Tragedy of Complexity

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Abstract

Complexity can create value. At the same time, understanding more complex goods requires more of an agent's attention. We show that equilibrium complexity is generally inefficient when agents face competing demands on their limited attention. Because attention allocation is hump-shaped in complexity, equilibrium complexity is distorted towards intermediate levels: well-understood goods are inefficiently complex, whereas less well-understood goods are oversimplified. We apply our model to financial institutions facing regulatory bodies and CEOs interacting with corporate divisions.

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1 Introduction

Economic activities differ vastly in their complexity. Some appear overly complex, like the rules set by financial regulators. Others appear oversimplified, for example when the media and politicians discuss policy issues without the required nuance. What these situations have in common is that an economic agent (a household, a financial institution, or a CEO) faces multiple demands on her limited attention. This paper develops a parsimonious model of the economic forces that determine equilibrium complexity in such settings.

The key premise of our model is that complexity can add value. This feature represents a departure from much of the existing literature, which has focused mainly on complexity as a means for obfuscation and, hence, a source of market power. At the same time, our model captures that when something is more complex—i.e., "not easy to understand or explain"[1]—reaching a certain depth of understanding requires more of the agents' limited attention, which invariably reduces the amount of attention agents can allocate to other goods.

Our analysis yields three main results. First, we show that, even though complexity can generate value, equilibrium complexity is generally inefficient. In contrast to models in which complexity is a means to obfuscate, we find that equilibrium complexity can, in some cases, be too low. The inefficiency is the consequence of an *attention externality*: When choosing the complexity of their goods, suppliers do not take into account that attention is a common resource. As a result, suppliers distort their complexity choice to increase the amount of attention the consumer allocates to their goods. They neglect the concomitant reduction in the value of other goods that receive less attention. Depending on the direction of the consumer's attention reallocation in response to changes in complexity, this attention externality can lead to too much or too little complexity.

Second, we characterize the consumer's attention reallocation in response to a change in the complexity of an individual good. Under relatively weak assumptions (mainly boundedness of

[1]The Britannica Dictionary (last retrieved October 18, 2024) https://www.britannica.com/dictionary/complex

the value of the good), the amount of attention a consumer allocates to a good is hump-shaped in the good's complexity: Goods of intermediate complexity attract the most attention from consumers. This hump shape leads to a distortion towards intermediate complexity. Therefore goods that are well understood in the social planner's solution are made more complex than they should be, whereas goods that are less well understood in the social planner's solution are made too simple.

Third, we show that the direction of the consumer's attention reallocation in response to a change in complexity depends on the own-price elasticity of demand for understanding a good. When the consumer's demand for understanding a good is inelastic (i.e., the own-price demand elasticity is less than one), the consumer reacts to an increase in the complexity of good i by increasing the attention allocated to that good, thereby creating an incentive for the supplier to raise complexity above the socially optimal level.

In our model, an agent with limited attention consumes goods from multiple suppliers and decides how much attention to pay to different goods given their complexities. We model limited attention by assuming that the consumer has a fixed time budget that she allocates across all goods. Suppliers have market power, which allows them to extract a share of the value generated by their goods. The main choice variable for suppliers is the complexity of their goods, which they set in a non-cooperative fashion.

The value generated by good i consists of two components. First, the value depends directly on the good's complexity. This assumption captures that, holding fixed the consumer's attention, a more complex good can be more valuable, for example, because of additional features or customization. Second, holding fixed a good's complexity, the value of good i is higher the more time the consumer devotes to understanding the good. Therefore, as in Becker (1965), the consumer's time acts as an input to the value of goods. This assumption captures that a deeper understanding of a good makes this good more valuable to the consumer.

The key to our model is the assumption that more complex goods require more attention to achieve the same depth of understanding. Specifically, we assume that, when the complexity of a good doubles, it takes the consumer twice as much time to attain the same depth of understanding. Accordingly, the consumer's understanding of a good equals the time spent on the good divided by the good's complexity. This formulation generates an intuitive trade-off: While a more complex good is potentially more valuable to the consumer, the consumer must pay more attention to reach the same depth of understanding.

When choosing the complexity of their goods, suppliers internalize that consumers respond by adjusting the amount of attention they allocate to the good. Because suppliers extract a fraction of the value generated by their goods, they have an incentive to distort the good's complexity in the direction that increases the amount of attention paid to it by the consumer. However, suppliers do not internalize that attention is a shared resource—an increase in attention paid to their goods necessarily corresponds to a decrease in attention paid to other goods. These other goods decrease in value, generating an *attention externality.* Our analysis reveals that the consumer's attention allocation is hump-shaped in complexity: Goods of intermediate complexity attract the most attention, whereas goods of low and high complexity attract less attention. This hump shape arises because the payoff from understanding a good is bounded. Simple goods are so easy to understand that the consumer has no incentive to devote much attention to them. In contrast, the consumer does not understand exceptionally complex goods well even if she devoted all her attention to them, leading her to give up learning about them.

Because of this hump-shape, relatively simple goods are located to the left of the peak of the attention curve. In this region, the consumer's attention allocation is increasing in complexity, incentivizing suppliers to make these goods overly complex. Since simple goods are relatively well understood, it is (perhaps counterintuitively) well-understood goods that are too complex in equilibrium. In contrast, relatively complex goods lie to the right of the peak of the attention curve. In this region, the consumer's attention allocation is decreasing in complexity. Suppliers then have an incentive to make these goods too simple so that less well-understood goods are "dumbed down" in equilibrium. Irrespective of whether the equilibrium distortion is towards too much or too little complexity, the shadow price of attention is higher than in the social planner's solution—the consumer is "pressed for time."

Dealing with the tragedy of complexity is not straightforward because consumer attention is generally not priced. The price of the good only eliminates the inefficiency in special cases when all attention is allocated after the consumer has paid for the good. Moreover, in many salient cases, including regulation and communication, the good is often not priced either. We discuss two ways to overcome the inefficiency in such non-price settings. The first option is to require suppliers to offer a full menu of complexities. In this case, the consumer will choose the optimal level of complexity. This solution necessitates regulation because each supplier has an incentive to deviate by not offering the optimal complexity as part of its menu. Alternatively, the consumer can eliminate the inefficiency if she can commit to spending a fixed amount of time on each good. Such a commitment eliminates the reallocation of attention in response to changes in complexity and, hence, the externality. However, this solution demands substantial sophistication from the consumer, who must choose (and commit to) the efficient amount of time spent on each good.

Our results generate predictions regarding the situations conducive to excess complexity. For example, firms and financial institutions often face competing demands on their attention both from regulators and from managing their business operations. They also have relatively large attention budgets and a strong incentive to understand relevant regulations in detail. In such cases, our model predicts that regulators tend to make regulation overly complex. We show that it would improve welfare if banks could choose from a menu of rules with different levels of complexity. In contrast, households with relatively limited attention budgets are unlikely to understand complicated policy issues deeply. According to our model, the media and politicians will oversimplify these issues. Finally, when corporate divisions communicate with CEOs, our model predicts that simple problems that should be easy to understand will often be expressed in an overly complex fashion. In contrast, inherently complicated issues are presented in an oversimplified manner. Our analysis points towards a solution: CEOs may want to commit to spending a fixed amount of time dealing with a particular division, limiting meetings and time spent on emails.

Related literature. By viewing time as an input to the value of consumption goods, our approach to modeling complexity builds on the classic work of Becker (1965). We extend this framework by introducing complexity choice. The choice of complexity affects the value of the good directly, but also changes how the consumer transforms her time into understanding the good. By assuming a limited time budget for the consumer, our framework captures that complexity is inherently tied to bounded rationality (Brunnermeier and Oehmke, 2009) and inattention (Gabaix, 2019). The constraint on the consumer's time serves a role similar to information processing constraints in models of rational inattention (see, e.g., Sims 1998, 2003). The interaction of complexity choice with the consumer's choice of attention allocation differentiates our work from models of quality choice, as analyzed by a literature going back to Spence (1975). Consistent with our findings, Gonçalves (2024) finds that time spent on a task is non-monotonic in complexity but does not investigate complexity choice, the main focus of this paper.

Our paper contributes to the growing literatures on complexity and competition for attention. The existing literature on complexity has predominantly focused on how firms can use complexity as a means to obfuscate a product's price or quality (see, e.g., Carlin, 2009; Carlin and Manso, 2010; Ellison and Wolitzky, 2012; Piccione and Spiegler, 2012; Spiegler, 2016). A common feature of these models is that complexity is inherently bad: It is used by firms to increase market power or to influence the consumer's purchasing decision to their advantage. In these settings, firms generally have an incentive to make products too complex. In contrast, in our model complexity has also a bright side, because complexity can increase the value of the underlying good. As a result, our model generates the novel prediction that equilibrium complexity can be too high or too low.

The literature on competition for attention has focused mainly on how limited consumer attention affects advertising levels and product market competition. Like our paper, this literature highlights externalities that can arise when attention is a common resource. In contrast to our paper, this literature has focused manipulating consumer attention allocation to reduce price competition, a channel that is absent in our paper. For example, in Anderson and de Palma (2012), competition for limited attention generates links across otherwise independent markets, as more attention in one market increases competition there but reduces attention and competition in other markets. In De Clippel, Eliaz and Rozen (2014), limited consumer attention introduces an additional dimension of competition across markets when consumers can search for a competitor's price only in some markets. Bordalo, Gennaioli and Shleifer (2016) show that consumer attention can be drawn to either price or quality, resulting in equilibria that are price- or quality-salient. In Eliaz and Spiegler (2011a,b) sellers seek to influence the options considered by consumers with limited attention and may use loss-making goods as attention grabbers. Hefti (2018) and Hefti and Liu (2020) investigate how limited attention affect pricing and advertising in models of horizontal product differentiation. In Johnen and Leung (2022), firms disclose excessively detailed information to distract consumers from making price comparisons.

As applications of our model, we highlight financial regulation and communication within firms. In the context of financial regulation, Hakenes and Schnabel (2014) develop a model of regulatory capture by sophistication. Asriyan, Foarta and Vanasco (2023) show that regulatory complexity can arise in situations in which the median voter has to approve a new regulation. In the context of communication within firms, Dessein, Galeotti and Santos (2016) study optimal organizational focus and communication structures in face of limited attention. Thakor and

Merton (2023) study the interaction between product complexity, transparency, and trust in firms.

Finally, complexity has also been studied in the context of financial securities. Carlin, Kogan and Lowery (2013) provide experimental evidence that complexity affects volatility, liquidity and trading efficiency. Célérier and Vallée (2017) and Ghent, Torous and Valkanov (2019) study complexity, promised yields, and realized returns in structured financial products. Ganglmair and Wardlaw (2017) investigate complexity in private loan agreements. Basak and Buffa (2017) analyze how more complex financial models can generate operational risk.

2 Model Setup

We consider an economy with N suppliers (he), who design (choose the complexity) of the good they supply.[2] A single representative consumer (she) consumes the goods designed by the suppliers. There is one supplier per good $i \in \{1, ..., N\}$. The supplier has an interest in the value of the good he designs. Specifically, we assume that the supplier's payoff is given by $\theta_i \cdot v_i$, while the payoff to the consumer is $(1 - \theta_i) \cdot v_i$, where $\theta_i \in (0, 1)$.[3] One interpretation of these payoffs is that v_i is the value of the good, which is split between the supplier and the consumer according to the weights θ_i (supplier) and $1 - \theta_i$ (consumer). This is the interpretation we use in the text below. An alternative interpretation is that both the consumer and supplier care about the value of the good but assign different utility weights.[4]

The key decision for each supplier is to choose the complexity c_i of the good he designs. While complexity has no direct cost (or benefit) for the supplier, complexity matters because it affects the value of the good. On the one hand, complexity can add value, for example,

[2]Our main model focuses on the symmetric case in which all suppliers choose complexity. However, it is sufficient for the main results that at least one of the suppliers chooses complexity strategically. The complexity of other goods could be fixed or set by the consumer, see Section 4.2.

