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Screening with Securities

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Abstract

A liquidity-constrained asset owner designs an asset-backed security to raise funds from an informed liquidity supplier. Information insensitive securities reduce the liquidity supplier's informational rents. The issuer *optimally* screens the liquidity supplier's private information by offering a menu of debt contracts with face values monotonically ordered in the liquidity supplier's valuation. We leverage this characterization to show that when the liquidity supplier's private information becomes more accurate (Lehmann [1988]), the issuer optimally offers debt contracts with smaller face values. Surprisingly, the concavity of debt on the asset's future cashflows implies that the issuer may benefit from trading with a more informed liquidity supplier. Our results challenge the idea that, when trading securities, the informed party should obtain an information sensitive security and suggest a novel rationale for the emergence of venture debt and the prevalence of collateralized lending.

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Keywords: Security Design, Informed Investors, Informational Rents, Information Sensitivity, Lehmann order.

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1 Introduction

Financial institutions facing liquidity constraints regularly sell securities to raise funds and meet their short-term obligations. Investors, who act as liquidity suppliers, have private information about their valuations for the securities' underlying assets. However, which securities should a liquidity-constrained asset owner sell when faced with informed investors? With some exceptions, the theoretical literature on security design has largely focused on the case where the issuer is an agent endowed with private information about the asset's future returns. However, in practice, there exist many important environments where the buyside possesses superior information about its valuation for the underlying assets.

Indeed, the heterogeneity of investors' valuations for financial assets is prevalent in financial markets.[1] This heterogeneity can originate from multiple sources. It may emerge, e.g., as a response to discriminating tax rules, as the result of asymmetric information among different market participants, as the output of heterogeneous technologies to process private and public information, or as the result of exposures to idiosyncratic, nontradeable risks. In this paper, we study how an asset owner can screen an investor's private information by optimally choosing the security design.

Consider a startup trying to raise funds from a venture capital (VC) fund. VCs specialize in funding and coaching similar projects, and they usually have superior information about the potential for growth and the future cashflows that could be generated.[2] Startups, on the other hand, struggle to raise cash to fund their initial operations and are therefore strongly liquidity-constrained. To raise liquid funds, entrepreneurs usually sell claims on future cashflows (i.e., securities) in exchange for cash. If the entrepreneur could design the securities to be sold to the VC to maximize the amount of funds raised, which security would she choose?

The main insights from the existing theoretical literature suggest that when trading securities, the informed party (usually the issuer) should keep an information-sensitive security, e.g., an equity stake or a call option (Nachman and Noe [1994], DeMarzo and Duffie [1999], Biais and Mariotti [2005]). Intuitively, these securities increase the issuer's skin in the game and are therefore a form to costly signal her private information. In this paper, we show that there exists a fundamental reason why the issuer should sell information-insensitive securities to informed investors. Specifically, these securities allow the issuer to optimally screen the

[1] Bagwell [1991] and Bagwell [1992] document investor heterogeneity in the context of stock repurchases; Bradley et al. [1988] find evidence of heterogeneity in the context of corporate acquisitions. Bernardo and Cornell [1997], in turn, extend the analysis to the case of complex derivatives.

[2] Several papers find evidence of VC firms repeatable skills. See, e.g., Kaplan and Schoar [2005] and Ewens and Rhodes-Kropf [2015]. The performance persistence might originate from access to networks (Hochberg et al. [2007]), high levels of industry experience (Hellmann and Puri [2002]), or screening skills (Sorensen [2007]).

liquidity supplier's private information and reduce their informational rents. This allows the issuer to maximize the funds raised from the sale.

To gain some intuition, consider the following simple example. There is an issuer (she) and a liquidity supplier (he) who can be of two types, namely, either H or L. Assume that H's beliefs about the asset's future cashflows allocate more weight to higher realizations (formally, H's beliefs dominate L's beliefs in the monotone likelihood ratio (MLR) order). This could be either because the asset is more productive when managed or monitored by type H, or simply because H is more optimistic about the asset's prospects. We show that when the issuer is subject to liquidity constrains (i.e., has a low discount factor), she is better off by selling debt instruments.

To more clearly understand, suppose that the issuer designs a menu of securities and respective payments, $\left\{\left(s^L, t^L\right), \left(s^H, t^H\right)\right\}$, where s^i represents the security and t^i represents the payment designed for type $i \in \{H, L\}$. The issuer can replicate the expected cashflows associated with s^L (where the expectation is computed according to L's beliefs) with a debt contract, s_d^L, which promises to pay a fixed amount $d > 0$ in the future and seniority if this amount is not met. If the face value of s_d^L is chosen so that L is indifferent between s^L and s_d^L, then H values s_d^L strictly less than s^L. Indeed, H assigns more weight, relative to L, to high cashflow realizations. For these realizations, however, s_d^L always offers the same flat payoff, d. In other words, s_d^L minimizes the amount to be paid when cash flows are high (i.e., the upside), which are also the states that H is more likely relative to L. Thus, by changing the original security s^L for the debt contract s_d^L, the issuer reduces the *informational rents* that she needs to leave to type H to prevent him from mimicking type L. The issuer can therefore increase the price charged to H for security s^H, t^H, without spoiling his incentives and, at the same time, increasing the funds raised.

We show that the heuristic described above is general in that it can be extended to the case with an *arbitrary* number of liquidity supplier's types. Using a replication argument, we show that any mechanism which induces all liquidity supplier types to truthfully reveal their private information can be dominated by another mechanism where all liquidity supplier types purchase debt securities. The optimal mechanism consists of a *menu of debt contracts*, with face values monotonically ordered in the liquidity supplier's type. That is, the more optimistic liquidity supplier types purchase larger amounts of debt. Our main technical insight is that any menu of securities (with their associated prices) can be dominated by modifying the securities for debt contracts, which guarantees that the incentive constraints are not spoiled either locally or globally. Indeed, with more than two types, the intuition provided above may fail as the incentives of low types to mimic high types are exacerbated when the latter are offered debt contracts. We show that the securities in any incentive compatible mechanism must satisfy a property analogous to the monotonicity condition in the one-dimensional case. We leverage this property to show that by correctly permuting the securities for debt contracts, all incentive constraints are in fact relaxed. This last observation allows the issuer to increase the price of the securities being sold and hence maximizes the amount of funds raised.

We note that the heuristic described above does not contradict the main insights in the security design literature that postulate that more informed agents should expose themselves relatively more to the asset's cashflows to signal their private information (Leland and Pyle [1977], Ross [1977], Myers and Majluf [1984], etc.). In fact, under the optimal mechanism, more optimistic liquidity supplier types purchase larger fractions of the underlying asset (i.e., debt contracts with higher face values), which obviously expose them to right-tail risk. Perhaps surprisingly, however, we find that exploiting the information sensitivity of securities to screen the liquidity suppliers' private information is completely ineffective and is strictly dominated by information-insensitive instruments.

Following a direct mechanism design approach, we extend the classical results in the optimal screening literature to environments in which the designer has access to the rich allocation space of financial securities, i.e., infinite-dimensional objects.[3] One of the technical challenges we face in extending the classical results to our environment is that we lose the structure usually assumed in those earlier models.[4] We show that in our environment, the optimal mechanism still satisfies many of the qualitative properties usually found in the classical literature. Indeed, the optimal allocation rule features the properties of (i) no distortion at the top, (ii) binding downward, local incentive constraints, and (iii) no rents at the bottom.

We provide a full characterization of the mechanism design problem with securities. The advantage of this approach vis-à-vis the former results is that it allows us to find analytical expressions for the issuer's expected revenue and the liquidity supplier's informational rents. We leverage these results to perform novel monotone comparative statics results relating the quality of the liquidity supplier's private signals to the issuer's optimal mechanism.

We show that when the liquidity supplier's private information about the asset's future cashflows becomes more accurate (Lehmann [1988]), the face value of the debt contract designed for each liquidity supplier type decreases. Intuitively, when the liquidity supplier's

[3]The classical literature focuses on the case where the issuer either decides whether to sell the whole asset to the liquidity supplier, potentially in a stochastic manner. This is equivalent to asset selloffs or equity stakes. Instead, we propose enriching the allocation space to encompass all types of securities (e.g., debt, options, arbitrary tranches, etc.).

[4]It is standard to assume that the buyer's payoff has increasing differences in the allocation and the buyer's type. This assumption, together with some regularity conditions, jointly imply that local incentive constraints imply the global constraints. There is no obvious extension of this property to the infinite-dimensional space of all securities.

information improves, the issuer is forced to give up more informational rents to the liquidity supplier. To minimize the incentives of high types to mimic low types, the issuer truncates the securities designed to the low types at a lower level. Perhaps surprisingly, however, the geometry of the optimal securities, i.e., the concavity of debt on asset future cashflows, implies that improving the accuracy of the liquidity supplier's information increases the his valuation for these securities. This effect is similar to a reduction in uncertainty for a risk-averse agent, which increases the utility she derives from the security. We show by means of an example that under some conditions, the second effect dominates, leading to the striking conclusion that the issuer may benefit from facing more informed investors. Our result reinforces the importance of the characterization of the optimal mechanism, which becomes crucial to understanding efficiency gains from increasing the transparency and the level of asymmetric information between different market participants.

We argue that accuracy (Lehmann [1988]) is an appealing notion of informativeness in our environment for three reasons. First, provided that signals have the MLR property, accuracy is less restrictive than the Blackwell order in that it compares more signal structures.[5] Furthermore, accuracy implies the standard notion of informativeness usually assumed in the information design literature.[6] Second, the concept of accuracy is tightly related to the idea of interdependence. When an experiment is more accurate, the comovement between fundamentals and signals becomes stronger. Finally, the Lehmann order compares experiments (i.e., conditional distributions) as opposed to comparing joint distributions. Intuitively, we want to change the quality of the liquidity supplier's information without changing the distribution of the asset's future cashflows or the distribution of liquidity supplier types, as this would completely change the environment. We show that starting from a fixed (marginal) distribution of the assets' cash flows, one can increase the accuracy of the liquidity supplier's private information while preserving the (marginal) distribution of the liquidity supplier's private signals. Combined with our first observation, this implies that increasing signals' accuracy increases the liquidity supplier's private information about the fundamentals without changing the quality of underlying asset or the ex-ante distribution of liquidity supplier types, thus making the comparative statics exercise coherent.

Interestingly, the prediction that liquidity-constrained asset owners use debt instruments to raise funds from informed investors is consistent with several empirical regularities. In the

[5] Indeed, provided that signals satisfy the MLRP, any two signals ordered according to Blackwell are also ordered according to Lehmann [1988]. When the state space is binary, both notions of informativeness coincide (see, e.g., Jewitt [2007]). More recently, Kim [2022] showed that Lehmann domination is closely related to the concept of quasigarbling, a generalization of information garbling.

[6] Fixing the prior example, if two experiments are ordered according to Lehmann, then the distribution of posterior estimates induced by the experiments' signals are ranked in convex order. See Proposition 2 below.

case of VC funding, our predictions are consistent with the emergence of venture debt. Indeed, according to Tykvová [2017], approximately one-third of the current venture-backed companies use debt instruments to raise funds. A plausible explanation for this new trend is that as VC funds become more competitive, they retain lower informational rents.[7] In the context of structured financial products, such as the market for mortgage-backed securities (MBS), buyers who are generally large investment banks, brokerage firms, and institutional investors have substantial expertise in valuing the securities issued through their knowledge of secondary market conditions and their access to proprietary valuation models.[8] Our predictions are broadly consistent with the prevalence of tranching in securitization, wherein investors are promised a fixed face value according to their preferences and payments contingent on the underlying assets' cashflows when the face value is not met. Finally, our predictions are also consistent with the experience of the Resolution Trust Corporation (RTC) and the FDIC, which are institutions in charge of disposing the assets of failed financial institutions. Using pooled assets auctions and securitized vehicles, they have dramatically increased the funds raised.

The rest of the paper is organized as follows. Below, we wrap up the introduction by discussing how our paper connects with the rest of the literature. Section 2 describes the primitives of the model. Section 3 contains the derivation of our first result establishing the optimality of *menus of debt contracts* and the characterization of the optimal mechanism. We further discuss the relation between securities' information sensitivity and the liquidity supplier's informational rents. In section 4, we explore how changes in the accuracy of the liquidity supplier's private signal affects the optimal contracts, the amount of funds raised, and the agents' payoffs. All omitted proofs are relegated to the Appendix.

Related Literature

This paper relates to several strands of the literature. First, it contributes to the broad literature on security design under asymmetric information (see, e.g., Nachman and Noe [1994], DeMarzo and Duffie [1999], Biais and Mariotti [2005], DeMarzo and Fishman [2007],

[7]Concomitant with the emergence of venture debt is the fact that in the last decade, VCs with a founder-friendly reputation have gained prominence (Ewens et al. [2018], Lerner and Nanda [2020]). Contrary to the former governance approach, which entailed the intensive monitoring of startups, VCs are adopting a hands-off approach, leaving much discretion to the entrepreneurs. Both regularities seem consistent with the idea that startups have greater bargaining power.

[8]Bernardo and Cornell [1997] analyze data from an auction of collateralized mortgage obligations (CMO) and find statistical evidence of a large dispersion in investors' valuation for the securities.

etc.).[9] We depart from those earlier models by assuming that when trading securities, it is the liquidity supplier who is endowed with private information. In the context of informed liquidity suppliers, DeMarzo et al. [2005] study general securities auctions with multiple ($N \geq 2$) bidders.[10] Following an indirect mechanism approach, they show that among all general symmetric mechanisms,[11] the first-price auction where buyers are restricted to bid call options (i.e., the buyer purchases a debt contract) maximizes the issuer's revenue. Our result that the optimal mechanism consists of a menu of debt contracts is consistent with their finding. We provide a full characterization of the *unrestricted* solution to the liquidity-constrained issuer's problem with a single bidder. Following a direct mechanism approach, we show that the optimality of the menus of debt contracts originates on the fact that debt minimizes the liquidity suppliers' informational rents and therefore allow the issuer to raise more funds. Our characterization is instrumental in performing novel comparative statics on the optimal contract as a function of the primitives of the problem, especially the extent of asymmetric information among the agents. Axelson [2007] studies security auctions with multiple bidders when the issuer is the liquidity-constrained, as in our environment. He finds that when the issuer restricts attention to a sealed-bid, uniform-price, K-units auction, debt is optimal. Liu [2016] follows a mechanism design approach similar to the one proposed in this paper and study optimal auctions when the issuer is constrained to sell equity securities. Liu and Bernhardt [2021] provide sufficient conditions under which equity plus cash auctions achieve optimality in the context of target-initiated takeovers. More recently, Yuan [2020] tackles a similar problem wherein multiple issuers compete by selling securities to informed liquidity suppliers. Building on the fact that competition among sellers leads to the winner's curse, she finds that there exists an equilibrium where all issuers sell debt securities. We show that the optimality of debt occurs even in the absence of competition. In our environment, the issuer has all the bargaining power and optimally designs a menu of debt contracts. We show that the optimality of debt originates from the fact that it allows the issuer to minimize informational rents.

Our paper is also related to the emerging literature on information and security design. Yang [2020] studies a security design problem wherein the liquidity supplier can acquire costly information. He shows that a debt contract is uniquely optimal and minimizes the incentives

[9] Some recent work within this areas of study include Malenko and Tsoy [2020] and Lee and Rajan [2018]. They explore optimal security design under robust requirements.

