FTG Working Paper Series

A Theory of Liquidity Spillover Between Bond and CDS Markets

by

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Working Paper No. 00072-00

Finance Theory Group

www.financetheory.com

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The Review of Financial Studies, Forthcoming

Abstract

I build a search model of bond and credit default swap (CDS) markets with endogenous investor participation and show that shorting bonds through CDS increases the liquidity and price of bonds. By allowing investors to trade the credit risk of bonds without trading the bonds, CDS introduction expands the set of feasible trades and attracts investors into the credit market. Because search is non-directed within the credit market, new investors also trade bonds and consequently increase their price and liquidity. My results suggest that naked CDS bans increased sovereigns' borrowing costs and thereby exacerbated the 2010–2012 European debt crisis.

Keywords: credit default swaps (CDS), search frictions, over-the-counter (OTC) markets, market liquidity, costly participation, CDS-bond basis, short-selling, credit risk.

*Princeton University. For many helpful comments, I thank Laurence Ales, Sylvain Chassang, Itay Golstein (the editor), Burton Hollifield, Stefano Sacchetto, Chris Telmer, Pierre-Olivier Weill, and two anonymous referees. I also thank Patrick Augustin, Luca Benzoni, Peter Feldhutter, Richard C. Green, Lars Kuehn, Pete Kyle, Laurence Lescourret, Artem Neklyudov, Chester Spatt, Randy Wright, Fernando Zapatero, Ariel Zetlin-Jones, Haoxiang Zhu, seminar participants at the CFTC, McGill University, BIS, Ohio State University, Penn State, University of Lausanne, AEI, University of Oklahoma, Notre Dame University, University of Calgary, FRB Dallas, participants at the AFA, NYU Stern Microstructure Meeting, EFA, the Southwest Search and Matching, FRB Chicago Summer Workshop, Carefin-Bocconi Conference, 10th Central Bank Microstructure Workshop, Early Career Women in Finance, Young Scholars Finance Consortium, EWFS, Annual Fixed Income Conference, and FDIC/UofM Annual Bank Research Conference. Send correspondence to Batchimeg Sambalaibat, Email: sbchimeg@gmail.com.

In this paper, I propose a novel theory of how short positions through derivatives affect the underlying asset. I do so in the context of bond and credit default swaps (CDS) markets.[1] Existing theories predict that derivatives fragment investors across the derivative and underlying markets and, as a result, attract liquidity away from the underlying market.[2] They predict this while keeping the aggregate number of investors fixed. I build a dynamic search model of bond and CDS markets and show that when the aggregate number of investors is instead endogenous, the results reverse. Short positions through CDS contracts increase the liquidity and price of the underlying bonds. I refer to this effect as a liquidity spillover effect.

I show the spillover effect with a continuous time dynamic search model that builds on Duffie, Garleanu, and Pedersen (2005, 2007) and Vayanos and Weill (2008). The credit market consists of two assets: a risky bond and a CDS contract with a cash flow based on the bond. In a CDS contract, the CDS seller pays the CDS buyer if the underlying bond defaults. A CDS buyer—who benefits if the bond defaults—is short the underlying credit risk. A CDS seller has the opposite long exposure. CDSs are in zero net supply, while bonds are in fixed supply. Investors have heterogenous private (high and low) values of bearing the credit risk associated with the bond. The difference in their private values generates a motive to participate in the credit market and trade the bond and CDS. Participating in the credit market, however, is costly as in Grossman and Miller (1988), Huang and Wang (2009), and Vayanos and Wang (2013). Investors, as a result, participate if the expected gains of doing so exceeds the fixed participation cost. Once they choose to participate, they search for potential counterparties and meet them at rates proportional to their endogenous masses. Search is thus non-directed within the credit risk market: investors search for both bond and CDS counterparties at the same time and trade with the counterparty they find first. Upon finding a counterparty, investors

[1]CDS contracts resemble an insurance protection against a firm or a government default. The CDS buyer pays the seller a premium until either the contract matures or a default (or a similar event) occurs. In return, the CDS seller pays the buyer a pre-agreed amount, referred to as the "notional" amount, in the event of default. The contract specifies the reference entity, the contract maturity, the notional amount, and the events that constitute a credit event.

[2]For example, Subrahmanyam (1991), Gorton and Pennacchi (1993), John et al. (2003), and Oehmke and Zawadowski (2015) show that stock index futures, security baskets, options, and CDS, respectively, reduce liquidity of the underlying asset market because some traders migrate to the derivative markets.

bargain over the price and trade. Finally, investors can get a valuation shock at any point in which case they unwind any positions they have (e.g., sell the bond they previously purchased) and exit the market. Put together, I extend the standard search models by, first, modeling CDS contracts and, second, by endogenizing the investors' participation decision and thereby their aggregate masses.

The main result I show is the liquidity spillover effect. It works as follows. CDS contracts allow investors to establish a short position by buying CDS.[3] The introduction of CDS, as a result, attracts into the credit market low private value investors, who in equilibrium seek to short credit risk by buying CDS. I refer to these investors that buy CDS without the underlying bonds as naked CDS buyers. The introduction of CDS also attracts a greater mass of long investors—high private value investors who want to trade the other side and long credit risk. Before, few of them participated in the credit market because they could only trade with bond sellers. Now they participate at a greater mass in response to the additional trading opportunity of selling CDS to naked CDS buyers. The larger mass of long investors, in turn, improves bond market liquidity. Once long investors enter, they do not just search and trade with naked CDS buyers. As buying bonds and selling CDS are economically similar positions, they search for bond sellers at the same time and trade with the counterparty they find first (search is non-directed). That is, the larger mass of long investors translates to a greater number of bond buyers. Bond sellers, in turn, spend less time searching for a bond buyer, have a greater bargaining power whey they do find a buyer, and consequently sell at a higher price. The result is a larger volume of bond trades, a smaller illiquidity discount in the bond price, and a higher bond price.[4]

A limiting case in which the supply of the bond is vanishingly small illustrates the mechanism more clearly.[5] When the bond is very rare, the probability of finding a

[3]For the baseline results, I assume that investors cannot physically short bonds. In an extension, I relax this assumption and show it is not a crucial assumption.

[4]The illiquidity discount in the bond price, as standard in search models, arises from search frictions. As bond sellers bargain with bond buyers, they concede to a price discount accounting for the time it takes to find another bond buyer. It is a price-based measure of bond market illiquidity or, equivalently, of the bond trading cost. One empirical proxy for the illiquidity discount is bid-ask spreads. An analogous illiquidity component arises in CDS spreads.

[5]While this is an extreme case and not necessary for my results, it is consistent with the fact that

bond seller—the only potential counterparty for long investors in the absence of CDS—approaches zero. The difficulty of finding a bond seller, moreover, puts long investors in a poor bargaining position when they do find a bond seller. Put together, for long investors the expected gains of participating in the credit market is vanishingly small and thereby does not justify the participation cost. No long investor, as a result, participates in the credit market. Since they are the only potential bond buyers, their absence in the credit market implies a perfectly illiquid bond market. The volume of bond trades is exactly zero and bond sellers face an infinite trading delay. The introduction of CDS indexed to the bond cash flow restores bond market liquidity. Now long investors are willing to incur the participation cost and enter the credit market because, thanks to naked CDS buyers, they face a nontrivial probability of finding a counterparty. Since CDS contracts do not require trading the underlying bond neither for the seller nor for the buyer of the contract, such CDS trades are not constrained by the supply of the bond. Long investors' presence in the credit market, in turn, revives the bond market. They search simultaneously for bond sellers and purchase the bond when such opportunities arrive. While bond trading is still constrained by its limited supply, the volume of trade is no longer zero but positive and bond sellers face a finite trading delay as opposed to an infinite delay. While my results do not depend on the bond supply being small, this limiting case clarifies the key ingredients necessary to generate the spillover effect: endogenous participation, search frictions, and non-directed search.[6]

This limiting case also clarifies that direct short-selling would not help relax the bond supply constraints. This is because short-selling requires trading the underlying asset. A short seller first has to search for a bond lender in the repo market to borrow a bond from. Then she has to search for a buyer in the bond market to sell the bond to. To unwind the short position, the investor has to again search for a bond, buy it, and return it to

in practice the size of the underlying assets is often smaller than the amount of derivative positions on those assets. For example, the total U.S. corporate debt averaged $6.3 trillion between 2008-2011; over the same period, the total notional amount of CDSs purchased referencing that debt was $12.4 trillion (SIFMA 2018b, DTCC 2014).

[6]Additionally, I focus on a parameter range that ensures an interior solution for the investors' participation rate. Such parameter range guarantees a large enough increase in the long investors' participation rate in response to the introduction of CDS and thereby helps generate the spillover effect. See the discussion in Section 3.2.

the bond lender. CDSs, in contrast, allow investors to trade the credit risk independent of the availability of the underlying bond. Thus, short-selling activity and the effect it can have on the underlying asset are inherently limited by the supply of the underlying asset.[7]

In a second set of results, I clarify the respective role of naked versus covered CDS buyers. Covered CDS buyers are investors who buy both the bond and the CDS. In reality, such investors buy CDS either as a hedge on their bonds or as a part of CDS-bond basis trades. I show that, first, covered CDS buyers reduce the volume of bond transactions. This is because some of the bond sellers—instead of selling their bond—buy CDS as a hedge on their bonds, hold on to their bond, and thereby become covered CDS buyers. Second, they do not affect neither the bond illiquidity discount nor the bond price and are thus redundant. Thus, covered CDS buyers do not generate the spillover effect.[8]

Finally, I highlight two testable asset pricing predictions of the model. First, both the bond yield and CDS spreads depend on three key components: the credit risk of the bond, the participation cost, and the (endogenous) illiquidity of the respective asset. These components are realistic and I discuss the relevant evidence in Sections 5.2-5.3. Second, in equilibrium, the CDS-bond basis—defined as the CDS spread minus the bond yield spread—depends on the relative liquidity of bond and CDS markets. It is negative if the bond market is less liquid than the CDS market and is positive otherwise. This result helps explain the stylized fact that the relative liquidity of bond and CDS markets helps explain a non-zero CDS-bond basis and that we observe both positive and negative CDS-bond bases in the data.[9]

[7] A search framework also implies that the total search cost of short-selling is significantly higher than that of synthetic shorts through CDS. This is because, unlike CDSs, short-selling requires multiple search stages and in each stage trading a bond in a potentially limited supply.

[8] The limiting case with a vanishing supply of the bond also clarifies this point. Holding covered CDS positions involves finding a bond in limited supply. Allowing covered CDS positions, as a result, does not help relax the bond supply constraints that deterred potential investors from credit markets.

[9] See Bühler and Trapp (2009), Nashikkar, Subrahmanyam, and Mahanti (2011), Kucuk (2010), Arce, Mayordomo, and Peña (2013), Augustin and Schnitzler (2021), and references in Footnote 13.

Related Literature

My paper contributes to the literature, first, by helping reconcile conflicting empirical findings on the effects of CDS on bonds. On the one hand, Ashcraft and Santos (2009) and Das, Kalimipalli, and Nayak (2014) document that CDSs are redundant: They have no effect on bond yields nor on price-based measures of bond market liquidity (e.g., the LOT and the Amihud illiquidity measures). Moreover, in Das, Kalimipalli, and Nayak (2014), CDSs are associated with a decrease in the bond turnover. On the other hand, a second body of work documents that CDS trading is associated with higher bond prices, narrower bid-ask spreads, and larger volumes of bond trades.[10] My results suggest that the first set of results arises from bond holders purchasing CDS as a hedge (i.e., covered CDS buyers). The common interpretation of the second set of results is that when investors can buy CDS as a hedge on their bonds, they are more willing to buy bonds. My model predicts that the second set of results instead arises from naked CDS buyers and the long investors they attract into both the CDS *and* the bond market, not from bond holders buying CDS as a hedge.[11] Recent micro-evidence on the nature of CDS buyers and sellers supports this interpretation. Acharya, Gunduz, and Johnson (2018) and Czech (2019) document that financial institutions with large portfolios of bonds and loans primarily sell CDS and thereby use CDS to expand their credit risk exposure, not to hedge and decrease their exposure.

Second, my results help explain how sovereign bond markets reacted to naked CDS bans. In October 2011, the European Union voted to ban naked CDS purchases—buying CDS without owning the underlying bonds—against EU governments. Using a difference-in-difference analysis, Sambalaibat (2019) documents that following the ban the liquidity of the underlying bonds deteriorated. Consistent with this evidence, banning naked CDS positions in my model reverses the spillover effect. Investors can no longer sell CDS because their counterparties are banned from buying CDS. Long investors exit the CDS market, but by exiting the CDS market, they also pull out from the bond market. The

[10]See, for example, Ismailescu and Phillips (2015), Massa and Zhang (2012), Shim and Zhu (2014), Nashikkar, Subrahmanyam, and Mahanti (2011), and Czech (2019).

[11]I explain in Section 5.1 why we may observe empirical findings consistent with both naked and covered CDS purchases. In short, how investors use CDS has evolved over time.

result is a decrease in bond market liquidity. Thus, preventing investors from shorting ultimately ends up banning investors who want to take the opposite side and long the underlying asset.

Third, the paper sheds light on the effect of synthetic short positions—positions that do not require trading the underlying asset. Most of the CDS literature focus on the effects of CDS when investors trade them in conjunction with the underlying bonds either as a hedge on the bonds or as a part of CDS-bond basis trades (Thompson 2007, Arping 2014, Bolton and Oehmke 2011, Sambalaibat 2012, and Parlour and Winton 2013). They thereby ignore the defining feature of CDSs that led to their proliferation—they allow investors to trade the bond issuer's credit risk without trading the bonds.

Finally, my paper's focus is related to Oehmke and Zawadowski (2015), who analyze the impact of CDS trading in a model with exogenous trading costs and exogenous aggregate number of investors.[12] I show that endogenizing the number of investors as well as bond and CDS trading costs reverses several implications of Oehmke and Zawadowski (2015). First, they show that CDS introduction crowds out trading in the bond market and, as a result, unambiguously decreases the trading volume of bonds. I instead show that the introduction of CDS can increase the volume of bond trades, consistent with the evidence from Nashikkar, Subrahmanyam, and Mahanti (2011) and Czech (2019). Second, their model predicts that the CDS-bond basis is always negative. This result arises from the assumption that the CDS trading cost is smaller than the bond trading cost. Yet, in the data we observe both negative and positive CDS-bond bases and a large cross-sectional and time series variation in the sign of the CDS-bond basis.[13] I show that the CDS-bond basis, in equilibrium, equals the difference between (endogenous) bond

[12]Another related paper with exogenous trading costs and fixed number of investors is Banerjee and Graveline (2014). In their model, derivatives are redundant if the underlying asset has an abundant supply. In my model, in contrast, derivatives are nonredundant even if the underlying asset has an abundant supply.

[13]Prior to the 2007-2009 crisis, the CDS-bond basis for corporate bonds was positive on average (Blanco, Brennan, and Marsh 2005, Nashikkar, Subrahmanyam, and Mahanti 2011, Mitchell and Pulvino 2012, Boyarchenko et al. 2018, Bai and Collin-Dufresne 2019). During the 2007-2009 crisis, the CDS-bond basis turned negative and more so for high-yield corporate bonds. The CDS-bond basis for sovereign bonds, on the other hand, is on average positive both during normal times and crises episodes (Fontana and Scheicher 2016, Kucuk 2010, Arce, Mayordomo, and Peña 2013). In fact, the sovereign CDS-bond basis increased and became even more positive during the 2007-2009 crisis and later during the Euro area sovereign debt crisis (Gyntelberg et al. 2017).

and CDS trading costs.[14] The CDS-bond basis, as a result, can be positive, negative, or zero in my model depending on the underlying credit risk and bond supply parameters. Lastly, in their model, CDS trading does not arise if the CDS trading cost exceeds the bond trading cost. In my model, CDS trading arises even if the CDS trading cost in equilibrium exceeds the bond trading cost. This prediction is consistent with the fact for some sovereign credit markets, for example, the bond market is more liquid than the CDS market, yet investors still trade CDS on those sovereign names.[15]

The paper is organized as follows. Section 1 presents the model environment, Section 2 characterizes the equilibrium and the bond price, and Section 3 presents the main result of the paper on the effect of naked CDS buyers. Section 4 shows the effects of covered CDS buyers as well as the combined effect of both covered and naked CDS buyers. Section 4 also allows investors to short-sell in addition to buying CDS and analyzes how this extension affects the main results of the paper. Finally, Section 5 connects the testable implications of the model (including asset pricing predictions) with the the empirical literature. Proofs are relegated to the appendices.

1 Model

Time is continuous and goes from zero to infinity. Agents are risk-averse, live infinitely, have idiosyncratic stochastic endowments, and can invest in a risk-free asset with return $r > 0$. They hold and trade bilaterally a risky bond and a CDS contract with a cash flow based on the risky bond. Finding someone to trade with involves search. Agents choose to participate in the credit market if doing so makes them better off. This is the model in a nutshell; the rest of this section elaborates.

[14] In particular, the CDS-bond basis equals the illiquidity component in CDS spreads minus the illiquidity component in bond yields.

[15] An example is the U.S. government bond market. The CDS bid-ask spread for CDS contracts referencing US sovereign bonds averaged 5.5 basis points in 2009-2012, while the bond bid-ask spread averaged less than 1 basis points over the same period (Sambalaibat 2019). Similarly, between 2008-2012, the daily trading volume of US Treasury securities averaged \$515 billion, while the daily notional amount of CDS contracts traded was \$0.1 billion (SIFMA 2018b, Sambalaibat 2019).