[3]For now, we simply assume this simple payoff structure. We provide a more detailed discussion of the payoff structure and other assumptions in Section 4.1.

[4]For arbitrary weights that do not add up to one, v_i can always be rescaled to recover the above setup.

when it arises as a byproduct of customization that caters the consumer's specific needs. On the other hand, realizing the full value of a more complex good requires attention from the consumer, who needs to devote time to understand a more complex good. The total value of a complex good therefore arises from the combination of its characteristics and the time the consumer allocates to the good. In this respect, our paper builds on classic work on time as an input into the utility derived from market goods pioneered by Becker (1965). Thus while our notion of complexity may also be interpreted as sophistication of the good in the sense that attention and understanding is needed to enjoy the good, it is clearly distinct from the traditional notion of quality which increases the value of the good without any additional time input from the consumer.

Formally, we capture these features of complexity by assuming that the value to the consumer of a unit of good i with complexity $c_i \geq 0$, having allocated $t_i \geq 0$ units of time, is given by

$$v_i\left(c_i, d_i\right), \qquad \text{where} \qquad d_i \equiv \frac{t_i}{c_i}. \tag{1}$$

Here, the first argument of $v(\cdot)$ captures the direct effect of complexity on the value of the good. The second argument captures the effect of the consumer's understanding on the value of the good. How well the consumer understands the good depends on the amount of attention she devotes to the good and the good's complexity. Note that, as the good's complexity increases, reaching a certain level of understanding requires more attention. Specifically, we assume that the consumer's depth of understanding is determined by the time spent on the good divided by the good's complexity. Therefore, a good that is twice as complex takes twice as much time to understand to the same extent. For example, a contract that is twice as long takes twice as much time to read and understand.

We make the following assumptions about the function v_i which we then discuss.

Assumption 1. *Defining $c_i^{max} \in (0, \infty]$, we assume that:*

(i) v_i *is twice continuously differentiable in* d_i *with* $\frac{\partial v_i}{\partial d_i} > 0$, $\frac{\partial^2 v_i}{\partial d_i^2} < 0$ *for* $d_i > 0$,

(ii) v_i *is twice continuously differentiable in* c_i *with* $\frac{\partial v_i}{\partial c_i} > 0$, $\frac{\partial^2 v_i}{\partial c_i^2} < 0$ *for* $c_i \in (0, c_i^{max})$,

(iii) v_i *is continuously differentiable in* c_i *with* $\frac{\partial v_i}{\partial c_i} = 0$ *for* $c_i \geq c_i^{max}$,

(iv) $\frac{\partial^2 v_i}{\partial c_i \partial d_i} \geq 0$,

(v) $v_i(c_i, d_i) \geq 0$ *and is bounded from above,*

(vi) *the limits of all above derivatives exist as* $c_i \to 0$ *and* $d_i \to 0$.

Part (i) means that the value of the good increases with the consumer's depth of understanding d_i with diminishing marginal return. Part (ii) and (iii) implies that *ceteris paribus* complexity raises the value of the good. However, as a good becomes more complex, this direct benefit of complexity exhibits diminishing marginal returns. At some point, the marginal direct effect of complexity on value could become zero.[5] Part (iv) implies that, all else equal, it is weakly more valuable to understand complicated goods. Part (v) states that goods have non-negative value even if the consumer pays no attention to them. This assumption simplifies the analysis because it guarantees that all goods are consumed in equilibrium. It also states that the value of each good is bounded. Specifically, goods have finite value even if they have infinite complexity or are infinitely well understood. Hence, suppliers cannot attain infinite value by increasing the complexity of their good, nor can consumers generate infinite value by understanding a good extremely well. As will become clear below, boundedness of value plays an important role in characterizing how consumer attention depends on complexity.

Given that the consumer consumes all goods, the key decision faced by the consumer is how much attention $t_i \geq 0$ to allocate to each of these goods. In making her decision, the consumer takes the complexity of each good as given, but takes into account that she receives a share $1 - \theta_i$ of the value v_i generated by good i.[6] The key constraint faced by the consumer

[5] In principle, the direct effect could even become negative. However, because the supplier would never choose a c_i in this region, the assumption that the marginal direct benefit of complexity is weakly positive is without loss of generality.

[6] We discuss the model's timing assumptions in more detail in Section 4.1

is that her attention is limited. Specifically, the consumer has a fixed amount of time T that she can allocate across the N goods. One interpretation of this limited attention constraint is that it introduces an element of bounded rationality, which makes complexity a meaningful concept, see Brunnermeier and Oehmke (2009).

3 The Tragedy of Complexity

In this section, we present the main conceptual result of our paper: Equilibrium complexity is generally inefficient and can be too high or too low. Given Assumption 1, the consumer receives positive utility from consuming good i even when paying no attention to it. It is therefore always weakly optimal to consume all N goods, thus from hereon we simply assume all goods are consumed. We formally define equilibrium and the planner's choice as follows.

Definition 1. Equilibrium *is given by:*

(i) good complexities c_i chosen simultaneously by each supplier i to maximize $\theta_i \cdot v_i$, and

(ii) time allocations t_i (for all $i \in \{1, ..N\}$) chosen by the consumer, taking all c_i's as given, to maximize $\sum_{i=1}^{N}(1-\theta_i) \cdot v_i$ subject to the attention constraint.

Definition 2. The planner's solution *is given by:*

(i) good complexities c_i (for all $i \in \{1, ..N\}$) chosen by the social planner to maximize total surplus $\sum_{i=1}^{N} v_i$, and

(ii) time allocations t_i (for all $i \in \{1, ..N\}$) chosen by the consumer, taking all c_i's as given, to maximize $\sum_{i=1}^{N}(1-\theta_i) \cdot v_i$ subject to the attention constraint.

Note that the planner chooses complexity anticipating privately optimal attention allocation decisions by consumers. The planner cannot allocate attention on the consumer's behalf.

We solve the model by backward induction. We first characterize the consumer's attention allocation problem for given good complexities. We then derive complexity chosen by competing suppliers and contrast them with those chosen by a benevolent social planner.

3.1 The Consumer's Problem

The consumer's maximization problem reduces to choosing the amount of attention she allocates to each good, taking as given the complexity c_i of each good,

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1-\theta_i) \cdot v_i \left(c_i, \frac{t_i}{c_i} \right), \tag{2}$$

subject to the constraint that total attention paid cannot exceed the attention budget,[7]

$$\sum_{i=1}^{N} t_i \leq T. \tag{3}$$

Note that in the maximization problem we have written out d_i as t_i/c_i to highlight the dependence of the depth of understanding on complexity choice.

Denoting by λ the Lagrange multiplier associated with the attention constraint, attention paid to good i satisfies the first-order condition

$$(1-\theta_i) \cdot \frac{\partial v_i \left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i} \right)}{\partial d_i} \cdot \frac{1}{c_i} \leq \lambda. \tag{4}$$

This first-order condition holds with equality when $t_i > 0$, in which case the consumer's marginal payoff from an additional unit of attention allocated to good i equals the shadow price of attention λ.

3.2 Equilibrium Complexity: The Supplier's Problem

We now turn to the supplier's choice of complexity. Supplier i's objective is to maximize its own payoff, which are given by a fraction θ_i of the value generated by good i, v_i. The supplier's

[7]By rewriting this constraint as $\sum_{i=1}^{N} \frac{t_i}{c_i} \cdot c_i \leq T$ (i.e., multiplying and dividing by c_i), we see that one can think of the attention constraint as a standard budget constraint in terms of the depth of consumer's understanding, $d_i = t_i/c_i$. In this interpretation, an increase in good i's complexity raises the price for understanding the good. The consumer's wealth is given by her endowment of time T.

only choice variable is the complexity c_i of his good. However, in choosing c_i, the supplier anticipates that the good's complexity affects the amount of attention that the consumer will allocate to the good. Like a Stackelberg leader, the supplier therefore internalizes that the attention the consumer pays to good i, $t_i(c_1, \ldots, c_N)$, is function of the chosen complexity, c_i. Accordingly, the supplier's objective function is given by

$$\max_{c_i} \theta_i \cdot v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right), \tag{5}$$

with the associated first-order condition

$$\theta_i \cdot \frac{d}{dc_i} v_i \left(c_i, \frac{t_i(c_1, \ldots, c_N)}{c_i} \right) \leq 0, \tag{6}$$

which holds with equality whenever $c_i > 0$.

Unless otherwise noted, we focus on the most interesting case in which both the first-order condition for c_i and t_i are binding. Taking the total derivative of the first-order condition (6), it can be rewritten as

$$\frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial c_i} = \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial d_i} \cdot \frac{t_i}{{c_i}^2} - \frac{\partial v_i \left(c_i, \frac{t_i}{c_i} \right)}{\partial d_i} \cdot \frac{1}{c_i} \cdot \frac{\partial t_i}{\partial c_i}. \tag{7}$$

This condition states that, from the supplier's perspective, the optimal level of complexity equates the marginal increase in value from additional complexity (the left-hand side of the optimality condition (7)) to the value reduction that arises from lower levels of depth of understanding holding the consumer's attention to the good constant (the first term on the right-hand side), net of the change in the good's value that arises from the consumer's change in attention paid to good i in response to an increase of the complexity of that good (the second term on the right-hand side). In equilibrium, this first-order condition holds for each supplier i.

The key observation is that suppliers take into account that changing the complexity of their good affects the amount of attention that the consumer will allocate to the good, as indicated by the partial derivative $\frac{\partial t_i}{\partial c_i}$ in the optimality condition (7). Crucially, suppliers perceive additional attention paid to their good in response to a change in complexity as a net gain, even though, in aggregate, changes in attention are merely a reallocation—any additional attention paid to good i would otherwise be allocated to goods offered by other suppliers. Because the supplier of good i is diverting attention away from other goods, competing suppliers engage in *attention grabbing*.

Substituting in the consumer's first-order condition (4), we can rewrite the supplier's optimality condition (7) in terms of the shadow price of attention λ.

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i}\left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right). \tag{8}$$

Expressing the supplier's first-order condition in this more concise fashion is useful when comparing the supplier's first-order condition to the social planner's optimality condition derived in the next section.

3.3 Optimal Complexity: The Social Planner's Problem

We now turn to the social planner's choice of good complexity. The key difference compared to the supplier's maximization problem described above is that the planner takes into account that the consumer allocates attention across *all* N goods. Therefore, the planner internalizes the effect of a change in the complexity of good i not only on the value of good i but also, via the consumer's attention reallocation, on all other goods $j \neq i$.

The planner chooses the complexity of the N goods so that they maximize total surplus,

$$\max_{c_1,\ldots c_N} \sum_{i=1}^{N} v_i\left(c_i, \frac{t_i(c_1,\ldots,c_N)}{c_i}\right). \tag{9}$$

Following the same steps as in the derivation of the supplier's first-order condition (including the assumption that the optimal complexity for each good is positive), the condition for the optimal complexity of good i, c_i^*, is given by

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial d_i} \cdot \frac{t_i}{c_i^2} - \sum_{j=1}^{N} \frac{\partial v_j\left(c_j, \frac{t_j}{c_j}\right)}{\partial d_j} \cdot \frac{1}{c_j} \cdot \frac{\partial t_j}{\partial c_i}. \tag{10}$$

This optimality condition highlights the difference between the social planner's solution and the suppliers' privately optimal complexity choice characterized by Equation 7. In particular, whereas the supplier of good i only takes into account the change in the valuation of good i that results from the reallocation of attention to or from good i, the planner takes into account the valuation changes resulting from the reallocation of attention across *all* goods, captured by $N-1$ additional partial derivatives $\frac{\partial t_j}{\partial c_i}$ on the right hand side. The supplier's privately optimal complexity choice therefore generally differs from the social planner's solution—the reallocation of attention from other goods to good i represents an externality that is not taken into account by the supplier of good i.

