.3), referring to them for the remainder of this section as "noise-trading economies."

Proposition 6 *Irrespective of whether we consider market orders or limit orders, in the risk-seeking economies liquidity increases when information becomes cheaper. In the noise-trading economies, liquidity decreases when information becomes cheaper if the marginal cost*

[11] As we can see in Appendix A.3, if we use risk neutrality there is no equilibrium, and thus to compare with risk seeking we now need some risk aversion.

Market orders: risk seekers versus noise traders

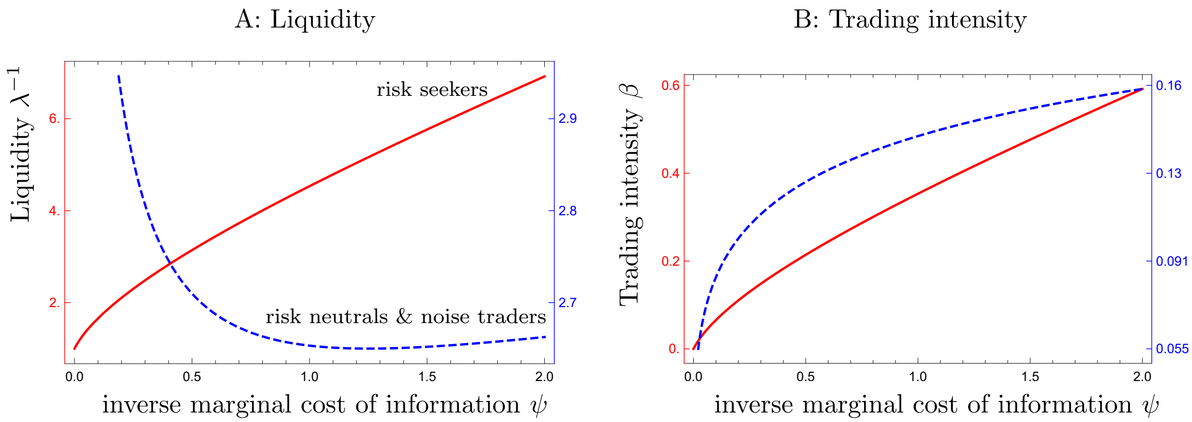

Figure 1: Liquidity (left) and trading intensity (right) as a function of how cheap it is to acquire information for two market-order economies: one with risk seekers, and one with noise traders. In each panel, the solid red curves show the risk-seeking economy, with scales on the left vertical axis; the dash-dotted blue curves show the noise-trading economy, with scales on the right vertical axis. For the risk-seeking economy the risk-preference parameter is $\delta = -1$, while for the noise-trading economy it is $\delta = 0$. For all economies, the precision of the dividend is $\tau_D = 1$, the number of rational traders is $N = 10$, and where used, the precision of the noise traders' demand is $\tau_\theta = 1$.

of information is sufficiently high.

It is important to note that, in all economies, the rational traders acquire more information when information becomes cheaper. The distinction we see in Proposition 6 thus arises because of how traders respond to increased precision, while the connection with information costs is meant both as a robustness check and as an additional empirical prediction. Moreover, in all economies, there are two effects of acquiring more information: one direct, and one indirect.

The direct effect makes liquidity increase. As traders acquire more information, the price endogenously trades closer to the fundamental, reducing the market impact of individual traders.

The indirect effect, however, makes liquidity decrease. As discussed at length above, holding the traders' information fixed, each trader internalizes the impact that he and other

Limit orders: risk seekers versus noise traders and random endowments

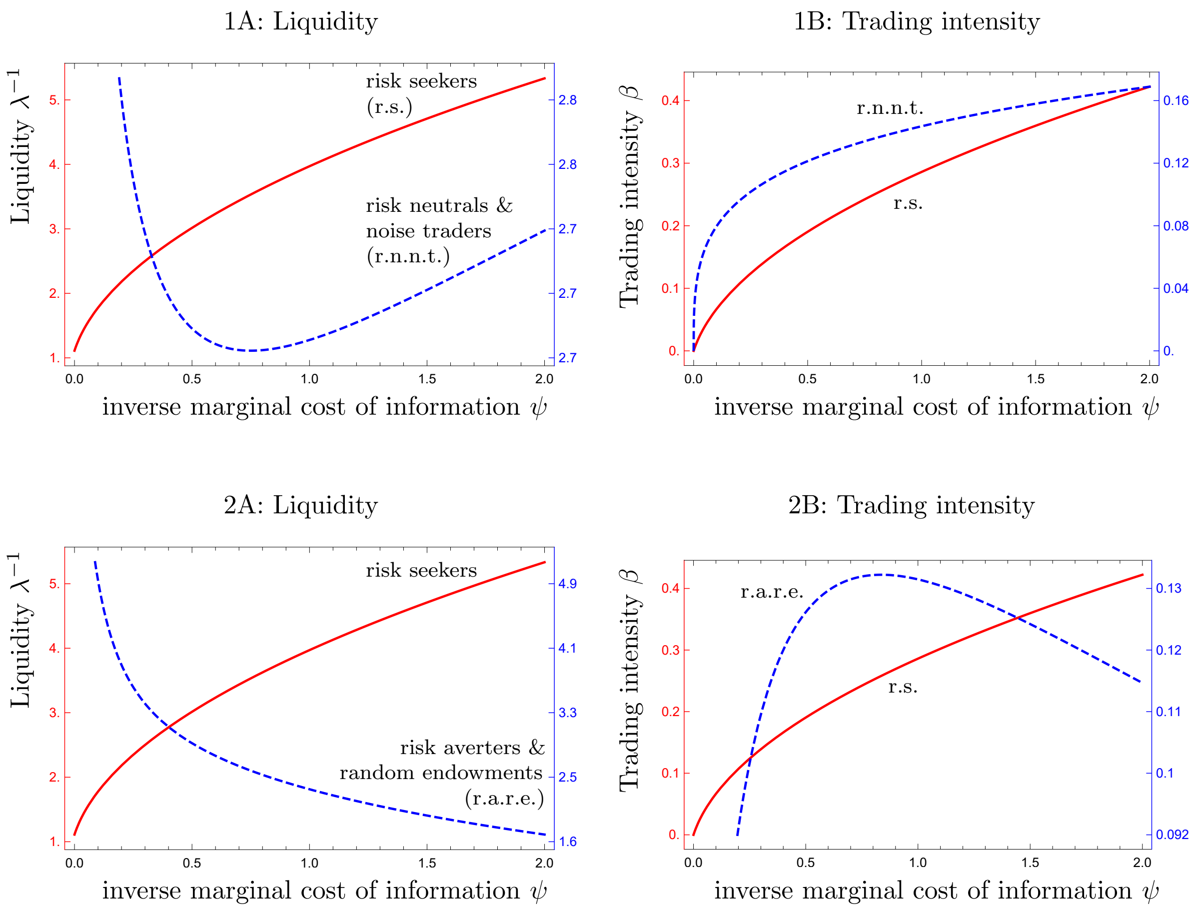

Figure 2: Liquidity (left column) and trading intensity (right column) in limit-order economies as a function of how cheap it is to acquire information. Each row compares risk seekers with either noise traders (top row) or random endowments (bottom row). In each panel, the solid red curves show the risk-seeking economy, with scales on the left vertical axis; the dash-dotted blue curves show the noise-trader and random-endowment economies, with scales on the right vertical axis. To facilitate comparison, the solid red curves show the same risk-seeking economy in both rows. For the risk seeking economy the risk-preference parameter is $\delta = -1$, for the noise-trader economy it is $\delta = 0$, and for the random-endowment economy it is $\delta = 1$. For all economies, the precision of the dividend is $\tau_D = 1$ and the number of rational traders is $N = 10$. Where used, the precision of the noise traders' demand is $\tau_\theta = 1$ and the precision of random endowments is $\tau_z = 1$.

traders have on the price, scaling back his trading to take everyone's market impact into account. Moreover, as traders acquire more information, the information revealed by prices becomes more precise, implying that individual traders must scale back their trading even more.

In the risk-seeking economies the direct effect dominates. When, however, we inject noisy demand into an economy—either in the form of noise traders or random endowments—the relationship between liquidity and acquired precision changes. As the noisy demand is exogenous, it does not respond to any changes in the economy. Behaving as extra noise, this demand makes the price respond less to the rational traders increasing their precisions. Weakening the direct effect, this injection of noise tips the balance of the two effects towards the indirect effect, making it strong enough to overcome the direct effect.[12] Liquidity, thus, may decrease when acquired precision increases, or equivalently, when information becomes cheaper.

As an illustration, Figures 1 and 2 compare the risk seeking economies with the noise-trading economies, with scaled magnitudes for each pair of curves. In Figure 1 we have liquidity and trading intensity for the market-order economies, with risk seeking in solid red and noise trading in dashed blue. In Figure 2 we have the same objects for the limit-order economies.

Trading intensities increase in ψ under risk neutrality (used in Figure 1 and in the top row of Figure 2) because the traders acquire more information as it becomes cheaper. Nonetheless, trading intensities are also concave in ψ because the traders scale back their trading as they acquire more information. Risk aversion (used in the bottom row of Figure 2) amplifies the scaling-back effect, making trading intensity decrease as the traders acquire more

[12] While the rational traders' incentives to trade aggressively may also increase when we inject noisy demand into the economy—as more noise masks the traders' information more—this is a secondary effect which does not mitigate that noisy demand adds noise to the price.

information for a wide range of ψ.

Overall, in the risk-seeking economies liquidity unequivocally increases as information becomes cheaper. In the noise-trading economies, however, the traders' strategic behavior dominates their willingness to trade more aggressively as they acquire more information, and all the more so when information is expensive, i.e. when ψ is small.

4.1 On hedging as a motive for trade

One of the ways that the existing literature circumvents the no-trade theorems is hedging against external risk of various types (Wang, 1994; Biais and Hillion, 1994; Dow and Gorton, 1994; Blume et al., 2006; Vives, 2008, and others). It is therefore reasonable to ask whether hedging concerns and risk seeking are economically distinct, at least at some level. I therefore revisit the equilibrium by adding hedging concerns, checking if trade ensues if the rational traders are risk averse.

As there is more than one way to introduce hedging, I focus on income risk for two reasons: First, it is intuitive that people would use financial assets as insurance against risk in their income. Second, standard arguments from consumption-based asset pricing show that an asset's holding value lies solely on how much it negates consumption shocks, and, as income risk translates to consumption risk, it can be argued that income risk may be the most economically interesting alternative motive for trade.

A caveat of examining how hedging may affect trade is that many hedging models use price-taking traders. My model, however, uses strategic traders. Thus, to place income hedging on an equal footing with my model, I embed hedging directly into an economy with strategic traders, while still excluding all pure noise trading. To simplify the analysis, I develop a model as similar as possible to the benchmark in Section 3.1, while changing the minimum number of ingredients necessary to capture hedging concerns.

More specifically, all traders have mean-variance utility. Trader n has additional income y_n in the liquidating period of the economy, where

$$y_n = -\phi D + \zeta_n, \tag{20}$$

and where ζ_n, $n = 1, \ldots, N$ are iid random variables, with distribution $\mathcal{N}\left(0, \tau_\zeta^{-1}\right)$, independently of D and of ε_n, $n = 1, \ldots, N$. The parameter ϕ captures the degree to which the asset can be used to hedge: if ϕ is zero then hedging is moot, while if ϕ is positive the asset can be used to insure against bad states of the world.

Trader n's liquidating wealth is

$$W_n = \pi_n + y_n = X_n(D - P) + y_n, \tag{21a}$$

where, as before, π_n is trader n's profit and X_n is his demand. The traders maximize their expected utility of wealth, so instead of (2) we now have

$$u\left(\pi_n + y_n; \mathcal{F}_n\right) = \mathbb{E}\left[\pi_n + y_n \middle| \mathcal{F}_n\right] - \frac{1}{2}\delta \mathrm{Var}\left(\pi_n + y_n \middle| \mathcal{F}_n\right), \tag{21b}$$

with the information set $\mathcal{F}_n$ containing only s_n.

I assume that precisions are exogenous and homogeneous—this further simplifies the analysis by allowing trading intensity to be the same for everyone. The rest of the economy is the same as in Section 3.1: market makers set prices according to their break-even condition, prices are linear in order flow, and demands are linear in signals. There is, however, a minor, albeit technical, difference in the exact functional form of linearity between this equilibrium and the one in Section 3.1. As we will see below, because traders want to hedge against income risk their demand functions contain a constant offset. To allow for this, instead of

(4), the right conjectures are

$$X_n = \alpha + \beta s_n \tag{22a}$$

and

$$P = \lambda \left[\sum_{n=1}^{N} (X_n - \alpha) \right]. \tag{22b}$$

As α is constant, it does not affect the information content of the total order flow, and it thus plays no role in price setting. We can therefore, as above, pin down λ by setting it equal to the projection coefficient of the dividend on the sum of orders.