1.1 Assets

The bond is a perpetuity that occasionally comes short of its promised cash flow. I define such occasions as a default. In particular, the bond has supply S, trades at price p_b, and has a cumulative cash flow process $D_{b,t}$ satisfying

$$dD_{b,t} = \delta dt - JdN_t. \tag{1}$$

In (1), $\delta > 0$ is the promised rate of the coupon flow, $\{N_t, t \geq 0\}$ is a Poisson counting process with an intensity parameter $\eta > 0$, and $J > 0$ is the size of the default. The process N_t counts the number of defaults in $[0, t]$, and its increment, dN_t, is 0 or 1. Thus, (1) says, in a small interval $[t, t + dt]$, with probability ηdt, the bond defaults and its cash flow decreases by J. Otherwise, it pays the coupon at the promised rate. Agents hold $\theta_b \in \{0, 1\}$ units of the bond. They cannot short bonds for now, but I relax this in Section 4.2.

In a CDS contract, the buyer pays a premium flow p_c to the seller; the seller, in return, pays the buyer J if the bond defaults. The CDS buyer's cumulative cash flow $D_{c,t}$, as a result, follows

$$dD_{c,t} = JdN_t. \tag{2}$$

Since this is perfectly negatively correlated with the bond cash flow, the CDS buyer has a short exposure to the underlying credit risk. Conversely, the CDS seller has a cash flow $(-JdN_t)$ that is positively correlated with the bond and is thus long credit risk. Herein, when I refer to a long or a short position, I will mean with respect to the underlying credit risk. Thus, a long position through the CDS market, for example, does not mean an investor has bought CDS but means she has sold CDS and is thus long exposed to the underlying default risk. I denote an agent's CDS position with $\theta_c \in \{\text{-}1, 0, 1\}$, where each denotes a short, a neutral, and a long position, respectively.

An investor terminates a CDS contract by paying their counterparty a fee. The fee is endogenous and is such that the nonterminating side is indifferent between (a) continuing the contract and (b) accepting the fee, searching for a new counterparty, and, upon a match, entering a new position. I assume that when the nonterminating side is indifferent, she accepts the fee and starts the process again. I denote by T_{S} and T_{B} the fees the seller and the buyer pay their respective counterparties.

Put together, the feasible portfolios are (1) bought the bond: $[\theta_b, \theta_c] = [1, 0]$, (2) sold CDS: $[\theta_b, \theta_c] = [0, 1]$, (3) bought CDS: $[\theta_b, \theta_c] = [0, \text{-}1]$, (4) no position: $[\theta_b, \theta_c] = [0, 0]$, and (5) bought both the bond and CDS: $[\theta_b, \theta_c] = [1, \text{-}1]$. In the last portfolio, CDS acts as a hedge on the bond. It is a covered CDS position. In Sections 1-3, I shut down covered CDS positions to isolate the effect of naked CDS positions. Then in Sections 4.1 and 5, I relax this assumption and allow covered CDS positions.

For tractability, I restrict the net position to $0 \leq |\theta_b + \theta_c| \leq 1$, which rules out a simultaneous long position in both assets ($[\theta_b, \theta_c] = [1, 1]$). If we relax this restriction, the CDS introduction would result in even more trading opportunities for investors and hence in a larger liquidity spillover effect. This restriction, as a result, puts a lower bound on the magnitude of the liquidity spillover effect and does not qualitatively affect the main results of the paper.

1.2 Agents

Agents have time preference rate β and CARA utility preferences with risk aversion parameter α: $u(C) = -e^{-\alpha C}$. Agent i's cumulative endowment process $e_{i,t}$ follows

$$de_{i,t} = \mu_e \rho_{i,t} dt + \rho_{i,t} \sigma_e (-dN_t) + \sqrt{1 - \rho_{i,t}^2} \sigma_e dZ_t, \tag{3}$$

where $\mu_e > 0$ and $\sigma_e > 0$ are constants, Z_t is a standard Brownian motion, and $\rho_{i,t}$ is the instantaneous correlation process between the bond cash flow and the agent's endowment process.[16] The processes $\{Z_t, \rho_{i,t}, N_t\}$ are pairwise independent. The correlation process $\rho_{i,t}$ is independent across agents and is a three-state Markov chain with states $\rho_{i,t} \in \{-\rho, 0, \rho\}$ where $\rho > 0$. Agents switch from the negative and positive correlation states

[16]The third term in (3), combined with the second term, helps ensures that the variance of the endowment is the same across agents. It is a simplifying specification that allows me to focus on the correlation between an agent's endowment and the bond as the key source of heterogeneity across agents. See Duffie, Gârleanu, and Pedersen (2007) and Vayanos and Weill (2008) for similar setups.

to the zero correlation state with Poisson intensities γ_d and γ_u, respectively. The intensity of switching from the zero correlation state to either the positive or negative correlation state is zero (the zero correlation state is thus an absorbing state). I will explain later in Section 2.1 that the zero correlation state serves as an exit state. Upon reverting to the zero correlation state, investors without a previously established position exit the credit market, while those with positions first unwind their positions and then exit.

The different correlation realizations across agents generate heterogenous private valuations for the underlying credit risk. As I show later in Section 2.2, an investor whose endowment is currently negatively correlated with the bond ($\rho_{i,t} = -\rho$) has the highest private valuation for the bond (hence, the most willing to buy it); those with an uncorrelated endowment ($\rho_{i,t} = 0$) have an intermediate valuation; and those with a positively correlated endowment ($\rho_{i,t} = \rho$) have the lowest valuation. This difference in valuations creates a motive for trade. In particular, a random change in an agent's valuation (due to a random change in her correlation) generates a need to trade and rebalance her portfolio. From hereon, I will refer to an agent with $\rho_{i,t} = -\rho$ as a high-valuation agent or "h" for short, with $\rho_{i,t} = 0$ as an average-valuation ("a") agent, and with $\rho_{i,t} = \rho$ as a low-valuation ("l") agent. I will denote the valuations with i where $i \in \{h, a, l\}$. Referring to agents according to their valuations is simpler than referring to their correlations.

To characterize the equilibrium later in Section 2, I group agents into types based on their current valuation and asset position. An agent of type $\tau = i[\theta_b, \theta_c]$ has valuation $i \in \{h, a, l\}$ and asset position $[\theta_b, \theta_c]$. For example, an agent of type $h[1, 0]$ is a high-valuation investor who owns a bond.

1.3 Agents' Decisions

Agents make two sets of decisions. First, they decide whether to participate in the credit market. At any point in time, fixed flows of agents F_h and F_l are born as high- and low-valuation agents, respectively. Each compares the expected profit (or, equivalently, the continuation value) of entering and trading in the credit market, $V_{i[0,0]}$ for $i \in \{h, l\}$, with the participation cost, O, and chooses to participate if the former exceeds the latter.

The subscript [0, 0] on the continuation value $V_{i[0,0]}$ denotes that investors initially do not hold any assets. Then, the equilibrium fraction of investors that choose to participate, ν_i, solves

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O \\ [0,1] \text{ if} & V_{i[0,0]} = O \\ 0 & V_{i[0,0]} < O, \end{cases} \tag{4}$$

for $i \in \{h, l\}$.[17] Investors' participation rates $\{\nu_h, \nu_l\}$ translate to endogenous flows of new high- and low-valuation investors entering the credit market ($\nu_h F_h$ and $\nu_l F_l$) and hence endogenous masses of high- and low-valuation investors ($\frac{\nu_h F_h}{\gamma_d}$ and $\frac{\nu_l F_l}{\gamma_u}$) active in the credit market. This approach of endogenizing the number of investors by endogenizing their participation decision is standard and follows Grossman and Miller (1988), Huang and Wang (2009) and Vayanos and Wang (2013). I explain in Section 2.1 how agents exit the credit market.

The participation cost, O, captures any costs that prevent full participation in the credit market by all agents at all times. They include human capital, regulatory, information acquisition, and financial capital costs. Vayanos and Wang (2013), for example, interpret the participation cost as costs of "buying trading infrastructure or membership of a financial exchange, having capital available on short notice, monitoring market movements, etc." (see also Huang and Wang (2009)). The participation cost can also be interpreted as the expected profit from alternative investment opportunities and hence the opportunity cost of participating in the credit market.

The second set of decisions agents make is, once inside the credit market, they choose their consumption, C_t, and their bond and CDS positions, $\{\theta_{b,t}, \theta_{c,t}\}$, to maximize their expected utility

$$\mathbb{E}\left[\int_0^\infty e^{-\beta t} u(C_t) dt\right], \tag{5}$$

[17]We can ignore the participation decision of average-valuation agents because, in equilibrium, the continuation value of an average-valuation agent is zero: $V_{a[0,0]} = 0$. Thus, for any positive participation cost, O, their participation rate is zero.

subject to the wealth process,

$$dW_t = (rW_t - C_t)\, dt + de_t + dD_{b,t}\theta_{b,t} - p_b d\theta_{b,t} + (p_c dt - dD_{c,t})\theta_{c,t}, \tag{6}$$

and the transversality condition, $\lim_{T\to\infty} \mathbb{E}[e^{-\beta T} e^{-\alpha r W_T}] = 0$.

1.4 Search and Bargaining

I adopt the standard non-directed random search and matching framework of Duffie, Garleanu, and Pedersen (2005, 2007) as follows. An agent randomly matches with another agent at Poisson arrival times with intensity parameter λ. The total volume of matches between any two agent types τ and τ', as a result, is $\lambda\mu_\tau\mu_{\tau'}$, where μ_τ and $\mu_{\tau'}$ are their respective masses.[18] Given the total volume, a type τ agent matches with a type τ' agent with total intensity $\frac{\lambda\mu_\tau\mu_{\tau'}}{\mu_\tau} = \lambda\mu_{\tau'}$. The corresponding expected search time, $\frac{1}{\lambda\mu_{\tau'}}$, as a result, has both an exogenous (λ) and an endogenous component ($\mu_{\tau'}$). Upon a match, if trading either the bond or CDS yields positive gains from trade and the resulting positions are feasible, they Nash-bargain and trade at mutually agreeable terms of trade (to be described in Section 2.3). Search is thus non-directed, and whether agents trade the bond or CDS with each other depends on their equilibrium trading strategies as I explain in Section 2.1. The matching intensity λ is exogenous for now. But in online Appendix H, I endogenize it and allow agents to optimally choose potentially different search intensities for bond versus CDS matches.[19] The main results remain qualitatively the same.

[18]Weill (2008) provides an empirical evidence that this functional form for the matching function fits well the trading patterns of OTC traded financial assets. A large body of work uses a matching function with the same functional form. See, for example, Vayanos and Wang (2007), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Hugonnier, Lester, and Weill (2014), Shen, Wei, and Yan (2015), Sambalaibat (2018), Neklyudov (2019), and Uslu (2019).

[19]Milbradt (2017) also provides a search-based model with endogenous search intensities.

2 Equilibrium and the Bond Price

I start this section by, as standard in the literature, conjecturing the agents' optimal trading strategies. Doing so helps characterize who are the bond and CDS buyers and sellers and their masses. I then characterize the agents' continuation values. Then, using the conjectured trading strategies and continuation values, I characterize prices, define the steady state equilibrium, and prove its existence in Proposition 2. Proposition 2 also proves that the conjectured trading strategies are indeed optimal. Since this section is tedious, readers wishing to see the main result may skim it and proceed to Section 3 which contains the main result of the paper.

2.1 Optimal Trading Strategies

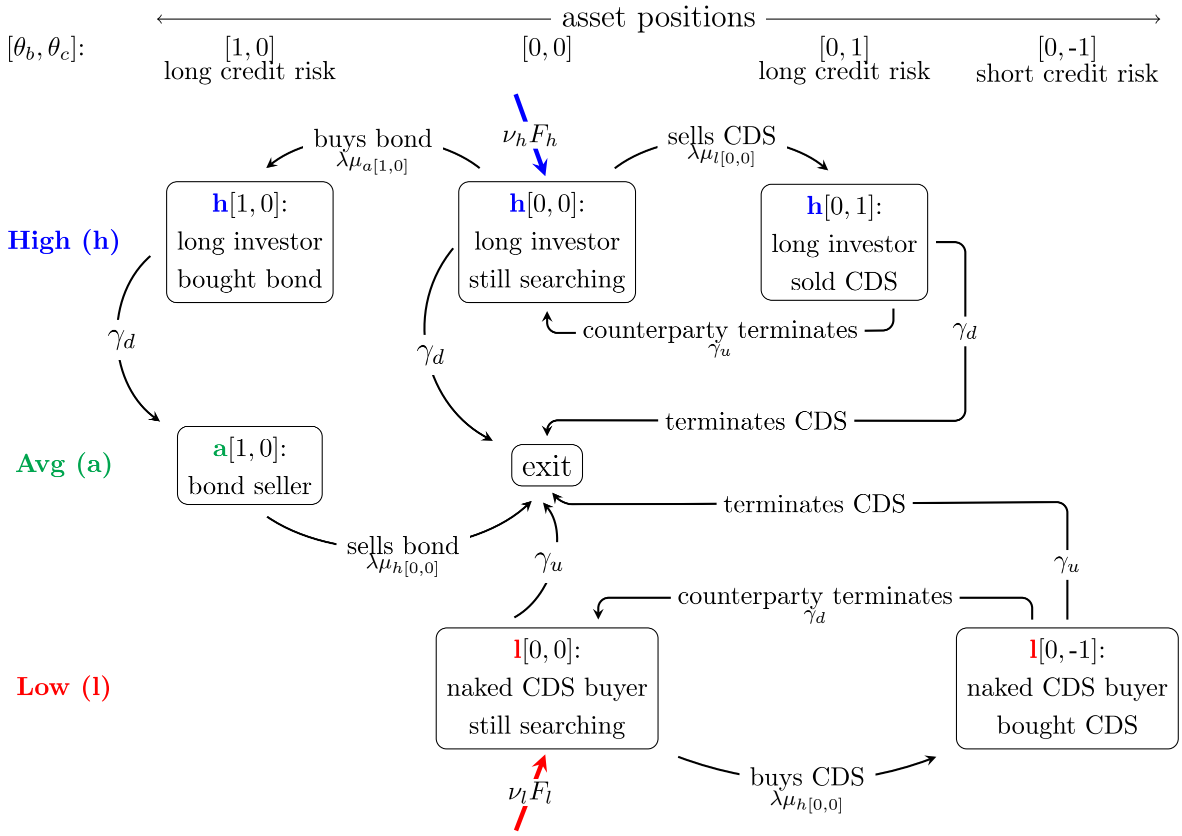
The investors' lives, in a nutshell, evolve as follows. High- or low-valuation investors choose to participate in the credit market and subsequently search for a counterparty. Upon finding a counterparty with whom trading is profitable, they bargain over the price, trade, and reach their optimal asset position. At any point, high- and low-valuation investors may get a valuation shock. If they do, they optimally exit the credit market. If the shock occurs before they were able to reach their optimal position, they exit immediately. But if it occurs after they have established a position, they unwind their position and then exit. The rest of this section elaborates and Figure 1 illustrates the discussion.

High-valuation investors who have chosen to participate in the credit market, $h[0,0]$, seek to long credit risk by either buying the bond or selling CDS. They search for both a bond seller and a CDS buyer and trade with the counterparty they find first.[20] The population of high-valuation investors, as a result, consist of investors who are at different stages in their search: those who have not yet established a position and are still searching ($h[0,0]$), those who have purchased the bond ($h[1,0]$), and those who have sold CDS ($h[0,1]$). The investors with the latter two positions have reached their optimal position. I will interchangeably refer to high-valuation investors as long investors.

[20]Although I assume that investors search for both bond and CDS counterparties, doing so is indeed optimal if investors optimize over search strategies: searching for bond counterparties, searching for CDS counterparties, or searching for both at the same time.

Figure 1: A Snapshot of Transitions Between Agent Types

The figure shows the transitions between agent types. Flows of $\nu_h F_h$ and $\nu_l F_l$ agents enter the credit market as new high- and low-valuation investors. High- and low-valuation investors revert to average-valuation with intensities γ_d and γ_u, respectively. An investor seeking a long position ($h[0,0]$) finds a counterparty in the bond and CDS markets with intensities $\lambda\mu_{a[1,0]}$ and $\lambda\mu_{l[0,0]}$, respectively. A bond seller ($a[1,0]$) finds a buyer with intensity $\lambda\mu_{h[0,0]}$. A trader seeking a short position by buying CDS ($l[0,0]$) finds a counterparty with intensity $\lambda\mu_{h[0,0]}$.



Low-valuation investors seek to short credit risk by buying CDS. They are the naked CDS buyers in the model (low-valuation investors do not own bonds in equilibrium). The population of low-valuation investors consist of investors who are searching to buy CDS ($l[0,0]$) and investors who bought CDS ($l[0,\text{-}1]$). The latter have reached their optimal position. I will interchangeably refer to low-valuation investors as short investors.

Investors who revert to the average-valuation state unwind and exit the market as follows. Once high- and low-valuation investors get a valuation shock and revert to the average-valuation state, their optimal position is no position, $[\theta_b, \theta_c] = [0, 0]$. They, as a result, optimally unwind any positions they have. Investors holding a bond ($a[1,0]$) unwind by becoming one of the bond sellers and searching for a bond buyer. When they find one, they sell and revert to $a[0,0]$-type agent. Investors with CDS exposures, on the other hand, revert to type $a[0,0]$ agent immediately by paying a fee and terminating their contract, while their counterparties accept the fee and start the search process again. Since the average-valuation state is an absorbing state, the investors that transition to a[0,0]-type permanently remain that type and thereby do not affect the rest of the economy again. Agents of type $a[0,0]$ also do not actively participate in the credit market (i.e., they neither hold an asset nor seek to hold one). I thus refer to the transition to the a[0,0]-type as exiting the credit market.