[10] They refer to auctions designed by the issuer as formal auctions. They also study games where the bidders have the freedom to bid with arbitrary securities, which they dub informal auctions.

[11] A general symmetric mechanism (GSM) is a symmetric incentive-compatible mechanism in which the highest type wins and pays a security chosen at random from a given set, S. The randomization can depend on the realization of types but not on the identity of the bidders.

to produce private information. In an informed issuer model, Daley et al. [2022] show that reducing the degree of asymmetric information between the issuer and the liquidity supplier leads the former to issue information-sensitive securities. Vanasco [2017], Szydlowski [2021] and Inostroza and Tsoy [2022] study the case where the issuer can design both the security and the information structure. Vanasco [2017] studies the case where issuer chooses both the security and the effort to improve the quality of the asset. She demonstrates that the adverse selection induced by the issuer's superior information mitigates the issuer's moral hazard problem when monitoring the quality of the pool of assets. Szydlowski [2021] shows that if the issuer's objective consists of raising a prespecified amount of funds, she is indifferent between all the securities yielding the same payoff. Inostroza and Tsoy [2022] show that when the issuer designs both the security and information structure, information-sensitive securities dominate debt instruments and pure equity maximizes the issuer's payoff.

The paper is also related to the literature on information orders and monotone comparative statics under uncertainty. Quah and Strulovici [2009] show that the Lehmann [1988] order is closely related to a natural order on utility functions, namely, the so-called interval dominance order. We show that the geometry of the optimal securities implies that informational rents are ranked according to the interval dominance order and leverage this to show that the optimal mechanism is monotone in the Lehmann [1988] order. Persico [2000] and Ganuza and Penalva [2010] study the effect of increasing the accuracy of the agents' signals in auctions. Kim [1995] and Jewitt [2007] study the effect of increasing the informativeness of agents' signals in moral hazard problems, whereas Dewatripont et al. [1999] do so using career concerns model. More recently, Mekonnen and Vizcaíno [2022] have studied comparative statics of agents' optimal distributions of actions in Bayesian games when the informativeness of their signals increases. To the best of our knowledge, ours is the first paper to perform comparative statics in the context of optimal security design.

2 The Model

2.1 Security Design

The economy consists of an issuer (she) and a liquidity supplier (he). The issuer owns a risky asset that delivers a stochastic cashflow $\boldsymbol{y} \in \mathbb{R}_+$. There are two periods, $t \in \{1, 2\}$. In period 1, the issuer may sell a claim $s(y)$ on the asset's period 2-cashflows to the liquidity supplier at a price p. In period 2, the asset delivers the stochastic cashflow $\boldsymbol{y} \in \mathbb{R}_+$, and the liquidity supplier obtains $s(y)$.

Securities. The issuer is free to design any arbitrary security satisfying limited liability and monotonicity in the asset's cashflows.[12] Therefore, the set of available securities is given by the following:[13]

$$\mathcal{S} \equiv \{s : \mathbb{R}_+ \to \mathbb{R}_+ \quad \text{s.t:} \quad \begin{array}{l} (\text{LL}):\ 0 \leq s(y) \leq y,\ \forall y \geq 0 \\ (\text{M}):\ s \text{ is nondecreasing}\}. \end{array}$$

Information. In period 1, the liquidity supplier observes his type, $\boldsymbol{\omega} \in \{\omega_n\}_{n=1}^{N}$, which is private information. We denote by $F(\omega, y)$ the joint distribution of $\boldsymbol{\omega}$ and $\boldsymbol{y}$. We denote by Φ the marginal distribution of ω; for notational ease, we let $\Phi_n \equiv \mathbb{P}\{\boldsymbol{\omega} \leq \omega_n\}$ and $\phi_n \equiv \mathbb{P}\{\boldsymbol{\omega} = \omega_n\}$. The conditional distribution of cashflows is ordered according to MLRP. That is, $F(y|\boldsymbol{\omega} = \omega_i) \succ_{\text{MLRP}} F(y|\boldsymbol{\omega} = \omega_j)$ for all $i, j \in \{1, ..., N\}$ with $i > j$. [14] We further assume that $\mathbb{E}(\boldsymbol{y}|\omega = \omega_N) < \infty$.

Preferences. The liquidity supplier is risk-neutral. The valuation, in monetary terms, of a future cashflow x, for liquidity supplier ω_n is given by

$$u(x, \omega_n) \equiv \varphi(\omega_n)\, x + \nu(\omega_n).$$

where $\varphi(\omega_i), \nu(\omega_i)$ are both nonnegative and nondecreasing. The case where $\varphi(\omega) = 1$ and $\nu(\omega) = 0$ for all ω implies that heterogeneity only arises from different degrees of optimism about future cashflows (i.e., *belief heterogeneity*). The case where φ or ν are not constant corresponds to the case wherein higher types have a more productive technology that yields larger returns (i.e., *payoff heterogeneity*).

Therefore, the expected utility of liquidity supplier ω_n from buying security $s(\cdot)$ at price p is given by the following:

$$\begin{aligned} \mathbb{E}(u(s, \omega_n)|\omega_i) - p &= \int_{\mathbb{R}_+} u_n(s(y))\mathrm{d}F(y|\omega_n) - p \\ &= \int_{\mathbb{R}_+} (\varphi_n s(y) + \nu_n)\mathrm{d}F(y|\omega_n) - p \end{aligned}$$

where we write, for brevity, $u_n(s) = u(s, \omega_n)$, $\varphi_n = \varphi(\omega_n)$, and $\nu_n = \nu(\omega_n)$.

[12] In the absence of monotonicity, the issuer has the option of requesting (risk free) credit to a third party to boost cashflows and thus decrease the amount owed to liquidity suppliers

[13] The literature usually imposes $y - \tilde{s}(y)$ nondecreasing. The issuer would otherwise have incentives to *burn* cashflows to increase her payoff. Our results do not require this assumption, but the optimal security does satisfy the property.

[14] This is equivalent to the conditional probability density function $f(y|\omega)$ satisfying log-supermodularity.

The issuer, on the other hand, has stringent liquidity needs and therefore has a low discount factor. For ease of exposition, we assume that she maximizes the amount of funds raised completely discounting future payoffs from asset's cashflows.

Mechanisms. Without loss of generality, we restrict attention to incentive-compatible, direct mechanisms. The issuer asks the liquidity supplier to report his type and offers and allocation and price given by $\mathcal{M} = \{s_n(y), p_n\}_{n=1}^N$, where s_n and p_n represent the security and the price offered to a liquidity supplier reporting ω_n, respectively. The issuer has all the bargaining power and designs $\mathcal{M}$.

Let $U_{\mathcal{M}}(\omega_i; \omega_j)$ represent the expected utility of a liquidity supplier whose true type is ω_j and reports ω_i under mechanism $\mathcal{M}$:

$$\begin{aligned} U_{\mathcal{M}}(\omega_i; \omega_j) &\equiv \int_{\mathbb{R}_+} u_j(s_i(y))\, \mathrm{d}F_j(y) - p_i \\ &= \mathbb{E}_j(u_j(s_i(y))) - p_i, \quad \forall i, j \in \{1, ..., N\}. \end{aligned}$$

We say that a mechanism $\mathcal{M}$ is *feasible* if it satisfies (i) individual rationality:

$$[\mathrm{IR}_i]: \quad U_{\mathcal{M}}(\omega_n; \omega_n) \geq 0, \quad \forall n \in \{1, ..., N\},$$

and (ii) incentive compatibility:

$$[\mathrm{IC}_{i,j}]: \quad U_{\mathcal{M}}(\omega_i; \omega_i) \geq U_{\mathcal{M}}(\omega_j; \omega_i), \quad \forall i, j \in \{1, ..., N\}.$$

The issuer, who has pressing liquidity needs, fully discounts period 2-cashflows and therefore maximizes revenue $\sum_{i=1}^{n} \phi_i p_i$ among all feasible mechanisms.[15]

3 The Optimality of Debt Menus

In this section, we show that the optimal feasible mechanism consists of a *menu of debt securities*. To prove this result, we first establish some basic properties that mirror standard results in the screening literature. At the optimal mechanism, (i) the worst type obtains a nil payoff (*no rents at the bottom*), (ii) the highest type purchases a pure equity claim (*no distortion at the top*), and (iii) all liquidity supplier types $\omega > \omega_1$ obtain (strictly) positive informational rents (*impossibility of full extraction*).

[15] The results below can be extended for small discount factors $\delta \in (0, 1)$. For clarity, we tackle the case where $\delta = 0$.

Proposition 1. *For any feasible mechanism $\mathcal{M} = \{s_n(\cdot), p_n\}_{n=1}^{N}$, the following properties are true:*

1. *For any $i, j \in \{1, ..., N\}$ with $i < j$, $U_{\mathcal{M}}(\omega_j; \omega_j) > U_{\mathcal{M}}(\omega_i; \omega_i) \geq 0$.*

2. *If $[IR_1]$ does not bind, then $\mathcal{M}$ is strictly dominated.*

3. *If $s_N(y) \neq y$ for all $y \in \mathbb{R}_+$, then $\mathcal{M}$ is strictly dominated.*

Because of the issuer's liquidity constraints, full extraction is *impossible* in our context. Indeed, property 1 in Proposition 1 implies that at any incentive-compatible and individually rational mechanism, all agents $\omega > \omega_1$ earn positive informational rents. This result contrasts with the famous results by Crémer et al. [1987] and Crémer and McLean [1988] showing that when the issuer's and liquidity supplier's information signals are correlated, the issuer can capture all the surplus. In our case, $\boldsymbol{y}$ and $\boldsymbol{\omega}$ are correlated; yet the structure of the problem prevents the issuer from appropriating the whole surplus. The assumption that the issuer faces stringent liquidity needs imposes two frictions that prevent her from capturing all the liquidity supplier rents. First, we assume that the issuer is liquidity constrained and therefore needs to raise cash with urgency in period 1 and cannot wait until period 2. Second, we assume that the issuer sells an *asset* and that therefore, there is the gradual resolution of uncertainty as the asset's cashflows $\boldsymbol{y}$ materialize later in the game, i.e., at $t = 2$. These two observations together imply that the price p paid by the liquidity supplier cannot depend on the future realizations of the asset's cashflows. Remarkably, either belief or payoff heterogeneity on their own are enough to imply the results above. That is, even if the type ω is noninformative about cashflows or irrelevant for the liquidity supplier's preferences over cashflows, there will be informational rents for all but the lowest type.

Properties 2 and 3 are standard. Property 2 follows from the fact that the lowest type, ω_1, obtains the lowest informational rents (property 1). If this type obtains positive rents under a given mechanism, the issuer can increase all prices $\{p_n\}_n$ by the same amount until $[\mathrm{IR}_1]$ binds. Finally, property 3 obtains from the fact that the highest type, ω_N, is the agent who values the asset the most. It is therefore optimal not to distort the allocation designed for him.

3.1 Relaxing Incentive Constraints with Debt

The next property is instrumental in proving the subsequent results.

Definition 1. [Single crossing from below/above] We say that a function $h : \mathbb{R}_+ \to \mathbb{R}$ satisfies the *single crossing from below* (SCFB) property if there exists $y_0 \in \mathbb{R}_{++}$ so that

$h(y) \leq 0$ for any $y < y_0$ and $h(y) \geq 0$ for any $y \geq y_0$. We say that h satisfies *strict single crossing from below* (SSCFB) if, in addition, the sets $\{y \in \mathbb{R}_+ : h(y) < 0\}$ and $\{y \in \mathbb{R}_+ : h(y) > 0\}$ have a positive (Lebesgue) measure. Similarly, we say that h satisfies *single crossing from above* (SCFA) or *strict single crossing from above* (SSCFA) if $-h$ satisfies SCFB or SSCFB, respectively.

Lemma 1. *Suppose that $(\boldsymbol{y}, \boldsymbol{\omega})$ satisfy the MLRP and that h satisfies the SCFB. If, for some $\omega' \in \Omega$, $\int_0^\infty h(y)\, dF(y|\omega') \geq 0$, then necessarily,*

$$\int_0^\infty h(y)\, dF(y|\omega'') \geq 0, \ \forall \omega'' > \omega'.$$

If h satisfies SSCFB, then the second inequality is strict.

We now show that downward incentive constraints are maximally relaxed by assigning a debt contract to the lowest type. Intuitively, among all securities that provide the same expected payoff (according to the beliefs of an arbitrary type), debt is the least preferred by higher types as it minimizes payments for high realizations of y (i.e., the upside). Therefore, debt contracts minimize the informational rents captured by high types.

Lemma 2. *Consider an arbitrary mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$. Suppose that there exists $i \in \{1, ..., N\}$ such that $[IC_{j,i}]$ holds for all $j > i$. Then, the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ with $(\hat{s}_n, \hat{p}_n) = (s_n, p_n)$ for all $n \neq i$, and $(\hat{s}_i(y), \hat{p}_i) = (\min\{y, D_i\}, p_i)$, with D_i defined such that $\mathbb{E}_i(\min\{\boldsymbol{y}, D_i\} - s_i) = 0$, satisfies $[IC_{j,i}]$ for all $j > i$. Moreover, whenever $s_i \neq \hat{s}_i$, $[IC_{j,i}]$ is slack for all $j > i$.*

Proof. Consider the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^N$ described above. We show that the new mechanism relaxes incentive compatibility constraints. In fact, for any $j > i$, type ω_j's payoff from mimicking type ω_i decreases under $\hat{\mathcal{M}}$. To see this more clearly, observe that $s_i - \hat{s}_i$ satisfies SCFB in y. This fact, coupled with the log-supermodularity of $f(y|\omega)$ implies that the conditions in lemma 1 apply. Hence,

$$\mathbb{E}_i(s_i - \hat{s}_i) = 0 \Rightarrow \mathbb{E}_j(s_i - \hat{s}_i) \geq 0,$$

with strict inequality whenever s_i is not a debt contract (i.e., $s_i \neq \hat{s}_i$ over a set with F-positive measure). This, in turn, implies that, for any $j > i$, type ω_j weakly prefers contract s_i over $\hat{s}_i$ as $\mathbb{E}_j(u_j(s_i)) > \mathbb{E}_j(u_j(\hat{s}_i))$. Thus, the incentive compatibility constraints $[\mathrm{IC}_{j,i}]$ are relaxed for all $j > i$ under $\hat{\mathcal{M}}$. □

Lemma 2 establishes that downward incentive constraints can be relaxed by allocating debt contracts to low types. This does not necessarily imply that debt contracts must be part of the optimal mechanism, as upward incentive constraints might be compromised by changing the securities for debt. In fact, as the steps leading to the proof of lemma 2 suggest, among all the securities s_i that provide type ω_i the same expected payoff $\mathbb{E}_i(s_i)$, the debt contract $\hat{s}_i(y) = \min\{y, D_i\}$ is the one that provides maximal incentives to mimic for types $h < i$. The next section shows that despite this potential conflict, the issuer optimally sells debt securities to *all* types. We note that for the lowest type, ω_1, for which there is no upward incentive constraints, lemma 2 implies that the optimal security must be debt.