As before, using the consumer's first-order condition (4), we can rewrite the social planner's optimality condition (10) in terms of the shadow price of attention λ, which yields

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta_i} \cdot \left(\frac{t_i}{c_i} - \frac{\partial t_i}{\partial c_i}\right) - \sum_{j \neq i} \frac{\lambda}{1-\theta_j} \cdot \frac{\partial t_j}{\partial c_i}. \tag{11}$$

The second term on the right hand side captures the attention externality on other goods $j \neq i$ that is neglected by the supplier of good i.

3.4 The Complexity Externality

A particularly simple case arises when the share of value going to the suppliers is equal across goods, $\theta_i = \theta$. In this case, the social planner's optimality condition reduces to

$$\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = \frac{\lambda}{1-\theta} \cdot \left(\frac{t_i}{c_i} - \sum_{j=1}^{N} \frac{\partial t_j}{\partial c_i}\right) = \frac{\lambda}{1-\theta} \cdot \frac{t_i}{c_i}, \tag{12}$$

where the second equality makes use of the fact that, when viewed across all goods, attention is merely reallocated (i.e., $\sum_{j=1}^{N} t_j = T$ implies that $\sum_{j=1}^{N} \frac{\partial t_j}{\partial c_i} = 0$). The comparison between the supplier's and the social planner's first-order conditions (given by equations (8) and (12), respectively) reveals that there is an externality in complexity choice. The supplier of good i has an incentive to deviate from the socially optimal level of complexity c_i^* whenever the attention grabbing effect is nonzero at c_i^*, i.e., $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} \neq 0$. When $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} > 0$, the supplier of good i has an incentive to increase the complexity of his good beyond the socially optimal level, whereas when $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} < 0$, the supplier of good i wants to decrease complexity below the socially optimal level. In both cases, the direction of the externality is driven by the desire to divert the consumer's attention away from other goods.

Therefore, the crucial step in determining the direction of the externality is signing the direction of the attention grabbing effect, $\frac{\partial t_i}{\partial c_i}$. To do so, it is useful to introduce some additional notation. First, at times it will be useful to write attention as a function of the good's own complexity and the shadow price of attention λ. We denote this function by $\tilde{t}_i(c_i, \lambda)$. Second, note that we can rewrite the value of good i in terms of attention t_i instead of the depth of understanding $d_i = t_i/c_i$. We denote this function by $\tilde{v}(c_i, t_i)$. We then obtain

Lemma 1. Attention Grabbing: Equivalence Results. *For any given vector of complexities $(c_1, \ldots, c_N)$, the attention grabbing effect $\frac{\partial t_i(c_i, c_{-i})}{\partial c_i}$ has the same sign as*

(i) the effect of good i's complexity on the shadow cost of attention, $\frac{\partial \lambda(c_i, c_{-i})}{\partial c_i}$;

(ii) the attention grabbing effect when keeping fixed the shadow cost of attention λ, $\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}\big|_{\lambda}$;

(iii) the complementarity (or substitutability) of attention and complexity, $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$.

Lemma 1 contains useful intuition for the attention externality. Part (i) states that a supplier has an incentive to increase complexity above the efficient level if, at the optimal level of complexity c_i^*, the shadow price of attention increases when complexity is increased. Part (ii) states that a supplier has an incentive to increase complexity above the efficient level, if attention paid to the good increases as a result of increased complexity, even if the shadow price of attention is held fixed. Finally, part (iii) of Lemma 1 states that there is an incentive for suppliers to inefficiently increase complexity when attention and complexity are complements at the optimal level of complexity c_i^*. In contrast, when attention and complexity are substitutes, suppliers have an incentive to decrease complexity below the optimal level.

While the result that equilibrium complexity is generally inefficient is seen most easily when $\theta_i = \theta$, it holds more generally, as stated in the following proposition.

Proposition 1. The Complexity Externality. *Starting from the social planner's solution* $(c_1^*, .., c_N^*)$*, the supplier of good* i

(i) has an incentive to increase complexity c_i *if* $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} > 0$*;*

(ii) has an incentive to decrease complexity c_i *if* $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} < 0$*;*

(iii) has no incentive to change complexity c_i *if* $\frac{\partial t_i}{\partial c_i}\big|_{c_i=c_i^*} = 0$.

Proposition 1 states that the complexity externality has the same sign as the attention grabbing effect. Locally, suppliers have an incentive to increase the complexity of their good beyond the optimal level if consumers respond by increasing the amount of attention they allocate to the good. In contrast, when an increase in complexity induces consumers to decrease the amount of attention they allocate to the good, suppliers have a local incentive to reduce the complexity of their good below the socially optimal level.

While complexity may be distorted upward or downward in equilibrium, irrespective of the direction of the equilibrium distortion, the supplier's objective is to divert attention away from other goods. This unambiguously raises the equilibrium shadow price of attention λ.

Proposition 2. The Consumer is Pressed for Time. *Suppose that an equilibrium exists and the complexity of at least one good differs from the social planner's choice. Then the equilibrium shadow price of attention λ^e strictly exceeds the shadow price of attention λ^* under the social planner's solution, $\lambda^e > \lambda^*$.*

The difference between equilibrium complexity and the social planner's solution has parallels with the classic tragedy of the commons. Like grass on a common grazing meadow, attention is a shared resource. However, in contrast to the classic tragedy of the commons, attention grabbing can manifest itself in too much or too little complexity, depending on whether "overcomplicating" or "dumbing down" leads to an increase in consumer attention paid to a particular good. Yet, whereas the complexity externality can go either way, the scarce resource—the consumer's attention—is always overused, irrespective of the direction of the externality. Specifically, competition for the consumer's attention implies that the shadow price of attention is higher in equilibrium than it would be under the social planner's solution, $\lambda^e \geq \lambda^*$, with strict inequality whenever $c_i^e \neq c_i^*$ for at least one good. In words, when supplier's compete for attention, the consumer feels more pressed for time than under the social planner's solution, irrespective of whether goods are too complex or too simple in equilibrium.

Thus the conventional tragedy-of-commons intuition holds for the fixed-supply common resource used by all goods, attention. The contribution of our paper is to show that the classic tragedy of commons with respect to consumer attention leads to equilibrium complexity that is generically inefficient and can be above or below the efficient the efficient complexity level—the *tragedy of complexity*.

3.5 The Attention Curve

As shown in Proposition 1, the decision to distort complexity depends on the direction of the consumer's attention reallocation in response to a change in complexity. The consumer's response depends on the attention curve $t_i(c_i, c_{-i})$, which captures how much attention the consumer allocates to good i if its complexity were c_i, keeping the complexity of all other goods c_{-i} fixed at their current levels.

We now characterize the shape of the attention curve $t_i(c_i, c_{-i})$. For tractability, we assume that complexity and depth of understanding enter the valuation of the good in either an additively separable or multiplicative fashion. The multiplicative specification captures the case in which there is a positive complementarity between complexity and the depth of understanding, $\frac{\partial^2 v_i}{\partial c_i \partial d_i} > 0$. The additively separable specification captures the case in which there is no such complementarity, $\frac{\partial^2 v_i}{\partial c_i \partial d_i} = 0$.

Assumption 2. *v_i is either additively separable*

$$v_i\left(c_i, d_i\right) = f_i(c_i) + g_i\left(d_i\right), \tag{13}$$

or multiplicative

$$v_i\left(c_i, d_i\right) = f_i(c_i) \cdot g_i\left(d_i\right). \tag{14}$$

To ensure that v_i continues to satisfy Assumption 1, we place the following restrictions on f_i and g_i.

Assumption 3. *For both additive and multiplicative $v_i\left(c_i, d_i\right)$:*

(i) $g_i(d_i)$ is twice continuously differentiable and $g_i'(d_i) > 0$, $g_i''(d_i) < 0$ for $d_i > 0$,

(ii) $f_i(c_i)$ is twice continuously differentiable and $f_i'(c_i) > 0$, $f_i''(c_i) < 0$ for $c_i \in (0, c_i^{max})$,

(iii) $f_i'(c_i) = 0$ for $c_i \geq c_i^{max}$,

(iv) $f_i(c_i) \geq 0$, $g_i(d_i) \geq 0$ and both are bounded from above,

(v) the limits of all above derivatives exist as $c_i \to 0$ and $d_i \to 0$.

Given Assumptions 2 and 3, we can now determine the sign of the complexity externality $\frac{\partial t_i}{\partial c_i}$ by differentiating the consumer's first-order condition (4). Specifically, holding fixed λ and taking the derivative with respect to c_i, for interior $c_i > 0$ we obtain

$$\left.\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}\right|_\lambda = \frac{g_i'}{g_i''} + \frac{t_i}{c_i} \tag{15}$$

for the additively separable case and

$$\left.\frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}\right|_\lambda = \frac{g_i'}{g_i''} + \frac{t_i}{c_i} - \frac{f_i' \cdot g_i'}{f_i \cdot g_i''} \cdot c_i \tag{16}$$

for the multiplicative case. Recall from Lemma 1 that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\big|_\lambda$ has the same sign as $\frac{\partial t_i}{\partial c_i}$. Hence, we have shown:

Lemma 2. *$\frac{\partial t_i}{\partial c_i} > 0$ if and only if*

(i) $\sigma_i(d_i) < 1$ for additive v_i

(ii) $\sigma_i(d_i) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right) < 1$ for multiplicative v_i

where $\sigma_i(d_i) \equiv -\frac{g_i'}{g_i'' d_i}$. For $\frac{\partial t_i}{\partial c_i} < 0$, the conditions are the opposite.

Lemma 2 can be interpreted in terms of the elasticity of demand for understanding a good. When the supplier of good i increases complexity, from the consumer's perspective this raises the price of understanding the good. When the demand for understanding is inelastic (i.e., the own-price elasticity is smaller than one, $\eta_i = \left|\frac{\partial d_i}{\partial c_i} \frac{c_i}{d_i}\right| < 1$) an increase in complexity – the price of a unit of understanding – reduces the demand for understanding the good less than one for one. This less than one-for-one reduction requires that the consumer increases the time spent on the good, so that $\frac{\partial t_i}{\partial c_i} > 0$. Conditions (i) and (ii) in Lemma 2 ensure that $\eta_i < 1$ in the additive and multiplicative case, respectively.

To characterize the shape of the attention curve $t_i(c_i, c_{-i})$, we first show that, if an individual supplier increases the complexity of its good, holding fixed the complexity of all other goods, the depth of understanding for this good unambiguously decreases. This means that, even if the consumer reacts by allocating more time to the good, the reallocation of attention only partially offsets the decrease in the depth of understanding.

Lemma 3. *The depth of understanding d_i resulting from the attention choice of the consumer is strictly decreasing in the good's complexity c_i for any $d_i > 0$.*

Remember that the payoff difference between understanding a good very well and not understanding a good is finite (i.e., g is bounded from above and below) and that the payoff difference between a very complex good and very simple good is also finite (i.e., f is bounded from above and below). We now show that under this relatively weak assumption of boundedness, the direction of the attention grabbing effect $\frac{\partial t_i}{\partial c_i}$ reverses sign at least once.