The above setup with three valuations and investors entering the credit market as high and low types and exiting as average types is a simple way to endogenize the aggregate masses of investors with different valuations and to model short positions. See Vayanos and Wang (2007), Vayanos and Weill (2008), Rocheteau and Weill (2011), and Afonso (2011) for similar setups (though not necessarily all with endogenous participation and short positions as in my model). I explain in Appendix C.1 why I need all three valuations.

In the above discussion, the assumption that investors unwind their CDS positions by terminating them is not crucial for my results. This is because the termination fees are endogenous and capture all the forces that would be present under alternative specifications on how investors unwind their CDS positions. Suppose, for example, that trader A has sold CDS to trader B and now wants to unwind her position. If finding a CDS seller is difficult, trader B in equilibrium will demand a large termination fee from trader A. Trader A, in turn, internalizes the difficulty of finding a CDS seller through the high termination fee she faces. An alternative approach would be to allow investors—instead of terminating their positions—to enter an offsetting position. In this case, trader A would have to search for a CDS seller willing to take over her side of the contract with trader B, and the difficulty of finding one would directly decrease trader A's utility. Thus, alternative assumptions on how agents unwind their CDS positions will yield similar results. I adopt the simplest one to model, which is outright terminations. CDS terminations are also realistic. Benos, Wetherilt, and Zikes (2013) document that among CDS contracts with an original maturity of 5 years, 83% are terminated within the first year of the contract and an additional 7% within two years of contract initiation.

Given the optimal positions and trading strategies, the equilibrium agent types are $\mathcal{T} \equiv \{h[0,0], h[1,0], h[0,1], a[1,0], l[0,0], l[0,\text{-}1]\}$. Of these, $a[1,0]$ and $h[0,0]$ are the actively searching bond sellers and buyers that make up the bond market. The bond trading volume, as a result, is

$$M_b \equiv \lambda \mu_{a[1,0]} \mu_{h[0,0]}.$$

Agent types $l[0,0]$ and $h[0,0]$ are the actively searching CDS buyers and sellers, respectively, and make up the CDS market. The CDS trading volume is thus

$$M_c \equiv \lambda \mu_{l[0,0]} \mu_{h[0,0]}.$$

Note that $h[0,0]$-type investors are both a bond buyer and a CDS seller at the same time.

2.2 Continuation Values

Given the conjectured optimal trading strategies, Proposition 1 characterizes the agents' continuation values, V_τ, that solve the agents' optimization problem (5).

Proposition 1. *Suppose that*

1. *$U(W,\tau)$ denotes the indirect utility of an agent of type τ with current wealth W.*
2. *An agent of type τ switches to type τ' with intensity $\gamma(\tau,\tau')$, where $\gamma(\tau,\tau')$ incorporates the conjectured optimal trading strategies.*
3. *$P(\tau,\tau')$ is the instantaneous payoff associated with buying and selling the bond:*

$$P(\tau,\tau') = \begin{cases} -p_b & \text{if } \tau = i[0,\theta_c] \text{ and } \tau' = i[1,\theta_c], \text{ where } i \in \{h,a,l\} \\ p_b & \text{if } \tau = i[1,\theta_c] \text{ and } \tau' = i[0,\theta_c], \text{ where } i \in \{h,a,l\} \\ 0 & \text{else.} \end{cases} \tag{7}$$

4. *Let $\bar{a} \equiv \frac{1}{r}\left(\frac{log(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2\right)$,*

$$x \equiv r\alpha\rho\sigma_e\eta J, \tag{8}$$

and

$$y \equiv \frac{r\alpha}{2}\eta J^2. \tag{9}$$

Then, solutions for $U(W,\tau)$ *are of the form* $U(W,\tau) = -e^{-r\alpha(W+V_\tau+\bar{a})}$*, where* V_τ *is given by*

$$\begin{aligned} rV_\tau = & \left(\delta - \eta J + x_\tau\right)\theta_b + \left(p_c - \eta J + x_\tau\right)\theta_c - y\left(\theta_b + \theta_c\right)^2 \\ & + \sum_{\tau' \in \mathcal{T}} \gamma(\tau,\tau') \frac{1}{r\alpha}\left(1 - e^{-r\alpha(V_{\tau'} - V_\tau + P(\tau,\tau'))}\right), \end{aligned} \tag{10}$$

$x_\tau = x$ *for a high,* $x_\tau = 0$ *for an average, and* $x_\tau = -x$ *for a low-valuation investor.*

The first row of (10) illustrates the difference in the agents' private valuations that generate the equilibrium trading strategies discussed in the previous section. Owning a bond ($[\theta_b, \theta_c] = [1, 0]$), for example, yields a flow utility of $\delta - (\eta J + y - x)$ to a high-valuation investor, $\delta - (\eta J + y)$ to an average-valuation investor, and $\delta - (\eta J + y + x)$ to a low-valuation investor, where δ is the coupon flow. The bracketed expressions in the flow utilities are the private costs of bearing credit risk: ηJ is the expected default loss; y is the baseline disutility associated with both long and short exposures to credit risk that arises due to agents' risk aversion; and x captures how this disutility varies across investors. Thus, high-, average-, and low-valuation investors have the smallest, average, and the largest private cost of bearing a long credit exposure, respectively. Similarly, a short position (buying CDS, $[\theta_b, \theta_c] = [0, \text{-}1]$) yields a flow utility of $-p_c + (\eta J - y - x)$ to a high-valuation investor, $-p_c + (\eta J - y)$ to an average-valuation investor, and $-p_c + (\eta J - y + x)$ to a low-valuation investor, where p_c is the CDS spread that the CDS buyer pays and the bracketed expressions are now the private benefits of buying insurance. Thus, buying CDS yields the most utility to a low-valuation investor, an average utility to an average-valuation investor, and the least utility to a high-valuation investor. The flow utilities of a long position through CDS are analogous.

To understand the second row of (10), consider as an example the continuation value of a high-valuation investor (i.e., a long investor) with no position ($[\theta_b, \theta_c] = [0, 0]$): $V_{h[0,0]}$. Suppose that the risk aversion parameter α is small. Then, in a small time interval

$[t, t+dt]$, it evolves as

$$V_{h[0,0]} = (1 - rdt)\Big(\gamma_d dt 0 + \lambda\mu_{a[1,0]}dt(V_{h[1,0]} - p_b) + \lambda\mu_{l[0,0]}dtV_{h[0,1]} \qquad (11)$$
$$+ (1 - \gamma_d dt - \lambda\mu_{a[1,0]}dt - \lambda\mu_{l[0,0]}dt)V_{h[0,0]}\Big).$$

It is a probability weighted average utility across four possible outcomes for a long investor. First, with probability $\gamma_d dt$, the long investor reverts to an average-valuation investor and exits the market, in which case her utility is zero. Second, with probability $\lambda\mu_{a[1,0]}dt$, she finds a bond seller and buys a bond, changing her continuation value to that of a bond owner ($V_{h[1,0]}$) minus the bond price. Third, with probability $\lambda\mu_{a[1,0]}dt$, she finds a CDS buyer and sells CDS, which changes her continuation value to that of a CDS seller ($V_{h[0,1]}$). With probability $(1 - \gamma_d dt - \lambda\mu_{a[1,0]}dt - \lambda\mu_{l[0,0]}dt)$, neither of these events occur, and she remains a long investor with no position. In the continuous time limit, (11) simplifies to

$$rV_{h[0,0]} = \gamma_d \left(0 - V_{h[0,0]}\right) + \lambda\mu_{a[1,0]} \left(V_{h[1,0]} - V_{h[0,0]} - p_b\right) + \lambda\mu_{l[0,0]} \left(V_{h[0,1]} - V_{h[0,0]}\right). \quad (12)$$

Mapping this back to (10) for small α, a long investor switches to either $a[0,0]$, $h[1,0]$, or $h[0,1]$-type upon (1) reverting to an average-valuation agent, (2) finding a bond seller, and (3) finding a CDS buyer, respectively. The associated intensities are $\gamma(\tau, a[0,0]) = \gamma_d$, $\gamma(\tau, h[1,0]) = \lambda\mu_{a[1,0]}$, and $\gamma(\tau, h[0,1]) = \lambda\mu_{l[0,0]}$, respectively. The continuation values of the other agent types are analogous.

Eq. (10) also illustrates how the continuation values incorporate flow payments (i.e., the CDS spread) versus one time up-front payments (i.e., the bond price). Selling CDS, for example, yields a high-valuation investor a flow utility of $p_c - (\eta J - x + y)$. The CDS seller's continuation value ($V_{h[0,1]}$) reflects the present value of this flow utility and thus incorporates the stream of CDS spreads she receives. Purchasing a bond, on the other hand, yields her a flow utility of $\delta - (\eta J - x + y)$. The continuation value of a bond owner ($V_{h[1,0]}$) reflects the present value of this flow utility only, not the one-time upfront payment she makes to buy the bond (i.e., the bond price). The bond price is instead captured separately by the payoff function $P(\tau, \tau')$ in (7).

In the rest of the paper, I assume that the risk aversion parameter (α) is small and linearize (10). See Duffie, Gârleanu, and Pedersen (2007) and Vayanos and Weill (2008) for similar approximations.

2.3 Bargaining and Terms of Trade

I model prices as arising from Nash-bargaining between buyers and sellers. The bond price, for example, is characterized as follows. The marginal benefit of buying the bond (i.e., the buyer's reservation value) is the difference between the expected utility of owning versus not owning the bond: $V_{h[1,0]} - V_{h[0,0]}$. The buyer's gains from trade is then $V_{h[1,0]} - V_{h[0,0]} - p_b$. Similarly, the seller's reservation value and her gains from trade are $V_{a[1,0]}$ and $p_b - V_{a[1,0]}$, respectively. Put together, the total gains from trade (ω_b) is the difference between the buyer and seller's reservation values:

$$\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}.$$

The bond price is such that the bond buyer extracts half of the gains from a bond trade:

$$V_{h[1,0]} - V_{h[0,0]} - p_b = \frac{1}{2}\omega_b, \tag{13}$$

while the seller extracts the other half. The bond price, as a result, is the midpoint between the buyer's and the seller's reservation values:

$$p_b = \frac{1}{2}V_{a[1,0]} + \frac{1}{2}(V_{h[1,0]} - V_{h[0,0]}). \tag{14}$$

I characterize the CDS spread (i.e., the CDS premium), p_c, analogously. In particular, it is implicitly defined such that the CDS seller captures half of the gains from a CDS trade:

$$V_{h[0,1]} - V_{h[0,0]} = \frac{1}{2}\omega_c, \tag{15}$$

where ω_c is the total gains from CDS trade:

$$\omega_c \equiv (V_{h[0,1]} - V_{h[0,0]}) + (V_{l[0,\text{-}1]} - V_{l[0,0]}).$$

The CDS buyer captures the other half. I characterize the termination fees similarly in Appendix A.

2.4 Equilibrium Definition and Existence

I analyze the steady state equilibrium. It is given by continuation values $\{V_\tau\}_{\tau\in\mathcal{T}}$, population measures $\{\mu_\tau\}_{\tau\in\mathcal{T}}$, prices $\{p_b, p_c\}$, termination fees $\{T_{\text{B}}, T_{\text{S}}\}$, and participation rates $\{\nu_h, \nu_l\}$ such that (i) the continuation values $\{V_\tau\}_{\tau\in\mathcal{T}}$ solve the agents' optimization problem (5), (ii) population masses equate the flow of agents switching into type $\tau \in \mathcal{T}$ to the flow of agents switching out of τ and solve (A10)-(A15), (iii) market clearing conditions (A16) and (A17) hold, (iv) bond and CDS prices $\{p_b, p_c\}$ arise from bargaining and solve (14) and (15), (v) participation rates $\{\nu_h, \nu_l\}$ solve (4), and (vi) termination fees $\{T_{\text{B}}, T_{\text{S}}\}$ solve (A7) and (A8). For the solution to be an equilibrium, the conjectured trading strategies have to be optimal.

Proposition 2 (Equilibrium Existence in the Environment with CDS)**.** *There exists an open set of parameter values such that a steady state equilibrium exists with an interior solution for the agents' participation rates:* $\nu_h \in (0,1)$ *and* $\nu_l \in (0,1)$.

Appendix B outlines the proof, while online Appendix E contains the full proof. The proof derives the parameter range under which Proposition 2 holds. It shows, for example, that the participation cost (O) has to be in an intermediate range for an interior participation rate of high-valuation agents, $\nu_h \in (0,1)$. If O is too low, all high-valuation investors participate in the credit market (i.e., $\nu_h = 1$). If O is too high, no high-valuation investor participates (i.e., $\nu_h = 0$). The proof of Proposition 2 also establishes a useful corollary: when Proposition 2 holds, the participation rate of high-valuation agents is also interior, $\nu_h \in (0,1)$, in the environment without CDS. This implies that the free entry condition, $V_{h[0,0]} = O$, holds in equilibrium both before and after CDS introduction.

In the remainder of the paper, I impose the implicit parameter restriction of Proposition 2 ensuring interior participation rates. As I explain in Section 3.2, this restriction is one of the key ingredients of the spillover effect. It ensures that the investors' participation rates can adjust sufficiently following the introduction of CDS. It rules out, for example, the participation cost (O) being too small, in which case the long investors' participation rate is stuck at a corner ($\nu_h = 1$) both before and after the introduction of CDS.

2.5 The Equilibrium Bond Price

Proposition 3 characterizes the bond price and establishes its three key determinants: credit risk, the participation cost, and bond market illiquidity.

Proposition 3. *The bond price is*

$$p_b = \frac{\delta}{r} - \underbrace{\frac{\eta J - x + y}{r}}_{\substack{\textit{credit risk} \\ \textit{discount}}} - \underbrace{\frac{(r+\gamma_d)O}{r}}_{\substack{\textit{participation cost} \\ \textit{discount}}} - \underbrace{\frac{d_b}{r}}_{\substack{\textit{illiquidity} \\ \textit{discount}}}, \tag{16}$$

where

$$d_b \equiv (\frac{1}{2}r + \gamma_d)\frac{(x - (r+\gamma_d)O)}{r + \gamma_d + \gamma_d \frac{1}{\left(\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}}\right)^{\frac{1}{2}}}}, \tag{17}$$

and, as defined in (8) and (9), $x = r\alpha\rho\sigma_e\eta J$ *and* $y = \frac{r\alpha}{2}\eta J^2$.

The first three terms of (16) capture the bond price in the absence of search frictions. The term $\frac{\delta}{r}$ is the present value of the bond coupon flow. The first discount arises from credit risk. It is the present value of, as explained on page 17, the high-valuation investors' private cost of bearing credit risk. The second discount arises from the participation cost. It shows that investors require a higher return as a compensation for the participation cost. These two discounts reflect costs from the perspective of high-valuation investors because in the absence of search frictions high-valuation investors upon a valuation shock sell their bond immediately to another high-valuation investor. Only high-valuation investors, as a result, own and price the bond in the absence of frictions.

The last discount, which I refer to as the illiquidity discount, arises from search frictions. Search frictions, by precluding bond sellers from instantaneously finding and trading with a bond buyer, generate a positive mass of bond sellers ($\mu_{a[1,0]}$). Bond sellers are average-valuation investors, who have a higher private cost of bearing credit risk than high-valuation investors. A positive mass of bond sellers, as a result, indicates a misallocation of bonds—that investors with a high private cost of bearing credit risk hold and price the bond. The illiquidity discount captures the price effect of this misallocation. It increases, first, with the difference between high- and average-valuation investors' private cost of bearing credit risk (x) net of the participation cost: $x - (r + \gamma_d)O$.[21] Second, it increases with the extent of the misallocation, captured by the ratio between the mass of bond sellers and the mass of high-valuation bond owners: ($\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}}$). The extent of the misallocation, in turn, increases with the expected time a bond seller takes to find a bond buyer ($\frac{1}{\lambda \mu_{h[0,0]}}$):

$$\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}} = \frac{1}{\lambda \mu_{h[0,0]}} \gamma_d, \tag{18}$$

where $\lambda \mu_{h[0,0]}$ is the intensity with which a bond seller finds a bond buyer. Thus, the worse the search frictions are, the larger the bond illiquidity discount is. The environments with and without the CDS market have the same functional form for the bond price as (16) but differ by the extent of the bond misallocation.[22]

After I present the main result in the next section, I revisit (16) in Section 5.2 and discuss its asset pricing implications. I also characterize and discuss the CDS spread.

3 The Main Result: The Liquidity Spillover Effect

In Section 3.1, I present the main result of the paper and its intuition. In Section 3.2, I explain the four key ingredients that generate the spillover effect: endogenous participation, the implicit parameter restriction of Proposition 2 ensuring interior participation rates, search frictions, and non-directed search.

[21] The parameter conditions of Proposition 2 ensure that $x - (r + \gamma_d)O > 0$ and hence $d_b \geq 0$.

[22] The bond misallocation does not have a closed form solution. But I explain in the next section how the introduction of CDS affects it.

3.1 The Main Result and The Intuition

Proposition 4 presents the main result of the paper: the introduction of CDS contracts and the resulting ability to short through naked CDS purchases increases the liquidity and price of the underlying bonds.

Proposition 4 (The Liquidity Spillover Effect)**.** *The introduction of CDS contracts:*

(i) increases the participation rate of high-valuation investors (ν_h) and the mass of bond buyers ($\mu_{h[0,0]}$),

(ii) decreases the mass of bond sellers ($\mu_{a[1,0]}$) and the bond illiquidity discount (d_b), and

(iii) increases the volume of bond trades (M_b) and the bond price (p_b).