Theorem 7 *If $\delta \geq 0$, no rational trading equilibrium exists. If $\delta < 0$, a unique rational trading equilibrium with the structure of* (22) *exists, in which*

$$\alpha = -\phi, \tag{23}$$

while β and λ are given by using homogeneous precisions in Theorem 2.

As we see, income hedging cannot generate trade among strategic risk-averse agents—at least not under standard assumptions. In contrast, and similarly to Section 3.1 above, we also see that risk seeking is again able to generate trade. This not only says that risk seeking is (vacuously) distinct from income hedging, but it also suggests that conclusions drawn from models with hedging-induced trade may be specific to price taking.

5 Conclusion

To have an inefficient market, no noise is necessary other than what is acquired by rational traders. What is necessary, however, is that traders like risk, not only enabling trade, but also providing enough noise for everyone to hide their information. Such risk seekers are rational because, even though their preferences are convex, their market impact ensures that their demand-choice problems are concave in equilibrium.

To convey the main intuition I use an economy with homogeneous risk seekers. One question that is thus left open is whether it is plausible to assume that *every* trader likes risk. While it would be good to explore what happens when risk seeking coexists with other risk attitudes, modeling this type of heterogeneity comes at the expense of tractability, even if we constrain preferences to be exponential. Be that as it may, Appendix B shows that equilibrium survives with a sole risk seeker, even when other traders are risk neutral or risk averse.

As my economy has one trading period and one asset, extending it to many periods and to many assets may yield new insights. Moreover, as I assume that all agents value assets in the same way, it may be interesting to study risk seeking with other valuation structures. Finally, I use preferences and information structures based on ingredients of workhorse models; it may thus be fruitful to pursue more general economic primitives (Xiao, 2019, is one example). I leave such questions for future work.

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I Appendix

Proof of Theorem 2. The first-order condition implies

$$X_n = \frac{\mathbb{E}\left[D - P_{-n}\middle|s_n\right]}{2\lambda + \delta \mathrm{Var}\left(D - P_{-n}\middle|s_n\right)}, \tag{24}$$

where $P_{-n} = \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} X_i$ is the price excluding the demand of trader n. Let

$$\rho = \frac{\sum_{i=1}^{N} \beta_i}{\sum_{i=1}^{N} \beta_i^2 \tau_i^{-1}}. \tag{25}$$

From (4b) we obtain

$$\lambda = \frac{\tau_D^{-1} \sum_{i=1}^{N} \beta_i}{\tau_D^{-1} \left(\sum_{i=1}^{N} \beta_i\right)^2 + \sum_{i=1}^{N} \beta_i^2 \tau_i^{-1}} \Rightarrow \frac{1}{\lambda} = \sum_{i=1}^{N} \beta_i + \frac{\tau_D}{\rho}, \tag{26}$$

which proves (5b). From (26) we also obtain the auxiliary relations

$$1 - \lambda \sum_{i=1}^{N} \beta_i = \frac{\tau_D \lambda}{\rho}, \tag{27a}$$

and

$$\tau_D \lambda^2 \sum_{i=1}^{N} \beta_i^2 \tau_i^{-1} = \lambda \sum_{i=1}^{N} \beta_i \left(1 - \lambda \sum_{i=1}^{N} \beta_i\right). \tag{27b}$$

Writing out the conditional moments in (24), matching coefficients with (4a) and rearranging, we obtain

$$\beta_n = \frac{\tau_n \left(1 - \lambda \sum_{i=1}^N \beta_i\right)}{2\lambda\tau_D + \lambda\tau_n + \delta\left[\left(1 - \lambda\sum_{i=1}^N \beta_i + \lambda\beta_n\right)^2 + (\tau_D + \tau_n)\lambda^2\left(\sum_{i=1}^N \beta_i^2\tau_i^{-1} - \beta_n^2\tau_n^{-1}\right)\right]}. \tag{28}$$

Conjecture that (5a) holds. To verify it, by using (27a) in the numerator of (28) it follows that the denominator of must be equal to $2\rho + \delta$, or, equivalently, that

$$\frac{\rho}{\tau_D}\tau_n + \frac{\rho}{\tau_D\lambda}\delta\left[\left(1 - \lambda\sum_{i=1}^N \beta_i + \lambda\beta_n\right)^2 + (\tau_D + \tau_n)\lambda^2\left(\sum_{i=1}^N \beta_i^2\tau_i^{-1} - \beta_n^2\tau_n^{-1}\right)\right] = \delta. \tag{29}$$

After carrying out the algebra, this is equivalent to two equations, one of which does not depend on τ_n, and another, which does. They are:

$$\frac{\rho}{\tau_D\lambda}\left[\left(1 - \lambda\sum_{i=1}^N \beta_i\right)^2 + \tau_D\lambda^2\sum_{i=1}^N \beta_i^2\tau_i^{-1}\right] = 1 \tag{30a}$$

and

$$\lambda\tau_n + \delta\left[2\lambda\beta_n\left(1 - \lambda\sum_{i=1}^N \beta_i\right) - \tau_D\lambda^2\beta_n^2\tau_n^{-1} + \tau_n\lambda^2\sum_{i=1}^N \beta_i^2\tau_i^{-1}\right] = 0. \tag{30b}$$

Equation (30a) holds already because of (27b) and (26). To make (30b) hold for any τ_n, by (5a) it suffices that

$$\lambda + \delta\left[2\lambda\frac{1}{2\rho + \delta}\left(1 - \lambda\sum_{i=1}^N \beta_i\right) - \tau_D\lambda^2\frac{1}{(2\rho + \delta)^2} + \lambda^2\sum_{i=1}^N \beta_i^2\tau_i^{-1}\right] = 0, \tag{31}$$

because then (30b) reduces to zero times τ_n. Multiplying (31) by $\rho\,(2\rho + \delta)^2/\lambda^2$, substituting (27a) into the resulting expression, and using (25) to express $\sum_{i=1}^N \beta_i^2\tau_i^{-1}$, we get that (31) is equivalent to

$$(\rho+\delta)\left[2\rho\left(2\tau_D+\sum_{i=1}^{N}\tau_i\right)+\delta\left(3\tau_D+\sum_{i=1}^{N}\tau_i\right)\right]=0. \tag{32}$$

Any ρ that satisfies (32) is a valid candidate solution for the equilibrium. This proves that in a trading equilibrium the β_n, $n=1,\ldots,N$ have the form (5a) for some constant ρ.

The solution for ρ is given by (32), but it also has to be consistent with the definition of ρ in (25). While (32) has two roots in ρ, only the solution $\rho=-\delta$ also satisfies (25).

The second-order condition is

$$2\lambda+\delta\mathrm{Var}\left(D-P_{-n}\middle|s_n\right)>0. \tag{33}$$

Combining (5a), (5b), and (6a) we have

$$\beta_n=\frac{\tau_n}{-\delta} \tag{34}$$

and

$$\lambda=\frac{-\delta}{\tau_D+\sum_{i=1}^{N}\tau_i}, \tag{35}$$

and thus the second-order condition becomes

$$\begin{aligned}2\lambda+\delta\mathrm{Var}\left(D-P_{-n}\middle|s_n\right)&=2\lambda+\delta\left(1-\lambda\sum_{\substack{i=1\\i\neq n}}^{N}\beta_i\right)^2\mathrm{Var}\left(D\middle|s_n\right)+\delta\lambda^2\sum_{\substack{i=1\\i\neq n}}^{N}\frac{\beta_i^2}{\tau_i}\\&=\delta\left[\frac{-2}{\tau_D+\sum_{i=1}^{N}\tau_i}+\frac{\tau_D+\tau_n}{\left(\tau_D+\sum_{i=1}^{N}\tau_i\right)^2}+\frac{\sum_{\substack{i=1\\i\neq n}}^{N}\tau_i}{\left(\tau_D+\sum_{i=1}^{N}\tau_i\right)^2}\right]=\frac{-\delta}{\tau_D+\sum_{i=1}^{N}\tau_i}>0.\end{aligned} \tag{36}$$

The information-acquisition problem of trader n is

$$\max_{\tau_n} \mathbb{E}\left[u\left(\pi_n; s_n\right)\right] - c(\tau_n). \tag{37}$$

Equations (2) and (24) together imply

$$\mathbb{E}\left[u\left(\pi_n; s_n\right)\right] = \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda + \delta \mathrm{Var}\left(D - P_{-n}\middle| s_n\right)\right]. \tag{38}$$

Using the optimal demand function, we can write the expected utility of trader n as

$$\mathbb{E}\left[u\left(\pi_n; s_n\right)\right] = \frac{1}{2}\frac{\tau_n^2}{\delta^2}\left(\frac{1}{\tau_D} + \frac{1}{\tau_n}\right)\left[2\lambda + \delta \mathrm{Var}\left(D - P_{-n}\middle| s_n\right)\right] = \frac{1}{-2\delta\tau_D}\frac{\tau_n\left(\tau_n + \tau_D\right)}{\tau_D + \sum_{i=1}^N \tau_i} \tag{39}$$

where the second equality is due to (36). If $\delta > 0$, then the traders are strictly better off without trading; if $\delta = 0$ no equilibrium exists because the traders' utility and their trading intensity diverges.

Taking the first order condition of trader n's information acquisition problem while holding every other trader's choice fixed we obtain

$$\frac{1}{-\delta\tau_D}\frac{\left(\tau_D + \tau_n\right)^2 + \left(\tau_D + 2\tau_n\right)\sum_{\substack{i=1\\i\neq n}}^N \tau_i}{\left(\tau_D + \tau_n + \sum_{\substack{i=1\\i\neq n}}^N \tau_i\right)^2} = \frac{\tau_n}{\psi}, \tag{40}$$

and setting $\tau_i = \tau$ for all i we get

$$\frac{\delta\tau_D}{\psi}N^2\tau^3 + \left(2\frac{\delta\tau_D^2}{\psi}N + 2N - 1\right)\tau^2 + \left(\frac{\delta\tau_D^2}{\psi} + N + 1\right)\tau_D\tau + \tau_D^2 = 0. \tag{41}$$

Equation (41) has a unique solution by Descartes' rule of signs. In particular, because $\delta < 0$ and $\tau_D > 0$, the only possibility for a multiple positive root requires that the coefficient of

τ^2 is positive and that the coefficient of τ is negative, but it is straightforward to show that this contradicts that N is positive. ■

Proof of Theorem 3. The profit of trader n is $\pi_n = X_n(D - P)$, and his utility is

$$u\left(\pi_n; s_n, P_{-n}\right) = X_n \mathbb{E}\left[D - P_{-n} \middle| s_n, P_{-n}\right] - \lambda_{-n} X_n^2 - \frac{1}{2}\delta X_n^2 \mathrm{Var}\left(D \middle| s_n, P_{-n}\right). \tag{42}$$

where P_{-n} stands for the price excluding his impact. The market clears deterministically,

$$\sum_{n=1}^{N} X_n = 0, \tag{43}$$

from which we obtain (9b) with $\lambda = \left(\sum_{n=1}^{N} \gamma_n\right)^{-1}$. Following Kyle (1989), it is straightforward to show that

$$X_n = \frac{\mathbb{E}\left[D \middle| s_n, P_{-n}\right] - P_{-n}}{2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}\right)}, \tag{44}$$

where P_{-n} is defined by

$$P = P_{-n} + \lambda_{-n} X_n, \tag{45}$$

with λ_{-n} being the slope of trader n's residual supply curve, and that

$$P_{-n} = \lambda_{-n}\left[D \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k + \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \varepsilon_k\right] \tag{46}$$

where

$$\lambda_{-n} = \frac{1}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \gamma_k}. \tag{47}$$

It is important to calculate expectations for each trader excluding their impact on the price (we otherwise get back to the conundrums in Grossman (1976), which rests on traders being price takers.) By the projection theorem we have

$$\mathbb{E}\left[D\middle|s_n, P_{-n}\right] = b_n s_n + c_n P_{-n}, \tag{48}$$

where

$$b_n = \tau_n \mathrm{Var}\left(D\middle|s_n, P_{-n}\right), \tag{49a}$$

$$c_n = \sum_{\substack{k=1\\k\neq n}}^{N} \gamma_k \frac{\sum_{\substack{k=1\\k\neq n}}^{N} \beta_k}{\sum_{\substack{k=1\\k\neq n}}^{N} \frac{\beta_k^2}{\tau_k}} \mathrm{Var}\left(D\middle|s_n, P_{-n}\right). \tag{49b}$$

and

$$\mathrm{Var}\left(D\middle|s_n, P_{-n}\right) = \left[\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^{N} \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^{N} \frac{\beta_k^2}{\tau_k}}\right]^{-1}. \tag{49c}$$