Proposition 3. General Shape of the Attention Curve. *Given Assumptions 1-3, the attention curve $t_i(c_i, c_{-i})$ is initially increasing from zero. At some point, it becomes decreasing and converges back to zero at $\bar{\bar{c}}_i \leq \infty$:*

(i) $\lim_{c_i \to 0} t_i(c_i, c_{-i}) = 0$

(ii) $\lim_{c_i \to 0} \frac{\partial t_i(c_i, c_{-i})}{\partial c_i} > 0$

(iii) there exists $c_i \in (0, \infty)$, *s.t.* $\frac{\partial t_i(c_i, c_{-i})}{\partial c_i} < 0$

(iv) $\lim_{c_i \to \infty} t_i(c_i, c_{-i}) = 0$

(v) $t_i(c_i, c_{-i}) = 0$ *for* $c_i > \bar{\bar{c}}_i$

(vi) $\bar{\bar{c}}_i < \infty$ *if and only if* $\lim_{d_i \to 0} g_i'(d_i) < \infty$

except for the degenerate case in which $t_i(c_i) = 0$ *for all* c_i*.*

Proposition 3 shows that under reasonably mild assumptions (mainly the boundedness of f and g), the attention curve has a generalized hump shape: It is first increasing in c_i

and, at some point, converges back to 0 for large c_i. We refer to this as a generalized hump shape because the derivative $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i}$ could change sign multiple times as c_i increases. We now impose regularity conditions on f_i an g_i which ensure that the attention curve has a single hump. Specifically, if $\sigma_i(d_i)$ and, in the case of multiplicative v, $\frac{f_i'(c_i)\cdot c_i}{f_i(c_i)}$ are strictly decreasing, then the attention grabbing effect changes sign exactly once, so that the attention curve becomes single-peaked.

Assumption 4. *$\sigma_i(d_i)$ is strictly decreasing in d_i for both multiplicative and additive v_i.*

Assumption 5. *In case of multiplicative v_i, $f(0) = 0$ and $\frac{f_i'(c_i)\cdot c_i}{f_i(c_i)}$ strictly decreasing for $c_i < c_i^{max}$ (for $c_i \geq c_i^{max}$ the function is flat)*

While Assumptions 4 and 5 do not have a simple intuitive interpretation, they are fulfilled by the most natural choices of f_i and g_i that satisfy Assumption 3. Example of such a functional form for $f(x)$ and $g(x)$ are: $\frac{\alpha\cdot x^\gamma}{\beta+x^\gamma}$ with $\alpha, \beta > 0$ and $\gamma \in (0, 1]$, and $\alpha\cdot\tanh(\beta\cdot x)$ with $\alpha, \beta > 0$.[8]

Using Assumptions 4 and 5, the shape of the attention curve is characterized by the following proposition.

Proposition 4. Single Peaked Attention Curve. *Given Assumptions 1-5, the attention curve $t_i(c_i, c_{-i})$ is single-peaked in c_i, i.e., there exist unique cutoffs $0 < \bar{c}_i < \bar{\bar{c}}_i \leq \infty$ such that*

(i) $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} > 0$ *for* $c_i < \bar{c}_i$

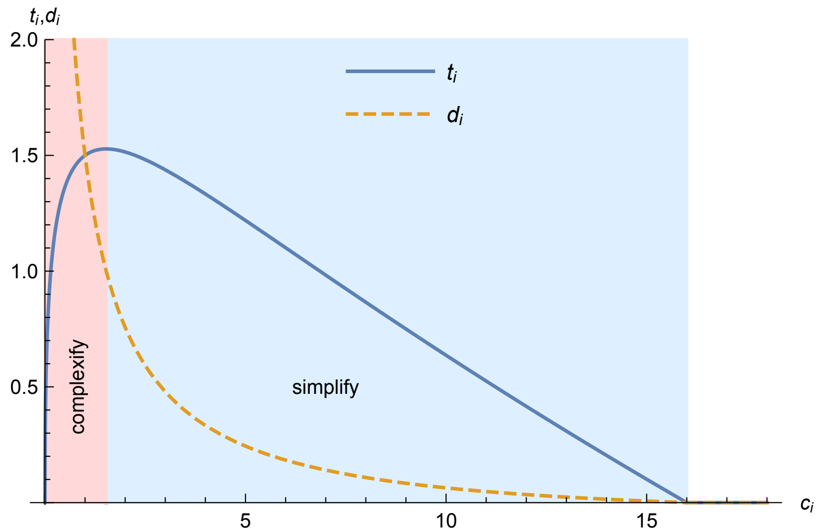
(ii) $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} < 0$ *for* $c_i \in (\bar{c}_i, \bar{\bar{c}}_i)$

except for the degenerate case in which $t_i(c_i) = 0$ for all c_i.

Figure 1 illustrates Proposition 4 by plotting the attention curve $t_i(c_i, c_{-i})$ as a function of the complexity of good i for a particular functional form for v_i (see Section 3.6), holding

[8]Also note that in Assumption 5 we assumed that both $\frac{f_i'(c_i)\cdot c_i}{f_i(c_i)}$ strictly decreasing and $f(0) = 0$ even though the former implies the latter. The reason we explicitly assumed $f(0) = 0$ is that it has a clear (and plausible) interpretation for multiplicative v_i: A good with zero complexity has no value, regardless of the amount of attention paid to it.

Figure 1: **Attention paid to a good as a function of its complexity**



This figure illustrates the consumer's attention choice $t_i(c_i, c_{-i})$ and the resulting depth of understanding $d_i(c_i, c_{-i})$ as a function of the complexity of good i c_i, holding fixed the level of other goods' complexities c_{-i}. Attention allocation is hump shaped: Initially, $t_i(c_i)$ is increasing, then decreasing, and at some point the consumer chooses to pay no attention to good i. Functional form given in Assumption 6 with parameters $N = 2$, $i = 1$, $\delta_1 = \delta_2 = 1$, $\alpha_1 = \alpha_2 = 1$, $\theta_1 = \theta_2 = 0.5$, $c_2 = 1$.

the complexity of all other goods fixed. Consumer attention follows a hump shape: For low levels of complexity up to $\bar{c}$, the curve is upward sloping. In this region, the consumer reacts to an increase in the complexity of good i by increasing attention paid to the good ($\frac{\partial t_i}{\partial c_i} > 0$). Therefore, the supplier of good i has an incentive to increase the complexity of the good. For higher levels of complexity (from $\bar{c}$ to $\bar{\bar{c}}$), there is a downward-sloping segment. In this region, the direction of the externality reverses: An increase in the complexity of good i leads to a reduction in attention paid to good i ($\frac{\partial t_i}{\partial c_i} < 0$). The supplier of good i then has an incentive to decrease the complexity of the good. Finally, for the specific functional form in this example, above some critical level of complexity ($c_i > \bar{\bar{c}}_i$) there is a region in which $t_i(c_i, c_{-i}) = 0$. In this region the consumer pays no attention to good i even though she still consumes the good. The consumer essentially gives up on learning about the good. Even with significant attention allocated to the good, the consumer would not understand the good well, so that it is better for the consumer to focus her attention on other goods.[9] In this region, the supplier does not have a local incentive to distort complexity.[10]

The hump shape illustrated in Figure 1 implies that the supplier of good i has an incentive to make goods that are relatively simple too complex and goods that are relatively complex too simple. Of course, whether a good is relatively simple or complex (i.e., whether it lies on the upward-sloping or downward-sloping segment of the attention curve) is an equilibrium outcome that depends on all the parameters of the model. Combining Propositions 1 and 4 with Lemma 2, we arrive at the following proposition which characterizes which goods are most likely to be too simple or too complex:

Proposition 5. Complexity and the Depth of Understanding. *If the consumer pays any attention to good i ($t_i > 0$), there exists a unique $\bar{d}_i \in (0, \infty)$ defined by:*

(i) $\sigma_i(\bar{d}_i) = 1$ *for additive* v_i*,*

(ii) $\sigma_i(\bar{d}_i) \cdot \left(1 - \frac{f_i'(c_i(\bar{d}_i)) \cdot c_i(\bar{d}_i)}{f_i(c_i(\bar{d}_i))}\right) = 1$ *for multiplicative* v_i*;*

such that the supplier has an incentive to

(i) overcomplicate goods that are relatively well understood in the social planner's solution $d_i > \bar{d}_i$*;*

(ii) oversimplify goods that are not well understood in the social planner's solution $d_i < \bar{d}_i$*.*

In the above, $c_i(d_i)$ *denotes the inverse function of the strictly decreasing function* $d_i(c_i)$*.*

Proposition 5 provides a simple way to determine whether a good is too complex or too simple relative to the social planner's solution. Because the consumer's depth of understanding d_i is decreasing in complexity (see Figure 1), goods that are well understood in the social planner's solution lie on the upward-sloping part of the attention curve $t_i(c_i)$. Therefore,

[9] Examples of this phenomenon include terms and conditions for online purchases and lengthy legal texts, which are classic instances of information overload (Brunnermeier and Oehmke, 2009).

[10] However, the supplier might have an incentive to reduce complexity a larger amount to $c_i < \bar{\bar{c}}_i$ so that the consumer pays attention to the good.

suppliers overcomplicate these goods. Conversely, goods that are less well understood in the social planner's solution are located on the downward-sloping part of the attention curve, giving the supplier an incentive to make these goods too simple.

3.6 Symmetric Equilibrium

In this section, we explicitly characterize equilibrium complexity for a specific functional form. We assume that v_i takes the following additively separable functional form and satisfies Assumptions 1-5:

Assumption 6. *Assume that v_i is additively separable and symmetric across goods, $v_i = f(c_i) + g(d_i)$ with*

$$f(c_i) = \begin{cases} \alpha \cdot c_i - c_i^2 & c_i \leq \frac{\alpha}{2} \\ \frac{\alpha^2}{4} & c_i > \frac{\alpha}{2} \end{cases} \tag{17}$$

$$g\left(d_i\right) = \delta \cdot \frac{d_i}{1 + d_i}. \tag{18}$$

In this specification, the parameter $\alpha > 0$ captures the direct benefit of complexity while $\delta > 0$ captures the importance of understanding the good. Given this functional form, we can solve for the equilibrium first-order conditions and characterize the resulting equilibrium in closed form.

Proposition 6. Equilibrium Distortion and the Complexity Paradox. *When the benefit of understanding a good δ is not too large, there exists a symmetric equilibrium in which all goods receive the same amount of attention $\frac{T}{N}$. The goods are too complex compared to the social planner's choice if and only if they are well understood in equilibrium*

$$d = \frac{T/N}{c} > 1. \tag{19}$$

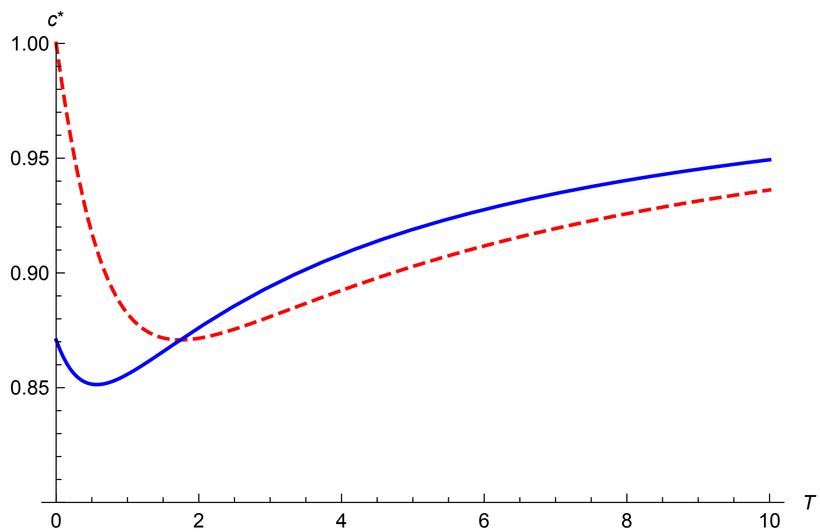
There exists a T^* such that equilibrium complexity is inefficiently high compared to the social planner's solution if and only if attention is abundant $T > T^*$.