I explain the intuition in three parts, focusing on the key channels. First, the introduction of naked CDS buyers increases the long investors' expected profit of participating in the credit market. To see this, suppose for simplicity that the parameter conditions are such that the total gains from bond and CDS trades are similar: $\omega_b = \omega_c$. Then, defining $\omega \equiv \omega_b = \omega_c$ and substituting (13) and (15) into (12), the flow value of the long investor's continuation value can be expressed as

$$(r + \gamma_d)V_{h[0,0]} = (\lambda\mu_{a[1,0]} + \lambda\mu_{l[0,0]})\frac{1}{2}\omega, \tag{19}$$

where $\mu_{a[1,0]}$ is the mass of bond sellers, $\mu_{l[0,0]}$ is the mass of naked CDS buyers, and $\frac{1}{2}\omega$ is the long investors' gains from trade upon a match with either a bond seller or a naked CDS buyer. Equation (19) shows that the long investors' expected profit upon entry depends on two components: (1) the probability of a match ($\lambda\mu_{a[1,0]} + \lambda\mu_{l[0,0]}$) and (2) the profit per match ($\frac{1}{2}\omega$).[23] The equivalent condition in the environment without CDS is

$$(r + \gamma_d)\hat{V}_{h[0,0]} = \lambda\hat{\mu}_{a[1,0]}\frac{1}{2}\hat{\omega}, \tag{20}$$

[23] In particular, $\lambda\mu_{a[1,0]} + \lambda\mu_{l[0,0]}$ is the intensity of a match arrival, while $(\lambda\mu_{a[1,0]} + \lambda\mu_{l[0,0]})dt$ is the probability of a match arrival in a small time interval $[t, t + dt]$.

where the possible matches are just bond sellers and the variables in hats denote their values in the environment without CDS. Comparing (19) and (20), the introduction of naked CDS buyers generates additional trading opportunities for long investors, increases their probability of a match, and thereby increases the long investors' expected profit of participating in the credit market.

Long investors respond to the increased trading opportunities as follows. They increase their participation rate and as they do so, not only their aggregate mass ($\frac{\nu_h F_h}{\gamma_d}$) but also the mass actively searching for a counterparty ($\mu_{h[0,0]}$) increases. As $\mu_{h[0,0]}$ increases, the competition between long investors increases, reducing the profit each can extract from their match ($\frac{1}{2}\omega$). Long investors continue to increase their participation until the decrease in their per match profit cancels the benefit of the higher probability of a match. In particular, since the participation rate of long investors in equilibrium is interior ($\nu_h \in (0,1)$), their participation expands until their expected profit equals the participation cost again: $V_{h[0,0]} = O$. The same free entry condition holds in the environment without CDS: $\hat{V}_{h[0,0]} = O$. Combining then the free entry conditions with (19)-(20), we can compare the long investors' expected profits across the environments with and without CDS:

$$\lambda \hat{\mu}_{a[1,0]} \frac{1}{2} \hat{\omega} = (\lambda \mu_{a[1,0]} + \lambda \mu_{l[0,0]}) \frac{1}{2} \omega. \tag{21}$$

Eq. (21) shows that, in equilibrium, long investors have a higher probability of a match but lower profit per match relative to the environment without CDS: $\lambda \mu_{a[1,0]} + \lambda \mu_{l[0,0]} > \lambda \hat{\mu}_{a[1,0]}$ and $\frac{1}{2}\omega < \frac{1}{2}\hat{\omega}$. The key force that generates these results is the increase in the mass of long investors ($\mu_{h[0,0]}$). Thus, the introduction of naked CDS buyers expands not only the mass of investors on the short side of the market (those seeking to either sell bonds or buy CDS, $\mu_{a[1,0]} + \mu_{l[0,0]}$) but also the mass of investors willing to take the other side and long credit risk ($\mu_{h[0,0]}$).

Parts (ii) and (iii) of Proposition 4 follow from the increase in the mass of long investors ($\mu_{h[0,0]}$). Since long investors trade as bond buyers, an increase in their mass implies that bond sellers find a buyer more quickly. As bond sellers sell faster, using (18), their mass and thereby the bond misallocation decreases. In particular, average-valuation investors hold a smaller fraction of the bond supply, leaving high-valuation investors with a larger fraction. The result is an increase in the volume of trade, a decrease in the illiquidity discount, and thereby an increase in the bond price.

3.2 Key Ingredients

The liquidity spillover effect relies on four key ingredients. The first is endogenous participation of investors and, in particular, that of long investors. Long investors are the counterparty to both bond sellers and naked CDS buyers. If their participation rate and hence their aggregate mass were fixed, the introduction of naked CDS buyers would just crowd out bond sellers and thereby exacerbate their search costs. Thus, introducing CDS but keeping the participation rate of long investors fixed reverses the spillover effect. Bond market liquidity deteriorates. If we interpret the results with fixed versus endogenous participation as partial versus general equilibrium effects of CDS, my results show that existing models focus on the partial equilibrium effects of derivatives when their general equilibrium effects can the opposite and more important.

The second ingredient is the parameter conditions of Proposition 2 that ensured an interior solution for the participation rate of long investors: $\nu_h \in (0, 1)$.[24] For the liquidity spillover effect to arise, the participation rate of long investors (ν_h) has to increase sufficiently in response to the CDS demand created by naked CDS buyers. If their participation cannot increase sufficiently (for example, if ν_h hits the corner value $\nu_h = 1$ too quickly), then the crowding out effect can dominate the spillover effect. For example, if long investors' participation is already 100% (i.e., $\nu_h = 1$) in the absence of CDS, then long investors' participation cannot increase further in response to CDS introduction. In this case, the introduction of CDS contracts (and hence naked CDS buyers) crowds out bond sellers and reduces the liquidity and price of bonds. Ensuring that the long investors' participation rate remains interior upon CDS introduction guarantees a sufficient increase in long investors' participation and thereby the spillover effect. In turn, for long investors' participation to remain interior, the participation cost (O) has to be

[24]This condition simplifies the proof of the spillover effect. It is a sufficient but not a necessary condition for the spillover effect (see Appendix B for a further discussion).

in an intermediate range: neither too large nor too small. If the participation cost (O) is zero, for example, then the introduction of CDS always crowds out bond trading.

The third ingredient for the spillover effect is search frictions. In a frictionless environment (i.e, $\lambda \to \infty$), CDS attracts additional long investors as before. But the increase in the aggregate mass of long investors does not affect bond market liquidity. The illiquidity discount is already zero, and the bond volume is the maximum possible. CDS contracts, as a result, are redundant. Thus, the broader message of the paper is that, in the presence of trading frictions, the introduction of securities that complete markets complements existing assets. In the absence of frictions, they are redundant. Similar results should arise with other frictions. Goldstein, Li, and Yang (2013) and Goldstein and Yang (2015), for example, highlight a similar complementarity theme in the context of multiple markets and multiple dimensions of information, respectively, using asymmetric information environments.

The last ingredient is the ability to search for both bond and CDS counterparties at the same time. Recall that the ability to also search for short investors increased the probability of trade and the bargaining position of long investors. Removing this ability (and thereby segmenting bond and CDS markets) cancels these effects and, with them, the reasons that long investors increased their participation rate in the first place. The spillover effect, as a result, does not arise.

The assumption that market participants incur the participation cost (O) and then search in both markets at the same time captures the idea that market participants use the same costly resources towards trading related assets. Suppose, for example, that the participation cost captures the cost of either information or trading such as the cost of subscription to a data service or an access to a communication or a trading platform. In reality, once market participants have paid for these services, they use them for both spot and derivative trading.[25] Similarly, if the cost reflects the cost of hiring traders, traders in reality have expertise in and trade in both spot and derivative instruments (Serdarevic

[25] An example is a subscription to the Bloomberg Terminal, the leading data and technology provider to financial institutions. It gives access to both bond and CDS data (Johnson 2013 and Johnson 2017). Its Instant Messaging service is the main communication platform for counterparties in OTC markets to contact each other and to negotiate both derivative and spot trades.

2010, DeChesare 2018, Acworth and Morrison 2017).[26] A trader, for example, may look for trading opportunities in the CDS market while waiting to hear back from bond counterparties.

4 Robustness Results

In this section, I provide additional robustness results. In Section 4.1, I allow covered CDS positions and show that, unlike naked CDS positions, they are price redundant. Then, I show that introducing naked CDS positions to a benchmark environment that allows covered CDS positions has the same effect as in Section 3. In Section 4.2, I relax the assumption that investors cannot short-sell and show that although naked CDS positions decrease the illiquidity discount and improve the volume of bond trades as in Section 3, their effect on the bond price as a whole can be ambiguous.

4.1 Covered CDS Positions

A covered CDS position is a position in which a bond holder buys CDS as a hedge on her bond ($[\theta_b, \theta_c] = [1, \text{-}1]$). Since investors with this position are both long and short the underlying credit risk, they also proxy arbitrageurs or CDS-bond basis traders. In Section 1, I had ruled out this position and assumed that the only way for investors to lower their credit exposure was to sell their bond. I now allow the position. As with naked CDS buyers, I assume that when a bond holder buys CDS, she splits the gains from trade equally with the CDS seller and denote with p_c^{cov} the CDS spread (i.e., the CDS premium) she pays. The full model is in online Appendix G.

Proposition 5 isolates the effect of covered CDS positions by shutting down naked CDS purchases and comparing the environments with and without covered CDS positions. It shows that covered CDS positions are price redundant. They do not affect neither the

[26] Financial institutions typically organize their trading desks (as defined in BIS (2016)) by major asset classes: equity, rates, credit, securitized products, municipals, currencies, and commodities. Then desks within each asset class trade both the cash and derivative instruments in that category (SIFMA 2018a, Cheung 2019). For example, credit trading desks as a group trade both bonds and related derivatives such as CDSs.

bond price nor the illiquidity discount. Their only effect is to reduce the volume of bond trade.

Proposition 5 (The Effect of Covered CDS)**.** *Introducing covered CDS positions to a benchmark environment with only bond trading does not affect the bond illiquidity discount* (d_b) *nor the bond price* (p_b) *but reduces the bond trading volume* (M_b)*.*

The intuition is as follows. When covered CDS positions are feasible, in equilibrium some of the bond sellers ($a[1,0]$)—instead of selling their bonds—hold on to their bond, buy CDS from long investors, and thereby become covered CDS buyers.[27] Conversely, some of the long investors instead of buying bonds from bond sellers sell them CDS. Long investors, moreover, are indifferent between selling CDS versus buying bonds from bond sellers. Bond sellers are also indifferent. This is because the gains from trade between any two investors arises from the difference between their private costs of bearing credit risk and is the same whether they trade the bond or CDS. Put together, unlike naked CDS positions, the feasibility of covered CDS positions does not affect neither the set of counterparties that long investors trade with nor the profits they derive from those counterparties. Covered CDS positions, as a result, do not affect the long investors' ex-ante participation incentives and hence their aggregate mass. In turn, the extent of search frictions bond sellers face, the bond illiquidity discount, and thereby the bond price remain the same. Thus, covered CDS positions are price redundant. They only reduce the bond trading volume because now only a portion of the matches between bond sellers and long investors result in a bond trade.[28]

I now reintroduce naked CDS positions and consider an environment with both covered and naked CDS positions. I present the full environment in online Appendix G.2. Proposition 6(i) isolates the marginal effect of naked CDS positions relative to a benchmark environment that allows covered CDS positions. It shows that allowing covered

[27] This implication is consistent with Massa and Zhang (2012), who document that CDS alleviates bond selling pressures.

[28] Since long investors and bond sellers, in equilibrium, are indifferent between trading the bond versus CDS, my model does not uniquely pin down the volume of CDS trades between them. I thus assume that a fixed $\pi \geq 0$ fraction of the matches between them ($\lambda \mu_{a[1,0]} \mu_{h[0,0]}$) result in a CDS trade and the rest in a bond trade.

CDS positions just changes the benchmark environment and that relative to this benchmark the marginal effect of naked CDS positions remains the same as in Section 3. Naked CDS positions increase bond market liquidity and thereby the bond price. The intuition is the same.

Proposition 6.

(i) *Introducing naked CDS positions to a benchmark environment that allows covered CDS positions decreases the bond illiquidity discount (d_b) and increases the bond trading volume (M_b) and the bond price (p_b).*

(ii) *Introducing both covered and naked CDS positions to a benchmark environment with just bond trading decreases the bond illiquidity discount (d_b), increases the bond price (p_b), but has an ambiguous effect on the bond trading volume (M_b).*

Proposition 6(ii) shows the combined effect of covered and naked CDS purchases. Since covered CDS purchases do not affect the illiquidity discount nor the bond price, the net effect of both positions on the bond price inherits the effect of naked CDS positions. They increase the bond price by decreasing the illiquidity discount. Their net effect on bond volume, however, is ambiguous and depends on the relative magnitudes of covered versus naked CDS positions. Empirically, for some markets, the amount of naked CDS purchases is at least twice as large as the amount of covered CDS purchases.[29] For such markets, the positive volume effect of naked CDS buyers likely outweighs the negative volume effect of covered CDS buyers.

In the above analysis, for simplicity I abstract from positive CDS-bond basis trades in which investors short bonds and at the same sell CDS ($[\theta_b, \theta_c]$ = [-1,1]). This is a

[29]While estimates of the magnitude of covered versus naked CDS purchases do not exist, for some markets we can put bounds on their relative magnitudes. Between 2008-2011, the total U.S. corporate debt outstanding averaged \$6.3 trillion, while the total notional amount of CDSs purchased referencing that debt averaged \$12.4 trillions over the same period (SIFMA 2018b, DTCC 2014). The total size of corporate debt (\$6.3 trillion) serves as an initial upper bound on the total amount of covered CDS purchases. Moreover, insurance companies, which are the largest holders of U.S. corporate debt, held about \$2.29 trillion of corporate debt on average between 2008-2011 but purchased only about \$0.02 trillion of CDS as of 2010 (NAIC 2011 and The Federal Reserve 2012). That is, at minimum \$2.27 trillion of corporate debt was not hedged with CDS. This further refines the upper bound on the amount of covered CDS purchases to be at most \$4.03 trillion. The remaining \$8.37 trillion of CDS purchases out of the \$12.4 trillion total serves as a lower bound on the amount of naked CDS purchases.

reasonable assumption. Hendershott, Kozhan, and Raman (2017) document that 1.8% of corporate bonds are sold short.[30] This serves as an upper bound on the number of positive basis positions. In contrast, the total amount of CDS purchased (both covered and naked) as a fraction of the corporate debt outstanding is 197%.[31] The much smaller upper bound on the number of positive basis trades suggests that any potential effects of this position will be insignificant relative to the effect of either covered or naked CDS positions. Consistent with these magnitudes, the credit derivative research group at J.P. Morgan—one of the five largest CDS market participants—writes that negative basis trades are more much common than positive basis trades (Elizalde, Doctor, and Saltuk 2009).

4.2 A Benchmark with Short-Selling and Naked CDS Effect

The results so far assume that investors cannot directly short-sell bonds. This is a realistic assumption as less than 2% of corporate bonds, for example, are sold short (Hendershott, Kozhan, and Raman 2017). It is nevertheless useful to analyze the effects of naked CDS positions if investors can already short-sell. So in this section, I relax the assumption that investors cannot short-sell and compare bond market liquidity and the bond price between two environments: (1) a benchmark environment in which short-selling is feasible, but CDS positions are not and (2) an environment in which both short-selling and naked CDS purchases are feasible. Online Appendix G.4 presents the full model. The results of this section are numerical and are illustrated in Figure 2 in online Appendix G.4.[32]

The short-selling part of the model follows Vayanos and Weill (2008), which to date is the most realistic model of short-selling. It works as follows. After purchasing the bond, long investors now lend their bond in a repo (i.e., a security lending) market and, as a result, earn a lending fee. On the other side of the repo transaction, short investors ($l[0,0]$)—in addition to searching for a CDS seller—search for a bond lender. After finding

[30] Nashikkar and Pedersen (2007) and Foley-Fisher, Narajabad, and Verani (2016) provide a similar evidence.

[31] See the numbers in Footnote 29.

[32] The model with both CDS and short-selling is complicated and involves solving, at minimum, a system of 23 equations and variables (10 value functions, 9 population masses, 2 participation rates, the CDS spread, and the lending fee). Thus, analytically showing any results is intractable.

one, they borrow the bond and short-sell it in the spot market. Parties meet in the repo market through search and, upon a match, negotiate over the lending fee. I denote with λ_r the exogenous search intensity in the repo market. An investor unwinds the short sale by first searching for and buying the bond in the spot market and then delivering it back to the bond lender. To unwind a bond loan, if her counterparty has not yet (short-) sold the bond, the lender recalls the bond, sells it, and exits. If the counterparty has already sold the bond, the lender walks away with the collateral that the short seller puts aside.

To disentangle the various channels, in this subsection I allow the search intensity, λ, to differ for bond versus CDS matches. Let λ_b and λ_c denote the search intensities governing the volume of bond and CDS transactions, respectively. Section 1 environment is a special case, where $\lambda_b = \lambda_c = \lambda$. The volume of bond and CDS transactions are then $M_b = \lambda_b \mu_{b,\mathrm{B}} \mu_{b,\mathrm{S}}$ and $M_c = \lambda_c \mu_{c,\mathrm{B}} \mu_{c,\mathrm{S}}$, where $\mu_{b,\mathrm{B}}$ and $\mu_{b,\mathrm{S}}$ are the masses of bond buyers and sellers, and $\mu_{c,\mathrm{B}}$ and $\mu_{c,\mathrm{S}}$ are the masses of CDS buyers and sellers.

Given this setup, short-selling arises in equilibrium under the following conditions. In the environment without CDS, it arises only if the bond supply is sufficiently large. This is because, first, short-selling requires trading the underlying bond both for long and short investors and is thus limited by the supply of the bond. Second, short-selling involves multiple stages of search that further reduce the gains from short-selling. Short (or low-valuation) investors, as a result, do not participate in the credit market unless the bond supply and the resulting short-selling opportunities are sufficiently large. In the environment with CDS, however, short-selling arises regardless of the bond supply. The ability to synthetically short through CDS contracts (which do not require trading the underlying bond) attracts short investors into the credit market. Once they choose to participate, they simultaneously search for physical short-selling opportunities. Put together, when the bond is in limited supply, introducing CDS simultaneously introduces short-selling. When the bond supply is large, short-selling arises in equilibrium both and after CDS introduction.