Corollary 1. *Any feasible mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^{N}$ for which there does not exist $D > 0$ such that $s_1 = \min\{y, D\}$ for (F-almost all) y (i.e., s_1 is not a debt contract), is strictly dominated by another feasible mechanism $\hat{\mathcal{M}} = \{\hat{s}_n, \hat{p}_n\}_{n=1}^{N}$ with $\hat{s}_1(y) = \min\{y, D_1\}$ for some $D_1 > 0$.*

3.2 Oriented Mechanisms

Definition 2. We say that a feasible mechanism $\mathcal{M} = \{s_i, p_i\}_{i=1}^{N}$ is *oriented* if, for any $j, k \in \{1, ..., N\}$, with $k > j$, $\mathbb{E}_k(s_k - s_j) \geq 0$.

The concept of orientation is closely related to the *monotonicity* condition in the screening literature. In the current environment, with a richer allocation space (i.e., the set of all monotone functions satisfying limited liability), the notion that higher types should obtain a larger allocation is captured by the requirement that higher types should prefer their securities according to their own beliefs, rather than the ones designed for lower types. That is, for any $k > j$, $\mathbb{E}_k(s_k - s_j) \geq 0$.

We argue that it is without loss of optimality to restrict attention to oriented mechanisms. To see this more clearly, consider an arbitrary feasible mechanism, $\mathcal{M} = \{s_n, p_n\}_{n=1}^{N}$, for which there exist j, k with $k > j$, $\mathbb{E}_k(s_k - s_j) < 0$. Then, $p_j > p_k$ since otherwise, the incentive constraint $[\mathrm{IC}_{k,j}]$ would be violated. This, in turn, means that there exists an alternative mechanism, $\hat{\mathcal{M}}$, which is identical to $\mathcal{M}$ except for the fact that it offers contract $(\hat{s}_k, \hat{p}_k) = (s_j, p_j)$ to type ω_k. In other words, $\hat{\mathcal{M}}$ deletes the contract offered to type ω_k under mechanism $\mathcal{M}$ and replaces it with the contract offered to type ω_j. Note that $\mathbb{E}_k(s_k - s_j) < 0$, together with the assumption that $F(y|\omega_k) \succ_{\mathrm{MLRP}} F(y|\omega_j)$ and the monotonicity of $u(x, \cdot)$ in ω, jointly imply that

$$U_{\hat{\mathcal{M}}}(\omega_k; \omega_k) = \mathbb{E}_k(u_k(s_j)) - p_j > \mathbb{E}_j(u_j(s_j)) - p_j \geq 0.$$

Therefore, the new mechanism satisfies [IR_k]. Furthermore, all other incentive constraints remain uncompromised under the new mechanism as the contract (s_j, p_j) was already available under $\mathcal{M}$. The new mechanism $\hat{\mathcal{M}}$ thus strictly raises more funds than $\mathcal{M}$. We note that in contrast to the standard argument wherein incentive compatibility alone implies monotonicity, in the current environment, orientation is a consequence of both incentive compatibility and optimality.

Lemma 3. [Oriented Mechanisms] *Any feasible mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$ that is not oriented is strictly dominated by another feasible, oriented mechanism.*

Equipped with the last result, we are now ready to present the main result of this section.

Theorem 1. *There exists an optimal mechanism, $\mathcal{M}^\star = \{s_n^\star, p_n^\star\}_{n=1}^N$, with the following properties. For any $n \leq N-1$, there exists $D_n > 0$ so that $s_n^\star(y) = \min\{y, D_n\}$ and $s_N^\star(y) = y$, for all y.*

We provide a sketch of the proof below, which is divided into 3 steps.

In Step 1, we show that for any oriented mechanism, when security s_i is a debt contract, then for any pair of types above i, $k > j > i$, the downward incentive constraints [$\text{IC}_{k,j}$] and [$\text{IC}_{j,i}$] imply the *global* downward constraint [$\text{IC}_{k,i}$]. The global downward incentive constraints are therefore redundant.

The intuition behind Step 1 is as follows. When s_i is a debt contract, then for any j, k with $k > j > i$, if type ω_j prefers his security s_j over s_i, then type ω_k must also prefer s_j over s_i. The standard argument used in the screening literature follows from the monotonicity of the allocation rule (which is implied by incentive compatibility) and the supermodularity of the liquidity supplier's payoff in both his type and the allocation. In the current setting, it is not clear what the correct notion should be of the supermodularity of the liquidity supplier's payoff in his type and the security, i.e., an infinite-dimensional object. Instead, we leverage the fact that when s_i is a debt contract, s_i crosses other securities from above. Lemma 1 then implies that for any s_j with $\mathbb{E}_j(s_j - s_i) \geq (>)\, 0$ (which is always true for oriented mechanisms), it must be the case that $\mathbb{E}_k(s_j - s_i) \geq (>)\, 0$. This in turn means that if type ω_j, according to his beliefs, does not mimic type ω_i, then neither does type ω_k.

In Step 2, we use the result obtained in Step 1 to show that when s_i is a debt contract, then the local downward incentive constraint [$\text{IC}_{i+1,i}$] must bind. The argument is obtained by contradiction. Suppose that for some $i \in \{1, ..., N-1\}$, [$\text{IC}_{i+1,i}$] is satisfied with slackness; then the argument in Step 1 implies that for any $k \geq i+1$, the incentive constraints [$\text{IC}_{k,i}$] are slack. The issuer can therefore increase the transfers p_k for all types $k \geq i+1$ without spoiling incentive compatibility. We conclude that at any undominated mechanism, constraint $[\text{IC}_{i+1,i}]$ must bind.

Finally, in Step 3, we show that for any oriented mechanism, $\mathcal{M}$, if we let $i+1 \leq N$ be the smallest type for whom his security s_{i+1} is not a debt contract, we can improve upon the issuer's payoff by swapping security s_{i+1} for the payoff-equivalent debt contract s_{i+1}^{D} (according to ω_{i+1}'s beliefs), without spoiling the upward incentive constraints, $[\text{IC}_{h,i+1}]$, for $h \leq i$. That is, if we let s_{i+1}^{D} be a debt contract with $\mathbb{E}_{i+1}\left(s_{i+1}^{D} - s_{i+1}\right) = 0$, the issuer can relax the downward incentive constraints without spoiling the upward incentive constraints. Indeed, we observe that

$$\begin{aligned} p_{i+1} - p_i &= \mathbb{E}_{i+1}\left(u_{i+1}\left(s_{i+1}\right) - u_{i+1}\left(s_i\right)\right) \\ &= \mathbb{E}_{i+1}\left(u_{i+1}\left(s_{i+1}^{D}\right) - u_{i+1}\left(s_i\right)\right) \\ &> \mathbb{E}_{i}\left(u_{i+1}\left(s_{i+1}^{D}\right) - u_{i+1}\left(s_i\right)\right), \end{aligned} \tag{1}$$

where the first equality follows from the result in Step 2, and the second equality is obtained by the construction of s_{i+1}^{D}. The inequality, in turn, follows from noting that the mechanism $\mathcal{M}$ is oriented. Therefore,

$$\mathbb{E}_{i+1}\left(s_{i+1}^{D}\right) = \mathbb{E}_{i+1}\left(s_{i+1}\right) > \mathbb{E}_{i+1}\left(s_i = \min\left\{\boldsymbol{y}, D_i\right\}\right).$$

This means that $s_{i+1}^{D} - s_i$ is nondecreasing, which, coupled with the MLRP assumption and the monotonicity of $u\left(s, \omega\right)$ in ω, implies the result.

Inequality (1) implies that type ω_i does not mimic type ω_{i+1} when the latter's security is replaced by its payoff-equivalent debt contract; hence, the upward incentive constraint $[\text{IC}_{i,i+1}]$ is satisfied. We show in the Appendix that a similar argument can be made to show that for any $h < i$, $[\text{IC}_{h,i+1}]$ is also satisfied in the new mechanism.

Theorem 1 shows that to maximize the funds raised from an informed investor, restricting attention to menus of debt contracts does not result in loss of optimality. However, restricting attention to debt securities substantially simplifies the problem. Indeed, similar to the classical screening problem (e.g., Mussa and Rosen [1978]), the allocation space is now captured by a single-dimensional variable, namely, the face value of the debt contract designed for each type.

Interestingly, theorem 1 further implies that exploiting the possibility of introducing securities with different degrees of information sensitivity in the menu is *ineffective* at screening the liquidity supplier's private information. In subsection 3.5, we formalize the intuition that the liquidity supplier's informational rents are monotone in the securities' information sensitivity.

3.3 Characterization of the Optimal Contract

Equipped with the result in theorem 1, we proceed to characterize the optimal mechanism with securities. The previous results imply that we can restrict attention to menus of debt securities, that is, mechanisms satisfying that, for any n, $s_n = \min\{y, D_n\}$ for all y, where $D_n > 0$ and is increasing in n, and $D_N = +\infty$. Moreover, Step 2 in the proof theorem 1 implies that, without loss, we can restrict attention to mechanisms satisfying that, for any $j > 1$, $[\mathrm{IC}_{j,j-1}]$ binds and, because of proposition 1, further satisfying $p_1 = \mathbb{E}(\min\{y, D_1\})$. Together, these properties imply the following characterization.

Lemma 4. *The issuer's problem can be reformulated as*

$$\begin{aligned} \max_{\{D_n\}_{n=1}^N} \quad & \sum_{n=1}^N \phi_n \int_0^\infty u_n(\min\{y, D_n\}) \left(1 - \left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f(y|\omega_{n+1}) - f(y|\omega_n)}{f(y|\omega_n)}\right)\right) \mathrm{d}F(y|\omega_n) \\ \text{s.t.} \quad & D_n \text{ nondecreasing in } n. \end{aligned} \tag{2}$$

3.3.1 (Ruling out) Bunching

As in the classical problem, there exist regularity conditions under which the optimal mechanism does not involve bunching. In this case, these conditions guarantee that face values D_n are strictly increasing in n. Whenever this is the case, the optimal face value D_n corresponds to the one obtained under the the pointwise solution to (2) disregarding the monotonicity constraint.

Assumption 1. *The inverse hazard rate $\frac{1-\Phi(\omega)}{\phi(\omega)}$ is nonincreasing in ω.*

Assumption 2. *The conditional distribution function $F(y|\omega)$ is convex in ω.*

Assumption 1 is standard in the literature of mechanism design (see, e.g., Myerson [1981]). Intuitively, it captures the idea that the relative mass of liquidity suppliers above a given type decreases and therefore the informational rents that need to be forgone decrease. Assumption 2, on the other hand, requires the differential

$$\Delta_n(y) \equiv 1 - F(y|\omega_{n+1}) - (1 - F(y|\omega_n))$$

to increase at a decreasing pace as n increases. Intuitively, this captures the idea that the informational rents associated with larger cashflow realizations y do not increase too fast as the liquidity supplier's valuation increases. This effect is reminiscent to the requirement that the marginal value of increasing the buyer's allocation to be concave in his type (see, for example, Fudenberg and Tirole [1991]). In our case, where the liquidity supplier's allocation is completely determined by the face value of his debt contract, D_n, the marginal value of increasing type ω_n's allocation is given by

$$\begin{aligned}\frac{\partial}{\partial D_n}\int_{\mathbb{R}_+} u_n\left(\min\{y, D_n\}\right)\mathrm{d}F\left(y|\omega_n\right) &= \varphi_n \frac{\partial}{\partial D_n}\left(\int_0^{D_n} y\mathrm{d}F\left(y|\omega_n\right) + D_n\left(1 - F\left(D_n|\omega_n\right)\right)\right) \\ &= \varphi_n \cdot \left(1 - F\left(D_n|\omega_n\right)\right).\end{aligned}$$

Thus, assumption 2 implies that the marginal value of increasing the liquidity supplier's allocation is concave in ω_n. Our next result shows that these two assumptions guarantee that the monotonicity constraint is nonbinding.

Lemma 5. *Suppose that assumptions 1 and 2 hold. Then, for every $n \in \{1, ..., N\}$, D_n^* is monotone in n and is implicitly characterized by the solution to*

$$\int_{D_n}^{\infty}\left(1 - \left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f\left(y|\omega_{n+1}\right) - f\left(y|\omega_n\right)}{f\left(y|\omega_n\right)}\right)\right) dF\left(y|\omega_n\right) = 0.$$

3.4 Continuum of types

In this section, we extend the analysis to case of a continuum of liquidity supplier types. We assume that $\Omega = [\underline{\omega}, \bar{\omega}] \subseteq \mathbb{R}_+$. For simplicity, we focus on the case where $u\left(x, \omega\right) = x$, i.e., where the liquidity supplier types are heterogeneous only in beliefs. The issuer designs a mechanism $\{s(\cdot|\omega), p\left(\omega\right)\}_{\omega\in\Omega}$ where, for each ω, $s\left[\omega\right] \in \mathcal{S}$ and $p\left(\omega\right) \in \mathbb{R}$. The payoff of a liquidity supplier of type ω who reports to be $\hat{\omega}$ is then given by

$$U_{\mathcal{M}}\left(\hat{\omega}; \omega\right) = \int_0^{\infty} s(y|\hat{\omega})\mathrm{d}F(y|\omega) - p\left(\hat{\omega}\right)$$

A mechanism is said to be *feasible* if it satisfies the *Incentive Compatibility* and *Individually Rational* constraints:

$$[\text{IC}]{:}\omega \in \arg\max_{\hat{\omega}\in\Omega} U_{\mathcal{M}}\left(\hat{\omega}; \omega\right), \ \forall \omega \in \Omega$$

$$[\text{IR}]{:}U_{\mathcal{M}}\left(\omega; \omega\right) \geq 0, \ \forall \omega \in \Omega.$$

The next theorem shows that the main qualitative properties found in the case of finitely many types extend to the current environment. The characterization below is instrumental for the comparative statics exercise we perform in the next section.

Theorem 2. *Suppose that Assumptions (1) and (2) hold. Further, asssume that,for all* $\omega \in \Omega$, $\mathbb{E}\left\{\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial \omega}}{f(\boldsymbol{y}|\omega)}|\omega\right\} < \infty$. *Then, the revenue maximizing mechanism is characterized by a menu of debt securities given by*

$$s^*(y|\omega) = \min\{y, D_*(\omega)\}$$

where $D_*(\omega)$ *is defined as the unique solution of*

$$\int_{D_*(\omega)}^{\infty} \left\{1 - \frac{1-\Phi(\omega)}{\phi(\omega)} \frac{\frac{\partial}{\partial \omega} f\left(y|\omega\right)}{f\left(y|\omega\right)}\right\} \mathrm{d}F\left(y|\omega\right) \quad = \quad 0. \tag{3}$$

3.5 Information Sensitivity and Informational Rents

Definition 3. We sat that security $s'' \in \mathcal{S}$ is more information sensitive than $s' \in \mathcal{S}$, if $s'' - s'$ has the SSCFB property.

Intuitively, more information sensitive securities increase the exposure of the security owner to the asset's cash flow realizations. That $s'' - s'$ has the SSCFB property implies that for low cash flow realizations $s''\left(y\right) < s'\left(y\right)$ the security owner has a smaller payoff under s'' than under s', whereas the opposite holds true, for high cash flow realizations.