Combining (45), (44), and (48) we get

$$X_n = \frac{b_n s_n + (c_n - 1)\left(P - \lambda_{-n} X_n\right)}{2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)} \Rightarrow X_n = \frac{b_n s_n - (1 - c_n)\, P}{\lambda_{-n}\left(1 + c_n\right) + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)}. \tag{50}$$

Matching coefficients in (9a) and (50) we obtain

$$\beta_n = \frac{b_n}{\lambda_{-n}\left(1 + c_n\right) + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)} \tag{51a}$$

and

$$\gamma_n = \frac{(1 - c_n)}{\lambda_{-n}\left(1 + c_n\right) + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)}. \tag{51b}$$

Conjecture that (10a) holds. Matching (10a) with (51a) we obtain

$$(\nu + 1)\rho = \frac{\lambda_{-n}\left(1 + c_n\right)}{\mathrm{Var}\left(D\middle|s_n, P_{-n}\right)}, \tag{52}$$

and the following relations as a consequence:

$$\sum_{k=1}^{N} \beta_k = \frac{\sum_{k=1}^{N} \tau_k}{(\nu+1)\rho+\delta}, \tag{53a}$$

$$\sum_{k=1}^{N} \frac{\beta_k^2}{\tau_k} = \frac{\sum_{k=1}^{N} \tau_k}{[(\nu+1)\rho+\delta]^2}, \tag{53b}$$

$$\operatorname{Var}\left(D \middle| s_n, P_{-n}\right) = \left[\tau_D + \sum_{k=1}^{N} \tau_k\right]^{-1}, \tag{53c}$$

$$\frac{c_n}{\operatorname{Var}\left(D \middle| s_n, P_{-n}\right)} = [(\nu+1)\rho+\delta] \sum_{\substack{k=1 \\ k\neq n}}^{N} \gamma_k. \tag{53d}$$

Using (52), (53c), and (53d) in (51b) we get

$$\gamma_n = \frac{\tau_D + \sum_{k=1}^{N} \tau_k - [(\nu+1)\rho+\delta] \sum_{\substack{k=1 \\ k\neq n}}^{N} \gamma_k}{(\nu+1)\rho+\delta}, \tag{54}$$

which yields

$$\sum_{k=1}^{N} \gamma_k = \frac{\tau_D + \sum_{k=1}^{N} \tau_k}{(\nu+1)\rho+\delta}. \tag{55}$$

Using (53c) and (53d) in (52) we get

$$\delta \sum_{\substack{k=1 \\ k\neq n}}^{N} \gamma_k + \tau_D + \sum_{k=1}^{N} \tau_k = 0, \tag{56}$$

which we can solve for γ_n by using (55). Doing so yields

$$\gamma_n = \sum_{k=1}^{N} \gamma_k + \frac{1}{\delta}\left(\tau_D + \sum_{k=1}^{N} \tau_k\right) = \left[\frac{1}{(\nu+1)\rho+\delta} + \frac{1}{\delta}\right]\left(\tau_D + \sum_{k=1}^{N} \tau_k\right). \tag{57}$$

The coefficients β_n and γ_n have been solved for all n; what remains to close the equilibrium is to pin down the values of ρ and ν. To do that, summing (57) we obtain

$$\sum_{k=1}^{N} \gamma_k = N \left[\frac{1}{(\nu+1)\rho+\delta} + \frac{1}{\delta} \right] \left(\tau_D + \sum_{k=1}^{N} \tau_k \right), \tag{58}$$

which, by comparison to (55), gives

$$\frac{1}{(\nu+1)\rho+\delta} = N \left[\frac{1}{(\nu+1)\rho+\delta} + \frac{1}{\delta} \right] \Rightarrow (\nu+1)\rho = -\delta \left(\frac{N-1}{N} + 1 \right). \tag{59}$$

Equation (59) is the solution of the equilibrium. Without loss of generality, we can set ρ as in (11a) and ν as in (11b). Rewriting (55) gives (10b), while combining (55), (11a) and (11b) gives (11c).

With homogeneous precisions ($\tau_n = \tau$ for all n) we get

$$\beta = \frac{N}{N-1} \frac{\tau}{-\delta} \tag{60}$$

and

$$\gamma = \frac{1}{N-1} \frac{\tau_D + N\tau}{-\delta}. \tag{61}$$

The coefficients β and γ are both positive if and only if $\delta < 0$. In addition, the (negative of the) traders' second-order condition is

$$2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n} \right) = \frac{2}{(N-1)\gamma} + \delta \frac{1}{N\tau + \tau_D} = -\delta \frac{1}{N\tau + \tau_D} > 0, \tag{62}$$

which holds if and only if $\delta < 0$. Moreover, by the law of iterated expectations, the first-order condition, Equation (62), and because

$$\mathbb{E}\left[X_n^2\right] = \mathbb{E}\left[\beta^2 s_n^2 - 2\beta\gamma s_n P + \gamma^2 P^2\right] = \beta^2 \frac{N-1}{\tau N} = \frac{\tau N}{\delta^2 (N-1)}, \tag{63}$$

the ex-ante utility of each trader is

$$\begin{aligned}\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] &= \mathbb{E}\left[X_n \mathbb{E}\left[D - P_{-n}\middle| s_n, P_{-n}\right] - \lambda_{-n} X_n^2 - \frac{1}{2}\delta X_n^2 \text{Var}\left(D\middle| s_n, P_{-n}\right)\right] \\ &= \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda_{-n} + \delta \text{Var}\left(D\middle| s_n, P_{-n}\right)\right] = \frac{1}{-2\delta(N-1)}\frac{\tau N}{\tau N + \tau_D}. \end{aligned} \tag{64}$$

Equation (64) shows that the agents trade profitably only if $\delta < 0$; with $\delta = 0$ the equilibrium does not exist because β and γ diverge, and if $\delta > 0$ everyone is strictly better off not trading.

To justify the equilibrium with homogeneous precisions, suppose that each trader n faces the information cost function in (3). His ex-ante utility is

$$\begin{aligned}\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] &= \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda_{-n} + \delta \text{Var}\left(D\middle| s_n, P_{-n}\right)\right] \\ &= \frac{1}{2}\left[\beta_n^2\left(\frac{1}{\tau_n} + \frac{1}{\tau_D}\right) - 2\beta_n\gamma_n \frac{\frac{\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\tau_D} + \frac{\beta_n}{\tau_n}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)} + \gamma_n^2 \frac{\frac{\left(\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\tau_D} + \frac{\beta_n^2}{\tau_n} + \sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)^2}\right] \\ &\quad \times \left\{\frac{2}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k} + \delta\left[\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}\right]^{-1}\right\}\end{aligned} \tag{65}$$

Because each trader k commits to β_k and γ_k after choosing τ_k, it follows that if we fix $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$, changes in the utility in (65) happen only through τ_n (with β_n and γ_n as functions of τ_n). We may thus take the first-order condition of (65) holding $(\beta_k, \gamma_k, \tau_k)$

for $k \neq n$ as constants. Doing so we get

$$\frac{\tau_n}{2\psi} = \frac{d\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right]}{d\tau_n} = \frac{1}{2\tau_D}\left(\frac{\sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}}{2\left(\sum_{\substack{k=1\\k\neq n}}^{N}\beta_k\right)^2 + \sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}\left[\delta\sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k + 2\left(\tau_n+\tau_D\right)\right]}\right)^2$$
$$\times\left\{\delta\left[\left(\sum_{\substack{k=1\\k\neq n}}^{N}\beta_k - \sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k\right)^2 + \tau_D\sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}\right] + 2\tau_D\sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k\right\}, \quad (66)$$

and setting $\tau_k = \tau$ and rearranging we get

$$\delta N^2(N-1)\tau^3 + 2\delta N(N-1)\tau_D\tau^2 + \left[\delta(N-1)\tau_D^2 + \psi N(N-2)\right]\tau + \psi(N-1)\tau_D = 0. \quad (67)$$

Because $\delta < 0$, the coefficients of the polynomial in (67) switch signs only once, either between the quadratic and the linear term, or between the linear and constant terms. In either case, by Descartes's rule of signs, (67) has a unique positive solution for τ. ■

Lemma I.1 *System* (12) *holds for any* τ_n*,* $n = 1, \ldots, N$*, and with either market orders or limit orders.*

Proof. The signal-to-noise ratio of the price is

$$\mathcal{Q} = \frac{\mathrm{Var}\left(D\right)}{\mathrm{Var}\left(\sum_{k=1}^{N}\frac{\beta_k}{\sum_{n=1}^{N}\beta_n}\varepsilon_k\right)} = \frac{\sum_{n=1}^{N}\tau_n}{\tau_D}. \quad (68)$$

By Theorem 2 for the market-order equilibria and Theorem 3 for the limit-order equilibria we have

$$\lambda\sum_{n=1}^{N}\beta_n = \frac{\sum_{n=1}^{N}\tau_n}{\tau_D + \sum_{n=1}^{N}\tau_n} = \frac{1}{1+\mathcal{Q}^{-1}} \quad (69a)$$

and

$$1-\lambda\sum_{n=1}^{N}\beta_n=\frac{\sum_{n=1}^{N}\tau_n}{\tau_D+\sum_{n=1}^{N}\tau_n}=\frac{\mathcal{Q}^{-1}}{1+\mathcal{Q}^{-1}}. \tag{69b}$$

Equation (12a) now follows from (69a) because

$$P=\lambda\sum_{n=1}^{N}\beta_n\left(D+\sum_{k=1}^{N}\frac{\beta_k}{\sum_{n=1}^{N}\beta_n}\varepsilon_k\right). \tag{70}$$

By (70)

$$D-P=\left(1-\lambda\sum_{n=1}^{N}\beta_n\right)D-\lambda\sum_{k=1}^{N}\beta_k\varepsilon_k, \tag{71}$$

and thus, by system (69) and Theorems 2 and 3,

$$\begin{aligned}\mathrm{Var}(D-P)&=\frac{1}{\tau_D}\left(\frac{\mathcal{Q}^{-1}}{1+\mathcal{Q}^{-1}}\right)^2+\lambda^2\sum_{n=1}^{N}\frac{\beta_k^2}{\tau_n}\\&=\frac{1}{\tau_D}\left(\frac{\mathcal{Q}^{-1}}{1+\mathcal{Q}^{-1}}\right)^2+\frac{\tau_D}{\sum_{n=1}^{N}\tau_n}\left(\frac{\sum_{n=1}^{N}\tau_n}{\tau_D+\sum_{n=1}^{N}\tau_n}\right)^2\\&=\frac{1}{\tau_D}\left(\frac{\mathcal{Q}^{-1}}{1+\mathcal{Q}^{-1}}\right)^2+\frac{1}{\tau_D}\left(\frac{1}{1+\mathcal{Q}^{-1}}\right)^2=\frac{1}{\tau_D}\frac{\mathcal{Q}^{-1}}{1+\mathcal{Q}^{-1}}.\end{aligned} \tag{72}$$

Rearranging (72) gives Equation (12b). ■

Proof of Lemma 4. The claims for the trading equilibria follow immediately from Theorem 2, Theorem 3, and equation (68).