Proposition 7. *The bond price in the presence of short sales is given by:*

$$p_b = \frac{\delta}{r} - \underbrace{\frac{\eta J - x + y}{r}}_{\substack{\text{credit risk}\\ \text{discount}}} - \underbrace{\frac{(r+\gamma_d)O}{r}}_{\substack{\text{participation}\\ \text{cost discount}}} - \underbrace{\frac{d_b}{r}}_{\text{illiquidity discount}} + \underbrace{\frac{(r+\lambda_b\mu_{b,\mathrm{B}})}{r}\frac{1}{2}\frac{\lambda_r\mu_{l[0,0]}\frac{1}{2}\omega_r}{r+\gamma_d+\lambda_b\mu_{b,\mathrm{B}}\frac{1}{2}}}_{\text{lending fee premium}}, \quad (22)$$

where d_b is given by (G88) and ω_r, defined by (G82), is the total gains from a repo transaction.

Proposition 7 shows that when investors short-sell in equilibrium, an additional determinant affects the bond price. It is the lending fee that the bond generates. Bond owners now earn an additional cash flow by lending their bond to short-sellers. The additional cash flow, captured by the last term in (22), increases the bond price.

The introduction of naked CDS positions to a benchmark environment in which short-selling is feasible affects the bond price through two channels. First, it creates the liquidity spillover effect. It lowers the illiquidity discount (which, in turn, drives the bond price up) and increases the bond trading volume. The intuition is analogous to Section 3. Trading CDS allows high- and low-valuation investors to bypass intermediate search processes that short-selling involves (for example, long investors first search for a bond to buy and then search for low-valuation investors to lend the bond to). This ability to directly enter CDS contracts with naked CDS buyers attracts a greater number of long investors into the credit market, who, in turn, search and trade at the same time with bond sellers and thereby create the spillover effect.

Second, the introduction of naked CDS buyers affects the lending fee component of the bond price. The direction of the change depends on parameter values. When the bond is in limited supply, CDS introduction, by simultaneously generating short-selling, creates the lending fee component. Together with the first effect, CDS introduction unambiguously increases the bond price. For other parameter values (for example, if the bond supply and the CDS market matching efficiency, λ_c, are very large), the ability to buy CDS lowers short investors' willingness to borrow the bond. The lending fee component, as a result, decreases, putting a downward pressure on the bond price. Since naked CDS purchases have the opposite effect on the bond price through the illiquidity component (the first channel), the overall price effect of naked CDS buyers is ambiguous. Thus, for some parameter values, the previous section's result that naked CDS buyers increase the bond price can reverse. Online Appendix G.4 elaborates on the results of this section.

5 Testable Model Predictions and Empirical Evidence

In Section 5.1, I discuss the empirical evidence consistent with the effects of naked and covered CDS positions. In Sections 5.2-5.4, I discuss the testable predictions of my model on the bond price, CDS spreads, and the CDS-bond basis, respectively, and connect them to the empirical evidence.

5.1 Evidence of the Effects of Naked and Covered CDS Buyers

My model helps explain how sovereign bond markets reacted to naked CDS bans. In October 2011, the European Union voted to ban naked CDS purchases—buying CDS without owning the underlying bonds—against EU government bonds.[33] Using a difference-in-difference analysis, Sambalaibat (2019) documents that following the European Union naked CDS ban, liquidity of sovereign bonds affected by the ban deteriorated. Consistent with this evidence, shutting down naked CDS purchases in my model reverses the liquidity spillover effect. Long investors can no longer sell CDS because their counterparties (naked CDS buyers) are banned from buying CDS. Long investors, as a result, scale back their overall credit market participation and, in doing so, pull out from the bond market. The result is a decrease in bond market liquidity. Thus, preventing investors from shorting ultimately drives away investors who want to take the opposite side and long the underlying asset.

My model also helps reconcile conflicting empirical findings on the effects of CDS. On the one hand, Ashcraft and Santos (2009) and Das, Kalimipalli, and Nayak (2014)

[33]It did so by allowing investors to buy CDS only if they held the underlying bonds. It thus prevented investors from purchasing CDS either to speculate or to hedge positions correlated with the sovereign. In the model, consistent with the actual ban, both would be considered a naked CDS purchase because the naked CDS buyer in the model does not hold the underlying bonds.

document that CDSs are redundant: They have no effect on bond yields and price-based measures of bond illiquidity (e.g., the Amihud illiquidity measure). Moreover, in Das, Kalimipalli, and Nayak (2014), CDSs are associated with a decrease in the bond turnover. On the other hand, a second body of work documents that CDS trading is associated with higher bond prices, narrower bid-ask spreads, and larger volumes of bond trades.[34] My results suggest that the first set of results arises from bond holders purchasing CDS as a hedge (i.e. covered CDS buyers). The common interpretation of the second set of results is that when investors can buy CDS as a hedge on their bonds, they are more willing to buy bonds. My model predicts that the second set of results instead arises from naked CDS buyers and the long investors they attract into both the CDS *and* the bond market, not from bond holders buying CDS as a hedge. Recent micro-evidence on the nature of CDS buyers and sellers supports this interpretation. Acharya, Gunduz, and Johnson (2018) and Czech (2019) document that financial institutions with large portfolios of bonds and loans primarily sell CDS and thereby use CDS to expand their credit risk exposure, not to hedge and decrease their exposure. This finding is consistent with my model implication that if most of the CDS purchases are naked, investors that buy bonds, instead of buying CDS, should primarily sell CDS.

One reason we may see evidence consistent with both covered and naked CDS purchases is that the CDS market has evolved over time. The first body of work analyzes the time series impact of CDS, comparing the same bonds right before and after CDSs first start trading on the bonds. The second body of evidence instead compares in the cross-section bonds with versus without CDS contracts. The before-and-after effect likely captures the CDS's effect in its initial stages of trading, while the cross-sectional analysis more likely captures CDS's effect when they are more mature and widely traded derivative. These results together with my model suggest that in the earlier years investors used CDS primarily for hedging the underlying bonds and loans; then as the CDS mar-

[34] Ismailescu and Phillips (2015), for example, document that CDS trading is associated with lower bond yields. Massa and Zhang (2012) and Shim and Zhu (2014) document that the presence of the CDS market is associated with lower bond yields and narrower bid-ask spreads. Nashikkar, Subrahmanyam, and Mahanti (2011) find that CDS trading is associated with not only lower bond yields but also a higher bond turnover. Acharya, Gunduz, and Johnson (2018) and Czech (2019) document that CDS trading is associated with greater bond trading activity.

ket developed, investors expanded their CDS trades beyond simple hedging to hedging correlated risks and to speculate. For evidence of such evolution of the CDS market, see Zabel (2008) and Tett (2010).

5.2 Bond Pricing Implications

In this section, I discuss the bond pricing implications of my model. In the environment with both covered and naked CDS trading, the bond price has the same functional form as (16), which I repeat here:

$$p_b = \frac{\delta}{r} - \underbrace{\frac{\eta J - x + y}{r}}_{\substack{\text{credit risk}\\ \text{discount}}} - \underbrace{\frac{(r+\gamma_d)O}{r}}_{\substack{\text{participation cost}\\ \text{discount}}} - \underbrace{\frac{d_b}{r}}_{\substack{\text{illiquidity}\\ \text{discount}}} . \tag{23}$$

The illiquidity discount, however, changes to

$$d_b = \left(\frac{1}{2}r + \gamma_d\right) \frac{(x - (r+\gamma_d)O)}{r + \gamma_d + \gamma_d \frac{1}{\left(\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}+\mu_{a[1,\text{-}1]}}\right)^{\frac{1}{2}}}}, \tag{24}$$

where now the efficient holders of the bond are not only high-valuation investors ($\mu_{h[1,0]}$) but also average-valuation bond owners who have bought CDS ($a[1,\text{-}1]$). As before, the extent of the bond misallocation is a function of the expected time that an average-valuation bond holder ($a[1,0]$) takes to find a long investor: $\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}+\mu_{a[1,\text{-}1]}} = \frac{1}{\lambda \mu_{h[0,0]}}\gamma_d$. Using (23), the expected return of the bond (or, equivalently, the bond yield) is:

$$\frac{\delta}{p_b} = r + \underbrace{\frac{\eta J - x + y}{p_b}}_{\substack{\text{risk premium due to}\\ \text{credit risk}}} + \underbrace{\frac{(r+\gamma_d)O}{p_b}}_{\substack{\text{risk premium due to}\\ \text{participation cost}}} + \underbrace{\frac{d_b}{p_b}}_{\substack{\text{risk premium due to}\\ \text{bond market illiquidity}}} . \tag{25}$$

The bond pricing equations (23) and (25) capture a broad set of empirical regularities. First, in the data, both illiquid and low credit quality bonds trade at a discount. For example, Bao, Pan, and Wang (2011), Chen, Lesmond, and Wei (2007), Dick-Nielsen, Lando, and Feldhütter (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012), among others, document that the illiquidity of corporate bonds is priced and significant, while the entire credit risk pricing literature quantifies the importance of default risk in explaining credit spreads (see Huang and Huang (2012) and references therein). These results are consistent with the credit risk and illiquidity discounts in (23) and their associated risk premia in (25). Second, a large literature documents that funding conditions of market participants matter for asset prices.[35] In particular, tighter funding conditions are associated with higher risk premia. In Appendix C.2, I illustrate with a simple model that the participation cost, O, can in part be interpreted as the investors' cost of capital.[36] Then, consistent with this literature, the third term in (25) implies that some of the bond risk premia arises from the investors' cost of capital.

The bond pricing equations also shed light on the credit spread puzzle. The credit spread puzzle is a well documented observation that a sizable portion of corporate bond yields cannot be explained by default risk (see Huang and Huang (2012) and references therein). My model shows that search frictions and participation costs (in particular, their interpretation as the investors' cost of capital) generate risk premia beyond the credit risk premium and thereby explain the non-default risk component.

5.3 CDS Pricing Implications

I now characterize the CDS spread (i.e., the CDS premium or price) and its determinants. I do so in an environment in which both covered and naked CDS positions are feasible and arise in equilibrium. I denote by p_c^{nak} and p_c^{cov} the CDS spreads that naked and covered CDS buyers trade at.[37]

[35]See, for example, Fontaine and Garcia (2012), Mitchell and Pulvino (2012), Wang et al. (2016), Siriwardane (2019), and He, Kelly, and Manela (2017).

[36]The idea is as follows. Suppose that investors maximize their expected profit across multiple asset classes, one of which consists of credit market instruments. Suppose also that they face a constraint in the spirit of Basel III capital requirements that the capital charges aggregated across all their positions cannot exceed 12.5 times their total equity capital (the multiplier 12.5 is set by the regulation). Then, their participation decision (4) resembles their first-order condition, and O captures the total capital cost of a credit market position: the shadow cost of investors' equity capital times the capital charge of a credit market position. As the investors' shadow cost of capital increases, so does O. In (25), the scaling by $(r + \gamma_d)$ converts the total cost to a per-period flow cost, while the normalization by p_b converts it into return units.

[37]In Sections 1-2, I denoted the CDS spread that naked CDS buyers pay by p_c. I now superscript it with *nak* to distinguish it from the spread paid by covered CDS buyers.

Proposition 8. *The CDS spreads that covered and naked CDS buyers pay are*

$$p_c^{cov} = (\eta J - x + y) + (r + \gamma_d)O + d_c^{cov}, \tag{26}$$

$$p_c^{nak} = (\eta J - x + y) + (r + \gamma_d)O + d_c^{nak}, \tag{27}$$

respectively, where d_c^{cov} *and* d_c^{nak} *are the CDS illiquidity premia:*

$$d_c^{cov} \equiv \left(\frac{1}{2}r + \gamma_d\right)\left[\frac{x - (r+\gamma_d)O}{r + \gamma_d + \gamma_d \frac{1}{\left(\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}+\mu_{a[1,-1]}}\right)^{\frac{1}{2}}}}\right], \tag{28}$$

$$d_c^{nak} \equiv \left(\frac{1}{2}r + \gamma_d\right)\left[\frac{2x - 2y - (r+\gamma_d)O}{r + \gamma_d + \gamma_u + (\gamma_d + \gamma_u) \frac{1}{\left(\frac{\mu_{l[0,0]}}{\mu_{l[0,-1]}}\right)^{\frac{1}{2}}}}\right], \tag{29}$$

and, as defined in (8) and (9), $x = r\alpha\rho\sigma_e\eta J$ *and* $y = \frac{r\alpha}{2}\eta J^2$*. The volume weighted average CDS spread,* $\overline{p}_c \equiv \frac{M_c^{cov}p_c^{cov} + M_c^{nak}p_c^{nak}}{M_c^{cov}+M_c^{nak}}$*, as a result is*

$$\overline{p}_c = \underbrace{\eta J - x + y}_{\substack{\text{premium due to} \\ \text{credit risk}}} + \underbrace{(r+\gamma_d)O}_{\substack{\text{premium due to} \\ \text{participation cost}}} + \underbrace{\overline{d}_c}_{\substack{\text{premium due to} \\ \text{CDS market} \\ \text{illiquidity}}}, \tag{30}$$

where

$$\overline{d}_c \equiv \frac{M_c^{cov}}{M_c^{cov} + M_c^{nak}} d_c^{cov} + \left(1 - \frac{M_c^{cov}}{M_c^{cov} + M_c^{nak}}\right) d_c^{nak} \tag{31}$$

is the volume weighted average illiquidity premium, and $M_c^{cov} \equiv \pi\lambda\mu_{a[1,0]}\mu_{h[0,0]}$ *and* $M_c^{nak} \equiv \lambda\mu_{l[0,0]}\mu_{h[0,0]}$ *are the volumes of covered and naked CDS purchases.*

The CDS pricing equation (30) establishes three key determinants of CDS spreads. The first is credit risk and is captured by $\eta J - x + y$ in (30). It is the long investors' private cost of bearing credit risk. The second determinant is the participation cost and is captured by $(r + \gamma_d)O$. Similar to the bond pricing equation, the first two terms reflect long investors' costs and together capture the CDS spread in the absence of search frictions. This is because long investors—who have the lowest private cost of bearing credit risk—are the only investors that bear credit risk and price CDS in the absence of search frictions. The third determinant of CDS spreads is CDS market illiquidity. It is captured by the third term in (30) that arises from search frictions. While low- and average-valuation investors search for a counterparty, they bear credit risk and thereby price CDS. Their high private costs of bearing credit risk increase CDS spreads. In Appendix C.3, I explain the difference between the expressions for d_c^{nak} and d_c^{cov} as well as the difference between d_c^{nak} and d_b.[38]

The CDS pricing equation (30) complements a large body of empirical work. First, the most documented determinant of CDS spreads is the default probability of the reference entity (Cossin et al. 2002 and Berndt et al. 2007). The premium due to credit risk captures this finding. Second, Siriwardane (2019), Wang et al. (2016), and Junge and Trolle (2015) document that CDS spreads reflect capital costs of CDS market participants. Interpreting the participation cost as the investors' cost of capital as discussed above, my model implies the same pattern—that a portion of the CDS spread is driven by the investors' cost of capital. Third, in contrast to the early CDS literature that treated CDS spreads as a pure measure of credit risk, a separate literature has emerged documenting that CDS market illiquidity explains an economically significant portion of CDS spreads.[39] The illiquidity premium captures the main result of this work.

The CDS pricing equations also address limitations of existing reduced-form CDS pricing models. First, existing work places exogenous restrictions on how CDS spreads depend on the fundamentals such as credit risk. Their quantitative estimates, as a result, depend on the imposed restrictions. My model instead provides a micro-founded model of how CDS spreads *should* depend on the underlying frictions (credit risk, the participation cost, and search frictions). Second, existing approaches, by ignoring the key determinants of CDS spreads, exaggerate the quantitative importance of the determinants that they do consider. A large body of work, for example, tries to quantify how much of the CDS

[38] The parameter conditions of Proposition 2 ensure that $x-(r+\gamma_d)O>0$ and $2x+2y-(r+\gamma_d)O>0$ and hence $d_c^{nak}\geq 0$ and $d_c^{cov}\geq 0$.

[39] See, for example, Tang and Yan (2007), Bongaerts, Jong, and Driessen (2011), Qiu and Yu (2012), Bühler and Trapp (2007), Chen, Fabozzi, and Sverdlove (2010), Badaoui, Cathcart, and El-Jahel (2013), and Junge and Trolle (2015).

spreads is due to credit risk versus credit risk premium. My model shows that incorporating the participation cost and CDS market illiquidity as additional determinants of CDS spreads should yield more accurate estimates of the relative importance of each determinant.

5.4 Testable Implications on The CDS-Bond Basis

In this section, I analyze the CDS-bond basis and its determinants. I define the basis as follows. Buying both the bond ($\theta_b = 1$) and the CDS ($\theta_c = -1$) yields an investor a constant cash flow equal to the contractual coupon flow, δ, minus the average CDS spread: $\delta - \overline{p}_c$. Since the upfront one-time cost of this position is p_b, its expected return is $\frac{\delta}{p_b} - \frac{\overline{p}_c}{p_b}$. The CDS-bond basis is the difference between the risk-free rate and the return of this position: $r - \left(\frac{\delta}{p_b} - \frac{\overline{p}_c}{p_b}\right)$. Rearranging it, we get the following expression for the CDS-bond basis:

$$basis \equiv \frac{\overline{p}_c}{p_b} - \left(\frac{\delta}{p_b} - r\right). \tag{32}$$

I refer to the basis computed with the average CDS spread as the "aggregate" CDS bond-basis. Using the average CDS spread characterizes the CDS-bond basis from the perspective of an econometrician who does not observe the CDS spreads paid by naked versus covered CDS buyers and who has to instead use the CDS spread averaged across all transactions.[40]

Proposition 9. *Using (25) and (30), the CDS-bond basis is given by:*

$$basis = \frac{1}{p_b}\left(\overline{d}_c - d_b\right), \tag{33}$$

where d_b and $\overline{d}_c$ are given by (24) and (31) and capture illiquidity of bond and CDS markets, respectively.