The result that equilibrium complexity is too high if $\frac{T/N}{c} > 1$ is the equilibrium counterpart to the local deviation described in Proposition 5, which states that suppliers have an incentive to make goods are too complex if they are well understood. A particularly interesting observation, illustrated in Figure 2, is that, for relatively high consumer attention budgets T, the equilibrium level of complexity (solid blue line) lies above that chosen by the planner (dashed red line). Therefore, complexity rises to inefficiently high levels precisely when information processing capacity grows. This result highlights a *complexity paradox*: Increased information processing capacity can be a source of excessive complexity in the economy because it induces individual suppliers to increase complexity above the efficient level. This finding echoes Ellison and Wolitzky (2012), who show that, in a search framework, reductions in consumer search costs are partially offset by increased obfuscation by firms.

Together with Figure 2, Proposition 6 also highlights the importance of distinguishing between absolute and relative complexity. For example, while a decrease in the attention budget T (or an increase in the number of goods N) leads to overly simple goods relative to the social planner's solution, it is not necessarily the case that goods become simpler in an absolute sense. Figure 2 illustrates that decreasing the attention budget T can lead to an increase in absolute complexity. The reason is that, when attention is severely limited (low T), no goods are well understood, so that it becomes optimal for suppliers to focus solely on the direct benefit of complexity.

Finally, note that Proposition 6 assumes that the benefit of understanding the good δ is not too large. This assumption ensures that the equilibrium is symmetric. When δ is large, an asymmetric equilibrium might arise: For high δ and small attention capacity T it can be optimal (both in equilibrium and in the social planner's solution) to choose complexities asymmetrically across otherwise ex-ante identical goods: One good is very simple ($c_i = 0$)

Figure 2: **Equilibrium and socially optimal complexity as a function of attention capacity T**



The figure shows equilibrium complexity (solid blue line) and the complexity chosen by the planner (dashed red line) as a function of the attention budget T. Equilibrium and social planner's complexity converge to the unconstrained optimal level of complexity of 1 as $T \to \infty$. Homogenous goods with parameters: $N = 2$, $\delta = 0.9$, $\alpha = 2$, $\theta = 0.5$.

and receives no attention, whereas the other good is complex ($c_j > 0$) and receives all of the consumer's attention. In this case, fundamentally similar goods can have very different levels of complexity.

4 Discussion, Extensions, and Applications

In this section, we provide a discussion of some of assumptions underlying our model, present some extensions of the model, and discuss a number of applications of the theory.

4.1 Discussion of Assumptions

In the analysis presented above, we made a number of assumptions to keep the model simple. One key simplifying assumption is that the supplier receives a fixed share θ_i of the value of the good. This assumption captures in reduced form that suppliers care about the value of the good they design. This could be because the good's value is split between the consumer and the supplier or, more generally, it could reflect that suppliers care intrinsically about the value of their good. In both cases, the crucial assumption is that the supplier can benefit from *some* of the increase in the value of the good that results when the consumer allocates more attention to it or simply values such goods more.

Another assumption of our model is that limited attention takes the form of a hard constraint on the consumer's time budget. This is meant to capture that the amount of time that a consumer can spend on analyzing and consuming goods is limited. This assumption is uncontroversial for individuals who have to allocate their time to understand several goods. In the case of companies, one may argue that, by employing more people or purchasing additional information technology, the company can relax its attention constraint. However, as long as increasing the attention budget is costly, the implications of such a setting would be similar to those under a hard attention constraint.[11]

Finally, our model contains a timing assumption: Complexity is chosen by the supplier before the consumer makes her choices. This results in market power for the supplier, similar to that of a Stackelberg leader. Here, the crucial assumption is not the specific timing, but that the consumer cannot choose (or shop around for) the complexity that she would prefer. In many markets this timing is realistic, given that goods and services are often designed before they reach the consumer, and that their complexity cannot be easily altered afterwards.

4.2 Extensions: Exogenous Complexity and Consumer-Chosen Complexity

Our baseline model has focused on the situation in which all suppliers choose the complexity of their goods. However, in some situations, the complexity of certain goods (or activities) may be fixed or chosen by the consumer rather than the supplier. For example, for some firms, the complexity of managing their own business might be determined by exogenous technological

[11] If the cost of additional attention capacity is fully born by the consumer but she only captures part of the social benefit, this introduces underinvestment in attention as an additional source of inefficiency not present in our model.

constraints or by internal processes set by the firm itself. In such cases, suppliers who set the complexity of their goods also compete for attention with goods or activities for which complexity is not set strategically.

A natural extension of the baseline model is one with three types of goods: First, there is at least one good for which the complexity is set by a strategic supplier. Second, there are goods with exogenously given complexity. Third, for some goods complexity is set by the consumer herself. The main insights of our paper continue to hold in this setting. Proposition 1, which shows the strategic supplier's incentive to distort the complexity of his good, holds for any complexity of other goods, regardless of how these are set. Proposition 4, which shows that the attention curve is hump-shaped, and Proposition 5, which shows that well understood goods will be overly complex, also take all other complexities as given and, therefore, the results do not depend on how the complexities of other goods are set.

In this extended model, one could also investigate whether goods for which the complexity is chosen by the consumer herself will be too complex or too simple compared to the first best in which all complexities (except the exogenously given ones) are set by the planner. To analyze this question, first note that, in the special case in which the fraction θ_i of the value v_i that accrues to the consumer is the same for all goods ($\theta_i = \theta$), the consumer's complexity choice coincides with that of the planner. In this case, the consumer maximizes the same objective function as the planner (see Equation (9)), up to a constant multiplier of θ. Thus, if the strategic suppliers were to choose the socially optimal levels of complexity, the consumer would choose the socially optimal level of complexity for the goods for which she has the choice.

To understand what happens when other suppliers choose their complexities strategically, first note that Proposition 2 also holds in this setting. Therefore, the privately optimal complexities chosen by strategic suppliers increase the shadow price of attention relative to the social planner's solution. Equation (12) implies that, if the shadow price of attention λ increases from its first-best level, the marginal value of attention $\frac{\partial v_i}{\partial c_i}$ for the goods for which the consumer chooses the complexity must increase compared to the first best, which implies that their complexity has to be lower. Therefore, the consumer reacts to the strategic complexity choices of suppliers by simplifying what they can (e.g., their own operations) in order to make up for the attention "lost" to goods supplied by strategic actors.

4.3 Applications

In this subsection, we discuss the implications of our model in the context of two practical applications, (i) regulation and (ii) communication. We then briefly review potential policy interventions that could lead to more efficient outcomes.

4.3.1 Financial Regulation

It is common for firms to spend a substantial amount of time to ensure they comply with regulation. Thus regulators compete for attention with other operations of the firms. In fact, in many cases, firms face multiple regulators, in which case regulators also compete for attention with each other. One prime example is the regulation of financial institutions, such as banks, insurance companies, and asset managers, where different market segments and activities fall under the reaches of different supervisors (e.g., the Federal Reserve, CFTC, SEC, OCC). Even within a single regulator, different parts of the regulatory framework are usually drafted by separate committees and subcommittees.

Regulators are primarily concerned with the impact of their regulation on their mandate, which is typically tied to a specific segment of the market. Therefore, they may increase the complexity of their own regulatory framework to increase surplus in their own market segment. Yet they ignore that in doing so they draw on financial institutions' limited attention to deal with all the other things they need to deal with, including their own operations and other regulations that apply to them. Hence, higher regulatory complexity in one market segment imposes a negative externality on the firms' other operations and other parts of the regulatory framework.

Our model implies that in such a situation the complexity of financial regulation will generally not coincide with the social optimum. Given the high stakes and the potential for fines, banks need to understand regulation well in order to comply. According to Proposition 5 and Proposition 6, this is likely to result in financial market regulation that is excessive complex.[12]

These results provide a novel angle to the ongoing debate on regulatory complexity (see, e.g., Gai et al., 2019). Specifically, the prediction that equilibrium regulatory complexity is likely to be too high when regulators design regulations without internalizing that they draw on a financial institution's limited attention budget is in line with recent evidence that has documented that the fragmented U.S. regulator landscape is indeed characterized by increasingly complex regulation. For example, Haldane and Madouros (2012) document increased complexity as measured by the word count of financial regulation. Herring (2018) documents that globally significant banks now have to meet an increasing number of regulatory capital requirements. Colliard and Georg (2021) develop a methodology to measure regulatory complexity similar to how the complexity of algorithms are measured in computer science.

The mechanism that leads to excessive regulatory complexity in our model is different from (and likely complementary to) other channels that have been proposed in the literature. For example, in Hakenes and Schnabel (2014), sophisticated banks can capture the regulator by complex arguments that the regulator does not fully understand. In Asriyan, Foarta and Vanasco (2023), a regulatory proposal has to be approved or rejected by the median voter. When the proposed regulation is complex, the decision to approve is made mainly based on

[12] Of course, the attention budget of a financial institution is not literally fixed. Rather, financial institutions react by expanding personnel to deal with excessively excessively regulation. The inefficiency in this case is the opportunity cost generated when the employees required to deal with bank regulation could be engaged in more productive activities.

the median voter's prior. From the supplier's perspective, complex regulation is optimal if, for example, there is strong demand for new regulation.

Applying the same rationale to everyday life, businesses and households confront similar issues: Laws and regulations (tax law, traffic rules, parking regulations, etc.) all draw on our limited attention. Therefore, equilibrium complexity is likely inefficient. Indeed, Zwick (2021) documents that the complexity of the corporate tax code leads to suboptimal behavior and has significant costs for firms.

4.3.2 Communication

The results of our model apply in any setting, in which suppliers of messages compete for the attention of a single party that has to pay attention to other issues, including other messages. One salient example is communication within a firm. Consider, for instance, a setting in which multiple division heads communicate with a single CEO. Each division head will care about the value to the CEO of the report sent by their particular division, ignoring that their report will affect how much time the CEO has left to deal with issues that arise in other divisions. If a more complex report induces the CEO to allocate more time to that division, division heads have an incentive to prepare reports that are overly complex. According to Proposition 5 and Proposition 6, this will especially be the case for issues that are well understood by the CEO. Analogously, if a simpler report induces the CEO to allocate more time, issues will be communicated in an oversimplified manner. Hence, our model predicts that simple issues that are easy to understand are blown up and communicated in an overly complex way, whereas difficult issues are communicated in an overly simple manner. This result also suggests that it can be efficient for CEOs to allocate a fixed amount of time to certain issue in order to counteract the attention externality.

Similar effects arise in other settings in which multiple actors communicate with a single attention-constrained individual or entity. Consider how politicians or lobby groups communicate their issues to citizens. When designing their messages, politicians and lobby groups have in mind mainly their own narrow issues and agendas, whereas citizens have to allocate their attention across the many separate issues that invariably affect them. Given the limited time citizens can devote to understanding these issues, intricate policy debates that are hard to understand will, in general, not be well understood. Proposition 5 and Proposition 6 then imply that politicians, lobby groups, and the media will oversimplify these issues when communicating to the public. For example, despite the apparent complexities, the question of whether the UK should leave the EU was often oversimplified to the UK's gross contribution to the EU budget. Correspondingly, smaller and relatively simple issues that are well understood by citizens are likely to be presented in an overly complicated fashion in order to attract attention.[13]

Irrespective of whether the equilibrium distortion features too much or too little complexity, Proposition 2 implies that the chosen complexity of messages leads to an inefficient burden on consumers' attention and, consequently, a welfare loss. Relative to the efficient allocation, CEOs will feel pressed for time and citizens as though they can barely keep up with issues.