In the information equilibria all precisions are equal to τ, which is given by either (41) or (67). Let $F_M(\tau,\delta)$ denote the left-hand side of (41) and let $F_L(\tau,\delta)$ denote the left-hand side of (67). For either $j=M,L$, the implicit function theorem gives

$$\frac{d\tau}{d\delta}=-\frac{\frac{\partial F_j}{\partial\delta}}{\frac{\partial F_j}{\partial\tau}}. \tag{73}$$

Using (41) for $j = M$ and (67) for $j = L$ shows that $\partial F_j/\partial\tau$ is negative because it is the slope at the unique positive root of a cubic polynomial with negative leading coefficient ($\delta < 0$). From (41) and (67) it also follows that $\partial F_j/\partial\delta$ is positive. Therefore,

$$\frac{d\tau}{d\delta} > 0, \tag{74}$$

from which it follows that τ decreases in $-\delta$. This proves that $Q = N\tau/\tau_D$ decreases in $-\delta$. As $\delta < 0$, the chain rule and (74) give

$$\frac{d}{d(-\delta)}\left(\frac{\tau}{-\delta}\right) = -\frac{d}{d\delta}\left(\frac{\tau}{-\delta}\right) = -\left(\frac{1}{-\delta}\frac{d\tau}{d\delta} + \tau\frac{1}{\delta^2}\right) < 0. \tag{75}$$

The claims for trading intensity and liquidity in the information equilibria now follow from Theorems 2 and 3. ■

Proof of Theorem 5. Here the market-clearing condition is

$$\sum_{n=1}^{N} X_n = \bar{\theta}, \tag{76}$$

where $\bar{\theta}$ is a constant. As $\bar{\theta} \neq 0$, we must now account for intercepts, so the demand conjecture of (9a) now becomes

$$X_n = \alpha + \beta s_n - \gamma P, \tag{77}$$

where the coefficients are homogeneous across the traders because τ is the same for all traders. Following the same methodology as in the proof of Theorem 3, and because having a non-zero constant supply does not affect the informational properties of the equilibrium, we obtain

$$X_n = \frac{\mathbb{E}\left[D\middle|s_n, \tilde{P}_{-n}\right] - P_{-n}}{2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, \tilde{P}_{-n}\right)}, \tag{78}$$

where

$$P = \lambda\left(N\alpha - \bar{\theta} + \sum_{k=1}^{N} \beta s_k\right), \tag{79}$$

$$P_{-n} = \lambda_{-n}\left[(N-1)\alpha - \bar{\theta} + \sum_{\substack{k=1\\k\neq n}}^{N} \beta s_k\right], \tag{80}$$

and where $\tilde{P}_{-n}$ is the stochastic component of $\tilde{P}_{-n}$, i.e.

$$\tilde{P}_{-n} = P_{-n} - \lambda_{-n}\left[(N-1)\alpha - \bar{\theta}\right]. \tag{81}$$

The definitions of λ and λ_{-n} remain the same. Equation (78) implies

$$X_n = \frac{b_n s_n + c_n \tilde{P}_{-n} - (P - \lambda_{-n} X_n)}{2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)} \Rightarrow X_n = \frac{b_n s_n - (1-c_n)\,P - c_n\lambda_{-n}\left[(N-1)\alpha - \bar{\theta}\right]}{\lambda_{-n}\left(1+c_n\right) + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)}. \tag{82}$$

Matching coefficients we obtain the same equations as in system (51), where due to the common noise η, the projection theorem now gives

$$b_n = \tau_\eta \tau_n \left[\tau_D \tau_\eta + (\tau_D + \tau_\eta)\left(\tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^{N} \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^{N} \frac{\beta_k^2}{\tau_k}}\right)\right]^{-1}, \tag{83a}$$

$$c_n = \tau_\eta \sum_{\substack{k=1\\k\neq n}}^{N} \gamma_k \frac{\sum_{\substack{k=1\\k\neq n}}^{N} \beta_k}{\sum_{\substack{k=1\\k\neq n}}^{N} \frac{\beta_k^2}{\tau_k}} \left[\tau_D \tau_\eta + (\tau_D + \tau_\eta)\left(\tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^{N} \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^{N} \frac{\beta_k^2}{\tau_k}}\right)\right]^{-1}, \tag{83b}$$

and

$$\mathrm{Var}\left(D \middle| s_n, P_{-n}\right) = \left[\tau_D + \frac{1}{\frac{1}{\tau_\eta} + \frac{1}{\tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}}\right]^{-1}. \tag{83c}$$

Under homogeneous precisions ($\tau_n = \tau$ for all n), using (83) in system (51) we get

$$\beta = \frac{1}{-\delta(N-1)\left(\frac{1}{\tau_\eta} + \frac{1}{N\tau}\right)} \tag{84a}$$

and

$$\gamma = \frac{\tau_D + \frac{1}{\frac{1}{\tau_\eta} + \frac{1}{N\tau}}}{-\delta(N-1)}. \tag{84b}$$

Matching coefficients in (9a) and (82) we obtain

$$\alpha = \frac{-c_n\lambda_{-n}\left[(N-1)\alpha - \bar{\theta}\right]}{\lambda_{-n}\left(1+c_n\right) + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}\right)}, \tag{85}$$

which, after carrying out the algebra, reduces to

$$\alpha = \frac{1}{N}\bar{\theta} \tag{86}$$

This proves that the equilibrium with market-clearing condition (76) exists, is unique, and that the solutions for β and γ do not depend on $\bar{\theta}$.

Next, from (65), (83c) and system (84) we get

$$
\begin{aligned}
\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] &= \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}\right)\right] \\
&= \frac{1}{2}\left[\beta^2\left(\frac{1}{\tau_D} + \frac{1}{\tau_\eta} + \frac{1}{\tau}\right) - 2\beta\gamma\frac{N\beta\left(\frac{1}{\tau_D} + \frac{1}{\tau_\eta}\right) + \frac{\beta}{\tau}}{N\gamma} + \gamma^2\frac{(N\beta)^2\left(\frac{1}{\tau_D} + \frac{1}{\tau_\eta}\right) + N\frac{\beta^2}{\tau}}{(N\gamma)^2}\right] \\
&\qquad \times\left\{\frac{2}{(N-1)\gamma} + \delta\left[\tau_D + \frac{1}{\frac{1}{\tau_\eta} + \frac{1}{N\tau}}\right]^{-1}\right\} \\
&= \frac{1}{-2\delta(N-1)N\tau}\frac{1}{\left(\frac{1}{\tau_D} + \frac{1}{\tau_\eta}\right)\left[\left(\frac{1}{\tau_D} + \frac{1}{\tau_\eta}\right)\tau_D + 1\right]}, \qquad (87)
\end{aligned}
$$

which increases in τ_η because $\delta < 0$.

On the other hand, elementary calculations show that

$$
H = \frac{1}{\bar{\theta}^2}\sum_{i=1}^{N} X_i^2 = \frac{1}{\bar{\theta}^2}\sum_{i=1}^{N}\left(\alpha + \beta\frac{N-1}{N}\varepsilon_i - \beta\frac{1}{N}\sum_{\substack{k=1\\k\neq i}}^{N}\varepsilon_k\right)^2. \qquad (88)
$$

By independence of ε_n, $n = 1, \ldots, N$, we get

$$
\begin{aligned}
\mathcal{H} = \mathbb{E}[H] &= \frac{1}{\bar{\theta}^2}\left\{N\alpha^2 + \left(\beta\frac{N-1}{N}\right)^2\mathbb{E}\left[\sum_{i=1}^{N}\varepsilon_i^2\right] + \sum_{i=1}^{N}\mathbb{E}\left[\frac{\beta^2}{N^2}\left(\sum_{\substack{k=1\\k\neq i}}^{N}\varepsilon_k\right)^2\right]\right\} \\
&= \frac{1}{\bar{\theta}^2}\left\{N\alpha^2 + \beta^2\left(\frac{N-1}{N}\right)^2 N\frac{1}{\tau} + \frac{\beta^2}{N}\sum_{\substack{k=1\\k\neq i}}^{N}\mathbb{E}\left[\varepsilon_k^2\right]\right\} \\
&= \frac{1}{\bar{\theta}^2}\left\{N\alpha^2 + \beta^2\frac{(N-1)^2}{N}\frac{1}{\tau} + \frac{\beta^2}{N}\frac{N-1}{\tau}\right\} = \frac{1}{N} + \frac{N-1}{\bar{\theta}^2}\frac{\beta^2}{\tau} \\
&= \frac{1}{N} + \frac{1}{\bar{\theta}^2\delta^2\tau\left(\frac{1}{\tau_\eta} + \frac{1}{N\tau}\right)^2}, \qquad (89)
\end{aligned}
$$

where the last equality follows from (84a). It follows that $\mathcal{H}$ increases in τ_η, so that risk sharing decreases in τ_η. ■

Proof of Proposition 6. Here I prove the comparative statics for the risk-seeking economies. Because the noise-trading economies are standard, I relegate their proofs to Section A of the Internet Appendix.

Market orders: Let $F(\tau,\psi)$ denote the left-hand side of (41). $F(\tau,\psi)$ decreases in τ in equilibrium because τ is the unique positive solution of a cubic polynomial with a negative leading coefficient ($\delta < 0$.) By the implicit function theorem we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial}{\partial\psi}F}{\frac{\partial}{\partial\tau}F} = \frac{\delta\tau_D\tau\left(\tau_D + N\tau\right)^2}{\psi^2\frac{\partial}{\partial\tau}F} > 0. \tag{90}$$

By (6b) we get that λ^{-1} depends on ψ only through τ, and thus by inspection it follows that liquidity increases in ψ.

Section A.1 of the Internet Appendix shows that a unique equilibrium with information acquisition exists if instead of risk seekers we have risk-neutral rational traders and noise traders. In this equilibrium, $d\tau/d\psi > 0$, but liquidity decreases in ψ if ψ is sufficiently low.

Limit orders: Let $F_p(\tau,\psi)$ denote the left-hand side of (67). As with $F(\tau,\psi)$, $F_p(\tau,\psi)$ decreases in τ in equilibrium because τ is the unique positive solution of a cubic polynomial with a negative leading coefficient. By the implicit function theorem we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial}{\partial\psi}F_p}{\frac{\partial}{\partial\tau}F_p} = -\frac{N(N-2)\tau + (N-1)\tau_D}{\frac{\partial}{\partial\tau}F_p} > 0, \tag{91}$$

as long as $N > 1$ (which is met without loss of generality because no trade happens with only one trader). By (11c), λ^{-1} increases in τ, and thus liquidity increases in ψ.

Section A.2 and A.3 of the Appendix extend the results of Section A.1 to observable prices—if we replace the risk seekers with risk-neutral rational traders and noise traders, or with risk-averse rational traders who have random share endowments, then $d\tau/d\psi > 0$ but $d\lambda^{-1}/d\psi < 0$ if ψ is sufficiently low. ■

Proof of Theorem 7. The utility of trader n is

$$u\left(\pi_n + y_n; \mathcal{F}_n\right) = X_n \mathbb{E}\left[D - P_{-n} \middle| s_n\right] - \lambda X_n^2 + \mathbb{E}\left[y_n \middle| s_n\right] \\ - \frac{1}{2} X_n^2 \delta \text{Var}\left(D - P_{-n} \middle| s_n\right) - \delta X_n \text{Cov}\left(D - P_{-n}, y_n \middle| s_n\right) - \frac{1}{2} \delta \text{Var}\left(y_n \middle| s_n\right), \quad (92)$$

from which we obtain the first-order condition

$$X_n = \frac{\mathbb{E}\left[D - P_{-n} \middle| s_n\right] - \delta \text{Cov}\left(D - P_{-n}, y_n \middle| s_n\right)}{2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)} \\ = \frac{\frac{\text{Cov}(D - P_{-n}, s_n)}{\text{Var}(s_n)} s_n - \delta \text{Cov}\left(D - P_{-n}, y_n \middle| \mathcal{F}_n\right)}{2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)} \quad (93)$$

where, as before, $P_{-n} = \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} X_i$ is the price excluding the demand of trader n. Because the conditional covariance of normal random variables is constant, it follows that

$$\alpha = -\delta \frac{\text{Cov}\left(D - P_{-n}, y_n \middle| s_n\right)}{2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)} \quad (94a)$$

and

$$\beta = \frac{\text{Cov}\left(D - P_{-n}, s_n\right)}{\text{Var}\left(s_n\right)\left[2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)\right]}. \quad (94b)$$

The return excluding the impact of trader n is

$$D - P_{-n} = D - \lambda\beta \sum_{\substack{i=1 \\ i \neq n}}^{N} \left(D + \varepsilon_i\right) = \left[1 - \lambda\beta(N-1)\right] D - \lambda\beta \sum_{\substack{i=1 \\ i \neq n}}^{N} \varepsilon_i. \quad (95)$$

By the law of total covariance,

$$\begin{aligned}
\text{Cov}\left(D - P_{-n}, y_n \middle| \mathcal{F}_n\right) &= \text{Cov}\left(D - P_{-n}, y_n\right) - \text{Cov}\left(\mathbb{E}\left[D - P_{-n}\middle| s_n\right], \mathbb{E}\left[y_n \middle| s_n\right]\right) \\
&= \text{Cov}\left(D - P_{-n}, y_n\right) - \text{Cov}\left(\frac{\text{Cov}\left(D - P_{-n}, s_n\right)}{\text{Var}\left(s_n\right)} s_n, \frac{\text{Cov}\left(y_n, s_n\right)}{\text{Var}\left(s_n\right)} s_n\right) \\
&= \text{Cov}\left(D - P_{-n}, y_n\right) - \text{Cov}\left(D - P_{-n}, s_n\right) \frac{\text{Cov}\left(y_n, s_n\right)}{\text{Var}\left(s_n\right)} \\
&= -\phi\left[1 - \lambda\beta(N-1)\right] \frac{1}{\tau_D + \tau} \qquad (96)
\end{aligned}$$

where the last equality follows from (95).