[40] In the model, no investor in equilibrium buys both the bond and CDS and at the same pays a CDS spread equal to the average CDS spread. However, as it will become clear in the rest of this section, defining the basis using the average CDS spread (which is the approach in existing empirical work) helps reconcile the stylized facts on the CDS-bond basis. The empirical studies referenced in this section primarily use Markit and CMA as the data sources for CDS prices. These databases provide an estimate of the average CDS price across all CDS trades and counterparties.

Proposition 9 implies three testable implications. First, a non-zero CDS-bond basis arises from heterogenous bond and CDS market illiquidity: $d_b \neq \overline{d}_c$. The basis is negative if the bond market is less liquid than the CDS market and is positive otherwise. The larger the difference between their illiquidity, the larger the magnitude of the basis. These results are consistent with Bühler and Trapp (2009), Nashikkar, Subrahmanyam, and Mahanti (2011), Kucuk (2010), Arce, Mayordomo, and Peña (2013), and Bai and Collin-Dufresne (2019), who document that a deterioration in CDS market liquidity and an increase in bond market liquidity (implying an increase in $\overline{d}_c$ and a decrease in d_b, respectively) increase the basis.

Second, the absolute magnitude of the basis should be positively and negatively correlated with the proportion of naked and covered CDS purchases, respectively. To see this, using (31), (33) can be expressed as

$$basis = \frac{1}{p_b} \frac{M_c^{cov}\left(d_c^{cov} - d_b\right) + M_c^{nak}\left(d_c^{nak} - d_b\right)}{M_c^{cov} + M_c^{nak}}.$$

By their definitions (24) and (28), $d_c^{cov} = d_b$. The aggregate CDS-bond basis, as a result, depends on the volume of naked CDS purchases and the difference between the illiquidity premium that naked CDS buyers face (d_c^{nak}) and the bond illiquidity discount (d_b):

$$basis = \frac{1}{p_b} \frac{M_c^{nak}}{M_c^{cov} + M_c^{nak}} \left(d_c^{nak} - d_b\right). \tag{34}$$

From (34), the larger the volume of naked CDS purchases (as a fraction of the aggregate CDS volume), the larger the absolute magnitude of the basis. Conversely, if covered CDS purchases constitute the entire CDS market (i.e., if the volume of naked CDS purchases is zero), the magnitude of the CDS-bond basis is zero. Oehmke and Zawadowski (2016) document that the magnitude of the CDS-bond basis is larger for debt issuers with larger total CDS purchases, controlling for debt outstanding. My model suggests that these CDS purchases are primarily naked CDS purchases, not covered CDS purchases.

Third, the absolute magnitude of the basis should be correlated with the dispersion in CDS spreads across different CDS buyers. To see this, the CDS spreads (26) and (27) imply $d_c^{nak} - d_c^{cov} = p_c^{nak} - p_c^{cov}$, where $d_c^{cov} = d_b$ by their definitions (24) and (28). Put together, we get $d_c^{nak} - d_b = p_c^{nak} - p_c^{cov}$. Substituting this in (34), (34) becomes

$$basis = \frac{1}{p_b} \frac{M_c^{nak}}{M_c^{cov} + M_c^{nak}} \left(p_c^{nak} - p_c^{cov}\right). \tag{35}$$

From (35), a zero basis implies that covered and naked CDS buyers trade at the same spread ($p_c^{nak} = p_c^{cov}$) and that the dispersion in CDS spreads is zero. A negative basis implies that naked CDS buyers buy more cheaply than covered CDS buyers: $p_c^{nak} < p_c^{cov}$. A positive basis implies the opposite. A large dispersion also suggests a large time series volatility of CDS spreads.[41] Thus, the absolute magnitude of the CDS-bond basis should be correlated with both the volatility and the cross-counterparty dispersion in CDS spreads.

Next, I explore when bond and CDS market illiquidity d_b and d_c^{nak} are initially the same in (34) (so that the CDS-bond basis is zero), what causes them to diverge, creating a non-zero basis.[42]

Proposition 10 (The Comparative Statics of the CDS-Bond Basis at Zero)**.** *Suppose the parameter conditions are such that $d_c^{nak} = d_b$ and hence the CDS-bond basis (34) is zero. Suppose also that F_l and γ_u are arbitrarily small, while their ratio $\frac{F_l}{\gamma_u}$ is bounded from above.[43] Then,*

(i) The CDS-bond basis decreases in the default size (J) and increases in the correlation between the agents' endowment and the bond cash flow (ρ) and in the agents' endowment risk (σ_e): $\frac{\partial}{\partial J}(basis)\big|_{basis=0} < 0$, $\frac{\partial}{\partial \rho}(basis)\Big|_{basis=0} > 0$, and $\frac{\partial}{\partial \sigma_e}(basis)\Big|_{basis=0} > 0$.

(ii) The default intensity (η), the participation cost (O), the bond supply (S), and

[41]The intuition is as follows. Suppose in a given trading period a naked CDS buyer buys CDS at p_c^{nak}. The following period, a covered CDS buyer purchases CDS at p_c^{cov}. Then, an econometrician who does not observe the buyers' identities or the nature of the purchases observes two different CDS spreads on the same reference entity. If the difference between p_c^{nak} and p_c^{nak} is large, the econometrician observes a large volatility over the two trading periods.

[42]Due to non-linearities in the model, how the basis changes with different parameters is ambiguous and depends on the parameter values. I thus establish only local effects when the basis is zero.

[43]Assuming small γ_u keeps the analysis tractable. Numerically, the results are qualitatively similar for small versus non-small γ_u.

the matching efficiency (λ) *do not affect the basis if the basis is already zero:* $\frac{\partial}{\partial z}\left(basis\right)\big|_{basis=0} = 0$, *where* $z \in \{\eta, O, S, \lambda\}$.

Proposition 10(i) provides two explanations for a non-zero basis. The first is the loss given default (J). The aggregate CDS-bond basis decreases and turns negative as the loss given default (J) increases or, equivalently, as the bond recovery rate decreases. This result suggests that the CDS-bond basis turned negative during the 2007-2009 crisis because, as documented in Jankowitsch, Nagler, and Subrahmanyam (2014), recovery rates plummeted during the crisis. It also suggests that the basis tends to be positive for sovereign issuers because investors expect higher recovery rates from sovereign than from corporate issuers. The evidence for recovery rates as a determinant of the sign of the basis, while acknowledged by market practitioners, is anecdotal at this point.[44] Thus, future work should focus more on understanding how recovery rates affect the CDS-bond basis. The second explanation for, say, a negative CDS-bond basis is a decrease in the difference in the private values across agents x (where $x = r\alpha\rho\sigma_e\eta J$) due to a decrease in either the correlation between the agents' endowment and the bond cash flow (ρ) or the endowment risk (σ_e).[45]

As Proposition 10(ii) shows, other model parameters—the default intensity (η), the participation cost (O), the bond supply (S), and the matching efficiency (λ)—do not determine the sign of the CDS-bond basis.

The parameters η, O, S, and λ instead determine the absolute magnitude of the basis once the basis is nonzero. In online Appendix G.2, I derive sufficient conditions under which increasing the default intensity (η) and the participation cost (O) amplifies the absolute value of the basis, while increasing the bond supply (S) and the matching

[44]The credit derivatives research group at J.P. Morgan, one of the largest market participants in the CDS market, write that "Traders and investors will change their recovery rate assumptions to reflect changing market conditions, especially in distressed environments [...] where recovery rates become very important for pricing credit risk" (Elizalde, Doctor, and Saltuk 2009). They also emphasize that in computing the CDS-bond basis one should use measures of the bond yield spread that reflect changes in the recovery rate (e.g., PECS) instead of the commonly used measures that do not (e.g., ASW and Z-spreads).

[45]The difference in private values (x)—which arises in my model from heterogenous exposure to the bond cash flow—could alternatively arise from differences in beliefs. If we interpret x as a measure of disagreement, Proposition 10 implies that greater disagreement should be associated with a more liquid bond market relative to the CDS market and hence with a more positive CDS-bond basis.

efficiency (λ) reduces it. The result on the default intensity helps explain the stylized fact that both in the cross-section and in the time series high credit risk debt issuers and time periods, respectively, are associated with larger absolute values of the CDS-bond basis.[46] The amplifying effect of the participation cost suggests that the losses incurred by CDS sellers and the subsequent increase in their capital costs (which is one way to interpret the participation cost, O) further widened the basis during the 2007-2009 crisis. Indeed, a common empirical explanation for the basis widening is an increase in the investors' funding cost (Elizalde, Doctor, and Saltuk 2009, Fontana 2011, Mitchell and Pulvino 2012, Arce, Mayordomo, and Peña 2013, Wang et al. 2016, Bai and Collin-Dufresne 2019). Finally, the bond supply result is consistent with Fontana and Scheicher (2016), who document that larger debt outstanding is associated with smaller bases. In online Appendix G.3, I provide more intuition for the comparative statics results of this section.

6 Conclusion

The point I make in this paper is simple. If we want to model and understand the effect of new financial instruments and mechanisms on existing ones, the number of investors that could potentially trade and use the instruments should be endogenous.

I make this point in the context of bond and CDS markets. I build a search model of bond and CDS markets and show that introducing short positions through CDS contracts (referred to as naked CDS purchases) attracts into credit markets not only investors who want to short the underlying credit risk but also investors who want to take the opposite side and long the underlying credit risk. In turn, long investors—for whom bond and CDS positions are economically similar positions—search and trade at the same time in the bond market. They do this to expand their trading opportunities and to alleviate their search frictions. The result is an increase in the number of bond buyers, bond market

[46]Before the 2007-2009 crisis, the CDS-bond basis was more positive for high yield corporates than for investment grade corporate names; then during the 2007-2009 crisis, the basis flipped sign and was more negative for high-yield than for investment-grade corporates (Mitchell and Pulvino 2012, Elizalde, Doctor, and Saltuk 2009, Fontana 2011, Bai and Collin-Dufresne 2019, Boyarchenko et al. 2018). Similarly, among sovereign issuers, the basis is more positive for high credit risk sovereigns (Gyntelberg et al. 2017). In the time series, both for corporates and sovereigns, the absolute value of the basis is larger during the respective crisis episodes (e.g. the 2007-2009 crisis and the 2010-2012 Euro area sovereign debt crisis).

liquidity, and the bond price. I refer to this effect as a liquidity spillover effect.

This insight applies beyond derivatives to any mechanism that expands the set of feasible allocations in the economy (tradable securities and contracts, trading mechanisms and venues, private currencies, etc.).

While the point I make is simple, the implications are important. Shutting down naked CDS positions in the model reverses the spillover effect and, as a result, decreases bond market liquidity. This result suggests that by banning naked CDS positions on sovereign bonds in 2011, regulators in Europe inadvertently decreased bond market liquidity, reduced bond prices, and thereby increased sovereigns' borrowing costs when they intended to achieve the opposite and quell a sovereign debt crisis.

Appendix

A Value Functions, Terms of Trade, Population Masses

In Sections 1-2 when I had only one type of a CDS buyer, I denoted the CDS premium, the termination fees, the transaction volume, and the gains from trade by p_c, $\{T_B, T_S\}$, M_c, and ω_c, respectively. In the rest of the paper, I superscript these variables with either *nak* or *cov* depending on whether they apply to transactions with naked versus covered CDS buyers.

Using (14) and (15) and that the CDS buyer extracts half of the gains from trade $V_{l[0,\text{-}1]} - V_{l[0,0]} = \frac{1}{2}\omega_c^{nak}$, the continuation values (10) simplify to

$$
\begin{aligned}
rV_{l[0,0]} &= \gamma_u(0 - V_{l[0,0]}) + \frac{M_c^{nak}}{\mu_{l[0,0]}}\frac{1}{2}\omega_c^{nak} && \text{(A1)}\\
rV_{h[0,0]} &= \gamma_d(0 - V_{h[0,0]}) + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c^{nak}}{\mu_{h[0,0]}}\frac{1}{2}\omega_c^{nak} && \text{(A2)}\\
rV_{h[1,0]} &= (\delta - \eta J) + x - y + \gamma_d(V_{a[1,0]} - V_{h[1,0]}) && \text{(A3)}\\
rV_{a[1,0]} &= (\delta - \eta J) - y + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b && \text{(A4)}\\
rV_{h[0,1]} &= p_c^{nak} - (\eta J - x) - y + \gamma_d(-T_{\text{S}}^{nak} - V_{h[0,1]}) && \text{(A5)}\\
rV_{l[0,\text{-}1]} &= -p_c^{nak} + (\eta J + x) - y + \gamma_u\left(-T_{\text{B}}^{nak} - V_{l[0,\text{-}1]}\right), && \text{(A6)}
\end{aligned}
$$

where ω_b and ω_c^{nak} are the total gains from a bond and a CDS transaction:

$$\omega_b = V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]},$$

$$\omega_c^{nak} = \left(V_{h[0,1]} - V_{h[0,0]}\right) + \left(V_{l[0,\text{-}1]} - V_{l[0,0]}\right).$$

Consider the fees the CDS counterparties pay to terminate their contracts. If a buyer terminates, the seller switches from a $h[0,1]$ type to $h[0,0]$, and the seller's utility decreases by $(V_{h[0,1]} - V_{h[0,0]})$. To make the seller indifferent, the buyer has to pay a fee equal to the decrease in the seller's utility:

$$T_{\text{B}}^{nak} = V_{h[0,1]} - V_{h[0,0]}. \tag{A7}$$

Analogously, a CDS seller (the long side) has to pay the CDS buyer (the short side):

$$T_{\text{S}}^{nak} = V_{l[0,\text{-}1]} - V_{l[0,0]}. \tag{A8}$$

The right-hand sides of (A7) and (A8) coincide with the gains from trade to each counterparty. Hence, both equal $\frac{1}{2}\omega_c^{nak}$.

Inflow-Outflow Equations

Given the conjectured trading strategies, the steady state masses are such that the flow of agents switching into a type equals the flow of agents switching out of that type. For example, the mass of $h[0,0]$ agents evolves as

$$\frac{\partial \mu_{h[0,0]}}{\partial t} = \overbrace{\nu_h F_h + \gamma_u \mu_{h[0,1]}}^{\text{inflow}} - \overbrace{\left(\gamma_d \mu_{h[0,0]} + \left(\lambda \mu_{a[1,0]} + \lambda \mu_{l[0,0]}\right) \mu_{h[0,0]}\right)}^{\text{outflow}}. \tag{A9}$$

In (A9), the flow of agents turning into $h[0,0]$-type are (1) the new high-valuation entrants, $\nu_h F_h$, and (2) long investors who had previously sold CDS but are now searching again because their counterparty terminated the contract, $\gamma_u \mu_{h[0,1]}$. The agents switching out of type $h[0,0]$ are those who (1) get a valuation shock, $\gamma_d \mu_{h[0,0]}$, (2) match with a bond seller, $\lambda \mu_{a[1,0]} \mu_{h[0,0]}$, and (3) match with a CDS buyer, $\lambda \mu_{l[0,0]} \mu_{h[0,0]}$. The steady state mass is characterized by $\frac{\partial \mu_{h[0,0]}}{\partial t} = 0$. That is, $\mu_{h[0,0]}$ is constant, and the inflow equals the outflow.

The inflow-outflow equations for the other agent types are analogous:

$$\text{long investor } h[0,0]: \quad \nu_h F_h + \gamma_u \mu_{h[0,1]} = \gamma_d \mu_{h[0,0]} + M_b + M_c^{nak} \tag{A10}$$

$$\text{naked CDS buyer } l[0,0]: \quad \nu_l F_l + \gamma_d \mu_{l[0,\text{-}1]} = \gamma_u \mu_{l[0,0]} + M_c^{nak} \tag{A11}$$

$$\text{bond owner } h[1,0]: \quad M_b = \gamma_d \mu_{h[1,0]} \tag{A12}$$

$$\text{bond seller } a[1,0]: \quad \gamma_d \mu_{h[1,0]} = M_b \tag{A13}$$

$$\text{sold CDS } h[0,1]: \quad M_c^{nak} = (\gamma_u + \gamma_d) \mu_{h[0,1]} \tag{A14}$$

$$\text{bought CDS } l[0,\text{-}1]: \quad M_c^{nak} = (\gamma_u + \gamma_d) \mu_{l[0,\text{-}1]}. \tag{A15}$$

Market Clearing

For the bond market to clear, the total mass of bond owners has to equal the bond supply:

$$\mu_{h[1,0]} + \mu_{a[1,0]} = S. \tag{A16}$$

For CDS market clearing, the number of CDSs sold has to equal the number of CDSs purchased:

$$\mu_{h[0,1]} = \mu_{l[0,\text{-}1]}. \tag{A17}$$

B Proofs

In this section, I provide an outline of Proposition 2 proof and the full proofs for Propositions 3 and 4. To save space, I relegate the rest of the proofs, including the full proof of Proposition 2, to the online Appendix.

To prove Proposition 2, I assume in the rest of the paper that the parameter conditions satisfy Assumption 1. Assumption 1 helps ensure that the investors' optimal positions and trading strategies are as described in Section 2.1.