We argue that the liquidity supplier's informational rents are closely related to the securities' *information sensitivity.* Indeed, the steps leading to the result in Theorem 2, imply that, for any feasible menu of securities $\left\{s\left[\omega\right]\right\}_{\omega \in \Omega}$, the issuer's objective can be expressed as

$$\int_{\Omega} \left(\underbrace{\mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right)|\omega\right\}}_{\text{Gains from trade}} - \underbrace{\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right) \mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right) \left(\frac{\frac{\partial}{\partial \omega} f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)|\omega\right\}}_{\text{Informational Rents}} \right) \mathrm{d}\Phi\left(\omega\right).$$

For any $\omega \in \Omega$, and any feasible mechanism $\mathcal{M} = \left\{s\left[\omega\right], p\left(\omega\right)\right\}_{\omega \in \Omega}$, let

$$I^{\mathcal{M}}\left(\omega\right) \equiv \left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right) \mathbb{E}\left\{s\left(\boldsymbol{y}|\omega\right) \left(\frac{\frac{\partial}{\partial \omega} f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)|\boldsymbol{\omega} = \omega\right\}$$

represent type ω's *informational rents* under mechanism $\mathcal{M}$. This expression represents the rents that need to be paid to every type above ω to not pretend to be type ω. When the liquidity supplier's type is commonly known, the issuer maximizes the expected value of the security sold to the liquidity supplier. In that case, the issuer optimally sells the whole asset to the liquidity supplier, who is the efficient holder of the asset, and extracts all rents from him. Instead, when the liquidity supplier has private information, the issuer needs to leave informational rents to the liquidity supplier, which entails distorting allocative efficiency. As theorem 1 dictates, the issuer is better off by holding on to the high realizations of the asset's cashflows despite the fact that she does not assign any value to them (recall that she fully discounts future realizations). Doing so allows her to minimize the informational rents she leaves to the highest types.

To ellaborate further, consider the standard screening problem, wherein a buyer purchases an asset from an issuer constrained to sell pure equity (i.e., the whole asset). In that environment, it is without loss to restrict attention to direct mechanisms specifying, for each type ω, the probability of trading, $\alpha(\omega) \in [0,1]$, and the transfers $p(\omega) \geq 0$. That environment is equivalent to an issuer restricted to selling equity stakes (e.g., stocks), i.e., $\left\{s^E(y|\omega) = \alpha(\omega)\cdot y\right\}_{\omega\in\Omega}$, at prices $\{p(\omega)\}_{\omega\in\Omega}$.[16]

Selling information sensitive securities (such as these equity stakes), however, leaves large informational rents to the liquidity supplier. The issuer can minimize these rents by reducing the information sensitivity of the securities in her menu. Indeed, consider any feasible mechanism $\mathcal{M}^E = \left\{s^E[\omega], p^E(\omega)\right\}_{\omega\in\Omega}$, with $s^E(y|\omega) = \alpha(\omega)\cdot y$, for all ω. Construct an alternative mechanism with information insensitive securities, $\mathcal{M}^{II} = \left\{s^{II}[\omega], p^{II}(\omega)\right\}_{\omega\in\Omega}$, where for each ω, $\mathbb{E}\left\{s^{II}(\boldsymbol{y}|\omega)|\omega\right\} = \mathbb{E}\left\{s^E(\boldsymbol{y}|\omega)|\omega\right\}$, and $s^{II}[\omega]$ is *less information sensitive* than $s^E[\omega]$; that is, $s^{II}[\omega] - s^E[\omega]$ is SSCFA. Then,

$$\begin{aligned}
I^{\mathcal{M}^{II}}(\omega) - I^{\mathcal{M}^E}(\omega) &= \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\frac{\partial}{\partial\omega}f(y|\omega)\mathrm{d}y \\
&= \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\lim_{\delta\downarrow 0}\left(\frac{f(y|\omega+\delta)-f(y|\omega)}{\delta}\right)\mathrm{d}y \\
&< 0,\ \forall\omega\in\Omega
\end{aligned}$$

where the inequality follows from the construction of $\left\{s^{II}[\omega]\right\}_{\omega\in\Omega}$ and the inequality is a direct implication of Lemma 1 which jointly imply that, for any $\delta > 0$,

$$\begin{aligned}
\int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\left(\frac{f(y|\omega+\delta)-f(y|\omega)}{\delta}\right)\mathrm{d}y &= \int_0^\infty \left(s^{II}(y|\omega) - s^E(y|\omega)\right)\frac{f(y|\omega+\delta)}{\delta}\mathrm{d}y \\
&< 0.
\end{aligned}$$

We conclude that, regardless of the liquidity supplier's private signal, informational rents are

[16] The proof of Example 1 contains the derivation of the optimal mechanism restricted to this class of securities.

strictly smaller when the issuer offers more informational insensitive secutiries. The fact that these latter securities are constructed keeping the liquidity supplier's valuation unchanged, then implies that the issuerstrictly benefits form her ability to design securities which are less sensitive to the liquidity supplier's information. This prediction stands in sharp contrast with the typical finding in the security design literature according to which, when trading, the informed agent obtains information sensitive securities.[17]

4 Information and Monotone Comparative Statics

Motivated by the important role of the liquidity supplier's private information in determining the optimal mechanism, we explore how changes in the quality of the liquidity supplier's private signal affects the issuer's optimal securities. To simplify the derivations, we focus henceforth on the case with a continuum of types discussed above.

We show below that as the liquidity supplier's private signal becomes more *accurate* (Lehmann [1988]), the issuer optimally offers debt instruments with smaller face values. Perhaps surprisingly, however, the geometry of the optimal securities, i.e., the concavity of debt on the asset's future cashflows, implies that improving the accuracy of the liquidity supplier's information increases the his valuation for these securities. This effect is similar to a reduction in uncertainty for a risk-averse agent, which increases his utility. The overall effect on both surplus and the agents' payoffs is thus ambiguous.

Below, we provide an example that showcases the novel effect described above and challenges some economic intuitions from the classical screening problem.

Example 1. Suppose that $\boldsymbol{\omega} \sim U[0,1]$ and that for $\theta \in [1/3, 1]$, $\boldsymbol{y}^{\theta}$ is constructed as follows. With probability θ, $\boldsymbol{y}^{\theta} = \boldsymbol{\omega}$, whereas with probability $1-\theta$, $\boldsymbol{y}^{\theta} \sim U[0,1]$, independent of $\boldsymbol{\omega}$. That is,

$$F^{\theta}(y|\omega) = \Pr\left\{\boldsymbol{y}^{\theta} \leq y | \boldsymbol{\omega} = \omega\right\} = \theta \cdot 1\{\omega \leq y\} + (1-\theta)\, y.$$

The following properties are true:

(a) For any $\theta'', \theta' \in (0,1)$, $\theta'' > \theta'$, $F^{\theta''}$ is more accurate (Lehmann [1988]) than $F^{\theta'}$.[18]

(b) Suppose the issuer is restricted to use linear securities, i.e., for all $\omega \in [0,1]$,

$$s[\omega] \in \mathcal{S}^{E} \equiv \{s \in \mathcal{S} : \exists \alpha > 0, s(y) = \alpha y, \forall y \in [0,1]\}.$$

[17] See, e.g., Gorton and Pennacchi [1990], Nachman and Noe [1994], DeMarzo and Duffie [1999], Biais and Mariotti [2005].

[18] The formal definition is provided in the next subsection.

Then, the (restricted) optimal mechanism is characterized by

$$\alpha_\theta^E(\omega) = 1\left\{\omega \geq \omega_\theta^E \equiv \max\left\{\frac{3\theta-1}{4\theta}, 0\right\}\right\}.$$

Furthermore, under this mechanism $\mathbb{E}\left\{p_\theta^E(\boldsymbol{\omega})\right\} = \frac{(1+\theta)^2}{16\theta}$.

(c) The issuer's optimal mechanism is a menu of debt contracts $\{\min\{y, D_\theta^*(\omega)\}\}_{\omega\in[0,1]}$ with face values characterized by

$$D_\theta^*(\omega) = \omega,\ \forall\omega \in [0,1], \forall\theta \in (0,1).$$

Under this mechanism, informational rents are 0 for all $\omega \in [0,1]$; Furthermore, $\mathbb{E}\{p_\theta^*(\boldsymbol{\omega})\} = \mathbb{E}\{\mathbb{E}\{\min(\boldsymbol{y},\boldsymbol{\omega})|\boldsymbol{\omega}\}\} = \frac{2+\theta}{6}$.

Example 1 underscores some fundamental differences with the case wherein the issuer is constrained to use linear instruments, a typical assumption in the screening literature. Under the restriction, the optimal mechanism consists of a posted price. The issuer sells the whole asset to all liquidity supplier types above the critical type ω_θ^E at price $\mathbb{E}\left\{\boldsymbol{y}|\omega_\theta^E\right\}$, leaving strictly positive informational rents to all types strictly above ω_θ^E. Furthermore, as the accuracy of the liquidity supplier's private information increases, the issuer increases her posted price destroying both surplus and the expected revenue.

In contrast, when the issuer can flexibly design the security, she optimally sells debt instruments to all liquidity supplier types. Interestingly, as we prove in the Appendix, under the optimal mechanism, the issuer leaves *no informational rents.* Using debt contracts allows the issuer to alleviate incentive compatibility issues and extract all the surplus generated by these securities. Strikingly, we show that as the liquidity supplier's private information becomes more accurate, the issuer *increases* her revenue. As we argue above and formally prove below, this counterintuitive result is a direct consequence of the geometry of the optimal securities. The concavity of debt implies that the liquidity supplier's valuation for these securities increases when she is faced with a reduction in uncertainty.

The example above is special in some dimensions. First, the optimal mechanism is invariant in θ. As we prove below in Theorem 3, the set of optimal face values is nondecreasing in the accuracy of the liquidity supplier's private signal. This provides a countervailing effect that reduces the issuer's expected revenue. Second, in the example, the issuer is able to extract all the rents associated with debt securities. This property need not extend to more general environments. We show that the informational rents have the single crossing property in the accuracy of the liquidity supplier's signal. This provides another countervailing effect that reduces the issuer's revenue. In Example 1, neither of the two effects are present, which leads to the surprising result that the issuer benefits and raises more funds when she faces a more informed liquidity supplier.

4.1 [Lehmann [1988]] Information Accuracy

Suppose that conditional on the liquidity supplier drawing a private type $\boldsymbol{\omega} = \omega$, the distribution of the asset's future cashflows $\boldsymbol{y}$ is drawn according to the kernel function $F(y|\omega) = \Pr\{\boldsymbol{y} \leq y|\boldsymbol{\omega} = \omega\}$. Following the tradition in the information literature, we refer to $F(y|\omega)$ as an *experiment*. The experiment F and the prior distribution of $\boldsymbol{\omega}$, Φ, uniquely determine the joint distribution

$$\mathbf{F}_{\Phi}(y, \omega) \equiv \int_{\Omega \times \mathbb{R}_+} F(y|\tilde{\omega}) \, 1\{\tilde{\omega} \leq \omega\} \, \Phi(\mathrm{d}\tilde{\omega}).$$

Throughout the analysis, we maintain the assumption that the distribution of $\mathbf{F}_{\Phi}(y, \omega)$ admits a density $\mathbf{f}_{\Phi}(y, \omega)$, which satisfies MLRP. We refer to the induced marginal distribution of the asset's cashflows as $\Psi(F, \Phi) = \mathrm{marg}_{\boldsymbol{y}} \mathbf{F}_{\Phi} \in \Delta\mathbb{R}_+$.

Below, we formally introduce a natural ordering to rank the amount of information embedded in the liquidity supplier's private type, $\boldsymbol{\omega}$, about the asset cashflows, $\boldsymbol{y}$.

Definition 4. [Lehmann [1988]] Consider two random variables $\boldsymbol{y}''$ and $\boldsymbol{y}'$ representing the asset's future cashflows, and let $F''(y|\omega)$ and $F'(y|\omega)$ be the experiments associated with them, respectively. We say that F'' is *more accurate* about $\boldsymbol{\omega}$ than F', which we write as $F'' \succeq_{\text{Lehmann}} F'$ if for any y,[19]

$$F''^{-1}(F'(y|\omega)\,|\omega) \text{ is nondecreasing in } \omega.$$

Accuracy has been used to perform monotone comparative statics in the context of auctions (Persico [2000] and Ganuza and Penalva [2010]), moral hazard (Kim [1995], Jewitt [2007]), and career concerns (Dewatripont et al. [1999]).

[19] An alternative and perhaps more fundamental definition states that any decision-maker with a (Bernoulli) utility function supermodular in the action and the underlying variable ω would prefer the information obtained by learning $\boldsymbol{y}'$ over the information obtained from $\boldsymbol{y}$. See Jewitt [2007] and Quah and Strulovici [2009] for a detailed discussion.

4.2 Accuracy, Supermodularity, and Informativeness

We argue that accuracy (Lehmann [1988]) is an appealing notion of informativeness in our environment for three reasons. First, provided that signals satisfy MLRP, accuracy is less restrictive than the Blackwell order in that it compares more signal structures.[20] Furthermore, as the next proposition shows, accuracy implies the standard notion of informativeness usually assumed in the information design literature. Second, the concept of accuracy is tightly related to the idea of interdependence. When an experiment is more accurate, the comovement between fundamentals and signals becomes stronger. Finally, the Lehmann order compares experiments (i.e., conditional distributions) as opposed to comparing joint distributions. Intuitively, we want to change the quality of the liquidity supplier's information without changing the distribution of liquidity supplier types Φ or the distribution of the asset's future cashflows Ψ. This is natural since changing the marginal Φ changes the relative likelihood of facing different liquidity supplier types, which directly changes the amount of informational rents that must be provided to the different liquidity supplier types. In turn, changing the marginal Ψ changes the quality of the underlying asset and hence the liquidity supplier's valuation for the securities offered. In what follows, then, we fix a given marginal distribution, Φ, and we study the effect of increasing the accuracy of experiment F, while keeping the marginal distribution of cashflows $\Psi(F, \Phi) = \text{marg}_{\boldsymbol{y}} \mathbf{F}_\Phi$ unchanged. The results described in our next proposition guarantee that as we increase the accuracy of the experiment F, we do not change the primitives of the issuer's problem beyond the effect induced via the accuracy of the liquidity supplier's information making the comparative statics exercise coherent.

Our next result summarizes the appeal of using accuracy as the appropriate ordering to compare the informativeness of different signals structures.

Proposition 2. *Consider an arbitrary marginal distribution $\Phi \in \Delta\Omega$, and suppose that $F'' \succeq_{Lehmann} F'$. Let $\mathbf{F}''_\Phi$ and $\mathbf{F}'_\Phi$ be the induced joint distributions and assume that $\Psi = marg_{\boldsymbol{y}} \mathbf{F}''_\Phi = marg_{\boldsymbol{y}} \mathbf{F}'_\Phi$. Then,*

(i) For any supermodular function $v(\omega, y)$,

$$\int_{\mathbb{R}_+ \times \Omega} v(y, \omega)\, \mathrm{d}\mathbf{F}''_\Phi(y, \omega) \geq \int_{\mathbb{R}_+ \times \Omega} v(y, \omega)\, \mathrm{d}\mathbf{F}'_\Phi(y, \omega).$$

In other words, $\mathbf{F}''_\Phi$ dominates $\mathbf{F}'_\Phi$ in the supermodular order, $\mathbf{F}''_\Phi \succeq_{spm} \mathbf{F}'_\Phi$.