The market impact λ is the projection coefficient of the dividend on the stochastic part of the aggregate order flow. Thus, the presence of the offset α does not matter, and

$$\lambda = \frac{\text{Cov}\left(D, \sum_{i=1}^N X_i\right)}{\text{Var}\left(\sum_{i=1}^N X_i\right)} = \frac{\tau_D^{-1} \sum_{i=1}^N \beta_i}{\tau_D^{-1}\left(\sum_{i=1}^N \beta_i\right)^2 + \sum_{i=1}^N \beta_i^2 \tau_i^{-1}} = \frac{\tau}{\beta\left(N\tau + \tau_D\right)}, \qquad (97)$$

from which we get

$$\lambda\beta = \frac{\tau}{N\tau + \tau_D}. \qquad (98)$$

The conditional variance of the return $D - P_{-n}$ is

$$\text{Var}\left(D - P_{-n} \middle| s_n\right) = (\lambda\beta)^2 \frac{N-1}{\tau} + \left[1 - \lambda\beta(N-1)\right]^2 \frac{1}{\tau_D + \tau} = \frac{1}{N\tau + \tau_D} \qquad (99)$$

Using (99) in (94b) becomes

$$\begin{aligned}
\beta &= \frac{\text{Cov}\left(D - P_{-n}, s_n\right)}{\text{Var}\left(s_n\right)\left[2\lambda + \delta \text{Var}\left(D - P_{-n}\middle| s_n\right)\right]} \\
&= \frac{\tau}{\tau + \tau_D} \frac{\left[1 - \lambda\beta(N-1)\right]}{2\lambda + \delta\left[(\lambda\beta)^2 \frac{N-1}{\tau} + \left[1 - \lambda\beta(N-1)\right]^2 \frac{1}{\tau + \tau_D}\right]}, \qquad (100)
\end{aligned}$$

and rearranging this gives

$$\beta = \frac{\left[1-\lambda\beta(N-1)\right]\tau - 2\left(\tau+\tau_D\right)\lambda\beta}{\delta\left(\tau+\tau_D\right)\left[\left(\lambda\beta\right)^2\frac{N-1}{\tau} + \left[1-\lambda\beta(N-1)\right]^2\frac{1}{\tau+\tau_D}\right]}, \tag{101}$$

where $\lambda\beta$ is given by (98). Thus, using (98) shows that

$$\beta = -\frac{\tau}{\delta}. \tag{102}$$

Substituting the first-order condition from (93) into (92) gives

$$u\left(\pi_n + y_n; \mathcal{F}_n\right) \;=\; \mathbb{E}\left[y_n \middle| s_n\right] + \frac{1}{2}X_n^2\left[2\lambda + \delta\mathrm{Var}\left(D - P_{-n}\middle| s_n\right)\right] - \frac{1}{2}\delta\mathrm{Var}\left(y_n\middle| s_n\right), \tag{103}$$

and taking expectations yields

$$\begin{aligned}\mathbb{E}\left[u\left(\pi_n + y_n; \mathcal{F}_n\right)\right] &= \frac{1}{2}\beta^2\mathbb{E}\left[s_n^2\right]\left[2\frac{\lambda\beta}{\beta} + \delta\mathrm{Var}\left(D - P_{-n}\middle| s_n\right)\right] - \frac{1}{2}\delta\mathrm{Var}\left(y_n\middle| s_n\right)\\ &= \frac{1}{2}\frac{\tau^2}{\delta^2}\left(\frac{1}{\tau_D}+\frac{1}{\tau}\right)\delta\left[-2\frac{1}{\tau}\frac{\tau}{N\tau+\tau_D} + \frac{1}{N\tau+\tau_D}\right] - \frac{1}{2}\delta\left(\frac{1}{\tau_\zeta} + \frac{q^2}{\tau_D+\tau}\right)\\ &= -\frac{1}{2}\frac{\tau^2}{\delta}\left(\frac{1}{\tau_D}+\frac{1}{\tau}\right)\frac{1}{N\tau+\tau_D} - \frac{1}{2}\delta\left(\frac{1}{\tau_\zeta} + \frac{q^2}{\tau_D+\tau}\right).\end{aligned} \tag{104}$$

By inspection, if $\delta > 0$, then the traders are strictly better off ex-ante without trading, whereas if $\delta = 0$ no equilibrium exists because the traders' utility and their trading intensity diverges. If however, $\delta < 0$, then (102) shows that $\beta > 0$, (98) shows that $\lambda > 0$, and (104) shows that the traders are strictly better off ex-ante if they do trade.

The second-order condition is

$$2\lambda + \delta\mathrm{Var}\left(D - P_{-n}\middle| s_n\right) > 0, \tag{105}$$

which cannot hold unless $\delta < 0$ because, by (98), (99) and (102),

$$2\lambda + \delta \mathrm{Var}\left(D - P_{-n}\middle| s_n\right) = \frac{-\delta}{N\tau + \tau_D}. \tag{106}$$

This establishes that, unless $\delta < 0$, no trading equilibrium exists in which the traders' second-order conditions are satisfied.

Finally, (94a), (96), (98), and (106) together give $\alpha = -\phi$. ■

Supplementary appendix

A note on the contents

This Appendix provides supplementary material for the main paper, with sections ordered to reflect the order of presentation in the main text.

Section A derives the models used for the comparison in Proposition 6.

Section B solves two trading equilibria with heterogeneous risk attitudes: one with market orders and one with limit orders. These models are meant as robustness checks, exhibiting the loss of tractability associated with heterogeneous versions of the models in the main text, while maintaining the main result.

A Models with noise traders or random endowments and information acquisition (Proposition 6)

A.1 Market orders, risk neutrals, and noise traders

The model is the same as that in Section 3.1, with two differences: first, the rational traders are risk-neutral, and second, there are noise traders. The aggregate demand of the noise traders is $\theta \sim \mathcal{N}(0, \tau_\theta)$, and it is independent of D and ε_n, $n = 1, \ldots, N$.

The demand conjecture for the rational traders is the same as in (4a), but the market maker's pricing rule is now

$$P = \mathbb{E}\left[D \middle| \sum_{n=1}^{N} X_n + \theta\right], \tag{A.1a}$$

and the price conjecture is

$$P = \lambda\left(\sum_{n=1}^{N} X_n + \theta\right). \tag{A.1b}$$

The profit for rational trader n is $\pi_n = X_n(D - P)$, and his utility is $\mathbb{E}\left[\pi_n \middle| s_n\right]$. This implies that trader n's optimal demand is

$$X_n = \frac{\mathbb{E}\left[D - P_{-n} \middle| s_n\right]}{2\lambda}. \tag{A.2}$$

Comparing (4a) with (A.2) we obtain

$$2\lambda\beta_n = \frac{\tau_n}{\tau_n + \tau_D}\left(1 - \lambda\sum_{\substack{i=1\\ i\neq n}}^{N}\beta_i\right), \tag{A.3}$$

from which it follows that

$$\lambda\beta_n = \frac{\tau_n}{\tau_n + 2\tau_D}\left(1 - \sum_{i=1}^{N}\lambda\beta_i\right), \tag{A.4}$$

and that

$$\sum_{n=1}^{N}\lambda\beta_n = \left(\sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)\left(1 - \sum_{i=1}^{N}\lambda\beta_i\right). \tag{A.5}$$

Solving (A.5) we get

$$\sum_{n=1}^{N}\lambda\beta_n = \left(\sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)\left(1 + \sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)^{-1}, \tag{A.6}$$

and plugging into (A.4) we obtain

$$\lambda\beta_n = \frac{\tau_n}{\tau_n + 2\tau_D}\left(1 + \sum_{i=1}^{N}\frac{\tau_i}{\tau_i + 2\tau_D}\right)^{-1}. \tag{A.7}$$

By (A.1) we have

$$\lambda = \frac{\mathrm{Cov}\left(D, \sum_{i=1}^{N} X_i + \theta\right)}{\mathrm{Var}\left(\sum_{i=1}^{N} X_i + \theta\right)} = \frac{\frac{1}{\tau_D}\sum_{i=1}^{N}\beta_i}{\frac{1}{\tau_D}\left(\sum_{i=1}^{N}\beta_i\right)^2 + \sum_{i=1}^{N}\frac{\beta_i^2}{\tau_i} + \frac{1}{\tau_\theta}}, \tag{A.8}$$

which implies

$$\lambda^2 = \tau_\theta \left(\frac{1}{\tau_D}\sum_{i=1}^{N}\lambda\beta_i - \frac{1}{\tau_D}\left(\sum_{i=1}^{N}\lambda\beta_i\right)^2 - \sum_{i=1}^{N}\frac{(\lambda\beta_i)^2}{\tau_i} \right). \tag{A.9}$$

Using (A.7) yields

$$\lambda^2 = \tau_\theta \frac{\sum_{i=1}^{N}\frac{\tau_i(\tau_i+\tau_D)}{\tau_D(\tau_i+2\tau_D)^2}}{\left(1+\sum_{i=1}^{N}\frac{\tau_i}{\tau_i+2\tau_D}\right)^2}. \tag{A.10}$$

By the law of iterated expectations, (A.1b), (A.2) and (A.10) it follows that

$$\mathbb{E}\left[\pi_n\right] = \lambda \mathbb{E}\left[X_n^2\right] = \frac{(\lambda\beta_n)^2}{\lambda}\frac{\tau_n+\tau_D}{\tau_n\tau_D} = \frac{\frac{\tau_n(\tau_n+\tau_D)}{\tau_D(\tau_n+2\tau_D)^2}}{\sqrt{\tau_\theta}\sqrt{\sum_{i=1}^{N}\frac{\tau_i(\tau_i+\tau_D)}{\tau_D(\tau_i+2\tau_D)^2}}}\left(1+\sum_{i=1}^{N}\frac{\tau_i}{\tau_i+2\tau_D}\right)^{-1}. \tag{A.11}$$

With the same cost function as for Theorem 2, taking the first-order condition with respect to τ_n and then setting $\tau_n = \tau$ gives, after squaring both sides, that

$$\begin{aligned}\tau_\theta N^3\tau^3\left(\tau+\tau_D\right)\left(\tau+2\tau_D\right)^2\left[(N+1)\tau+2\tau_D\right]^4 \\ -\psi^2\tau_D\left[\left(6N^2-N-3\right)\tau^2+2\left(2N^2+5N-4\right)\tau_D\tau+4(2N-1)\tau_D^2\right]^2 = 0\end{aligned} \tag{A.12}$$

Condition (A.12) is the equilibrium condition for information acquisition with symmetric precisions. It is a tenth-order polynomial in τ, and it can be shown that is has a unique root by Descartes' rule of signs. (By inspection, the positive term contributes positive powers in descending orders of ten to three. The negative term contributes negative powers in descending orders of four to zero. In particular, the constant is always negative, which guarantees existence, and the polynomial contains powers from both of the two terms only for powers four and three. If N is large enough, those powers are both positive, while if ψ is large enough, those powers are both negative. The owers of orders two, one and zero are always negative, and thus we have a unique positive solution.)

Let $H(\tau, \psi)$ denote the left-hand side of (A.12). Noting that $H(\tau, \psi)$ is increasing in τ in equilibrium (the equilibrium condition in (A.12) has a unique positive root in τ , and because its leading coefficient is positive it must cross zero from below), we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial H}{\partial \psi}}{\frac{\partial H}{\partial \tau}} = \frac{2\psi\tau_D \left[\left(6N^2 - N - 3\right)\tau^2 + 2\left(2N^2 + 5N - 4\right)\tau_D\tau + 4(2N-1)\tau_D^2 \right]^2}{\frac{\partial H}{\partial \tau}} > 0. \tag{A.13}$$

From (A.10) we have that in equilibrium

$$\frac{1}{\lambda^2} = \frac{\tau_D}{\tau_\theta N} \frac{\left[(N+1)\tau + 2\tau_D\right]^2}{\tau\left(\tau + \tau_D\right)}. \tag{A.14}$$

and therefore liquidity depends on ψ only though its effect on τ, so that

$$\frac{d}{d\psi}\left(\frac{1}{\lambda^2}\right) = \frac{d}{d\tau}\left(\frac{1}{\lambda^2}\right)\frac{d\tau}{d\psi} \tag{A.15}$$

where by (A.14), we have

$$\frac{d}{d\tau}\left(\frac{1}{\lambda^2}\right) = \frac{\tau_D^2}{\tau_\theta N} \frac{\left[(N+1)\tau + 2\tau_D\right]}{\tau\left(\tau + \tau_D\right)} \left[(N-3)\tau - 2\tau_D\right]. \tag{A.16}$$

If $N \leq 3$, the above is always negative and thus liquidity decreases in ψ. If $N > 3$, whether liquidity is increasing or decreasing in ψ thus boils down to the sign of

$$(N-3)\tau - 2\tau_D \tag{A.17}$$

in equilibrium.