4.4 Addressing the Tragedy of Complexity

To conclude this section, we briefly discuss ways of addressing the tragedy of complexity. The standard way to address inefficiencies arising from a common pool problem is to price the shared resource. However, in most cases, it is not possible to price the consumer's attention directly. In some instances, the price of the good can help overcome the inefficiency. However, this requires that all consumer attention is allocated after the price of the good has been

[13]A similar dynamics is present in educational institutions in which multiple professors teach classes to the same body of students. Professors choose the level of complexity of the material, and students have to allocate their limited study time across classes. While PhD students arguably have a large attention capacity and tend to understand complex issues well, MBA students have less time for classes. Proposition 6 implies that, in such settings, MBA students will face overly simple classes as simplicity is the way to attract their attention. PhD students, on the other hand, will face inefficiently complex classes. This account is in line with perception by many instructors we have talked to.

determined. If some of the consumer's attention is sunk by the time the price of the good is set, the inefficiency remains (in the form of a hold-up problem) even when the good is priced.

In many salient cases (including the applications to regulation and communication), even the good itself is not priced. We discuss two ways to overcome the inefficiency in such non-price settings. One potential intervention to alleviate the inefficiency is to ensure the consumer (who determines the allocation of attention) can choose the complexity of the goods she consumes. Indeed, in the symmetric case, when the fraction of value extracted by the consumer θ is equal across goods, the consumer's objective function (2) and the social planner's objective (9) are identical up to a scaling factor. Therefore, in this case, the consumer would choose the socially optimal level of complexity. In contrast, when the value extracted by the consumer differs across goods, the consumer's objective function differs from the social planner's objective.

However, even in the symmetric case, in which consumer choice could alleviate the inefficiency, individual suppliers may have an incentive to deviate. Following the logic of Proposition 1, if all other suppliers offer the efficient level of complexity as part of their menu, an individual supplier has an incentive to deviate and not offer the efficient level of complexity the consumer would like to choose. Therefore, coordinated action and binding regulation is needed.

An alternative way to restore efficiency, based on Proposition 2, is to commit to allocate a fixed amount of time to each good. For example, CEOs may want to commit to spending a fixed amount of time dealing with a particular division. Such a commitment eliminates the reallocation of attention in response to changes in complexity and, hence, the attention externality. In this case, the supplier's first-order condition coincides with the social planner's, restoring efficiency. However, we note that this solution demands substantial sophistication from the consumer, who must choose (and commit to) the efficient amount of time spent on each good.

5 Conclusion

In many settings, competing suppliers draw on the limited attention that an economic agent allocates across multiple goods and activities. Examples include financial institutions facing regulatory bodies and CEOs interacting with corporate divisions.

This paper shows that, in such situations, equilibrium complexity is generally inefficient, even though complexity itself can be value-enhancing. The reason for this inefficiency is an attention externality. When choosing complexity, suppliers do not take into account that consumer attention is a common resource shared across goods. Suppliers distort complexity to increase attention paid to their goods, thereby diverting attention away from other goods or activities. Depending on the consumer's reaction to an increase in complexity—does the consumer devote more or less time when a good becomes more complex?— this leads to *too much or too little complexity* in equilibrium.

Our analysis shows that, under reasonably general conditions, consumer attention is hump-shaped in complexity. Hump-shaped attention allocation implies that firms overcomplicate goods that are well-understood in the social planner's solution. Conversely, firms oversimplify goods that, according to the planner, do not need to be well understood by consumers. Finally, equilibrium complexity is more likely to be excessive when attention is abundant. Therefore, while advances in information processing capacity help consumers deal with existing complexity, they also make it more likely that equilibrium complexity is excessive.

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A Proofs

Proof of Lemma 1.

First note that raising the shadow cost of attention leads to less attention being paid to all goods. Rewriting the FOC for attention allocation (4) in terms of $\tilde{v}$ when it is binding yields

$$(1-\theta_i) \cdot \frac{\partial \tilde{v}_i\left(c_i, t_i(c_i, \lambda)\right)}{\partial t_i} = \lambda, \tag{A1}$$

taking total derivative with respect to λ:

$$\frac{\partial \tilde{t}_i\left(c_i,\lambda\right)}{\partial \lambda} = \frac{1}{1-\theta_i}\cdot\frac{1}{\frac{\partial^2 \tilde{v}_i\left(c_i,\tilde{t}_i(c_i,\lambda)\right)}{\partial \tilde{t}_i^2}} < 0, \tag{A2}$$

where we used $\tilde{v}_j(c_j,\tilde{t}_j)=v_j(c_j,\frac{t_j}{c_j})$ and $\frac{\partial^2 v_j(c_j,t_j)}{\partial {t_j}^2} = \frac{\partial^2 v_j\left(c_j,\frac{t_j}{c_j}\right)}{\partial\left(\frac{t_j}{c_j}\right)^2}\cdot\frac{1}{c_j^2} < 0$, which holds by Assumption 1.

Equation (4) implicitly defines the attention allocated to good i, t_i, as $\tilde{t}_i\left(c_i,\lambda\right)$. Attention grabbing $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i}$ can then be written as:

$$\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} = \frac{d\tilde{t}_i(c_i,\lambda(c_i,c_{-i}))}{dc_i} = \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\cdot\frac{\partial \lambda(c_i,c_{-i})}{\partial c_i}, \tag{A3}$$

where the first term is the effect of c_i on t_i keeping λ fixed, while the second term captures the indirect effect through the shadow price of attention λ.

The equilibrium shadow price $\lambda(c_i,c_{-i})$ is implicitly defined by the binding attention constraint

$$T = \sum_j t_j = \sum_j \tilde{t}_j\left(c_j,\lambda\right). \tag{A4}$$

Without a specific functional form for v we cannot express $\lambda(c_i,c_{-i})$ explicitly. However, we can take a total derivative of (A4) with respect to c_i to get:

$$0 = \frac{\partial \tilde{t}_i\left(c_i,\lambda\right)}{\partial c_i} + \sum_{j=1}^{N}\frac{\partial \tilde{t}_j\left(c_j,\lambda\right)}{\partial \lambda}\cdot\frac{\partial \lambda}{\partial c_i}, \tag{A5}$$

from which it follows that

$$\frac{\partial \lambda}{\partial c_i} = \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}. \tag{A6}$$

Plugging this into (A3) yields

$$\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} = \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} + \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\cdot\frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}\cdot\left[1 - \frac{-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}}{\sum_{j=1}^{N} - \frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}\right] \tag{A7}$$

The second term is positive as $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1, N\}$ (see (A2)). Thus $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i}$, proving the equivalence of (i) and (iii). By the same argument, it is obvious from (A6) that $\frac{\partial \lambda}{\partial c_i}$ also has the same sign as $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i}$, proving the equivalence of (i) and (ii).

We now turn to the equivalence of (i) and (iii). We first rewrite the consumer's problem (2) in terms of $\tilde{v}$:

$$\max_{t_1,..t_N} \sum_{i=1}^{N} (1-\theta_i) \cdot \tilde{v}_i\left(c_i, t_i\right), \tag{A8}$$

which is maximized subject to the attention constraint (3). This yields the counterpart of Equation (4), which for interior solutions for t_i can be written as

$$\frac{1}{1-\theta_i} \cdot \lambda = \frac{\partial \tilde{v}_i\left(c_i, t_i\right)}{\partial t_i}. \tag{A9}$$

Differentiating this expression with respect to c_i (keeping all c_j, $j \neq i$, fixed), we obtain:

$$\frac{1}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} + \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial c_i} \tag{A10}$$

where we take into account that t_i is a function of c_i and λ, $\tilde{t}_i(c_i, \lambda)$ and that λ is a function of all c_i.

From (A2) we know that $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial t_i^2} \cdot \frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} = 1$, so that:

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}. \tag{A11}$$

Using (A6) to substitute $\frac{\partial \lambda}{\partial c_i}$, we then obtain

$$\frac{\theta_i}{1-\theta_i} \cdot \frac{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}}{\sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i} + \frac{1}{\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i}, \tag{A12}$$

which can be rearranged to yield

$$\frac{\frac{\theta_i}{1-\theta_i} \cdot \left(-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\right) + \sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}}{\left(-\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda}\right) \cdot \sum_{j=1}^{N} -\frac{\partial \tilde{t}_j(c_j,\lambda)}{\partial \lambda}} \cdot \frac{\partial \tilde{t}_i(c_i, \lambda)}{\partial c_i} = \frac{\partial^2 \tilde{v}_i\left(c_i, t_i\right)}{\partial c_i \partial t_i}. \tag{A13}$$

From (A2) we know that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial \lambda} < 0$ for all $i \in \{1,..N\}$, which implies that $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i}$ and $\frac{\partial^2 \tilde{v}_i(c_i,t_i)}{\partial c_i \partial t_i}$ have the same sign.

□

Proof of Proposition 1.

The incentives of supplier i to change the level of complexity c_i of good i starting from the social planner's optimum $(c_1^*, ...c_N^*)$ depend on the difference between the supplier's first-order condition (8) and that of the planner (11), both evaluated at the social planner's choice $(c_1^*, ...c_N^*)$. We rewrite the difference between the right-hand side of the two first-order conditions as

$$\sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial t_j(c_1^*, ..., c_N^*)}{\partial c_i} = \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..., c_N^*))}{dc_i} = \frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} \cdot \sum_{j \neq i} \frac{\lambda^*}{1-\theta_j} \cdot \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda}. \tag{A14}$$

The first step in (A14) uses the fact that we can rewrite the consumer's attention choice $t_j(c_1^*, ..c_N^*)$ as a function of the complexity of good j and the shadow cost of attention λ, $\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))$. The second step applies the chain rule, $\frac{d\tilde{t}_j(c_j^*, \lambda(c_1^*, ..c_N^*))}{dc_i} = \frac{\partial \tilde{t}_j(c_j^*, \lambda^*)}{\partial \lambda} \cdot \frac{\partial \lambda(c_1^*, ..c_N^*)}{\partial c_i}$ and moves the first (constant) term outside of the summation sign. λ^* denotes the shadow cost of attention at the social planner's solution, $\lambda(c_1^*, ..c_N^*)$.

Therefore, from (A14) we see that the externality is negative if $\frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i} > 0$, meaning that the social planner's optimum must entail a lower level of c_i, which in turn increases the left hand side of (11) (due to the decreasing benefits of complexity we have assumed). We show that $\frac{\partial \lambda(c_1^*, ..., c_N^*)}{\partial c_i}$ has the same sign as $\frac{\partial t_i(c_1^*, ..c_N^*)}{\partial c_i}$ in the proof of Lemma 1.

□

Proof of Proposition 2.

As shown in Proposition 1, the direction of the complexity distortion is determined by the sign of $\frac{\partial t_i(c_i, c_{-i})}{\partial c_i}$. Lemma 1 shows that this distortion has the same sign as $\frac{\partial \lambda}{\partial c_i}$. Therefore, suppliers have an incentive to distort complexity in the direction that raises the shadow price of attention λ. Thus starting from the social planner's solution and allowing suppliers to follow a best-response strategy, in every step of the iteration the shadow cost of attention weakly increases (strictly if $\frac{\partial t_i(c_i, c_{-i})}{\partial c_i} > 0$). Thus $\lambda^e > \lambda^*$ whenever equilibrium complexity and the social planner's solution do not coincide (which is the case only if $\frac{\partial t_i(c_i, c_{-i})}{\partial c_i} = 0$ for all suppliers at the social planner's solution).

□

Proof of Lemma 2.