[20] Indeed, provided that signals satisfy the MLRP, any two signals ordered according to Blackwell are also ordered according to Lehmann [1988]. When the state space is binary, both notions of informativeness coincide (see, e.g., Jewitt [2007]). More recently, Kim [2022] showed that Lehmann domination is closely related to the concept of quasigarbling, a generalization of information garbling.

(ii) $Cov_{\mathbf{F}''_{\Phi}}(\boldsymbol{\omega}, \boldsymbol{y}) \geq Cov_{\mathbf{F}'_{\Phi}}(\boldsymbol{\omega}, \boldsymbol{y})$.

(iii) Let $\boldsymbol{z}'' \equiv \mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$ *and* $\boldsymbol{z}' \equiv \mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$, *and denote by* H'' *and* H' *the respective cumulative distribution functions of* $\boldsymbol{z}''$ *and* $\boldsymbol{z}'$, *respectively. Then, for any convex function* $\gamma : \mathbb{R}_+ \to \mathbb{R}$,

$$\int \gamma(z)\, \mathrm{d}H''(z) \geq \int \gamma(z)\, \mathrm{d}H'(z).$$

In other words, $\boldsymbol{z}''$ *dominates* $\boldsymbol{z}'$ *in the convex order,* $\mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega}) \succeq_{cvx} \mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$.

Proposition 2 shows the appeal of using accuracy to rank the informativeness of different experiments. First, as an intermediate step, we recall that the Lehmann order, which compares experiments, is tightly related to the supermodular order, which in turn compares joint distributions. As explained by Meyer and Strulovici [2012], the fact that $\mathbf{F}''_{\Phi} \succeq_{\mathrm{spm}} \mathbf{F}'_{\Phi}$ implies that the degree of interdependence of $(\boldsymbol{y}, \boldsymbol{\omega})$ is larger under $\mathbf{F}''_{\Phi}$ than under $\mathbf{F}'_{\Phi}$. This means that we can interpret increments in accuracy as changes in the joint distribution of $(\boldsymbol{y}, \boldsymbol{\omega})$, which increase their degree of correlation while keeping their marginal distributions constant. Furthermore, as we show below, this property allows us to compare how the liquidity supplier's valuation for debt securities changes as we increase the accuracy his private signal.

Finally, Proposition 2 also shows that the concept of accuracy is closely related to the classical notion of informativeness in the information economics literature. Claim (iii) shows that when F'' is more accurate than F', then the random variable capturing the posterior estimates induced by learning the realization of $\boldsymbol{\omega}$ under F'', $\mathbb{E}_{\mathbf{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$ is a mean-preserving spread of the analogous random variable capturing the posterior estimates under F', $\mathbb{E}_{\mathbf{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$. In other words, when the accuracy of the experiment improves, the liquidity supplier's private information becomes more informative in the classical sense about the asset's future cashflows.

4.3 Information and Monotone Comparative Statics

In this section, we leverage the geometry of the optimal mechanism, i.e., the remarkable feature that it consists of a menu of debt contracts, to show that as the liquidity supplier's private information becomes more accurate, (i) the liquidity supplier's valuation for any debt security increases, and (ii) the issuer sells *smaller* securities, i.e., debt contracts with smaller face values. To the best of our knowledge, this is the first paper to perform monotone comparative statics using information orders in the context of security design.

Our first result formalizes the novel effect described above, according to which increasing the accuracy of the experiment, F, implies that from the point of view of the liquidity supplier, the distribution of cashflows becomes less risky. The concavity of debt on the asset's future cashflows then implies that, fixing a given security, $s(y) = \min\{y, D\}$, more accurate information improves the liquidity supplier's valuation for that security and hence the gains from trade. In other words, despite the risk neutrality of the liquidity supplier's utility function, the geometry of debt implies that increasing the accuracy of his private signal has an effect similar to a reduction in uncertainty for a risk-averse agent.

Formally, we prove below that, fixing an arbitrary menu of debt contracts (with face values monotonically ordered as implied by incentive compatibility), the gains from trade increase as the liquidity supplier's private signal becomes more accurate.

Proposition 3. *Consider an arbitrary menu of debt contracts characterized by the set of (monotone) face values* $\{D(\omega)\}_{\omega\in\Omega}$*. Suppose that* $F'' \succeq_{Lehmann} F'$*; then,* $\mathbb{E}_{\mathbf{F}''_{\Phi}}\{\min\{\boldsymbol{y}, D(\boldsymbol{\omega})\}\} \geq \mathbb{E}_{\mathbf{F}'_{\Phi}}\{\min\{\boldsymbol{y}, D(\boldsymbol{\omega})\}\}$*.*

The result is a direct consequence of property (i) in Proposition 2, above. Note that, for any nonnegative, nondecreasing function $D(\cdot)$, the function

$$\psi(y,\omega) \equiv \min\{y, D(\omega)\}$$

is supermodular in (y,ω). For any $\omega'' > \omega'$, the monotonicity of $D(\cdot)$ implies that

$$\psi(y,\omega'') - \psi(y,\omega') = \min\{y, D(\omega'')\} - \min\{y, D(\omega')\}$$

is nondecreasing in y. The fact that accuracy implies the supermodular order (part (i) in proposition (2)) implies that if $F'' \succeq_{\text{Lehmann}} F'$, then

$$\int_{\mathbb{R}_+\times\Omega} \min\{y, D(\omega)\}\, \mathrm{d}\mathbf{F}''_{\Phi}(y,\omega) \geq \int_{\mathbb{R}_+\times\Omega} \min\{y, D(\omega)\}\, \mathrm{d}\mathbf{F}'_{\Phi}(y,\omega).$$

Therefore, for any arbitrary menu of debt contracts with face values characterized by $D(\cdot)$, the liquidity supplier's ex ante valuation for the menu of securities increases as his information becomes more accurate.

Proposition 3 assumes a fixed menu of debt securities. However, as the accuracy of the liquidity supplier's private signal becomes more accurate, the issuer optimally responds by changing the face values of the debt securities. Our next result shows that the issuer's optimal menu monotonically decreases as we increase the accuracy of the liquidity supplier's signal.

Theorem 3. *Suppose Assumptions (1) and (2) hold. If* $F'' \succeq_{Lehmann} F'$*, then the respective optimal mechanisms under each experiment, characterized by the sets of face values* $\{D'_*(\omega)\}_{\omega\in\Omega}$ *and* $\{D''_*(\omega)\}_{\omega\in\Omega}$*, satisfy* $D'_*(\omega) \geq D''_*(\omega)$*, for all* $\omega \in \Omega$*.*

The formal proof is in the Appendix. We provide the intuition for the result here. Fix a marginal distribution Φ and an experiment F. Consider increasing the face value characterizing the contract designed for type ω, $D(\omega)$, by $\epsilon > 0$ small. The effect of such a variation on the issuer's revenue is approximately given by

$$\epsilon \frac{\partial}{\partial D(\omega)} \mathbb{E}\left(p(\boldsymbol{\omega}); F\right) = \epsilon\phi(\omega)\left(1 - F\left(D(\omega)|\omega\right)\right) - \epsilon\left(1 - \Phi(\omega)\right) \frac{\partial}{\partial \omega}\left(1 - F\left(D(\omega)|\omega\right)\right). \quad (4)$$

Indeed, with probability $\phi(\omega)$, the issuer faces liquidity supplier ω and obtains the additional revenue from increasing the contract $D(\omega)$ to $D(\omega) + \epsilon$ and selling at the fair valuation of type ω. The additional revenue is thus captured by $\epsilon \cdot (1 - F(D(\omega)|\omega))$. In turn, to prevent higher types $\tilde{\omega} > \omega$ from mimicking type ω, they need to be compensated with additional informational rents. The type right next to type ω (type "$\omega + \mathrm{d}\omega$") observes a differential increment of his utility from mimicking in $\epsilon \cdot \frac{\partial}{\partial \omega}(1 - F(D(\omega)|\omega))$. The issuer needs to give up informational rent to all types above ω. Thus, the loss in revenue equates to $\epsilon \cdot (1 - \Phi(\omega)) \frac{\partial}{\partial \omega}(1 - G(D(\omega)|\omega))$.

The main theoretical insight of the proof is that because of the geometry of debt, the marginal incentive to increase the face value of a debt security, captured by $\frac{\partial}{\partial D(\omega)}\mathbb{E}(p(\boldsymbol{\omega}); F)$, has the single crossing property in the Lehmann order. That is, if for some experiment F', the issuer does not have an incentive to increase $D(\omega)$ (i.e., $\frac{\partial}{\partial D(\omega)}\mathbb{E}(p(\boldsymbol{\omega}); F') \leq 0$), then for any $F'' \succeq_{\text{Lehmann}} F'$, the issuer does not have incentives to raise $D(\omega)$ (i.e., $\frac{\partial}{\partial D(\omega)}\mathbb{E}(p(\boldsymbol{\omega}); F'') \leq 0$).

Put differently, when the experiment linking cashflows $\boldsymbol{y}$ and the liquidity supplier's information $\boldsymbol{\omega}$ becomes more accurate, informational rents become too expensive. To prevent higher types from mimicking lower types, the optimal contracts designed for the latter must be distorted to a larger extent. This means that for each liquidity supplier type ω, the face value of the contract designed for him, $D(\omega)$, is weakly smaller when the accuracy of his signal is higher.

4.4 Information, Efficiency, and Revenue

A natural conjecture about the consequences of Theorem 3 is that more information asymmetry, as captured by a more accurate private signal, is detrimental to efficiency. Indeed, as the liquidity supplier becomes more informed, the issuer sells smaller securities to reduce informational rents. The liquidity supplier is the efficient holder of the asset's future cashflows (as she is more patient than the issuer); therefore, efficiency might be compromised. However, as argued above in Proposition 3, there exists a novel effect associated with the geometry of the optimal securities. The overall effect is thus ambiguous and depends on the underlying distributions Φ and experiment F.

To formally see this, consider the liquidity supplier's ex ante expected payoff at the optimal mechanism when the accuracy of the liquidity supplier's signal is captured by experiment F. That is,

$$\begin{aligned}\mathbb{E}\left(\min\left\{\boldsymbol{y}, D_{*}\left(\boldsymbol{\omega}; F\right)\right\}\right) &= \int_{\Omega}\int_{0}^{\infty}\min\left\{y, D_{*}\left(\omega; F\right)\right\}\mathrm{d}F\left(y|\omega\right)\mathrm{d}\Phi\left(\omega\right) \\ &= \int_{\Omega}\left\{\int_{0}^{D_{*}(\omega;F)} y\mathrm{d}F\left(y|\omega\right) + D_{*}\left(\omega; F\right)\left(1 - F\left(D_{*}\left(\omega; F\right)|\omega\right)\right)\right\}\mathrm{d}\Phi\left(\omega\right) \\ &= \int_{\Omega}\left\{\int_{0}^{D_{*}(\omega;F)}\left(1 - F\left(y|\omega\right)\right)\mathrm{d}y\right\}\mathrm{d}\Phi\left(\omega\right). \end{aligned} \tag{5}$$

Increasing the accuracy of experiment F has two effects. On the one hand, as implied by Theorem 3, when F becomes more accurate, $D_{*}\left(\omega; F\right)$ decreases for all ω. This is the direct effect of selling smaller securities, which reduces the gains from trade. However, increasing the accuracy of F increases the liquidity supplier's valuation for concave securities.

In general, the overall effect of increasing asymmetric information among the issuer and the liquidity supplier has an ambiguous effect on the gains from trade.

Similarly, the effect of increasing the accuracy of the liquidity supplier's private signal is ambiguous in the expected revenue. Again, increasing accuracy leads the issuer to optimally (a) sell smaller securities (Theorem 3) and (b), for a fixed security, to give up higher informational rents. However, the concavity of debt implies that (c) the liquidity supplier's valuation of the security increases.

Lemma 6. *Let $\mathbb{E}\left(p_{*}\left(\boldsymbol{\omega}; F\right)\right)$ be the issuer's expected funds raised when the liquidity supplier's private information is parametrized by experiment F (with pdf f). Then,*

$$\mathbb{E}\left(p_{*}\left(\boldsymbol{\omega}; F\right)\right) = \int_{\Omega}\int_{0}^{D_{*}(\omega;F)}\left(1 - F\left(y|\omega\right)\right)\left\{1 - \left(\frac{1 - \Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}\left(1 - F\left(y|\omega\right)\right)}{1 - F\left(y|\omega\right)}\right\}\mathrm{d}y\mathrm{d}\Phi\left(\omega\right). \tag{6}$$

Equation (6) summarizes all the effects described above. Effect (a) is captured by the integration limit of the inner integral. For any ω, $D_{*}\left(\omega; F\right)$ decreases with a more informative experiment, together with the fact that[21]

$$\left(1-F\left(y|\omega\right)\right)\left\{1-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right)}{1-F\left(y|\omega\right)}\right\}\geq 0,\forall y\in\left[0,D_{*}\left(\omega;F\right)\right],$$

jointly imply that the amount of funds raised decreases with informativeness. Effect (b), in turn, is obtained from the fact that

$$1-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right)}{1-F\left(y|\omega\right)}$$

is strictly decreasing in the informativeness of F (see the arguments establishing Theorem 3). Finally, the countervailing effect (c) is a consequence of the concavity of debt, which implies that for any given menu of debt contracts with face values captured by $D(\cdot)$, the gains from trade, i.e.,

$$\mathbb{E}\left(\min\left\{\boldsymbol{y},D\left(\boldsymbol{\omega}\right)\right\}\right)=\int_{\Omega}\left\{\int_{0}^{D(\omega)}\left(1-F\left(y|\omega\right)\right)\mathrm{d}y\right\}\mathrm{d}\Phi\left(\omega\right),$$

are monotone in the informativeness of F. As a result, the overall effect of the amount of information on the funds raised is also ambiguous.

As suggested by Example 1, this ambiguous effect on both gains from trade and revenue does not manifest in the classical screening problem wherein the issuer is constrained to sell pure equity, a linear instrument on the asset's cashflows. The new countervailing effect materializes in our environment with flexible security design because of the geometry of the optimal security.

We further note that this effect is quite different from previous results. First, it is distinct from the celebrated *linkage principle*. (Milgrom and Weber [1982]), which arises in common value auctions, and according to which the seller of the asset prefers to reduce the extent of asymmetric information to minimize the winner's curse. In our case, with a single liquidity

[21] To see this more clearly, let

$$\zeta\left(y,\omega\right)\equiv\left(1-F\left(y|\omega\right)\right)-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right).$$

The definition of $D_{*}\left(\omega;F\right)$ implies that $\zeta\left(D_{*}\left(\omega;F\right),\omega\right)=0$, for all $\omega\in\Omega$. Moreover,

$$\begin{aligned}\frac{\partial}{\partial y}\zeta\left(y,\omega\right) &= -f(y|\omega)+\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\partial}{\partial\omega}f(y|\omega).\\ &< -f(y|\omega)\left(1-\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right)<0,\ \forall y<D_{*}\left(\omega;F\right).\end{aligned}$$

supplier, the linkage principle does not apply. Further, Ottaviani and Prat [2001] discuss two reasons why a seller screening a buyer has incentives to reduce the degree of asymmetric information between them. First, contrary to our comparative statics exercise wherein we change the quality of the liquidity supplier's private signal, they assume that any information publicly revealed to the buyer is also observed by the seller. This means that public announcements dilute the informational advantage of the buyer, which helps the seller increase the revenue. Second, in contrast to our assumption that prices are paid in period 1 and cannot be made contingent on the cashflow realizations, their seller can contract on the information publicly revealed. This last effect is reminiscent of the well-known result in Crémer and McLean [1988] who show that a seller benefits from having access to a signal that correlates with the buyer's private information. Our result, in contrast, is a consequence of the geometry of the optimal securities. We argue that accounting for this effect is crucial for welfare analysis and should not be ignored when designing regulations.