Assumption 1.

(i) $x - (r + \gamma_d)O > 0$,

(ii) $2x - 2y - (r + \gamma_d)O - (r + \gamma_u)O > 0$,

(iii) $x - 2y - (r + \gamma_d)O < 0$,

(iv) $x - 2y - (r + \gamma_u)O < 0$.

To see the intuition for Assumption 1, consider first the gains from CDS trade. The gains from a naked CDS trade (i.e., the CDS buyer does not own the bond) between high- and average-valuation investors is proportional to $x - 2y - (r + \gamma_d)O$. This is negative by Assumption 1. That is, the difference in their valuations (x) is too small relative to the disutility both sides incur ($2y$) and the participation cost, $(r + \gamma_d)O$. The lack of gains from trade ensures that (a) an average-valuation investor without a bond does not buy CDS from a high-valuation investor, and (b) once a CDS buyer (initially, a low-valuation investor) switches to an average-valuation, she prefers to unwind the CDS position that she has with a high-valuation investor than to remain a CDS buyer. It is analogous between average- and low-valuation agents. The gains from CDS trade between them is proportional to $x - 2y - (r + \gamma_u)O$, which is negative. This ensures that (a) an average-valuation investor does not sell CDS to a low-valuation investor, and (b) once a CDS seller (a high-valuation investor) switches to an average-valuation investor, she prefers to unwind her long position than to remain a CDS seller. The gains from a naked CDS trade exists only between high- and low-valuation investors: $2x - 2y - (r+\gamma_d)O - (r+\gamma_u)O > 0$. The difference in their valuations, $2x = x - (-x)$, is large enough that it outweighs the total disutility, $2y$, and the participation costs, $(r + \gamma_d)O + (r + \gamma_u)O$.

For bond transactions, the gains from trade between high- and average-valuation investors is proportional to the difference in their valuations, x, minus the participation cost: $x - (r + \gamma_d)O$. The term y does not appear because bond transactions only transfer risk between agents and do not create new credit risk exposures. The expression $x - (r + \gamma_d)O$ is positive by Assumption 1. Thus, a high-valuation bond owner upon switching to an average-valuation investor prefers to unwind and sell her bond to a high-valuation investor.

Outline of Proposition 2 Proof. Proposition 2 proof consists of five main steps. In step 1, I show that the equilibrium conditions narrow down to a set of five equations and five unknowns $\{\mu_{h[0,0]}, V_{h[0,0]}, V_{l[0,0]}, \nu_h, \nu_l\}$:

$$(r + \gamma_d)V_{h[0,0]} - \lambda \frac{\gamma_d S}{\left(\gamma_d + \lambda\mu_{h[0,0]}\right)} \frac{1}{2} \frac{x - (r + \gamma_d)V_{h[0,0]}}{r + \gamma_d + \lambda\mu_{h[0,0]}\frac{1}{2}} - \lambda \frac{(\gamma_d + \gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d + \gamma_u + \lambda\mu_{h[0,0]}} \frac{1}{2} \frac{2x - 2y - (r + \gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda\mu_{h[0,0]}\frac{1}{2}} = 0 \tag{B1}$$

$$V_{l[0,0]} = \frac{1}{r + \gamma_u} \lambda\mu_{h[0,0]} \frac{1}{2} \frac{2x - 2y - (r + \gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda\mu_{h[0,0]}\frac{1}{2}} \tag{B2}$$

$$\nu_h F_h = \gamma_d\mu_{h[0,0]} + \gamma_d \frac{\lambda\mu_{h[0,0]} S}{\left(\lambda\mu_{h[0,0]} + \gamma_d\right)} + \gamma_d\lambda\mu_{h[0,0]} \frac{\frac{\nu_l F_l}{\gamma_u}}{\gamma_u + \gamma_d + \lambda\mu_{h[0,0]}} \tag{B3}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O \\ [0,1] \text{ if} & V_{i[0,0]} = O \\ 0 & V_{i[0,0]} < O \end{cases} \qquad \text{for } i \in \{h, l\}.$$

In step 2, I show that $V_{h[0,0]}$ decreases in ν_h. I also derive the parameter condition that ensures that an interior solution for ν_h exists. This condition, in turn, can be recast as bounds on the participation cost, O. These implicit bounds form the first set of bounds on O. Put together, I show that the solution for ν_h is unique, positive, and interior.

In step 3, using the result from step 2 that the participation rate of high-valuation investors, ν_h, is given by an interior solution, (B1) and (B2) become

$$(r+\gamma_d)O - \lambda\frac{\gamma_d S}{\left(\gamma_d+\lambda\mu_{h[0,0]}\right)}\frac{1}{2}\frac{x-(r+\gamma_d)O}{r+\gamma_d+\lambda\mu_{h[0,0]}\frac{1}{2}} \\ - \lambda\frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda\mu_{h[0,0]}}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O}{r+\gamma_d+\gamma_u+\lambda\mu_{h[0,0]}\frac{1}{2}} = 0 \tag{B4}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O}{r+\gamma_d+\gamma_u+\lambda\mu_{h[0,0]}\frac{1}{2}}. \tag{B5}$$

Equations (B4) and (B5) together define $V_{l[0,0]}$ as an implicit function of the participation rate of low-valuation investors, ν_l: $V_{l[0,0]}(\nu_l)$. I show that $V_{l[0,0]}(\nu_l)$ strictly increases in ν_l. Similar to Step 2, I also derive the parameter condition that ensures that an interior solution for ν_l exists. This condition implicitly characterizes a second set of bounds on O and together with the bounds from step 2 form the final bounds on O: $O \in (\underline{O}, \overline{O})$. Put together, these results ensure that a unique interior solution exists for the participation rate of low-valuation investors, ν_l. They also imply that two corner solutions exist: $\nu_l = 0$ and $\nu_l = 1$.

In step 4, I show that—taking the participation rates as given—the rest of the equilibrium variables are uniquely determined, and the population masses and the gains from trade are, in addition, positive. In step 5, I show that all the conjectured optimal trading strategies are indeed optimal.

The above outline shows that three equilibria exist, each with a different participation rate of low-valuation investors: a unique interior solution $\nu_l \in (0,1)$ and two corner solutions ($\nu_l = 0$, $\nu_l = 1$). For a given level of ν_l, the solution for the participation rate of high-valuation investors, however, is unique and interior. The fact that $\nu_l = 0$ is one of the solutions shows that even if CDS trading is feasible, investors may not trade CDS in equilibrium. Since the paper is about the effect of CDS, I contrast the equilibria with CDS (i.e., $\nu_l > 0$, whether it is an interior or a corner solution) to the environment in which I shut down the CDS market (or, equivalently, to the equilibrium with $\nu_l = 0$). The marginal effect of CDS is qualitatively the same for both the interior, $\nu_l \in (0,1)$, and the corner, $\nu_l = 1$, levels of the participation rate. Thus, the equilibrium multiplicity due to the different participation rates of low-valuation agents is unimportant.

The above results also imply that the participation rate of high-valuation investors, ν_h, is interior in the environment without CDS. In the environment with CDS, the participation rate of high-valuation investors is interior even if low-valuation investors in equilibrium do not participate in the credit market ($\nu_l = 0$). In turn, the equilibrium with no low-valuation investors is identical to the environment without CDS.

Ensuring that the participation rate of high-valuation investors is interior both before and after CDS introduction, while not necessary for the main results in Proposition 4, simplifies the analyses in the paper. In search models with exogenous participation, given the conjectured trading strategies, the system of equations characterizing the population masses does not depend on the value functions and can be solved on its own. Then, the value functions are a linear system of equations of the population masses. The conjecture-and-verify method, as a result, simplifies the analyses and proofs by decoupling the system of equations into two sets. Endogenizing participation, however, reverses this decoupling. The population masses depend on the participation rates, but the participation rates depend on the value functions, which, in turn, depend on the population masses. All three sets of variables have to be solved simultaneously. Thus, the model with endogenous participation is significantly more complicated. Focusing on the interior solution helps simplify the analysis. ■

Proof of Proposition 3 (The Bond Price). Recall that the bond price is the average of the bond buyer and the seller's reservation values:

$$p_b = \frac{1}{2}\left(V_{h[1,0]} - V_{h[0,0]}\right) + \frac{1}{2}(V_{a[1,0]}). \tag{B6}$$

Adding and subtracting $\frac{1}{2}\left(V_{h[1,0]} - V_{h[0,0]}\right)$ to the right-hand side, (B6) can be arranged as

$$p_b = \left(V_{h[1,0]} - V_{h[0,0]}\right) - \frac{1}{2}(V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}).$$

Using the definition of the gains from trade $\omega_b = V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}$, this becomes

$$p_b = \left(V_{h[1,0]} - V_{h[0,0]}\right) - \frac{1}{2}\omega_b. \tag{B7}$$

Using (A2)-(A3) (the continuation values of $h[1,0]$ and $h[0,0]$), the reservation value of a bond buyer is

$$V_{h[1,0]} - V_{h[0,0]} = \frac{\delta}{r} - \frac{\eta J - x + y}{r} - \frac{(r+\gamma_d)O}{r} - \frac{\gamma_d \omega_b}{r}. \tag{B8}$$

The first term, $\frac{\delta}{r}$, is the present value of the bond coupon flow. The second term is the high-valuation investor's private cost of bearing credit risk. The third term arises from the outside option of the high-valuation investor. The last term arises due to illiquidity. The higher the intensity with which a high-valuation investor reverts to an average-valuation investor and hence a bond seller, the less he is willing to buy the bond. By how much he is less willing to buy is ω_b or, equivalently, the change in his reservation value as a result of reverting to a seller.

Substituting (B8) into (B7), we get

$$p_b = \frac{\delta}{r} - \frac{\eta J - x + y}{r} - \frac{(r+\gamma_d)O}{r} - \frac{\gamma_d \omega_b}{r} - \frac{1}{2}\omega_b. \tag{B9}$$

The last term captures the fact that search frictions, by precluding competition, gives rise to bargaining. The bond buyer, in particular, extracts rents equal to half of the total rents, ω_b. Combining (A2)-(A4), substituting in M_b and M_c^{nak}, using $V_{h[0,0]} = O$, and simplifying, we get

$$(r+\gamma_d)\omega_b = x - (r+\gamma_d)O - \lambda\mu_{h[0,0]}\frac{1}{2}\omega_b. \tag{B10}$$

From (B10),

$$\omega_b = \frac{x - (r+\gamma_d)O}{r + \gamma_d + \lambda\mu_{h[0,0]}\frac{1}{2}}. \tag{B11}$$

Substituting this into (B9), we get

$$p_b = \frac{\delta}{r} - \frac{\eta J - x + y}{r} - \frac{(r+\gamma_d)O}{r} - \frac{(\frac{1}{2}r+\gamma_d)}{r}\frac{x-(r+\gamma_d)O}{r+\gamma_d+\lambda\mu_{h[0,0]}\frac{1}{2}}. \tag{B12}$$

The proof of Proposition F.1 in online Appendix F shows that as $\lambda \to \infty$, $\lambda\mu_{h[0,0]} \to \infty$. Hence, the bond price in the absence of search frictions is given by the first three terms in (B12). ■

Proof of Proposition 4 (The Liquidity Spillover Effect). I start by showing that the expected rents a long investor extracts from trading in the bond market, $\lambda\mu_{a[1,0]}\frac{1}{2}\omega_b$, has to be smaller in the equilibrium with CDS than in the equilibrium without CDS. To see this, the value function of a long investor is given by

$$(r+\gamma_d)V_{h[0,0]} = \lambda\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda\mu_{l[0,0]}\frac{1}{2}\omega_c^{nak}. \tag{B13}$$

The first term is the expected rents a long investor extracts from trading in the bond market. It is the intensity of finding a counterparty in the bond market times the gains from trade from a bond transaction. The second term is the analogous expected gains from trade in the CDS market. Since the high-valuation agents' participation rate is an interior solution with and without CDS, (B13) with and without CDS are

$$(r+\gamma_d)O = \lambda\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda\mu_{l[0,0]}\frac{1}{2}\omega_c^{nak}$$
$$(r+\gamma_d)O = \lambda\hat{\mu}_{a[1,0]}\frac{1}{2}\hat{\omega}_b,$$

respectively, where the variables in hats denote their values in the counterfactual environment without CDS. Since $\lambda\mu_{l[0,0]}\frac{1}{2}\omega_c^{nak} > 0$, and the left-hand sides are the same, it has to be that: $\lambda\hat{\mu}_{a[1,0]}\frac{1}{2}\hat{\omega}_b > \lambda\mu_{a[1,0]}\frac{1}{2}\omega_b$.

The decrease in the expected rents implies a decrease in the mass of bond sellers, which, in turn, implies the rest of the results. Combining (A16) and (A12), we get

$$\lambda\mu_{a[1,0]}\mu_{h[0,0]} = \gamma_d(S - \mu_{a[1,0]}). \tag{B14}$$

Equations (B11) and (B14) define $\mu_{h[0,0]}$ and ω_b as implicit functions of $\mu_{a[1,0]}$. Using (B11) and (B14), $\mu_{h[0,0]}$ and ω_b change with $\mu_{a[1,0]}$ as

$$\frac{\partial\mu_{h[0,0]}}{\partial\mu_{a[1,0]}} = -\frac{\gamma_d + \lambda\mu_{h[0,0]}}{\lambda\mu_{a[1,0]}}$$

$$\frac{\partial\omega_b}{\partial\mu_{a[1,0]}} = \frac{(\gamma_d+\lambda\mu_{h[0,0]})\frac{1}{2}\omega_b}{\mu_{a[1,0]}(r+\gamma_d+\lambda\mu_{h[0,0]}\frac{1}{2})}.$$

Thus, $\mu_{h[0,0]}$ decreases in $\mu_{a[1,0]}$, while ω_b increases in $\mu_{a[1,0]}$. Then, the expected rents a long investor extracts from trading in the bond market, $\lambda\mu_{a[1,0]}\frac{1}{2}\omega_b$, as an implicit function of $\mu_{a[1,0]}$, increases in $\mu_{a[1,0]}$. As a result, $\mu_{a[1,0]}$ has to be smaller in the equilibrium with CDS than in the equilibrium without CDS: $\mu_{a[1,0]} < \hat{\mu}_{a[1,0]}$. In turn, this implies that: $\omega_b < \hat{\omega}_b$ and $\mu_{h[0,0]} > \hat{\mu}_{h[0,0]}$. Since the illiquidity discount d_b just depends on $\mu_{h[0,0]}$, we have: $d_b < \hat{d}_b$. From (B14), a decrease in $\mu_{a[1,0]}$ implies an increase in the bond volume: $M_b > \hat{M}_b$. From (B3), an increase in $\mu_{h[0,0]}$ requires an increase in ν_h especially since ν_l changes from zero to a positive value in the presence of CDS. ■

C Discussion

C.1 The Need for All Three (High, Average, Low) Valuations

To see why I need all three valuations as well as entry and exit in and out of the credit market, consider, first, an environment with just two valuations (say, high- and low-valuation agents) and no entry and exit. In such environment, in the absence of CDS, the optimal position for the low-valuation investor is no position. Investors buy the bond as a high-valuation investor and sell when they become a low-valuation investor. When we introduce CDS, the optimal position for the low-valuation investor is now a short position. They, as a result, go one asset position further and buy CDS after they sell their bond. But because the number of investors of each valuation is fixed, allowing CDS and hence short positions deteriorates bond market liquidity. The point of the paper is to, instead, show that investors' participation incentives change in response to CDS. I thus need to endogenize the aggregate number of investors of each valuation.

A simple way to endogenize the number of investors is to endogenize their entry and exit in and out of the credit market. Exiting the credit market, in turn, is optimal under two conditions. First, investors cannot exit the market with an existing position. To ensure then that investors unwind their existing positions, their valuation has to change to a valuation whose terminal optimal position is no position. Second, once their valuation changes, they cannot have an incentive to wait to switch to another valuation instead of exiting the market. That is, the valuation they switch to has to be an absorbing valuation.

Consider then a model with two valuations where investors enter the credit market as a high-valuation investor, switch to a low-valuation investor at some point, and, once they switch, permanently remain a low-valuation investor. In this environment, in the absence of short positions, investors buy the bond upon entering the credit market; when they switch to a low-valuation, they sell and permanently exit the market. In the presence of CDS, high-valuation investors still go long, but low-valuation agents now want to short (as their terminal optimal position). This implies that exiting the market is no longer optimal for low-valuation agents. They, instead, remain in the market and short, resulting in an infinite mass of short investors. Put together, an environment with two valuations allows short positions in the absence of entry and exit, allows entry and exit in the absence of short positions, but does not allow both short positions and entry and exit.

C.2 A Cost of Capital Interpretation of The Participation Cost

To see how the participation cost (O) can be interpreted as the cost of equity capital, suppose that each of the newly born investors solves the following problem

$$\max_{\{\nu_C,\nu_K\}} V_C * \nu_C + V_K * \nu_K, \tag{C15}$$

where $V_C = V_{i[0,0]}$ is the expected dollar profit (in present value terms) of one additional trade in credit market instruments (C), V_K is the expected profit of another (unmodeled) asset class labeled "K", and ν_C and ν_K are the number of positions in each asset class. The expected profit of a credit market position accounts for costs of the trade including, for example, the bond price in a bond purchase, search costs associated with both establishing and reversing a position, and the expected investment horizon.

Investors maximize (C15) subject to a Basel III style risk-based regulatory capital constraint that says the total equity capital of the investor has to be at least as large as 8% of their risk-weighted assets (RWA):

$$8\% \left(\nu_C k_C + \nu_K * k_K\right) \leq \text{E}, \tag{C16}$$

where k_C and k_K are the capital charges associated with each asset class, E is the equity capital of the investor, and 8% is set by the regulation (BIS 2016). According to Basel III capital requirements, the capital charge for credit instruments is the loss upon default (which equals J in my model) times a risk weight that reflects the probability of default: $k_C = \eta J$. Basel III capital requirements, as a result, imply the same capital charge for both bond and CDS positions.