Using the chain rule

$$\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i} = \frac{\partial^2 v(c_i,t_i/c_i)}{\partial c_i \partial t_i} = \frac{1}{c_i}\cdot\frac{\partial^2 v(c_i,t_i/c_i)}{\partial c_i \partial d_i} - \frac{1}{c_i^2}\cdot\frac{\partial v(c_i,t_i/c_i)}{\partial d_i} - \frac{t_i}{c_i^3}\cdot\frac{\partial^2 v(c_i,t_i/c_i)}{\partial d_i^2}. \tag{A15}$$

From Lemma 1 we know that $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i}$ and $\frac{\partial^2 \tilde{v}(c_i,t_i)}{\partial c_i \partial t_i}$ have the same sign. Thus $\frac{\partial t_i}{\partial c_i} > 0$ if and only if

$$-\frac{\frac{\partial v_i}{\partial d_i}}{\frac{\partial^2 v_i}{\partial d_i^2}\cdot d_i}\cdot\left(1-\frac{\frac{\partial^2 v_i}{\partial c_i \partial d_i}\cdot c_i}{\frac{\partial v_i}{\partial d_i}}\right) < 1, \tag{A16}$$

and $\frac{\partial t_i}{\partial c_i} < 0$ if and only the opposite is true. Plugging in v_i for the two functional forms yields the result stated in the proposition. □

Proof of Lemma 3. By definition:

$$\frac{dd_i}{dc_i} = \frac{d}{dc_i}\left[\frac{t_i(c_i)}{c_i}\right] = \frac{\partial t_i}{\partial c_i}\cdot\frac{1}{c_i} - \frac{t_i}{c_i^2}. \tag{A17}$$

Thus if $\frac{\partial t_i}{\partial c_i} < 0$ then $\frac{\partial d_i}{\partial c_i} < 0$ trivially, so we only need to focus on the case of $\frac{\partial t_i}{\partial c_i} > 0$.

Taking the total derivative $\frac{d}{dc_i}$ (allowing d_i to adjust endogenously) of the first order condition of attention allocation (4) for $t_i > 0$ (and thus $d_i > 0$), we get

$$(1-\theta_i)\cdot\left[\frac{\partial^2 v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial c_i \partial d_i} + \frac{\partial^2 v_i\left(c_i,\frac{t_i}{c_i}\right)}{\partial d_i^2}\cdot\frac{\partial d_i}{\partial c_i}\right] = \frac{\partial \lambda}{\partial c_i}\cdot c_i + \lambda. \tag{A18}$$

Rearranging we get:

$$\frac{\partial d_i}{\partial c_i} = \frac{\frac{1}{1-\theta_i}\cdot\left[\frac{\partial \lambda}{\partial c_i}\cdot c_i + \lambda\right] - \frac{\partial^2 v_i(c_i,d_i)}{\partial c_i \partial d_i}}{\frac{\partial^2 v_i(c_i,d_i)}{\partial d_i^2}}. \tag{A19}$$

Case 1: additive v_i

Plugging $v_i(c,d) = f_i(c) + g_i(d)$ into (A19) we arrive at:

$$\frac{\partial d_i}{\partial c_i} = \frac{\frac{1}{1-\theta_i}\cdot\left[\frac{\partial \lambda}{\partial c_i}\cdot c_i + \lambda\right]}{g_i''(d_i)}. \tag{A20}$$

We know that $\lambda > 0$ always and $g_i''(d_i) < 0$ by definition. From Lemma 1 we know that $\frac{\partial \lambda}{\partial c_i} > 0$ if $\frac{\partial t_i}{\partial c_i} > 0$, thus it follows that $\frac{\partial d_i}{\partial c_i} < 0$ if $\frac{\partial t_i}{\partial c_i} > 0$, which was the case we had to prove.

Case 2: multiplicative v_i

Plugging $v_i(c,d) = f_i(c) \cdot g_i(d)$ in the first order condition for attention allocation (4) we get

$$(1-\theta_i) \cdot f_i(c_i) \cdot g_i'(d_i) = \lambda \cdot c_i \tag{A21}$$

Plugging $v_i(c,d) = f_i(c) \cdot g_i(d)$ into (A19) and using the above for λ we arrive at:

$$\frac{\partial d_i}{\partial c_i} = \frac{f_i'(c_i) \cdot g_i'(d_i) - \frac{1}{c_i} \cdot f_i(c_i) \cdot g_i'(d_i) - \frac{1}{1-\theta_i} \cdot \frac{\partial \lambda}{\partial c_i} \cdot c_i}{-f(c_i) \cdot g_i''(d_i)}. \tag{A22}$$

We know that $f(c_i) \cdot g_i''(d_i) < 0$ by definition. From Lemma 1 we know that $\frac{\partial \lambda}{\partial c_i} > 0$ if $\frac{\partial t_i}{\partial c_i} > 0$ thus a sufficient (but not necessary) condition for $\frac{\partial d_i}{\partial c_i} < 0$ is $f_i'(c_i) \cdot g_i'(d_i) - \frac{1}{c_i} \cdot f_i(c_i) \cdot g_i'(d_i) < 0$, which simplifies to $f_i'(c_i) \cdot c_i < f_i(c_i)$ given that by assumption $g_i'(d_i) > 0$. As f_i is a concave increasing function drawing a line with slope $f_i'(c_i)$ through the point $(c_i, f(c_i))$ on crosses the perpendicular axis at $f(c_i) - c_i \cdot f_i'(c_i)$. Since $f_i(0) \geq 0$ by Assumption 1 and f_i is strictly concave, $f(c_i) - c_i \cdot f_i'(c_i) > 0$, thus $f_i'(c_i) \cdot c_i < f_i(c_i)$ holds for all $c_i > 0$. □

Proof of Proposition 3.

We first prove $\lim_{c_i \to 0} t_i = 0$ by contradiction: Assume the opposite, i.e. that there exists a strictly decreasing infinite series $\lim_{n\to\infty} c_{i,n} = 0$ such that $\liminf_{n\to\infty} t_i(c_{i,n}) = \underline{t} > 0$. Define a new function $\tilde{t}_i = \frac{t_i}{2}$, with this new function $\liminf_{n\to\infty} \frac{\tilde{t}_i(c_i,n)}{c_i,n} = \infty$ as well, thus as $c_i \to 0$ there is no loss in good i's value but at least $\frac{\underline{t}}{2}$ can be reallocated to other goods, yielding extra value of at least $\frac{t}{2} \cdot \underline{\lambda}$ where $\underline{\lambda}$ is the shadow cost of attention if no attention was paid to good i. Thus the original $t_i(c_i)$ function was not an optimal choice of the consumer.

We now show that $\lim_{c_i \to \infty} t_i = 0$: By Assumption 1, we know that $v_i(c_i, d_i)$ is bounded from above and below thus $\lim_{d_i \to 0} d_i \cdot v_i(c_i, d_i) = 0$. Using l'Hôpital's rule:

$$\lim_{d_i\to 0} v_i(c_i,d_i) = \lim_{d_i\to 0} \frac{d_i \cdot v_i(c_i,d_i)}{d_i} = \lim_{d_i\to 0} \frac{v(c_i,d_i) + d_i \cdot \frac{\partial v_i(c_i,d_i)}{\partial d_i}}{1} = \lim_{d_i\to 0} v_i(c_i,d_i) + \lim_{d_i\to 0} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i \tag{A23}$$

implying $\lim_{d_i\to 0} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i = 0$ for any given c_i. Now we show that this convergence is uniform over any c_i: For additive v_i, we have $\lim_{d_i\to 0} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i = \lim_{d_i\to 0} g'(d_i) \cdot d_i$ which does not even depend on c_i so is trivially uniformly convergent. In case of multiplicative v_i, we have $\lim_{d_i\to 0} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i = \lim_{d_i\to 0} f(c_i) \cdot g'(d_i) \cdot d_i$ which again is also uniformly convergent as $f(c_i)$ is bounded from both below and above. Note that $\lim_{c_i\to\infty} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i$ also exists for all d_i: For additive v_i, we have $\lim_{c_i\to\infty} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i = g'(d_i) \cdot d_i$, while for multiplicative v_i we have $\lim_{c_i\to\infty} \frac{\partial v_i(c_i,d_i)}{\partial d_i} \cdot d_i = g'(d_i) \cdot d_i \cdot \lim_{c_i\to\infty} f(c_i)$, where the latter limit exists since $f(c_i)$ is increasing and bounded from above. Thus we can use the Moore-Osgood theorem, see Taylor (2012):

$$\lim_{\substack{c_i\to\infty \\ d_i\to 0}} d_i \cdot \frac{\partial v\left(c_i, d_i\right)}{\partial d_i} = \lim_{c_i\to\infty} \lim_{d_i\to 0} \frac{\partial v_i\left(c_i, d_i\right)}{\partial d_i} \cdot d_i = 0 \tag{A24}$$

Since $t_i \in [0, T]$ by definition, we have $\lim_{c_i\to\infty} d_i(c_i) = \lim_{c_i\to\infty} \frac{t_i}{c_i} = 0$ and thus:

$$\lim_{c_i\to\infty} d_i \cdot \frac{\partial v\left(c_i, d_i(c_i)\right)}{\partial d_i} = \lim_{\substack{c_i\to\infty \\ d_i\to 0}} d_i \cdot \frac{\partial v\left(c_i, d_i\right)}{\partial d_i} = 0 \tag{A25}$$

Using the FOC of attention allocation (4) for any given c_i it follows that:

$$\lim_{c_i\to\infty} d_i(c_i) \cdot \frac{\partial v\left(c_i, d_i(c_i)\right)}{\partial d_i} = \lim_{c_i\to\infty} \frac{t_i(c_i)}{c_i} \cdot \frac{\partial v\left(c_i, d_i(c_i)\right)}{\partial d_i} = \frac{1}{1-\theta_i} \cdot \lim_{c_i\to\infty} t_i(c_i) \cdot \lambda = 0. \tag{A26}$$

Since λ is bounded from below by $\underline{\lambda} > 0$ that would prevail if no attention was allocated to good i, we conclude $\lim_{c_i\to\infty} t_i = 0$.

We now show that $\lim_{c_i\to 0} \frac{\partial t_i}{\partial c_i} > 0$: Note that from Lemma 3 d_i is strictly decreasing in c_i, thus $\lim_{c_i\to 0} d_i > 0$ unless $t_i(c_i) = 0$ for all c_i. Using $\lim_{c_i\to 0} t_i = 0$, by l'Hôpital's rule:

$$\lim_{c_i\to 0} d_i = \lim_{c_i\to 0} \frac{t_i}{c_i} = \lim_{c_i\to 0} \frac{\frac{\partial t_i}{\partial c_i}}{1} = \lim_{c_i\to 0} \frac{\partial t_i}{\partial c_i} > 0.$$

We now show $t_i(c_i, c_{-i}) = 0$ for $c_i > \bar{\bar{c}}_i$: Note that all statements for $c_i > \bar{c}_i$ assume that the first order condition (4) defining t_i is binding and thus $t_i > 0$. Now assume that (4) is not binding for certain $c_i > 0$ and denote the lim inf of such c_i's by $\bar{\bar{c}}_i$. Since v_i is continuously differentiable it follows that $t_i(c_i)$ defined with equality in (4) is continuous. Since $\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} < 0$ for all $c_i > \bar{c}_i$, it follows that $\bar{\bar{c}}_i > \bar{c}_i$ and that (4) cannot be binding for any $c_i > \bar{\bar{c}}_i$ either. Thus $t_i(c_i) = 0$ for $c_i \geq \bar{\bar{c}}_i$ and thus

$\frac{\partial \tilde{t}_i(c_i,\lambda)}{\partial c_i} = 0$ for $c_i > \bar{\bar{c}}_i$. From (4) and that $f_i(c_i)$ is increasing and bounded from above it follows that $\bar{\bar{c}}_i < \infty$ if and only if $\lim_{d_i \to 0} g_i'(d_i) < \infty$.

We now show there exists a $c_i > 0$ such that $\frac{\partial t_i}{\partial c_i} < 0$: First note that since $t_i \geq 0$ and $\lim_{c_i \to 0} \frac{\partial t_i}{\partial c_i} > 0$, there exists a $\tilde{c}_i > 0$ such that $t_i(\tilde{c}_i, c_{-i}) > 0$. Second note that $t_i(c_i, c_{-i})$ is defined by (4): Since v_i is continuously differentiable in c_i, it must be the case that $t_i(c_i, c_{-i})$ is continuous in c_i. Now assume by contradiction that $\frac{\partial t_i}{\partial c_i} \geq 0$ always, in that case $\liminf_{c_i \to \infty} t_i(c_i, c_{-i}) \geq t_i(\tilde{c}_i, c_{-i}) > 0$ which directly contradicts the above result that $\lim_{c_i \to \infty} t_i = 0$.