Our next result provides a sufficient condition relating the marginal distribution Φ and the accuracy of experiment F under which more accurate experiments lead to a lower amount of funds raised.

Lemma 7. *Suppose that for all y, the function*

$$\zeta\left(y,\omega;F\right)\equiv\left(1-\Phi\left(\omega\right)\right)\left(1-F\left(y|\omega\right)\right)$$

has increasing differences in ω and accuracy. Then, for any $F''\succeq_{Lehmann}F'$, $\mathbb{E}\left(p_*\left(\boldsymbol{\omega};F''\right)\right)\leq\mathbb{E}\left(p_*\left(\boldsymbol{\omega};F'\right)\right)$.

From the derivation above, we know that

$$\begin{aligned}\mathbb{E}\left(p_*\left(\boldsymbol{\omega};F\right)\right) &= \int_\Omega\int_0^{D_*(\omega;F)}\left\{1-F\left(y|\omega\right)-\left(\frac{1-\Phi\left(\omega\right)}{\phi\left(\omega\right)}\right)\frac{\partial}{\partial\omega}\left(1-F\left(y|\omega\right)\right)\right\}\mathrm{d}y\mathrm{d}\Phi\left(\omega\right). \\ &= \int_\Omega\int_0^{D_*(\omega;F)}-\frac{\partial}{\partial\omega}\zeta\left(y,\omega;F\right)\mathrm{d}y\mathrm{d}\omega\end{aligned}$$

Assuming that $\zeta\left(y,\omega;F\right)$ has increasing differences in ω and accuracy is equivalent to stating that effect (b) dominates effect (c). When $\zeta\left(y,\omega;F\right)$ has this property, the amount of funds raised $\mathbb{E}\left(p_*\left(\boldsymbol{\omega};F\right)\right)$ decreases with the accuracy of F.

Intuitively, for each type ω, $-\frac{\partial}{\partial\omega}\partial\zeta\left(y|\omega,F\right)$ represents the issuer's marginal incentive to increase the face value of type ω's debt contract accounting for the informational rents that have to be given up to all types above ω. Indeed, from equation (4), we have

$$\frac{\partial}{\partial D(\omega)}\mathbb{E}\left(p\left(\boldsymbol{\omega}\right);F\right)=-\frac{\partial\zeta\left(y,\omega;F\right)}{\partial\omega}.$$

The increasing difference assumption then guarantees that as the liquidity supplier's private signal becomes more informative, the issuer's virtual valuation (that is, gains from trade minus informational rents) grows smaller, thereby reducing the issuer's expected revenue. When this is the case, facing a more informed liquidity supplier hurts the issuer's ability to raise liquid funds.

5 Conclusions

We study how the strategic choice of securities can help liquidity-constrained asset owners raise liquid funds from informed investors. We show, perhaps counterintuitively, that exploiting the information sensitivity of the securities within the menu is generally ineffective at screening the liquidity suppliers' private information and is dominated by simple menus of debt contracts. We show that information-insensitive securities allow the issuer to minimize the liquidity suppliers' informational rents and therefore allow the former to raise larger amounts of funds.

Furthermore, our contribution is methodological. We show how, using the logic of a well-designed replication argument and a suitable generalization of monotonicity, the standard tools in the screening literature can be extended to the rich environment of all possible securities, i.e., infinite-dimensional objects. Finally, we show that one of the advantages of our direct mechanism approach vis-a-vis former results is to provide a characterization of the liquidity supplier's informational rents within the optimal mechanism, which allows us to perform novel comparative statics exercises with respect to the primitives of the environment (e.g., the extent of agents' asymmetric information). We show that as the liquidity supplier becomes more informed, the issuer optimally designs a menu of smaller contracts. Interestingly, the overall effect on surplus and the agents' payoff is ambiguous. Our results underscore the fact that the geometry of the optimal securities is crucial for welfare analysis and should be accounted for when designing financial regulation.

The results in this paper are worth extending in several directions. The analysis assumes a single liquidity supplier. It would be interesting to generalize our direct mechanism approach to the case with multiple bidders and provide a general characterization of the liquidity suppliers' informational rents in such a case.[22]

[22]DeMarzo et al. [2005] follow an indirect approach and find the optimal solution within a fairly large

In our paper, we show how the optimal mechanism with securities change as we change the liquidity supplier's exogenous private information. It is reasonable to conjecture that in practice, asset owners can manipulate the information asymmetry with respect to the liquidity supplier. What is the optimal mechanism by which to sell securities when the issuer can design both the security and the information structure? Inostroza and Tsoy [2022] make progress in this direction and show that an issuer who can also design signal realizations prefers to sell information sensitive securities.

Appendix A: Proofs Section 3 (Optimality of Debt Menus)

Proof of Proposition 1. To see (1), note that, for $i, j \in \{1, ..., N\}$ with $i < j$,

$$\begin{array}{rcl} U_{\mathcal{M}}(\omega_j; \omega_j) = \mathbb{E}_j(u_j(s_j)) - p_j & \geq & U_{\mathcal{M}}(\omega_i; \omega_j) \\ & = & \mathbb{E}_j(u_j(s_i)) - p_i \\ & > & \mathbb{E}_i(u_i(s_i)) - p_i. \\ & \geq & 0 \end{array}$$

where the first inequality follows from $[\mathrm{IC}_{j,i}]$, the second inequality is implied by FOSD (in turn implied by MLRP) and the monotonicity of $u_n(x)$ in n, and the last inequality follows from $[\mathrm{IR}_i]$.

Next, let

$$\xi_{\mathcal{M}} \equiv \min_{n \in \{1,...,N\}} U_{\mathcal{M}}(\omega_n; \omega_n) = U_{\mathcal{M}}(\omega_1; \omega_1).$$

where the equality is a consequence of property (1). Next, define $\tilde{\mathcal{M}} = \{\tilde{s}_n(\cdot), \tilde{p}_n\}_{n=1}^N$ as follows. Let $\tilde{s}_n \equiv s_n$, and $\tilde{p}_n \equiv p_n + \xi_{\mathcal{M}}$ for all $n \in \{1, ..., N\}$. Increasing the prices by the same amount for all types does not spoil incentive compatibility. Moreover, increasing the transfers by $\xi_{\mathcal{M}} = U_{\mathcal{M}}(\omega_1; \omega_1) > 0$ implies that individual rationality constraints are satisfied as well. The new mechanism raises more funds than the original and therefore strictly dominates it.

Finally, to see (3), consider the mechanism $\hat{\mathcal{M}} = \{\hat{s}_n(\cdot), \hat{p}_n\}_{n=1}^N$ where $\hat{s}_j \equiv s_j$ and $\hat{p}_j \equiv p_j$ for all $j \in \{1, ..., N-1\}$, $\hat{s}_N(y) \equiv s_N(y) + \epsilon(y - s_N(y))$ and $\hat{p}_N \equiv p_N + \epsilon\varphi_N \mathbb{E}_N(y - s_N(y))$, for an arbitrary $\epsilon \in (0, 1)$. Under the new mechanism $\hat{\mathcal{M}}$, the utility of type ω_N remains unchanged, whereas the utility of any other type ω_j who chooses to report ω_N is strictly smaller than under $\mathcal{M}$. In fact,

set of mechanisms. However, their solution, i.e., first-price auction in call-options, can be dominated with nonstandard mechanisms.

$$
\begin{aligned}
U_{\hat{\mathcal{M}}}\left(\omega_N;\omega_j\right) &= \mathbb{E}_j\left(u_j\left(\hat{s}_N\right)\right)-\hat{p}_N \\
&= \mathbb{E}_j\left(u_j\left(s_N\right)\right)-p_N+\epsilon\underbrace{\left(\varphi_j\mathbb{E}_j\left(y-s_N\left(y\right)\right)-\varphi_N\mathbb{E}_N\left(y-s_N\left(y\right)\right)\right)}_{<0} \\
&< U_{\mathcal{M}}\left(\omega_N;\omega_j\right),
\end{aligned}
$$

where the inequality obtains from (i) that $y-s_N(y)$ is monotone and (ii) $[\boldsymbol{y}|\boldsymbol{\omega}=\omega_N]\succ_{\text{MLRP}}[\boldsymbol{y}|\boldsymbol{\omega}=\omega_j]$. Therefore, the new mechanism $\hat{\mathcal{M}}$ is feasible and is strictly preferred by the issuer since $\sum_i \hat{p}_i\phi_i > \sum_i p_i\phi_i$. $\square$

Proof of Theorem 1

We divide the proof of the theorem into 3 steps. Step 1 shows that when, for a given type ω_i, the security s_i is debt, then for any $k>j>i$, the local downward incentive constraints [IC$_{k,j}$] and [IC$_{j,i}$] imply the global downward constraint [IC$_{k,i}$]. Step 2 builds on this result to show that, in this case, the incentive constraint [IC$_{i+1,i}$] must necessarily bind at the optimal mechanism. Step 3 finally proves that, starting from a mechanism where some of the securities are not debt, one can construct an alternative mechanism where the nondebt securities are changed by their payoff equivalent debt contract and that this does not compromise upward incentive constraints while relaxing the binding downward incentive constraints.

Step 1. We show that any oriented mechanism, when for some $i\in\{1,...,N-2\}$, the security s_i is debt, then for any $k>j>i$, the local downward incentive constraints [IC$_{k,j}$] and [IC$_{j,i}$] imply the downward global constraint [IC$_{k,i}$]. This means that global downward constraints are redundant and can be ignored.

Proposition 4. [LOCAL CONSTRAINTS IMPLY GLOBAL CONSTRAINTS] *Let $\mathcal{M}=\{s_i,p_i\}_{i=1}^N$ be an oriented mechanism. Assume that, for some $i<N$, $s_i=\min\{y,D_i\}$ for ($\lambda-$almost all) y, with $D_i>0$. Suppose that, for any $i,j,k\in\{1,...,N\}$ with $k>j>i$, (i) [$IC_{j,i}$], (ii) [$IC_{k,j}$], and (iii) [IR_i] jointly hold; then, [$IC_{k,i}$] must also hold.*

Proof. That incentive constraint [IC$_{j,i}$] holds implies that

$$
\mathbb{E}_j\left(u_j\left(s_j\right)\right)-p_j \geq \mathbb{E}_j\left(u_j\left(s_i\right)\right)-p_i. \tag{7}
$$

Next, we show that

$$
\mathbb{E}_k\left(s_j-s_i\right) \geq \mathbb{E}_j\left(s_j-s_i\right). \tag{8}
$$

To see this, note first that $\mathcal{M}$ is oriented only if $\mathbb{E}_j(s_i - s_j) \leq 0$. Next, let $\gamma \geq 1$ be implicitly defined by

$$\mathbb{E}_j(\gamma s_i - s_j) = 0. \tag{9}$$

That s_i is a debt contract, together with the fact that $\gamma \geq 1$, imply that $\gamma s_i - s_j$ satisfies SCFA. This last observation, together with the fact that $[\boldsymbol{y}|\boldsymbol{\omega} = \omega_k] \succ_{\text{MLRP}} [\boldsymbol{y}|\boldsymbol{\omega} = \omega_j]$, jointly imply that

$$\mathbb{E}_k(\gamma s_i - s_j) \leq 0. \tag{10}$$

As a result, we conclude that

$$\begin{aligned}\mathbb{E}_j(s_i - s_j) - \mathbb{E}_k(s_i - s_j) &= \underbrace{\mathbb{E}_j(\gamma s_i - s_j)}_{=0 \text{ from (9)}} - \underbrace{\mathbb{E}_k(\gamma s_i - s_j)}_{\leq 0 \text{ from (10)}} \\ &+ (\gamma - 1) \times \underbrace{(\mathbb{E}_k(s_i) - \mathbb{E}_j(s_i))}_{\geq 0 \text{ from MLRP}} \\ &\geq 0,\end{aligned}$$

which proves the inequality in (8).

Combining inequalities (7) and (8), and using the fact that $\mathbb{E}_j(s_j - s_i) \geq 0$ (by orientation), we then obtain that type ω_k weakly prefers the contract designed for type ω_j over the one designed for type ω_i. That is,

$$\mathbb{E}_k(u_k(s_j)) - p_j \geq \mathbb{E}_k(u_k(s_i)) - p_i. \tag{11}$$

Finally, the fact that the local incentive constraint $[\text{IC}_{k,j}]$ is also satisfied implies that

$$\mathbb{E}_k(u_k(s_k)) - p_k \geq \mathbb{E}_k(u_k(s_j)) - p_j. \tag{12}$$

Combining (11) and (12), we thus conclude that type ω_k weakly prefers his contract over the one designed for type ω_i. That is,

$$\mathbb{E}_k(u_k(s_k)) - p_k \quad \geq \quad \mathbb{E}_k(u_k(s_i)) - p_i,$$

and hence $[\text{IC}_{k,i}]$ is satisfied. $\square$

This completes the proof of Step 1. $\blacksquare$

Step 2. We leverage on the result in Step 1 to show that, when s_i is a debt contract, then it is without loss to restrict attention to mechanisms for which the local downward incentive constraints $[\mathrm{IC}_{i+1,i}]$ bind.