Maximizing (C15) subject to (C16), we get

$$\max_{\{\nu_C, \nu_K\}} V_C * \nu_C + V_K * \nu_K - o(0.08\nu_C k_C + 0.08\nu_K k_K - E),$$

where o is the Lagrange multiplier on the constraint (C16). That is, o is the shadow cost of equity capital or, equivalently, the return on equity. Using $V_C = V_{i[0,0]}$ and $k_C = \eta J$, the FOC with respect to ν_C is

$$V_{i[0,0]} = o(0.08\eta J).$$

Defining O as

$$O \equiv o(0.08\eta J),$$

O is the total capital cost of a credit market position. It is the return on equity (o) times the capital requirement of a credit market position ($0.08\eta J$). Thus, in equilibrium, investors increase their allocation to credit market instruments until the expected profit from a credit market position equals the total capital cost of a credit market position or, equivalently, until the expected profit net of regulatory capital costs is zero.

C.3 The Bond and CDS Illiquidity Components

The intuition for the difference between the illiquidity components of covered versus naked CDS spreads, d_c^{cov} and d_c^{nak}, is as follows. Substituting $\frac{\mu_{a[1,0]}}{\mu_{h[1,0]}+\mu_{a[1,\text{-}1]}} = \frac{1}{\lambda\mu_{h[0,0]}}\gamma_d$ and $\frac{\mu_{l[0,0]}}{\mu_{l[0,\text{-}1]}} = \frac{1}{\lambda\mu_{h[0,0]}}(\gamma_d + \gamma_u)$ into (28) and (29), d_c^{cov} and d_c^{nak} become:

$$d_c^{cov} \equiv \left(\frac{1}{2}r + \gamma_d\right)\left[\frac{x - (r+\gamma_d)O}{r + \gamma_d + \lambda\mu_{h[0,0]}\frac{1}{2}}\right], \tag{C17}$$

$$d_c^{nak} \equiv \left(\frac{1}{2}r + \gamma_d\right)\left[\frac{2x - 2y - (r+\gamma_d)O}{r + \gamma_d + \gamma_u + \lambda\mu_{h[0,0]}\frac{1}{2}}\right]. \tag{C18}$$

The expressions in square brackets are the gains from the respective transactions or, equivalently, the total rents generated by search frictions. The difference between d_c^{cov}

and d_c^{nak} arises from the difference between the gains from naked vs. covered CDS transactions. The expressions in square brackets are the present value of the flow gains from trade, with two adjustments. The flow gains from trade is the difference between the CDS counterparties' private costs of bearing credit risk: x in a covered and $2x - 2y$ in a naked CDS transaction.[47] The first adjustments reflect how fast the counterparties' valuations change, eliminating the difference in their valuations and hence the gains from trade. In a covered CDS transaction, only the CDS seller (a high-valuation investor) can get a valuation shock. In a naked CDS transaction, both the CDS seller (a high-valuation investor) and the CDS buyer (a low-valuation investor) can get a valuation shock. These events occur with total intensities γ_d and $\gamma_d + \gamma_u$, respectively. The higher the intensities, the lower the gains from trade. The second set of adjustments, that also reduces the gains from trade, arises from the counterparties' outside options. The CDS seller's outside option is summarized by $(r + \gamma_d)O$ in the numerator. It is the equilibrium flow value of his continuation value $(r + \gamma_d)V_{h[0,0]}$. The CDS buyer's outside option is captured by the last term in the denominators of (C17) and (C18). That is, the CDS buyers' outside option improves with the mass of potential CDS sellers ($\mu_{h[0,0]}$).

The intuition for the difference between the bond illiquidity discount,

$$d_b = \left(\frac{1}{2}r + \gamma_d\right)\left[\frac{x - (r + \gamma_d)O}{r + \gamma_d + \lambda\mu_{h[0,0]}\frac{1}{2}}\right], \tag{C19}$$

and the naked CDS illiquidity premium (d_c^{nak}) is analogous to the above intuition.

The intuition for the difference between the bond and covered CDS illiquidity terms (d_b and d_c^{cov}) also works analogously. Since bond and covered CDS transactions involve the same pair of counterparties (a high-valuation investor $h[0,0]$ and an average-valuation bond owner $a[1,0]$), their gains from trade and hence d_b and d_c^{cov} are the same.

[47] The extra x in the CDS transaction with a naked CDS buyer reflects the fact that the endowment of the naked CDS buyer is more correlated with the bond than that of the covered CDS buyer. The minus $2y$ arises because a naked CDS purchase creates two new risky positions (one for the buyer and one for the seller of CDS) each involving a disutility (y). A covered CDS purchase, in contrast, only shifts the risk from the CDS buyer to the CDS seller and, as a result, does not affect the total disutility across the counterparties.

References

Acharya, V., Y. Gunduz, and T. Johnson. 2018. Bank use of sovereign CDS in the eurozone crisis: Hedging and risk incentives. Working paper.

Acworth, W., and J. Morrison. 2017. An interview with Vanguard's Sam Priyadarshi. MarketVoice, FIA. `www.fia.org-marketvoice-articles-interview-vanguards-sam-priyadarshi`.

Afonso, G. 2011. Liquidity and congestion. *Journal of Financial Intermediation* 20 (3): 324–360.

Arce, O., S. Mayordomo, and J. I. Peña. 2013. Credit risk valuation in the sovereign CDS and bonds markets: Evidence from the Euro area crisis. *Journal of International Money and Finance* 35:124–145.

Arping, S. 2014. Credit protection and lending relationships. *Journal of Financial Stability* 10:7–19.

Ashcraft, A., and J. Santos. 2009. Has the CDS market lowered the cost of corporate debt? *Journal of Monetary Economics* 56 (4): 514–523.

Augustin, P., and J. Schnitzler. 2021. Disentangling types of liquidity and testing limits-to-arbitrage theories in the CDS–bond basis. *European Financial Management* 27:120–146.

Badaoui, S., L. Cathcart, and L. El-Jahel. 2013. Do sovereign credit default swaps represent a clean measure of sovereign default risk? A factor model approach. *Journal of Banking & Finance* 37 (7): 2392–2407.

Bai, J., and P. Collin-Dufresne. 2019. The determinants of the CDS-bond basis during the financial crisis of 2007-2009. *Financial Management* 48 (2): 417–439.

Banerjee, S., and J. J. Graveline. 2014. Trading in derivatives when the underlying is scarce. *Journal of Financial Economics* 111 (3): 589–608.

Bao, J., J. Pan, and J. Wang. 2011. The illiquidity of corporate bonds. *Journal of Finance* 66 (3): 911–946.

Benos, E., A. Wetherilt, and F. Zikes. 2013. The structure and dynamics of the UK credit default swap market. Bank of England Financial Stability Paper 25, Bank of England.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz. 2007. Measuring default risk premia from default swap rates and EDFs. Working paper.

BIS. 2016. Minimum capital requirements for market risk. The Basel Framework. `www.bis.org/bcbs/publ/d457.htm`.

Blanco, R., S. Brennan, and I. Marsh. 2005. An empirical analysis of the dynamic relation between investment grade bonds and credit default swaps. *Journal of Finance* 60 (5): 2255–2281.

Bolton, P., and M. Oehmke. 2011. Credit default swaps and the empty creditor problem. *Review of Financial Studies* 24 (8): 2617–2655.

Bongaerts, D., F. D. Jong, and J. Driessen. 2011. Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market. *Journal of Finance* 66 (1): 203–240.

Boyarchenko, N., P. Gupta, N. Steele, and J. Yen. 2018. Trends in credit basis spreads. *Federal Reserve Bank of New York Economic Policy Review* 24 (2).

Bühler, W., and M. Trapp. 2007. Credit and liquidity risk in bond and CDS markets. Working paper.

— . 2009. Explaining the bond-CDS basis: The role of credit risk and liquidity. Working paper.

Chen, L., D. Lesmond, and J. Wei. 2007. Corporate yield spreads and bond liquidity. *Journal of Finance* 62 (1): 119–149.

Chen, R.-R., F. Fabozzi, and R. Sverdlove. 2010. Corporate credit default swap liquidity and its implications for corporate bond spreads. *The Journal of Fixed Income* 20 (2): 31–57.

Cheung, E. 2019. Sales and trading: Roles and asset classes. `www.wallstreetprep.com/knowledge/sales-and-trading-roles-and-asset-classes/`.

Cossin, D., T. Hricko, D. Aunon-Nerin, and Z. Huang. 2002. Exploring for the determinants of credit risk in credit default swap transaction data: Is fixed income data sufficient to evaluate credit risk? Working paper.

Czech, R. 2019. Credit default swaps and corporate bond trading. Working paper.

Das, S., M. Kalimipalli, and S. Nayak. 2014. Did CDS trading improve the market for corporate bonds? *Journal of Financial Economics* 111 (2): 495–525.

DeChesare, B. 2018. Fixed income trading: The definitive guide. `www.mergersandinquisitions.com/fixed-income-trading/`.

Dick-Nielsen, J., D. Lando, and P. Feldhütter. 2012. Corporate bond liquidity before and after the onset of the subprime crisis. *Journal of Financial Economics* 103 (3): 471–492.

DTCC. 2014. Trade Information Warehouse, Table 2: Single Name Reference Entity Type by Buyer of Protection. `dtcc.com/en/repository-otc-data.aspx`.

Duffie, D., N. Gârleanu, and L. Pedersen. 2005. Over-the-counter markets. *Econometrica* 73 (6): 1815–1847.

— . 2007. Valuation in over-the-counter markets. *The Review of Financial Studies* 20 (6): 1865–1900.

Elizalde, A., S. Doctor, and Y. Saltuk. 2009. Bond-CDS Basis Handbook. Europe Credit Derivatives Research, J.P.Morgan.

Foley-Fisher, N., B. Narajabad, and S. Verani. 2016. Securities lending as wholesale funding: Evidence from the U.S. life insurance industry. NBER Working Paper 22774, National Bureau of Economic Research.

Fontaine, J.-S., and R. Garcia. 2012. Bond liquidity premia. *Review of Financial Studies* 25 (4): 1207–1254.

Fontana, A. 2011. The negative CDS-bond basis and convergence trading during the 2007/09 financial crisis. Working paper.

Fontana, A., and M. Scheicher. 2016. An analysis of Euro area sovereign CDS and their relation with government bonds. *Journal of Banking & Finance* 62:126–140.

Friewald, N., Jankowitsch, and M. Subrahmanyam. 2012. Illiquidity or credit deterioration: A study of liquidity in the US corporate bond market during financial crises. *Journal of Financial Economics* 105 (1): 18–36.

Goldstein, I., Y. Li, and L. Yang. 2013. Speculation and hedging in segmented markets. *Review of Financial Studies* 27 (3): 881–922.

Goldstein, I., and L. Yang. 2015. Information diversity and complementarities in trading and information acquisition. *Journal of Finance* 70 (4): 1723–1765.

Gorton, G., and G. Pennacchi. 1993. Security baskets and index-linked securities. *The Journal of Business* 66 (1): 1–27.

Grossman, S., and M. Miller. 1988. Liquidity and market structure. *Journal of Finance* 43 (3): 617–633.

Gyntelberg, J., P. Hordahl, K. Ters, and J. Urban. 2017. Arbitrage costs and the persistent non-zero CDS-bond basis: Evidence from intraday Euro area sovereign debt markets. BIS Working Papers 631, BIS.

He, Z., B. Kelly, and A. Manela. 2017. Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics* 126 (1): 1–35.

Hendershott, T., R. Kozhan, and V. Raman. 2017. Short selling and price discovery in corporate bonds. *Journal of Financial and Quantitative Analysis* 55 (1): 77–115.

Huang, J., and J. Wang. 2009. Liquidity and market crashes. *Review of Financial Studies* 22 (7): 2607–2643.

Huang, J.-Z., and M. Huang. 2012. How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies* 2 (2): 153–202.

Hugonnier, J., B. Lester, and P.-O. Weill. 2014. Heterogeneity in decentralized asset markets. Working paper.

Ismailescu, I., and B. Phillips. 2015. Credit default swaps and the market for sovereign debt. *Journal of Banking and Finance* 52:43–61.

Jankowitsch, R., F. Nagler, and M. Subrahmanyam. 2014. The determinants of recovery rates in the US corporate bond market. *Journal of Financial Economics* 114 (1): 155–177.

John, K., A. Koticha, M. Subrahmanyam, and R. Narayanan. 2003. Margin rules, informed trading in derivatives, and price dynamics. Working paper.

Johnson, S. 2013. *Debt Markets and Analysis.* Hoboken, NJ: Bloomberg Press.

— . 2017. *Derivatives Markets and Analysis.* Hoboken, NJ: Bloomberg Press.

Junge, B., and A. Trolle. 2015. Liquidity risk in credit default swap markets. Swiss Finance Institute Research Paper 13-65.

Kucuk, U. 2010. Non-default component of sovereign emerging market yield spreads and its determinants: Evidence from the CDS market. *Journal of Fixed Income* 19 (4): 44–66.

Lagos, R., and G. Rocheteau. 2009. Liquidity in asset markets with search frictions. *Econometrica* 77 (2): 403–426.

Massa, M., and L. Zhang. 2012. CDS and the liquidity provision in the bond market. Working paper.

Milbradt, K. 2017. Asset heterogeneity in over-the-counter markets. Working paper.

Mitchell, M., and T. Pulvino. 2012. Arbitrage crashes and the speed of capital. *Journal of Financial Economics* 104 (3): 469–490.

NAIC. 2011. Insights into the insurance industry's credit default swaps exposure. Capital Markets Special Report, The NAIC's Capital Markets Bureau.

Nashikkar, A., and L. Pedersen. 2007. Corporate bond specialness. Working paper.

Nashikkar, A., M. Subrahmanyam, and S. Mahanti. 2011. Liquidity and arbitrage in the market for credit risk. *Journal of Financial and Quantitative Analysis* 46 (3): 627–656.

Neklyudov, A. 2019. Bid-ask spreads and the over-the-counter interdealer markets: Core and peripheral dealers. *Review of Economic Dynamics* 33:57–84.

Oehmke, M., and A. Zawadowski. 2015. Synthetic or real? The equilibrium effects of credit default swaps on bond markets. *Review of Financial Studies* 28 (12): 3303–3337.

— . 2016. The anatomy of the CDS market. *Review of Financial Studies* 30 (1): 80–119.

Parlour, C., and A. Winton. 2013. Laying off credit risk: Loan sales versus credit default swaps. *Journal of Financial Economics* 107 (1): 25–45.

Qiu, J., and F. Yu. 2012. Endogenous liquidity in credit derivatives. *Journal of Financial Economics* 103 (3): 611–631.

Rocheteau, G., and P.-O. Weill. 2011. Liquidity in frictional asset markets. *Journal of Money, Credit and Banking* 43 (2): 261–282.

Sambalaibat, B. 2012. Credit default swaps as sovereign debt collateral. Working paper.

— . 2018. Endogenous specialization and dealer networks. Working paper.

— . 2019. Naked CDS bans and the bond market: Empirical evidence. Working paper.

Serdarevic, M. 2010. Barriers falling between traders. *Financial Times* (London, UK). www.ft.com/content/5cd8db6c-0f6a-11df-a450-00144feabdc0.

Shen, J., B. Wei, and H. Yan. 2015. Financial intermediation chains in a search market. Working paper.

Shim, I., and H. Zhu. 2014. The impact of CDS trading on the bond market: Evidence from Asia. *Journal of Banking & Finance* 40:460–475.

SIFMA. 2018a. Global capital markets and financial institutions primer. SIFMA Insights Primers Series.

— . 2018b. US Fixed Income Securities Statistics. www.sifma.org/resources/archive/statistics.

Siriwardane, E. 2019. Limited Investment Capital and Credit Spreads. *Journal of Finance* 74:2303–2347.

Subrahmanyam, A. 1991. A theory of trading in stock index futures. *The Review of Financial Studies* 4 (1): 17–51.

Tang, D. Y., and H. Yan. 2007. Liquidity and credit default swap spreads. Working paper.

Tett, G. 2010. *Fool's Gold: The Inside Story of J.P. Morgan and How Wall St. Greed Corrupted Its Bold Dream and Created a Financial Catastrophe*. New York: Free Press.

The Federal Reserve. 2012. The Financial Accounts of the United States (Z.1). `www.federalreserve.gov/releases/z1`.

Thompson, J. 2007. Credit risk transfer: To sell or to insure. Working paper.

Uslu, S. 2019. Pricing and liquidity in decentralized asset markets. *Econometrica* 87 (6): 2079–2140.

Vayanos, D., and J. Wang. 2013. Market Liquidity — Theory and Empirical Evidence. *Handbook of the Economics of Finance* 2 (B): 1289–1361.

Vayanos, D., and T. Wang. 2007. Search and endogenous concentration of liquidity in asset markets. *Journal of Economic Theory* 136 (1): 66–104.

Vayanos, D., and P.-O. Weill. 2008. A search-based theory of the on-the-run phenomenon. *Journal of Finance* 63 (3): 1361–1398.

Wang, X., Y. Wu, H. Yan, and Z. Zhong. 2016. Funding liquidity shocks in a natural experiment: Evidence from the CDS Big Bang. Working paper.

Weill, P.-O. 2008. Liquidity premia in dynamic bargaining markets. *Journal of Economic Theory* 140 (1): 66–96.

Zabel, R. 2008. Credit default swaps: From protection to speculation. *Pratt's Journal of Bankruptcy Law* 4 (6): 546–552.