□

Proof of Proposition 4.

Most of Proposition 4 is simply a repetition of Proposition 3. We simply present the additional proof here.

Now we show that $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} > 0$ for $c_i < \bar{c}_i$ and $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} < 0$ for $c_i \in (\bar{c}_i, \bar{\bar{c}}_i)$: By Assumption 4, $\sigma_i(d_i)$ is strictly decreasing in d_i and by Lemma 3 d_i is strictly decreasing in c_i, thus we have that $\sigma_i(d_i(c_i))$ is strictly increasing in c_i. Also by Assumption 3 $f' > 0$ and $f'' < 0$ and by Assumption 5 we know that $\frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}$ is strictly decreasing in c_i so $\sigma_i(d_i(c_i)) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right)$ is strictly increasing in c_i. Since we have shown that $\lim_{c_i \to 0} \frac{\partial t_i}{\partial c_i} > 0$, Lemma 2 implies that $\lim_{c_i \to 0} \sigma_i(d_i(c_i)) < 1$ for additive v_i and $\lim_{c_i \to 0} \sigma_i(d_i(c_i)) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right) < 1$ for multiplicative v_i. By Proposition 3 there is a $c_i \in (0, \infty)$ at which $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} < 0$ thus by Lemma 2 at that specific c_i we have $\sigma_i(d_i(c_i)) > 1$ for additive v_i and $\sigma_i(d_i(c_i)) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right) > 1$ for multiplicative v_i. Since $\sigma_i(d_i(c_i))$ is strictly increasing in c_i for additive v_i and $\sigma_i(d_i(c_i)) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right)$ is strictly increasing in c_i for multiplicative v_i, thus there is a unique cutoff $\bar{c}_i \in (0, \infty)$ defined by $\sigma_i(d_i(\bar{c}_i)) = 1$ for additive v_i and $\sigma_i(d_i(\bar{c}_i)) \cdot \left(1 - \frac{f_i'(\bar{c}_i) \cdot \bar{c}_i}{f_i(\bar{c}_i)}\right) = 1$ for multiplicative v_i. From Lemma 2 it then follows that $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} > 0$ for $c_i < \bar{c}_i$ and $\frac{\partial t_i(c_i,c_{-i})}{\partial c_i} < 0$ for $c_i \in (\bar{c}_i, \bar{\bar{c}}_i)$. Note that we also used the continuity of $\sigma_i(d_i(c_i))$ and $\sigma_i(d_i(c_i)) \cdot \left(1 - \frac{f_i'(c_i) \cdot c_i}{f_i(c_i)}\right)$ in c_i for the above proof which follows from f_i and g_i being continuously differentiable for $c_i > 0$ and also $d_i(c_i)$ being continuous.

□

Proof of Proposition 6.

First, we derive the equilibrium FOC's in a symmetric equilibrium. For expositional purposes we allow for all v_i's to be different. In case of an interior solution, the consumer's first-order condition for a given shadow price of attention λ yields:

$$\tilde{t}_i(c_i, \lambda) = \sqrt{\frac{c_i \cdot \delta_i \cdot (1-\theta_i)}{\lambda}} - c_i. \tag{A27}$$

Plugging this expression into the (binding) attention constraint (3), we can express λ as:

$$\lambda = \frac{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k)}\right)^2}{\left(\sum_{j=1}^{N} c_j + T\right)^2}. \tag{A28}$$

Substituting this expression back into Equation (A27) we obtain

$$t_i(c_1, ..c_N) = \frac{\sqrt{c_i \delta_i (1-\theta_i)}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k)}} \cdot \left(\sum_{j=1}^{N} c_j + T\right) - c_i. \tag{A29}$$

Partially differentiating with respect to c_i yields

$$\frac{\partial t_i(c_1, ..c_N)}{\partial c_i} = \frac{\sum_{j \neq i} \sqrt{c_j \delta_j (1-\theta_j)}}{\left(\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k)}\right)^2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{c_i}} \cdot \sqrt{\delta_i (1-\theta_i)} \cdot \left(\sum_{j=1}^{N} c_j + T\right) + \frac{\sqrt{c_i \delta_i (1-\theta_i)}}{\sum_{k=1}^{N} \sqrt{c_k \delta_k (1-\theta_k)}} - 1. \tag{A30}$$

Imposing symmetry ($c_i = c, \ \forall i$), this implies that

$$\frac{\partial t}{\partial c_i} = \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right) \tag{A31}$$

and

$$\lambda = \frac{\delta \cdot (1-\theta)}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{A32}$$

Plugging these into Equations (8) and (12), using that fact that under the assumed functional form $\frac{\partial v_i\left(c_i, \frac{t_i}{c_i}\right)}{\partial c_i} = f'(c)$, and observing that in symmetric equilibrium all goods get the same amount of attention $t = \frac{T}{N}$, we then arrive at the following equations.

In a symmetric equilibrium the equilibrium first-order condition is

$$\alpha - 2 \cdot c = \left[\frac{T}{N \cdot c} - \frac{N-1}{2 \cdot N} \cdot \left(\frac{T}{N \cdot c} - 1\right)\right] \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}, \tag{A33}$$

whereas the social planner's first order condition is

$$\alpha - 2 \cdot c = \frac{T}{N \cdot c} \cdot \frac{\delta}{c \cdot \left(1 + \frac{T}{N \cdot c}\right)^2}. \tag{A34}$$

Comparing (A33) with (A34) shows that equilibrium complexity c is higher than the social planner's choice of complexity if and only if $c < \frac{T}{N}$.

Second, we show that there exists a $\delta^* > 0$, such that for $\delta < \delta^*$ both the social planner's solution and the equilibrium outcome are symmetric.

We first calculate a strictly positive lower bound for δ^* that is sufficient to ensure symmetric complexity choices, both under the social planner's solution and in equilibrium. First note that the maximum value v that can be derived from a good is when the complexity is chosen to be the first best (unconstrained) optimum $c_i = \frac{\alpha_i}{2}$ and depth of understanding is chosen to be infinite $\frac{t_i}{c_i} = \infty$. Therefore, v_i can be bounded form above,

$$v_i \left(c_i, \frac{t_i}{c_i} \right) < v_i \left(\frac{\alpha_i}{2}, \infty \right) = \left(\delta_i + \frac{\alpha_i^2}{4} \right). \tag{A35}$$

With N ex-ante symmetric goods, if all goods have the same amount of complexity and therefore receive the same amount of attention from the consumer, v_i can be bounded from below because if $c = \frac{\alpha}{2}$ is not optimal, then v must be higher under the optimal complexity choice:

$$v_i \left(c_i, \frac{t_i}{c_i} \right) > v \left(\frac{\alpha}{2}, \frac{T/N}{\alpha/2} \right) = \frac{\alpha^3 N + 2T \left(\alpha^2 + 4\delta \right)}{4\alpha N + 8T} \tag{A36}$$

To ensure that the social planner's choice of complexity is symmetric, it has to be the case that the social planner's first-order condition holds with equality for all goods. In particular, the planner must not have an incentive to set the complexity of one of the goods to zero. This is satisfied as long as

$$N \cdot v \left(c_s^*, \frac{t_{s,s}^*}{c_s^*} \right) > (N-1) \cdot v \left(c_a^*, \frac{t_{s,a}^*}{c_a^*} \right) + 1 \cdot v\left(0, \infty\right), \tag{A37}$$

where c_s^* is the social planner's optimum in the symmetric case and c_a^* in the asymmetric one (in which one of the goods has zero complexity but other goods are symmetric in complexity). From Equations

(A35) and (A36), a sufficient condition for a symmetric solution is that

$$N \cdot \frac{\alpha^3 N + 2T\left(\alpha^2 + 4\delta\right)}{4\alpha N + 8T} > (N-1) \cdot \left(\delta + \frac{\alpha^2}{4}\right) + \delta, \tag{A38}$$

which simplifies to

$$\delta < \frac{\alpha(\alpha N + 2T)}{4N^2}. \tag{A39}$$

Note that this condition holds for small enough δ and is more likely to be violated when the attention capacity T is small. The condition holds for all $T > 0$ if $\delta < \frac{\alpha^2}{4 \cdot N}$. This is the sufficient (but clearly not necessary) condition for the social planner's solution to be symmetric.

Now we show that for low enough δ there exists a symmetric equilibrium in which all suppliers choose the complexity c_s^e. First note that from Equation (A33) it follows that if $\delta \to 0$ then $c_s^e \to \frac{\alpha}{2}$ from below. Thus for δ close enough to 0 it must hold that $\frac{\alpha}{4} < c_s^e < \frac{\alpha}{2}$. Because $v(.,.)$ is increasing in both arguments for $c < \frac{\alpha}{2}$, we can bound the supplier's payoff in a symmetric equilibrium from below. Specifically,

$$v\left(c_s^e, \frac{t_s^e}{c_s^e}\right) > v\left(\frac{\alpha}{4}, \frac{T/N}{\alpha/2}\right) = \frac{3}{16} \cdot \alpha^2 + \delta \cdot \frac{\frac{2 \cdot T}{N \cdot \alpha}}{1 + \frac{2 \cdot T}{N \cdot \alpha}}. \tag{A40}$$

It remains to be shown that the supplier does not want to deviate from the symmetric equilibrium to producing a good with zero complexity (and thus infinite depth of understanding by the consumer). This requires that

$$v\left(c_s^e, \frac{t_s^e}{c_s^e}\right) > v(0, \infty) = \delta. \tag{A41}$$

Using Equation (A40) it suffices to show that

$$\frac{3}{16} \cdot \alpha^2 + \delta \cdot \frac{\frac{2 \cdot T}{N \cdot \alpha}}{1 + \frac{2 \cdot T}{N \cdot \alpha}} > \delta, \tag{A42}$$

which holds for any $T > 0$ if $\delta < \frac{3}{16} \cdot \alpha^2$. Thus we have shown that for small enough δ, there exists a symmetric equilibrium.

Third, we show that, starting from a set of parameters for which symmetric complexity choices coincide under the social planner's solution and in equilibrium, the comparative statics stated in the proposition hold. The separating hyperplane for which the optimal and equilibrium levels of complexity are the same happens at critical attention level T at which all goods have complexity $c = \frac{T}{N}$ (see

Equation (19)). Plugging this into Equation (A34) yields a quadratic equation for T, for which the unique solution that corresponds to a maximum of the social welfare function is (this can be checked by signing the second order condition):

$$T^{crit} = \frac{N}{4}\left(\alpha + \sqrt{\alpha^2 - 2\delta}\right). \tag{A43}$$

Note that this critical T only exists if $\delta \leq \frac{\alpha^2}{2}$. Equation (A43) defines a separating hyperplane in the parameter space. By continuity of the equilibrium complexity in the underlying parameters, all we have to check is whether there is too much complexity on one side of the hyperplane, arbitrarily close to the hyperplane itself.

To prove that complexity is inefficiently large if attention is abundant, we take the total derivative of the two first order conditions (A33) and (A34) with respect to T. Substituting $c = \frac{T}{N}$ (at $c^e = c^*$) and solving for $\frac{dc^e}{dT}$ and $\frac{dc^*}{dT}$ yields:

$$\frac{dc^e}{dT} = \frac{\delta - \delta N}{\delta N(N+1) - 16T^2}, \tag{A44}$$

$$\frac{dc^*}{dT} = 0. \tag{A45}$$

We note that $\frac{dc^e}{dT} > \frac{dc^*}{dT}$ holds at $T = T^{crit}$ if $\delta < \frac{\alpha^2}{2}$ (i.e., if T^{crit} exists). □