Corollary 2. *Consider any oriented mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$ for which there exists some $j \in \{1, ..., N-2\}$ such that (i) $s_j = \min\{y, D_j\}$ for ($\lambda-$almost all) y, with $D_j > 0$, and (ii) $[IC_{j+1,j}]$ does not bind. Then, $\mathcal{M}$ is strictly dominated by another feasible and oriented mechanism for which $[IC_{j+1,j}]$ does bind.*

Proof. Suppose by contradiction that there exists some $j \in \{1, ..., N-2\}$ such that $s_j = \min\{y, D_j\}$ for ($\lambda-$almost all) y, with $D_j > 0$, for which $[\mathrm{IC}_{j+1,j}]$ is satisfied with slackness. The steps establishing proposition 4 then imply that, for any k, i with $k \geq j+1 > i$, $[\mathrm{IC}_{k,i}]$ is also slack. That is,

$$\forall k, i \text{ with } k \geq j+1 > i, U_{\mathcal{M}}(\omega_k; \omega_k) - U_{\mathcal{M}}(\omega_i; \omega_k) > 0. \tag{13}$$

The issuer can then construct an alternative mechanism $\mathcal{M}^+ = \left\{s_i^+, p_i^+\right\}_{i=1}^N$ that strictly dominates $\mathcal{M}$. In fact, for any $l \in \{1, ..., N\}$, let $s_l^+ \equiv s_l$. For any $h \leq j$, let $p_h^+ \equiv p_h$, and for any $h > j$, let $p_h^+ \equiv p_h + \epsilon$, where $\epsilon > 0$ is small and chosen so that incentive constraints are not spoiled. Note that, for any $k'', k' \in \{j+1, ..., N\}$, with $k'' \geq k'$,

$$\mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}^+\right)\right) - p_{k''}^+ = \mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}\right)\right) - p_{k''} \geq \mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}\right)\right) - p_{k'} = \mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}^+\right)\right) - p_{k'}^+,$$

and therefore $[\mathrm{IC}_{k'',k'}]$ trivially holds. That, for any $i'', i' \in \{1, ..., j\}$, $[\mathrm{IC}_{i'',i'}]$ holds, follows from the facts that $\mathcal{M}$ is feasible and that, for any $i < j$, $\left(s_i^+, p_i^+\right) = (s_i, p_i)$. Finally, that under $\mathcal{M}^+$, for any k, i with $k \geq j+1 > i$, $[\mathrm{IC}_{k,i}]$ holds for any

$$\epsilon \in \left(0, \min_{k,i:k\geq j+1>i}\left(U_{\mathcal{M}}(\omega_k; \omega_k) - U_{\mathcal{M}}(\omega_i; \omega_k)\right)\right)$$

follows from the observation in (13). □

This completes the proof of Step 2. ■

Step 3. We finally show that for any oriented mechanism $\mathcal{M}$, if we let $i+1 \leq N$ be the smallest type for whom his security s_{i+1} is not debt, we can improve upon the issuer's payoff by swapping security s_{i+1} for the payoff-equivalent (according to ω_{i+1}'s beliefs) debt contract s_{i+1}^D without spoiling upward incentive constraints $[\mathrm{IC}_{h,i+1}]$, for $h \leq i$.

First, we prove an intermediate result.

Proposition 5. [Pooling]*Let $\mathcal{M} = \{s_i, p_i\}_{i=1}^N$ be a feasible mechanism. Assume that, for some $i < N$, $s_i = \min\{y, D_i\}$ with $D_i > 0$, for ($\lambda-$almost all) y. Suppose that, for some*

$j > i$, $[IC_{i,j}]$ and $[IC_{j,i}]$ are both binding. If $s_i(y) \neq s_j(y)$ over a set with positive λ-measure, then there exists another mechanism $\hat{\mathcal{M}} = \{\hat{s}_i, \hat{p}_i\}_{i=1}^N$ with $\hat{s}_i = \hat{s}_j$ that strictly dominates $\mathcal{M}$.

Proof. That $[\mathrm{IC}_{i,j}]$ and $[\mathrm{IC}_{j,i}]$ are both binding implies that

$$\varphi_i \mathbb{E}_i(s_i - s_j) = p_i - p_j = \varphi_j \mathbb{E}_j(s_i - s_j). \tag{14}$$

Assume next that $s_i \neq s_j$ over a set with positive λ-measure. We show that, necessarily, $\mathbb{E}_i(s_i - s_j) > 0$. Suppose by contradiction that $\mathbb{E}_i(s_i - s_j) \leq 0$. This implies that there must exist $\gamma \geq 1$ so that

$$\mathbb{E}_i(\gamma s_i - s_j) = 0. \tag{15}$$

The fact that s_i is a debt contract, together with the assumption that $s_i \neq s_j$, then jointly imply that $\gamma s_i - s_j$ satisfies SSCFA, and therefore, lemma 1 implies that

$$\mathbb{E}_j(\gamma s_i - s_j) < 0. \tag{16}$$

Thus,

$$\begin{aligned}
\mathbb{E}_i(s_i - s_j) - \mathbb{E}_j(s_i - s_j) &= \underbrace{\mathbb{E}_i(\gamma s_i - s_j)}_{=0 \text{ from } (15)} - \underbrace{\mathbb{E}_j(\gamma s_i - s_j)}_{<0 \text{ from } (16)} \\
&+ (\gamma - 1) \times \underbrace{(\mathbb{E}_j(s_i) - \mathbb{E}_i(s_i))}_{>0 \text{ from MLRP}} \\
&> 0,
\end{aligned}$$

which contradicts equation (14). Thus, $\mathbb{E}_i(s_i - s_j) > 0$.

Assume then that $s_i \neq s_j$ over a set with positive λ-measure and that $\mathbb{E}_i(s_i - s_j) > 0$. Equation (14) then implies that (a) $\mathbb{E}_j(u_j(s_i - s_j)) > 0$ and (b) $p_i > p_j$. This, in turn, means that there exists an alternative mechanism, $\hat{\mathcal{M}}$, identical to $\mathcal{M}$ except for the fact that it offers contract $(\hat{s}_j, \hat{p}_j) = (s_i, p_i)$ to type ω_j. In other words, $\hat{\mathcal{M}}$ deletes the contract offered to type ω_j under the mechanism $\mathcal{M}$ and replaces it by the debt contract offered to type ω_i. Clearly, the new mechanism $\hat{\mathcal{M}}$ is still feasible as the contract (s_i, p_i) was already available under $\mathcal{M}$. Moreover, $\hat{\mathcal{M}}$ strictly dominates $\mathcal{M}$ as $\hat{p}_j = p_i > p_j$ and $\hat{p}_l = p_l$ for any $l \neq j$. □

Next, consider a candidate oriented mechanism $\mathcal{M} = \{s_n, p_n\}_{n=1}^N$. Proposition 1 and Lemmas 2 - 4 jointly imply that it is without loss of optimality to restrict attention to mechanisms satisfying (A) $s_1 = \min\{y, D_1\}$ for ($\lambda-$almost all) y; (B) $s_N = y$ for ($\lambda-$almost all) y ; (C) $p_1 = \mathbb{E}_1(s_1)$; (D) for any j, k with $j < k$, $\mathbb{E}_k(s_k - s_j) \geq 0$ (lemma 3). Assume further that there exists $i \geq 1$, so that (E) for any $h \leq i$, $s_h = \min\{y, D_h\}$ for ($\lambda-$almost all) y, and, by means of corollary (2, that (F) for any $h \leq i$, [IC$_{h+1,h}$] binds. Finally suppose that s_{i+1} is not a debt contract. We show that we can construct a new oriented mechanism $\tilde{\mathcal{M}} \equiv \{\tilde{s}_h, \tilde{p}_h\}$ that strictly dominates $\tilde{\mathcal{M}}$. In fact, for any $h \neq i+1$, let $(\tilde{s}_h, \tilde{p}_h) \equiv (s_h, p_h)$, and let $\tilde{s}_{i+1} \equiv \min\{y, D_{i+1}\}$ where D_{i+1} is such that $\mathbb{E}_{i+1}(\tilde{s}_{i+1} - s_{i+1}) = 0$ and $\tilde{p}_{i+1} \equiv p_{i+1}$. Note that the fact that, under $\mathcal{M}$, [IC$_{i+1,i}$] binds implies that

$$\tilde{p}_{i+1} = p_{i+1} = \mathbb{E}_{i+1}(u_{i+1}(s_{i+1}) - u_{i+1}(s_i)) + p_i = \mathbb{E}_{i+1}(u_{i+1}(\tilde{s}_{i+1}) - u_{i+1}(\tilde{s}_i)) + \tilde{p}_i. \quad (17)$$

Claim 1. *Under the new mechanism $\tilde{\mathcal{M}}$, $[IR_{i+1}]$ and $[IC_{i+1,h}]$ hold for any $h \neq i+1$.*

Proof. The proof follows from noting that, by construction, for any $h \neq i+1$, $(\tilde{s}_h, \tilde{p}_h) = (s_h, p_h)$ and the fact that the new contract of type ω_{i+1} makes him indifferent between the new and the former contract. That is, $U_{\tilde{\mathcal{M}}}(\omega_{i+1}; \omega_{i+1}) = U_{\mathcal{M}}(\omega_{i+1}; \omega_{i+1})$. *q.e.d.*

Claim 2. *Under the new mechanism $\tilde{\mathcal{M}}$, for any $h \leq i$, $[IC_{h,i+1}]$ holds.*

Proof. Property (D) above, together with the construction of $\tilde{\mathcal{M}}$ and the fact that $\mathcal{M}$ is oriented, jointly imply that, for any $h \leq i$,

$$\mathbb{E}_{i+1}(\tilde{s}_{i+1} - \tilde{s}_h) = \mathbb{E}_{i+1}(s_{i+1} - s_h) \geq 0,$$

and therefore that $D_{i+1} \geq D_h$. This observation, in turn, implies that $\tilde{s}_{i+1} - \tilde{s}_h$ is nondecreasing and, as a result, for any $h \leq i$,

$$\begin{aligned}
\mathbb{E}_h(u_h(\tilde{s}_{i+1}) - u_h(\tilde{s}_h)) &= \mathbb{E}_h(u_h(\tilde{s}_{i+1}) - u_h(\tilde{s}_i)) + \mathbb{E}_h(u_h(\tilde{s}_i) - u_h(\tilde{s}_h)) \\
&\leq \mathbb{E}_{i+1}(u_{i+1}(\tilde{s}_{i+1}) - u_{i+1}(\tilde{s}_i)) + \mathbb{E}_h(u_h(\tilde{s}_i) - u_h(\tilde{s}_h)) \\
&= \tilde{p}_{i+1} - \tilde{p}_i + \mathbb{E}_h(u_h(\tilde{s}_i) - u_h(\tilde{s}_h)) \\
&\leq \tilde{p}_{i+1} - \tilde{p}_i + \tilde{p}_i - \tilde{p}_h \\
&= \tilde{p}_{i+1} - \tilde{p}_h
\end{aligned}$$

where the first inequality follows from MLRP, the second equality follows from using the result in equation (17), the second inequality obtains from the fact that, for any $h \leq i$, [IC$_{h,i}$] holds, which is inherited from the feasibility of $\mathcal{M}$. As a result, under the mechanism $\tilde{\mathcal{M}}$, for any $h \leq i$, [IC$_{h,i+1}$] holds. *q.e.d.*

Claim 3. *Under the new mechanism $\tilde{\mathcal{M}}$, for any $k > i+1$, $[IC_{k,i+1}]$ holds with slackness.*

The construction of $\tilde{\mathcal{M}}$, together with the fact that $s_{i+1} - \tilde{s}_{i+1}$ satisfies SSCFB (recall that

$\tilde{s}_{i+1}$ is a debt contract), jointly imply that by virtue of lemma 1, for any $k > i+1$,

$$\mathbb{E}_k\left(s_{i+1} - \tilde{s}_{i+1}\right) > 0. \tag{18}$$

The fact that, under $\mathcal{M}$, $[IC_{k,i+1}]$ *holds* thus implies that

$$\begin{aligned}
\mathbb{E}_k\left(u_k\left(\tilde{s}_k\right)\right) - \tilde{p}_k &= \mathbb{E}_k\left(u_k\left(s_k\right)\right) - p_k, \\
&\geq \mathbb{E}_k\left(u_k\left(s_{i+1}\right)\right) - p_{i+1} \\
&> \mathbb{E}_k\left(u_k\left(\tilde{s}_{i+1}\right)\right) - \tilde{p}_{i+1},
\end{aligned}$$

which proves the claim. *q.e.d.*

Claims (1)-(3) then jointly imply that $\tilde{\mathcal{M}}$ is feasible. Furthermore, Claim (3) together with the steps establishing Lemma 4 jointly imply that, for any k, h with $k > i+1 \geq h$, $[\text{IC}_{k,h}]$ does not bind. That is,

$$\forall k, h \text{ with } k > i+1 \geq h, U_{\tilde{\mathcal{M}}}\left(\omega_k; \omega_k\right) - U_{\tilde{\mathcal{M}}}\left(\omega_h; \omega_k\right) > 0. \tag{19}$$

This observation implies that, for any $k > i+1$, we can increase the transfers $\tilde{p}_k$ and still respect feasibility.

Rigorously, we can construct yet another feasible mechanism $\mathcal{M}^+ = \{s_n^+, p_n^+\}_{i=1}^N$ that *strictly* dominates $\tilde{\mathcal{M}}$. In fact, for any $n \in \{1, ..., N\}$, let $s_n^+ \equiv s_n$, for any $h \leq i+1$, let $p_h^+ \equiv \tilde{p}_h$, and for any $h > i+1$, let $p_h^+ \equiv \tilde{p}_h + \epsilon$, where $\epsilon > 0$ is small and chosen so that

$$\epsilon \in \left(0, \min_{k,h: k > i+1 \geq h}\left(U_{\mathcal{M}}\left(\omega_k; \omega_k\right) - U_{\mathcal{M}}\left(\omega_i; \omega_k\right)\right)\right)$$

Note that, for any $k'', k' \in \{i+2, ..., N\}$,

$$\mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}^+\right)\right) - p_{k''}^+ = \mathbb{E}_{k''}\left(u_{k''}\left(s_{k''}\right)\right) - p_{k''} \geq \mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}\right)\right) - p_{k'} = \mathbb{E}_{k'}\left(u_{k'}\left(s_{k'}^+\right)\right) - p_{k'}^+,$$

and therefore $[\text{IC}_{k'',k'}]$ trivially holds. That, for any $h'', h' \in \{1, ..., i+1\}$, $[\text{IC}_{h'',h'}]$ holds, follows from the fact that $\tilde{\mathcal{M}}$ is feasible and the fact that, for any $h \leq i+1$, $\left(s_h^+, p_h^+\right) = \left(s_h, p_h\right)$. Finally, that under $\mathcal{M}^+$, for any k, h with $k > i+1 > h$, $[\text{IC}_{k,h}]$ holds, follows from the choice of ϵ and the observation in (13). This completes Step 3 and formally proves the Theorem. $\square$

Proof of Lemma 4.

The result in theorem 1 implies that we can restrict attention to mechanisms with only debt securities, that is, mechanisms satisfying that, for any n, $s_n = \min\{y, D_n\}$ for all y, where $D_n > 0$ and is increasing in n, and $D_N = +\infty$. Moreover, Step 2 in the proof theorem 1 implies that we can restrict attention, without loss, to mechanisms satisfying that, for any $j > 1$, $[\mathrm{IC}_{j,j-1}]$ binds. Finally, because of proposition 1, we assume that $p_1 = \mathbb{E}(\min\{y, D_1\})$. Together, these properties imply that

$$\mathbb{E}_j\left(u_j\left(\min\{y, D_j\}\right)\right) - p_j = \mathbb{E}_j\left(u_j\left(\min\{y, D_{j-1}\}\right)\right) - p_{j-1},$$

and therefore, for any $j > 1$,

$$\begin{aligned} p_j &= \mathbb{E}_j\left(u_j\left(\min\{y, D_j\}\right)\right) - \mathbb{E}_j\left(u_j\left(\min\{y, D_{j-1}\}\right)\right) + p_{j-1} \\ &= \left(\sum_{n>1}^{j} \mathbb{E}_n\left(u_n\left(\min\{y, D_n\}\right)\right) - \mathbb{E}_n\left(u_n\left(\min\{y, D_{n-1}\}\right)\right)\right) + \mathbb{E}_1\left(u_1\left(\min\{y, D_1\}\right)\right). \end{aligned}$$

This further implies that the amount of funds raised is given by[23]

$$\begin{aligned} \sum_{j=1}^{N} p_n \phi_n &= \mathbb{E}_1\left(u_1\left(\min\{y, D_1\}\right)\right) + \sum_{j>1} \phi_j \sum_{n>1}^{j} \mathbb{E}_n\left(u_n\left(\min\{y, D_n\}\right)\right) - \mathbb{E}_n\left(u_n\left(\min\{y, D_{n-1}\}\right)\right) \\ &= \mathbb{E}_1\left(u_1\left(\min\{y, D_1\}\right)\right) + \sum_{n>1}^{N}\left(\sum_{j=n}^{N} \phi_j\right) \mathbb{E}_n\left(u_n\left(\min\{y, D_n\}\right)\right) - \mathbb{E}_n\left(u_n\left(\min\{y, D_{n-1}\}\right)\right). \\ &= \sum_{n=1}^{N} \phi_n \int_0^\infty u_n\left(\min\{y, D_n\}\right)\left(1 - \left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f\left(y|\omega_{n+1}\right) - f\left(y|\omega_n\right)}{f\left(y|\omega_n\right)}\right)\right) \mathrm{d}F\left(y|\omega_n\right). \end{aligned}$$

As a result, the issuer's problem reduces to find an increasing sequence $(D_n)_{n=1}^{N}$ to maximize

$$\sum_{n=1}^{N} p_n \phi_n = \sum_{n=1}^{N} \phi_n \int_0^\infty u_n\left(\min\{y, D_n\}\right)\left(1 - \left(\frac{1-\Phi_n}{\phi_n}\right)\left(\frac{f\left(y|\omega_{n+1}\right) - f\left(y|\omega_n\right)}{f\left(y|\omega_n\right)}\right)\right) \mathrm{d}F\left(y|\omega_n\right),$$

as claimed.□

Proof of Lemma 5.