Let τ_c be the value of τ for which (A.17) is zero, that is, $\tau_c = 2\tau_D/(N-3)$. It suffices to derive conditions for $\tau < \tau_c$, because then

$$(N-3)\tau - 2\tau_D < 0, \tag{A.18}$$

which implies that liquidity decreases in τ (and thus liquidity also decreases in ψ.) To wit, because the equilibrium condition in (A.12) has a unique positive root in τ, it suffices to check when $H(\tau_c, \psi) > 0$, which implies that $\tau_c > \tau$ in equilibrium. We have

$$H(\tau_c, \psi) = \frac{256\tau_D^5 N^2 (N-1)^4}{(N-3)^{10}} \left[32\tau_\theta \tau_D^5 N(N-1)(N-2)^2 - (N-3)^6 \psi^2\right], \tag{A.19}$$

which is positive for small enough ψ.

A.2 Limit orders, risk neutrals, and noise traders

The model is the same as in Section 3.2, but now the traders are risk neutral ($\delta = 0$) and there are noise traders, whose aggregate demand is $\theta \sim \mathcal{N}(0, \tau_\theta)$, independently of D and ε_n, $n = 1, \ldots, N$.

The derivation follows along the lines of the proof of Theorem 3; I thus highlight the differences. The market clears stochastically,

$$\sum_{n=1}^{N} X_n + \theta = 0, \tag{A.20}$$

from which we obtain

$$P = \lambda \left(D \sum_{n=1}^{N} \beta_n + \sum_{k=1}^{N} \beta_k \varepsilon_k + \theta \right). \tag{A.21}$$

The definitions of λ and λ_{-n} are algebraically the same as before. Following Kyle (1989), P_{-n} can be written as

$$P_{-n} = \lambda_{-n} \left[D \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k + \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \varepsilon_k + \theta \right]. \tag{A.22}$$

As before, by the projection theorem

$$\mathbb{E} \left[D \middle| s_n, P_{-n} \right] = b_n s_n + c_n P_{-n} \tag{A.23}$$

but now

$$b_n = \frac{\tau_n}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \right)^2}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}, \tag{A.24a}$$

$$c_n = \sum_{\substack{k=1 \\ k \neq n}}^{N} \gamma_k \frac{\frac{\sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \right)^2}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}, \tag{A.24b}$$

and

$$\mathrm{Var}\left(D \middle| s_n, P_{-n}\right) = \frac{1}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}} \tag{A.24c}$$

Matching coefficients in the demand conjecture and the demand functions implied by the equilibrium we obtain

$$\beta_n = \frac{b_n}{\lambda_{-n}\left(1 + c_n\right)} \tag{A.25a}$$

and

$$\gamma_n = \frac{\left(1 - c_n\right)}{\lambda_{-n}\left(1 + c_n\right)}. \tag{A.25b}$$

Under homogeneous precisions ($\tau_n = \tau$ for all n) we get

$$\beta = \sqrt{\tau}\sqrt{\frac{N-2}{\tau_\theta N(N-1)}} \tag{A.26}$$

and

$$\gamma = \frac{N\tau + 2\tau_D}{N\sqrt{\tau}}\sqrt{\frac{N-2}{\tau_\theta N(N-1)}}. \tag{A.27}$$

To justify the equilibrium with homogeneous precisions, suppose that each trader n faces the information cost function in (3). His ex-ante utility is

$$\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] = \mathbb{E}\left[X_n^2\right]\lambda_{-n}$$

$$= \left[\beta_n^2\left(\frac{1}{\tau_n} + \frac{1}{\tau_D}\right) - 2\beta_n\gamma_n\frac{\frac{\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\tau_D} + \frac{\beta_n}{\tau_n}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)} + \gamma_n^2\frac{\frac{\left(\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\tau_D} + \frac{\beta_n^2}{\tau_n} + \sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)^2}\right.$$

$$\times \left[\frac{1}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \gamma_k} \right] \quad \text{(A.28)}$$

Because each trader k commits to β_k and γ_k after choosing τ_k, it follows by inspection of (A.24) and (A.25), that if we fix $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$, changes in the utility in (A.28) happen only through τ_n (with β_n and γ_n as functions of τ_n.) We may thus take the first-order condition of (A.28) holding $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$ as constants. Doing so, and setting $\tau_k = \tau$ we get

$$\frac{\sqrt{N-2}}{N\sqrt{\tau_\theta N(N-1)}} \frac{N(N-3)\left(\sqrt{\tau}\right)^2 + 2(N-1)\tau_D}{\sqrt{\tau}\left(N\left(\sqrt{\tau}\right)^2 + 2\tau_D\right)^2} = \frac{\left(\sqrt{\tau}\right)^2}{2\psi}. \quad \text{(A.29)}$$

Rearranging (A.29) we have

$$\sqrt{\tau_\theta N^7(N-1)}\left(\sqrt{\tau}\right)^7 + 4\sqrt{\tau_\theta N^5(N-1)}\tau_D\left(\sqrt{\tau}\right)^5 + 4\sqrt{\tau_\theta N^3(N-1)}\tau_D^2\left(\sqrt{\tau}\right)^3$$
$$- 2\psi N(N-3)\sqrt{N-2}\left(\sqrt{\tau}\right)^2 - 4\psi\tau_D\sqrt{N-2} = 0. \quad \text{(A.30)}$$

By inspection it follows that the coefficients of the polynomial in (A.30) switch signs only once, between the cubic and the quadratic term and thus, by Descartes's rule of signs, (A.30) has a unique positive solution for $\sqrt{\tau}$. (Note that equilibrium exists only if $N > 3$.)

Let $H_p(\sqrt{\tau}, \psi)$ denote the left-hand side of (A.30). Noting that $H_p(\sqrt{\tau}, \psi)$ is increasing in $\sqrt{\tau}$ in equilibrium (the equilibrium condition in (A.30) has a unique positive root in $\sqrt{\tau}$, and because its leading coefficient is positive it must cross zero from below), we obtain

$$\frac{d\sqrt{\tau}}{d\psi} = -\frac{\frac{\partial H_p}{\partial \psi}}{\frac{\partial H_p}{\partial \sqrt{\tau}}} = \frac{2\sqrt{N-2}\left(N(N-3)\left(\sqrt{\tau}\right)^2 + 2\tau_D(N-1)\right)}{\frac{\partial H_p}{\partial \tau}} > 0, \quad \text{(A.31)}$$

because $N > 3$. By the chain rule, and because $\lambda^{-1} = N\gamma$,

$$\frac{d\lambda^{-1}}{d\psi} = \frac{d\lambda^{-1}}{d\sqrt{\tau}} \frac{d\sqrt{\tau}}{d\psi} = \sqrt{\frac{N-2}{\tau_\theta N(N-1)}} \frac{N\tau - 2\tau_D}{\tau} \frac{d\sqrt{\tau}}{d\psi}, \tag{A.32}$$

so that $d\lambda^{-1}/d\psi < 0$ if and only if

$$N\tau - 2\tau_D \tag{A.33}$$

is negative in equilibrium. Let τ_{pc} be the value of τ for which (A.17) is zero, that is, $\tau_{pc} = 2\tau_D/N$. Because the equilibrium condition in (A.30) has a unique positive root in $\sqrt{\tau}$ and it crosses zero from below, it suffices to show that $H_p\left(\sqrt{\tau_{pc}}, \psi\right) > 0$, which implies that $\tau < \tau_{pc}$ in equilibrium. We have

$$H_p(\sqrt{\tau_{pc}}, \psi) = 8\tau_D \left[4\sqrt{2(N-1)\tau_\theta\tau_D^5} - \sqrt{(N-2)^3}\psi\right], \tag{A.34}$$

which is positive for small enough ψ and negative for large enough ψ. This implies that $d\lambda^{-1}/d\psi < 0$ for small enough ψ, and $d\lambda^{-1}/d\psi > 0$ for large enough ψ.

A.3 Limit orders, risk averters, and random endowments

Following Kyle (1989), I decompose the price function into the price excluding trader n's impact and a term that captures his demand curve. Here the latter term has two components,

$$P = P_{-n} + \lambda_{-n}X_n + \mu_{-n}z_n, \tag{A.35}$$

with μ_{-n} playing the same role as the slope of the residual supply curve, specifically extended to accommodate stochastic endowments. Adapting the methodology of Kyle (1989) to stochastic endowments, we have

$$P_{-n} = \lambda_{-n} \left[\sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k s_k + \sum_{\substack{k=1 \\ k \neq n}}^{N} (\kappa_k - 1)\, z_k \right], \tag{A.36}$$

where, in order have (A.36) consistent with (A.35), we must have $\mu_{-n} = -\lambda_{-n}$ and

$$\lambda_{-n} = \frac{1}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \gamma_k}. \tag{A.37}$$

Before I establish the comparative static in the information equilibrium, I first show that a trading equilibrium exists if we allow the traders in Diamond and Verrecchia (1981) to have market impact.

Theorem A.1 *Under homogeneous precisions, if $N > 2$ and $\delta > 0$, a unique rational trading equilibrium with random endowments exists, as long as $\delta^2 > N\tau\tau_z/(N-2)$.*

Proof of Theorem A.1. From the first-order condition we obtain

$$X_n = \frac{\mathbb{E}\left[D - P_{-n} \middle| \mathcal{F}_n\right] + 2\lambda_{-n} z_n}{2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| \mathcal{F}_n\right)} \tag{A.38}$$

and by the projection theorem we obtain

$$\mathbb{E}\left[D \middle| s_n, P_{-n}, z_n\right] = b_n s_n + c_n P_{-n} + f_n z_n, \tag{A.39}$$

where

$$b_n = \tau_n \mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right) \tag{A.40a}$$

$$c_n = \frac{\sum_{\substack{k=1\\k\neq n}}^N \beta_k \sum_{\substack{k=1\\k\neq n}}^N \gamma_k}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \sum_{\substack{k=1\\k\neq n}}^N \frac{(\kappa_k-1)^2}{\tau_z}} \mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right), \tag{A.40b}$$

$$f_n = 0, \tag{A.40c}$$

and

$$\mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right) = \left[\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \sum_{\substack{k=1\\k\neq n}}^N \frac{(\kappa_k-1)^2}{\tau_z}}\right]^{-1}. \tag{A.40d}$$

Matching coefficients in (17) and (A.38) we obtain

$$\beta_n = \frac{b_n}{\lambda_{-n}(1+c_n) + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right)} \tag{A.41a}$$

$$\gamma_n = \frac{1-c_n}{\lambda_{-n}(1+c_n) + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right)}, \tag{A.41b}$$

and

$$\kappa_n = \frac{(1+c_n)\lambda_{-n}}{\lambda_{-n}(1+c_n) + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}, z_n\right)}. \tag{A.41c}$$

Under homogeneous precisions, $\beta_n = \beta$, $\gamma_n = \gamma$, and $\kappa_n = \kappa$ for all n. Combining systems (A.40) and (A.41) we can solve for $\kappa - 1/\gamma$ and β/γ independently of γ, and then use the resulting expression to get γ. We obtain the following unique nontrivial solution:

$$\frac{\kappa-1}{\gamma} = -\delta \frac{\frac{\delta^2}{\tau\tau_z} + N}{\left(\frac{\delta^2}{\tau\tau_z} + N\right)\tau + \left(\frac{\delta^2}{\tau\tau_z} + 1\right)\tau_D}, \tag{A.42a}$$

$$\frac{\beta}{\gamma} = \frac{\tau}{-\delta}\frac{\kappa-1}{\gamma} = \tau \frac{\frac{\delta^2}{\tau\tau_z} + N}{\left(\frac{\delta^2}{\tau\tau_z} + N\right)\tau + \left(\frac{\delta^2}{\tau\tau_z} + 1\right)\tau_D}, \tag{A.42b}$$

and

$$\gamma = \frac{N\tau + \tau_D}{-\delta(N-1)} + \frac{2\frac{\delta}{\tau_z}}{\left(\frac{\delta^2}{\tau\tau_z} + 1\right)} + \frac{\frac{\delta}{\tau\tau_z}\tau_D}{\frac{\delta^2}{\tau\tau_z} + N}. \tag{A.42c}$$

Evaluating the traders' second-order condition with this solution we get

$$2\lambda_{-n} + \delta \text{Var}\left(D \middle| s_n, P_{-n}, z_n\right) > 0 \Leftrightarrow \frac{\delta}{\frac{\delta^2}{\tau\tau_z}(N-2) - N} \frac{N\left(\frac{\delta^2}{\tau\tau_z} + 1\right)^2}{\frac{\delta^2}{\tau\tau_z}(\tau + \tau_D) + (N\tau + \tau_D)} > 0. \tag{A.43}$$

The second-order condition cannot be strictly satisfied if $N \leq 2$ or if $\delta = 0$. If, however, $N > 2$ and $\delta > 0$, the second-order condition is strictly satisfied if and only if

$$\delta^2 > \frac{N}{N-2}\tau\tau_z. \tag{A.44}$$

■

Next I show that liquidity decreases in ψ when ψ is small enough. Using (A.42c) we obtain

$$\frac{d}{d\tau}\gamma = \frac{-G(\tau)}{\delta(N-1)\left(\delta^2 + N\tau\tau_z\right)^2\left(\delta^2 + \tau\tau_z\right)^2}, \tag{A.45}$$

where

$$G(\tau) = \tau^4\tau_z^4 N^3 + 2\tau^3\tau_z^3\delta^2 N^2(N+1) + \tau^2\tau_z^2\delta^2 N\left[(N-1)\tau_D\tau_z - \left(N^2 - 6N - 1\right)\delta^2\right] +$$
$$2\tau\tau_z\delta^4 N\left[(N-1)\tau_D\tau_z - (N-3)\delta^2\right] + \delta^6\left[N(N-1)\tau_D\tau_z - (N-2)\delta^2\right]. \tag{A.46}$$

It follows that $\lambda^{-1} = N\gamma$ decreases in τ, at least for small enough τ. For example,

$$\left.\frac{d}{d\tau}\gamma\right|_{\tau=0} = -\frac{N-2}{\delta(N-1)} - \frac{N\tau_D\tau_z}{\delta^3}, \tag{A.47}$$

which is negative if $\delta > 0$, and thus by continuity λ^{-1} decreases if τ is small enough.

To show that λ^{-1} decreases in ψ for small enough ψ, by the chain rule it suffices to show that $d\tau/d\psi > 0$ in the information equilibrium. Repeating the analysis for trader n's ex-ante utility as in the proof of Theorem 3, adapting it to this economy, it follows that

$$\mathbb{E}\left[u_n\right] = \frac{1}{2}\left[2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}, z_n\right)\right]\mathbb{E}\left[X_n^2\right] + \mathbb{E}\left[z_n P_{-n}\right] - \lambda_{-n}\mathbb{E}\left[z_n^2\right], \tag{A.48}$$

where

$$2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}, z_n\right) = \frac{2}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k} + \frac{\delta\left[\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \sum_{\substack{k=1\\k\neq n}}^N \frac{(\kappa_k-1)^2}{\tau_z}\right]}{(\tau_D+\tau_n)\left[\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \sum_{\substack{k=1\\k\neq n}}^N \frac{(\kappa_k-1)^2}{\tau_z}\right] + \left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2} \tag{A.49}$$

and

$$\mathbb{E}\left[X_n^2\right] = \frac{\gamma_n^2\left(\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_D}\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2 + \sum_{\substack{k=1\\k\neq n}}^N \frac{(\kappa_k-1)^2}{\tau_z}\right)}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k\right)^2\left(\frac{\gamma_n}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k}+1\right)^2} + \frac{\beta_n^2\left(\frac{1}{\tau_D}+\frac{1}{\tau_n}\right)}{\left(\frac{\gamma_n}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k}+1\right)^2} + \frac{\left(\frac{\gamma_n}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k}+\kappa_n\right)^2}{\tau_z\left(\frac{\gamma_n}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k}+1\right)^2}. \tag{A.50}$$

Substituting (A.41) into the above, taking the first-order condition with respect to τ_n while holding the quantities that do not depend on it constant, then using the solution with homogeneous precisions from above, and then setting $\tau_{-n} = \tau$ gives

$$\psi\Big\{\delta^6 N^2\tau_D + \delta^4\tau_z\left[2(N-2)^2(N-1)\tau^2 + N(4N^2-7N+6)\tau\tau_D + (N-2)(N-1)N\tau_D^2\right]$$
$$+\,\delta^2 N\tau\tau_z^2\left[4(N-2)^2(N-1)\tau^2 + \left(N^3-2N^2+4\right)\tau\tau_D - 2(N-1)\tau_D^2\right]$$
$$-\,N^2\tau^2\tau_z^3\left[2(N-1)(2N-3)\tau^2 + \left(N^2-2\right)\tau\tau_D + (N-1)\tau_D^2\right]\Big\}$$
$$-\,\delta N^2(N-1)\tau_D\tau_z\tau\left[\delta^2(\tau+\tau_D) + \tau\tau_z(N\tau+\tau_D)\right]^2 = 0. \quad \text{(A.51)}$$

I now show that (A.51) has a unique positive solution that satisfies the second-order condition. Rewriting (A.51) as a fixed-point mapping we get

$$\tau = \frac{\psi}{\delta N^2(N-1)\tau_D\tau_z\left[\delta^2(\tau+\tau_D) + \tau\tau_z(N\tau+\tau_D)\right]^2}$$
$$\Big\{\delta^6 N^2\tau_D + \delta^4\tau_z\left[2(N-2)^2(N-1)\tau^2 + N(4N^2-7N+6)\tau\tau_D + (N-2)(N-1)N\tau_D^2\right]$$
$$+\,\delta^2 N\tau\tau_z^2\left[4(N-2)^2(N-1)\tau^2 + \left(N^3-2N^2+4\right)\tau\tau_D - 2(N-1)\tau_D^2\right]$$
$$-\,N^2\tau^2\tau_z^3\left[2(N-1)(2N-3)\tau^2 + \left(N^2-2\right)\tau\tau_D + (N-1)\tau_D^2\right]\Big\} \quad \text{(A.52)}$$

At $\tau = 0$ the right-hand side of (A.52) is

$$\psi\frac{\delta^2 N + (N-1)(N-2)\tau_D\tau_z}{N(N-1)\tau_D^2\tau_z} > 0 \quad \text{(A.53)}$$

and at $\tau \to \infty$ the right-hand side of (A.52) is

$$-\frac{\psi(4N-6)}{N^2\tau_D} < 0 \quad \text{(A.54)}$$

because $N > 2$. By continuity it follows that the right-hand side of (A.52) as a function of τ crosses the 45° line at least once in $[0, \tau]$. Because (A.44) must hold in equilibrium, and because, as we will see shortly, the right-hand side of (A.52) increases in δ holding τ fixed, it suffices to show that (A.52) has a unique solution when

$$\delta_0 = \sqrt{\frac{N\tau\tau_z}{N-2}} \tag{A.55}$$

for the following reason. If (A.52) has a unique solution with $\delta = \delta_0$, then it cannot have more than one solution for values of $\delta > \delta_0$, because that would imply that the right-hand side of (A.52) decreases in δ for some values of τ, which is a contradiction. Setting $\delta = \delta_0$ in (A.52) we obtain

$$\tau = \psi \frac{4N\tau}{(N-2)\left(N\tau + \tau_D\right)^2}, \tag{A.56}$$

which has a unique positive solution for τ.

To complete the argument for uniqueness, it remains to show that the right-hand side of (A.52) increases in δ holding τ fixed. The derivative the right-hand side of (A.52) with respect to δ is

$$\begin{aligned}
\tau = {} & \frac{2\psi\delta}{\delta N^2(N-1)\tau_D\tau_z\left[\delta^2\left(\tau+\tau_D\right)+\tau\tau_z(N\tau+\tau_D)\right]^3} \\
& \Big\{\delta^6 N^2\tau_D\left(\tau+\tau_D\right) + 3\delta^4 N^2\tau\tau_z\tau_D\left(N\tau+\tau_D\right) + \delta^2\tau\tau_z^2\tau_D\Big[\left(3N^4+12N^3-40N^2+44N-16\right)\tau^2 \\
& \qquad + N\left(N^3+4N^2-8N+6\right)\tau\tau_D + 2N\left(N-1\right)^2\tau_D^2\Big] \\
& \quad + N\tau^2\tau_z^3\Big[4N\left(N-1\right)^3\tau^3 + \left(N^4+12N^3-40N^2+44N-16\right)\tau^2\tau_D \\
& \qquad + \left(3N^3-2N^2-4N+4\right)\tau\tau_D^2 + 2N\left(N-1\right)^2\tau_D^3\Big]\Big\},
\end{aligned} \tag{A.57}$$

which is positive for $N > 2$ by inspection.

Finally, let $H_e(\tau, \psi)$ denote the left-hand side of (A.51). Using (A.51) to simplify the partial derivative of $H_e(\tau, \psi)$ with respect to ψ we have

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial H_e}{\partial \psi}}{\frac{\partial H_e}{\partial \tau}} = \frac{\delta N^2(N-1)\tau_D \tau_z \tau \left[\delta^2(\tau + \tau_D) + \tau\tau_z(N\tau + \tau_D)\right]^2}{-\psi \frac{\partial H_e}{\partial \tau}}. \tag{A.58}$$

Holding ψ fixed, the equilibrium condition in (A.51) has a *unique* positive root in τ, and because, by inspection, it is a fifth-order polynomial with a negative leading coefficient, it must cross zero from above, implying that the slope of (A.51) at its root is negative. This shows that holding ψ fixed, $H_e(\tau, \psi)$ is increasing in τ in equilibrium. It follows that $\partial H_e / \partial \tau < 0$, and that $d\tau / d\psi > 0$.

B Solutions of models with mixed risk attitudes

B.1 Market-order equilibrium with one risk seeker and one risk averter

The model here is the same as in Section 3.1, except that there are two traders, one risk averse and one risk seeking. The risk aversion parameter of the risk-averse trader is $\delta_{RA} = \delta > 0$, and the risk aversion parameter of the risk-seeking trader is $\delta_{RS} = -\delta < 0$. The signal precision is the same for both traders and equal to τ.

Let the risk averter's trading intensity be β_{RA} and the risk seeker's trading intensity be β_{RS}. Deriving the equilibrium as above gives the following equations:

$$\begin{aligned}
&\delta\beta_{RA}^5\tau^2(\tau+\tau_D)^2 + \beta_{RA}^4\tau^3(\tau+\tau_D)(2\delta\beta_{RS}+\tau+\tau_D) \\
&\quad + \beta_{RA}^3\beta_{RS}\tau^2\left[\delta\beta_{RS}\left(2\tau^2+3\tau\tau_D+2\tau_D^2\right)+\tau(\tau+\tau_D)(3\tau+2\tau_D)\right] \\
&\quad + \beta_{RA}^2\beta_{RS}^2\tau^3\left[2\delta\beta_{RS}(\tau+2\tau_D)+\tau(3\tau+5\tau_D)\right] \\
&\quad + \beta_{RA}\beta_{RS}^3\tau^2\left[\delta\beta_{RS}\left(\tau^2+\tau\tau_D+\tau_D^2\right)+\tau\left(\tau^2+\tau\tau_D+2\tau_D^2\right)\right] - \beta_{RS}^4\tau^3\tau_D(\tau+\tau_D) = 0,
\end{aligned} \tag{B.1a}$$

$$\begin{aligned}
&-\delta\beta_{RS}^5\tau^2(\tau+\tau_D)^2 + \beta_{RS}^4\tau^3(\tau+\tau_D)(-2\delta\beta_{RA}+\tau+\tau_D) \\
&\quad + \beta_{RS}^3\beta_{RS}\tau^2\left[-\delta\beta_{RA}\left(2\tau^2+3\tau\tau_D+2\tau_D^2\right)+\tau(\tau+\tau_D)(3\tau+2\tau_D)\right] \\
&\quad + \beta_{RS}^2\beta_{RS}^2\tau^3\left[-2\delta\beta_{RA}(\tau+2\tau_D)+\tau(3\tau+5\tau_D)\right] \\
&\quad + \beta_{RS}\beta_{RS}^3\tau^2\left[-\delta\beta_{RA}\left(\tau^2+\tau\tau_D+\tau_D^2\right)+\tau\left(\tau^2+\tau\tau_D+2\tau_D^2\right)\right] - \beta_{RA}^4\tau^3\tau_D(\tau+\tau_D) = 0.
\end{aligned} \tag{B.1b}$$

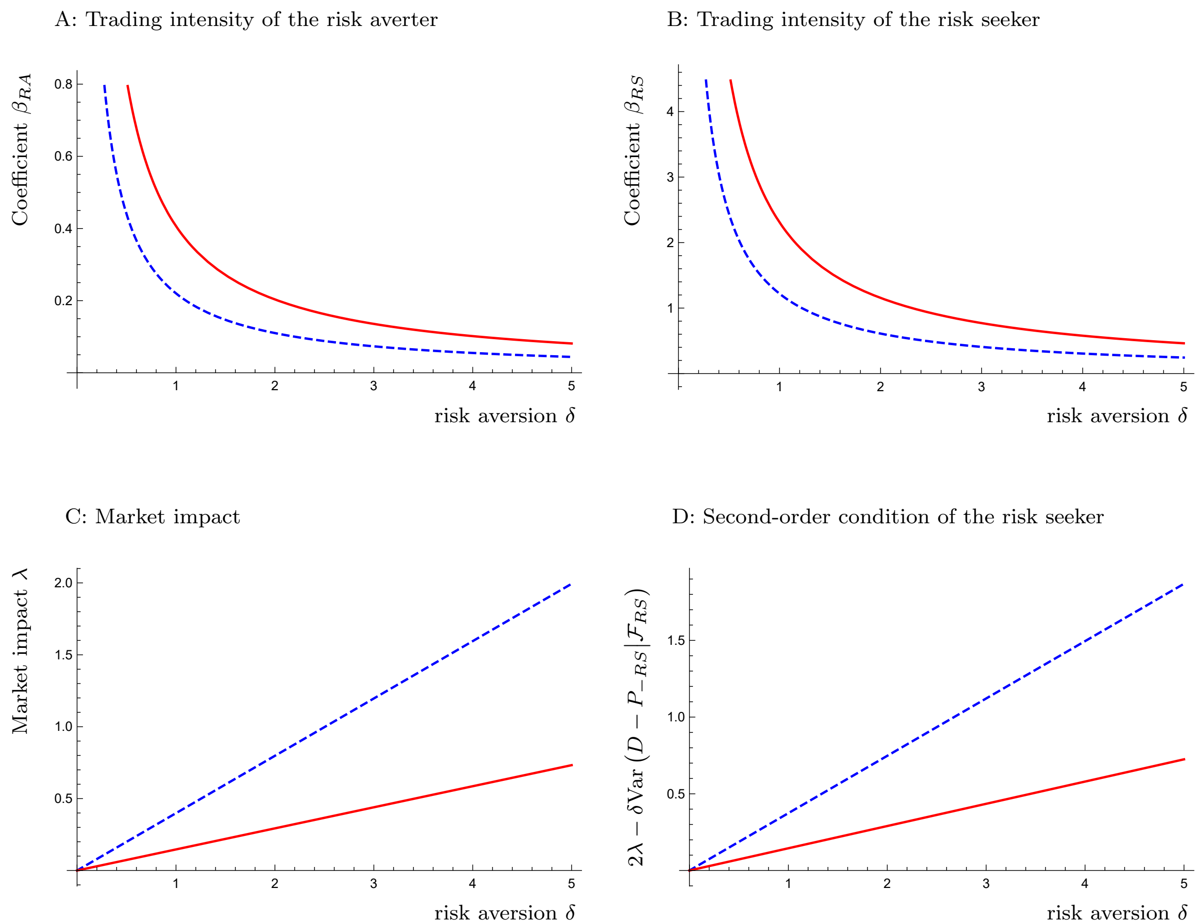

Figure 3: Solution of market-order equilibrium with one risk seeker and one risk averter as a function of risk aversion δ, with risk seeking parameter equal to $-\delta$. The dashed blue curves show trading intensities, market impact, and the risk seeker's second-order condition with signal precision set to $\tau = 1$, while the solid red curves show the same quantities with signal precision set to $\tau = 5$. The precision of the dividend is $\tau_D = 1$.

The second-order condition of the risk seeker is satisfied if and only if

$$2\lambda + \delta_{RS} \text{Var}\left(D - P_{-RS} \middle| s_{RS}\right) > 0, \tag{B.2}$$

while it is always satisfied for the risk averter as long as an equilibrium exists. Figure 3 shows such an equilibrium, under the values $\tau_D = 1$, $0 \leq \delta \leq 5$, with $\tau = 1$ and $\tau = 5$.

B.2 Limit-order equilibrium with one risk-seeking trader and many risk-neutral traders

The model I present here is similar to Kyle (1989), but without noise traders, without uninformed traders, and without risk aversion. There are $N+1$ traders in total. Every trader $n = 1, \ldots, N+1$, observes the price and a signal as in (1), under the simplifying assumption that τ_n is the same for all n. The utility of trader n is

$$u(\pi_n; s_n) = \mathbb{E}\left[\pi_n \middle| s_n\right] - \frac{1}{2}\delta_n \text{Var}\left(\pi_n \middle| s_n\right), \tag{B.3}$$

where, as in the main text, the profit for trader n is $\pi_n = X_n(D-P)$. The first N traders are risk neutral (so that $\delta_n = 0$, $n = 1, \ldots, N$) and the Nth trader likes risk ($\delta_n = \delta < 0$).

I assume that the price function is linear, and that the demand function of trader n is

$$X_n = \beta_n s_n - \gamma_n P. \tag{B.4}$$

The market clears deterministically. We have, in particular, that

$$\sum_{n=1}^{N+1} X_n = 0, \tag{B.5}$$

which implies that

$$P = \lambda \left[\left(\sum_{n=1}^{N+1} \beta_n\right) D + \sum_{n=1}^{N+1} \beta_n \varepsilon_n\right], \tag{B.6}$$

where

$$\lambda = \left(\sum_{k=1}^{N+1} \gamma_k\right)^{-1}. \tag{B.7}$$

Moreover, following Kyle (1989), it is straightforward to show that

$$X_n = \frac{\mathbb{E}\left[D - P \middle| s_n, P\right]}{\lambda_{-n} + \delta_n \mathrm{Var}\left(D - P \middle| s_n, P\right)}, \tag{B.8}$$

where λ_{-n} is the slope of the residual supply curve for trader n, given as

$$\lambda_{-n} = \left(\sum_{\substack{k=1 \\ k \neq n}}^{N+1} \gamma_k \right)^{-1}. \tag{B.9}$$

Let the demand coefficients of the risk-neutral traders be β_{RN} and γ_{RN}, and the demand coefficients of the risk seeker be β_{RS} and γ_{RS}. Moreover, let

$$\bar{\tau}_D = (N+1)\tau + \tau_D. \tag{B.10}$$

Deriving the conditional moments in (B.8) and matching coefficients with (B.4), gives, after some algebra, the following equations:

$$\beta_{RS} = N\beta_{RN} \frac{\tau\gamma_{RN}}{\beta_{RN}\bar{\tau}_D + N\gamma_{RN}(\tau + \delta\beta_{RN})} \tag{B.11}$$

and

$$\gamma_{RS} = N\gamma_{RN} \frac{\beta_{RN}\bar{\tau}_D - N\tau\gamma_{RN}}{\beta_{RN}\bar{\tau}_D + N\gamma_{RN}(\tau + \delta\beta_{RN})}, \tag{B.12}$$

where the coefficients of the risk-neutral traders are given as the solution to the system

$$\begin{aligned} \beta_{RN}^3 \left[\delta\gamma_{RN}N + \bar{\tau}_D\right]^3 (N-1)(\bar{\tau}_D - \tau) + \beta_{RN}^2\gamma_{RN}\left[\delta\gamma_{RN}N + \bar{\tau}_D\right]^2 (N-1)N\tau(3\bar{\tau}_D - \tau) \\ + \beta_{RN}\gamma_{RN}^2 \left[\delta\gamma_{RN}N + \bar{\tau}_D\right]\left[\delta\gamma_{RN}N(N-1) + \left(3N^2 - 1\right)\bar{\tau}_D\right] N\tau^2 \\ + \gamma_{RN}^3\left[-\delta\gamma_{RN}N + (N^2-1)\bar{\tau}_D\right] N^2\tau^3 = 0, \end{aligned} \tag{B.13a}$$

and

$$
\begin{aligned}
\beta_{RN}^{3}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)^{2}(N-1)\left(\bar{\tau}_{D}-\tau\right)\left[\delta\gamma_{RN}N(N-2)+2\left(N-1\right)\bar{\tau}_{D}\right] \\
-\beta_{RN}^{2}\gamma_{RN}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)(N-1)N\tau \\
\cdot\left\{\delta^{2}\gamma_{RN}^{2}N^{2}(N-1)+\delta\gamma_{RN}N\left[(2N+3)\bar{\tau}_{D}-2\tau\right]+2\bar{\tau}_{D}\left(2\bar{\tau}_{D}-\tau\right)\right\} \\
-\beta_{RN}\gamma_{RN}^{2}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)N^{2}\tau^{2}\left[\delta\gamma_{RN}N(N-1)^{2}+2\left(N^{2}+N-1\right)\bar{\tau}_{D}\right] \\
+\delta\gamma_{RN}^{4}N^{5}\tau^{3}=0. \quad \text{(B.13b)}
\end{aligned}
$$

The second-order condition of each trader is satisfied if and only if

$$2\lambda_{-n}+\delta_{n}\mathrm{Var}\left(D-P\middle|s_{n},P\right)>0. \tag{B.14}$$

For the risk-neutral traders this is equivalent to

$$(N-1)\gamma_{RN}+\gamma_{RS}>0, \tag{B.15}$$

while for the risk seeker it is equivalent to

$$\frac{2}{N\gamma_{RN}}+\frac{\delta}{\bar{\tau}_{D}}>0. \tag{B.16}$$

Figure 4 shows the only equilibrium for which the second-order conditions of all traders are satisfied, under the values $\tau_D=1$, $\tau=1$, $2\leq N\leq 30$, with $\delta=-1$ and $\delta=-2$.

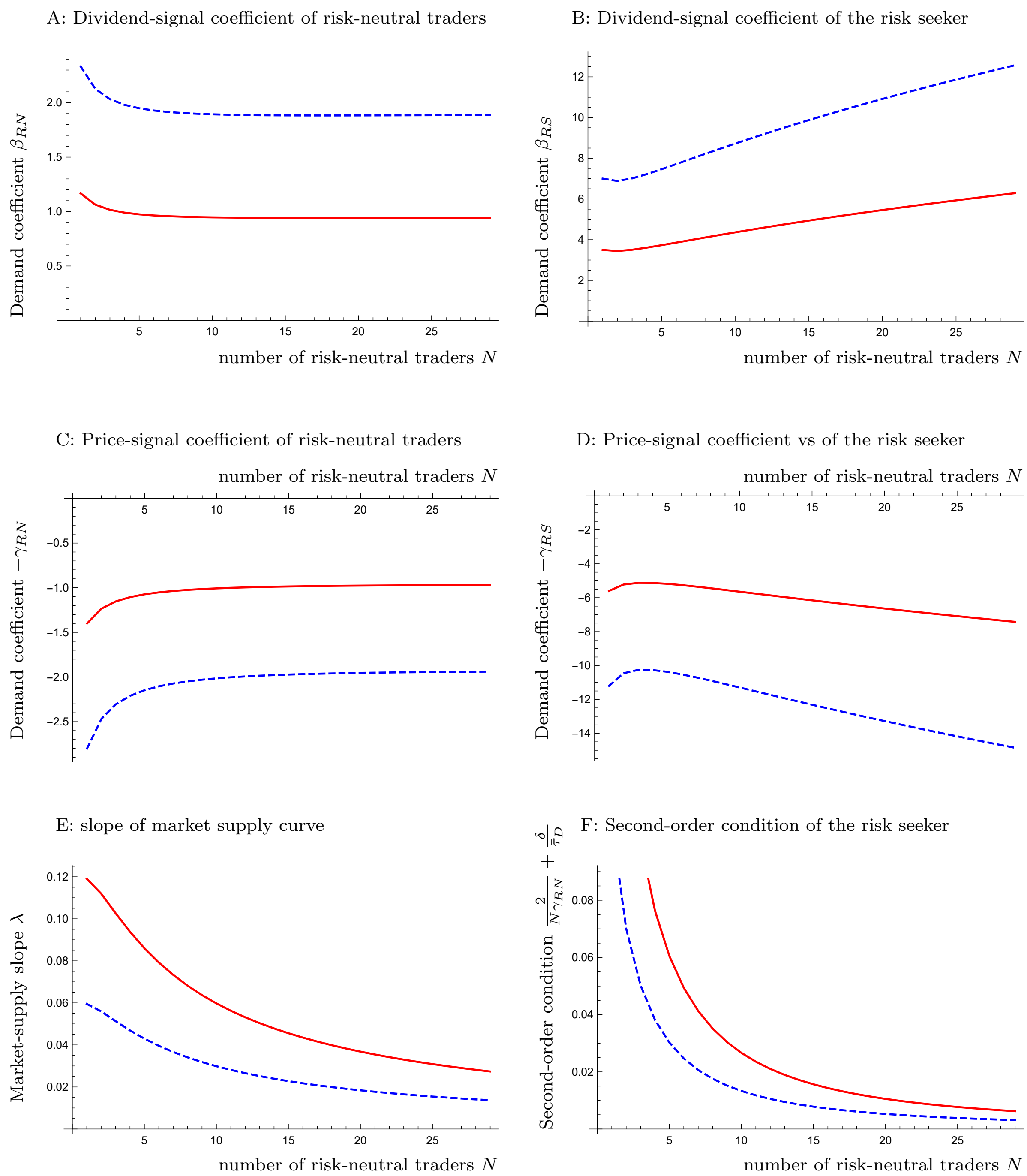

Figure 4: Solution of limit-order equilibrium with one risk seeker and N risk-neutral traders and observable prices. The dashed blue curves show demand coefficients, the slope of the market supply curve (λ), and the risk seeker's second-order condition with risk aversion set to $\delta = -1$, while the solid red curves show the same quantities with risk aversion set to $\delta = -2$. The precision of the dividend is $\tau_D = 1$, and the precision of the signal noise is $\tau = 1$ for all traders.