Assumptions 1 and 2 jointly imply that, for each n, the pointwise-optimal solution of the problem (that is, disregarding the monotonicity condition in (2)), given by

[23] In the formula, with abuse notation and let $D_N = +\infty$ (claim 3 in Proposition 1).

$$D_n^* \equiv \arg\max_D \quad \int_{\mathbb{R}_+} u_n \left(\min\left\{y, D\right\}\right) \left(1 - R\left(y, \omega_n\right)\right) \mathrm{d}F\left(y|\omega_n\right), \tag{20}$$

is monotone in n. This follows from noting that under assumptions 1 and 2, the objective is supermodular (y, ω_n). Indeed, note that the fact that $u_n(x) = \varphi_n x + \nu_n$, implies that

$$\begin{aligned}
\frac{\partial}{\partial D} \int_0^\infty u_n \left(\min\left\{y, D\right\}\right) \left(1 - R\left(y, \omega_n\right)\right) \mathrm{d}F\left(y|\omega_n\right) &= \varphi_n \int_D^\infty \left(1 - R\left(y, \omega_n\right)\right) \mathrm{d}F\left(y|\omega_n\right), \\
&= \varphi_n \left(1 - F\left(y|\omega_n\right) - \int_D^\infty R\left(y, \omega_n\right) \mathrm{d}F\left(y|\omega_n\right)\right)
\end{aligned}$$

When assumptions (1) and (2) hold, the last expression becomes monotone in ω_n. Hence, the pointwise-optimal solution D_n^* is necessarily nondecreasing (Milgrom and Shannon [1994]). This rules out the possibility of bunching.□

Appendix B: Proof of Theorem 2 (Continuum of types)

First note that, because $\mathbb{E}\left\{\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial \omega}}{f(\boldsymbol{y}|\omega)}\right\} < \infty$, for any feasible mechanism $\mathcal{M} = \left\{s\left[\omega\right], p\left(\omega\right)\right\}_{\omega \in \Omega}$, there exists an integrable function $b : \Omega \to \mathbb{R}$, satisfying

$$\left|\frac{\partial}{\partial \omega} U_{\mathcal{M}}\left(\tilde{\omega}, \omega\right)\right| = \left|\mathbb{E}\left\{s\left(\boldsymbol{y}|\tilde{\omega}\right) \frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial \omega}}{f\left(\boldsymbol{y}|\omega\right)}\right\}\right| \leq b\left(\omega\right), \ \forall \tilde{\omega}, \omega.$$

The arguments in Milgrom and Segal [2002] then imply that, because $U_{\mathcal{M}}\left(\hat{\omega}; \omega\right)$ is absolutely continuous in ω, $U_{\mathcal{M}}\left(\omega, \omega\right)$ is absolutely continuous. Furthermore,

$$U_{\mathcal{M}}\left(\omega, \omega\right) = \int_{\underline{\omega}}^{\omega} \left(\int_0^\infty s(y|\tilde{\omega}) \frac{\partial}{\partial \omega} f(y|\tilde{\omega}) \mathrm{d}y\right) \mathrm{d}\tilde{\omega}, \text{ for } \Phi - \text{almost all } \omega.$$

This means that we can rewrite the issuer's payoff as

$$\begin{aligned}
\mathbb{E}\left(p\left(\boldsymbol{\omega}\right)\right) &= \int_\Omega \left\{\int_0^\infty s(y|\omega)\mathrm{d}F(y|\omega) - \left(\int_{\underline{\omega}}^{\omega} \int_0^\infty s(y|\tilde{\omega}) \frac{\partial}{\partial \omega} f(y|\tilde{\omega}) \mathrm{d}y\right) \mathrm{d}\tilde{\omega}\right\} \mathrm{d}\Phi\left(\omega\right) - U_{\mathcal{M}}\left(\underline{\omega}, \underline{\omega}\right) \\
&= \int_\Omega \int_0^\infty \left(s(y|\omega) \left(1 - \left(\frac{1 - \Phi\left(\omega\right)}{\phi\left(\omega\right)}\right) \frac{\frac{\partial}{\partial \omega} f(y|\omega)}{f(y|\omega)}\right)\right) \mathrm{d}F(y|\omega)\mathrm{d}\Phi\left(\omega\right) - U_{\mathcal{M}}\left(\underline{\omega}, \underline{\omega}\right),
\end{aligned}$$

where the second equality obtains from integration by parts.

Proposition 6. *Consider an arbitrary mechanism* $\mathcal{M} = \{s(\cdot|\omega), p(\omega)\}_{\omega\in\Omega}$ *satisfying*

$$\left(\int_0^\infty (s(y|\omega) - s(y|\hat{\omega})) \frac{\partial}{\partial\omega} f(y|\omega)\, dy\right) \cdot (\omega - \hat{\omega}) \geq 0,\ \forall \omega, \hat{\omega} \in \Omega. \tag{21}$$

Then, $\mathcal{M}$ *is incentive compatible.*

Proof. Consider an arbitrary mechanism $\mathcal{M} = \{s(\cdot|\omega), p(\omega)\}_{\omega\in\Omega}$ satisfying (21). For any $\omega, \hat{\omega} \in \Omega$, let

$$Q(\hat{\omega}, \omega) \equiv U_{\mathcal{M}}(\omega, \omega) - U_{\mathcal{M}}(\hat{\omega}, \omega).$$

Note that, for any $\hat{\omega}$, $Q(\hat{\omega}, \cdot)$ is absolutely continuous. Moreover, $Q(\hat{\omega}, \hat{\omega}) = 0$ for all $\hat{\omega} \in \Omega$. This implies that, for any $\omega, \hat{\omega} \in \Omega$,

$$\begin{aligned}
Q(\hat{\omega}, \omega) &= Q(\hat{\omega}, \omega) - Q(\hat{\omega}, \hat{\omega}) \\
&= \int_{\hat{\omega}}^{\omega} \frac{\partial Q(\hat{\omega}, z)}{\partial\omega} \mathrm{d}z \\
&= \int_{\hat{\omega}}^{\omega} \left\{ \frac{\mathrm{d}}{\mathrm{d}\omega} U_{\mathcal{M}}(\omega, \omega)\bigg|_{\omega=z} - \frac{\partial}{\partial\omega} U_{\mathcal{M}}(\hat{\omega}, \omega)\bigg|_{\omega=z} \right\} \mathrm{d}z \\
&= \int_{\hat{\omega}}^{\omega} \left\{ \int_{\mathbb{R}_+} (s(y|z) - s(y|\hat{\omega})) \frac{\partial}{\partial\omega} f(y|z) \mathrm{d}y \right\} \mathrm{d}z \\
&\geq 0,
\end{aligned}$$

where the inequality follows from (21). We thus conclude that $U_{\mathcal{M}}(\omega, \omega) \geq U_{\mathcal{M}}(\hat{\omega}, \omega)$, for any $\omega, \hat{\omega} \in \Omega$. □

The strategy of the proof consists in ignoring constraint (21) and finding, for each ω, the security $s^*(\cdot|\omega)$ which pointwises maximize $\mathbb{E}(p(\boldsymbol{\omega}))$. We then show that, when (1) and (2) hold, the securities $\{s^*(\cdot|\omega)\}_{\omega\in\Omega}$ satisfy constraint (21).

Lemma 8. *Any security* $s(\cdot|\omega) \in \mathcal{S}$ *is weakly dominated by a debt contract.*

Proof. For any ω, let

$$k(y, \omega) \equiv 1 - \left(\frac{1 - \Phi(\omega)}{\phi(\omega)}\right) \frac{\frac{\partial}{\partial\omega} f(y|\omega)}{f(y|\omega)}.$$

We want to maximize

$$\mathbb{E}(p(\boldsymbol{\omega})) = \int_{\Omega} \int_0^\infty s(y|\omega) k(y, \omega) \mathrm{d}F(y|\omega) \mathrm{d}\Phi(\omega).$$

The fact that $f(y|\omega)$ satisfies MLRP implies that $k(\cdot,\omega)$ satisfies the SSCFA property. Let $y_0(\omega)$ be the unique solution to $k(y_0(\omega),\omega)=0$. From the definition of $D_*(\omega)$ in 3, we must necessarily have that $y_0(\omega) > D_*(\omega)$, for all $\omega \in \Omega$. The constraint that all securities in $\mathcal{S}$ are nondecreasing, together with the fact that $k(y,\omega) < 0$ for all $y < y_0(\omega)$, jointly imply that, any security $\tilde{s}(\cdot|\omega)$ in $\mathcal{S}$ which fails to be constant to the right of $y_0(\omega)$ is dominated by the security

$$s^{\#}(y|\omega) \equiv \tilde{s}(y|\omega)1\{y \leq y_0(\omega)\} + \tilde{s}(y_0(\omega)|\omega)1\{y > y_0(\omega)\}.$$

Finally, the fact that $k(y|\omega) > 0$ for all $y < y_0(\omega)$ implies that any security $\bar{s}(y|\omega) \in \mathcal{S}$ is weakly dominated by the security

$$s^{\#\#}(y|\omega) = \min\{y, \bar{s}(y_0(\omega)\} \cdot 1\{y \leq \bar{s}(y_0(\omega)|\omega)\} + \bar{s}(y|\omega)1\{y > \bar{s}(y_0(\omega)|\omega)\}.$$

This proves the lemma.□

Lemma 8 implies that, for any ω, debt securities pointwise maximize $\mathbb{E}(p(\boldsymbol{\omega}))$. The designer then chooses $\{D(\omega)\}_{\omega\in\Omega}$ to maximize

$$\mathbb{E}(p(\boldsymbol{\omega})) = \int_{\Omega}\left\{\int_0^{D(\omega)} yk(y,\omega)\mathrm{d}F(y|\omega) + D(\omega)\int_{D(\omega)}^{\infty} k(y,\omega)\mathrm{d}F(y|\omega)\right\}\mathrm{d}\Phi(\omega).$$

Note that when (1) and (2) hold, the function

$$\chi(\omega, D(\omega)) \equiv \int_0^{D(\omega)} yk(y,\omega)\mathrm{d}F(y|\omega) + D(\omega)\int_{D(\omega)}^{\infty} k(y,\omega)\mathrm{d}F(y|\omega)$$

is supermodular. Indeed,

$$\begin{aligned}
\frac{\partial^2}{\partial\omega\partial D(\omega)}\chi(\omega, D(\omega)) &= \frac{\partial}{\partial\omega}\int_{D(\omega)}^{\infty} k(y,\omega)\mathrm{d}F(y|\omega) \\
&= \frac{\partial}{\partial\omega}\left\{1 - F(D(\omega)|\omega) - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\partial}{\partial\omega}(1 - F(D(\omega)|\omega))\right\} \\
&= \underbrace{\frac{\partial}{\partial\omega}(1 - F(D(\omega)|\omega))}_{>0\text{ (FOSD)}} - \underbrace{\frac{\partial}{\partial\omega}\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)}_{<0\text{ (Assumption1)}} \cdot \underbrace{\frac{\partial}{\partial\omega}(1 - F(D(\omega)|\omega))}_{>0\text{ (FOSD)}} \\
&\quad - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\cdot\underbrace{\frac{\partial^2}{\partial\omega^2}(1 - F(D(\omega)|\omega))}_{<0\text{ (Assumption2)}} \\
&> 0.
\end{aligned}$$

Topkis Theorem then implies that the value of $D(\omega)$ that maximizes (pointwise) $\chi(\omega, D(\omega))$, $D_*(\omega)$, must be increasing in ω. This further implies that the constraint (21) is satisfied, and therefore the set of pointwise optimal secuirities is feasible.

This completes the proof of the Theorem. □

Appendix C: Additional Proofs

Proof of Example 1. We start by showing that θ orders the experiments in the Lehmann sense.

Claim 1. For any $\theta'' > \theta'$, $F^{\theta''} \succeq_{\text{Lehmann}} F^{\theta'}$.

Proof. For any $\theta \in [0,1]$, let $F^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta} > z\right) \equiv \mathbb{P}\left\{\boldsymbol{\omega} \leq \omega||\boldsymbol{y}^{\theta} > z\right\}$. Following Theorem 1 in Athey and Levin [2018], it is enough to prove that, for any $u : [0,1] \to \mathbb{R}$ satisfying SCFB,

$$\int_0^1 u(\omega)\,\mathrm{d}F^{\theta'}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta'} > z\right) \geq 0 \Rightarrow \int_0^1 u(\omega)\,\mathrm{d}F^{\theta''}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta''} > z\right) \geq 0. \tag{22}$$

First, note that, for any $\theta \in [0,1]$, and any $z \in [0,1]$,

$$\begin{aligned}
F^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta} > z\right) &= \frac{\mathbb{P}\left\{\boldsymbol{y}^{\theta} > z|\boldsymbol{\omega} \leq \omega\right\}\omega}{\int_0^1 \left(1 - F^{\theta}(z|\tilde{\omega})\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega \int_0^{\omega}\left(1 - F^{\theta}(z|\tilde{\omega})\right)\mathrm{d}\tilde{\omega}}{\int_0^1 \left(1 - F^{\theta}(z|\tilde{\omega})\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega\left(\omega - \int_0^{\omega}\left(\theta \cdot 1\{\tilde{\omega} \leq z\} + (1-\theta)z\right)\mathrm{d}\tilde{\omega}\right)}{1 - \int_0^1\left(\theta \cdot 1\{\tilde{\omega} \leq z\} + (1-\theta)z\right)\mathrm{d}\tilde{\omega}} \\
&= \frac{\omega\left(\omega - (1-\theta)z\omega - \int_0^{\omega}\left(\theta \cdot 1\{\tilde{\omega} \leq z\}\right)\mathrm{d}\tilde{\omega}\right)}{1-z} \\
&= \frac{\omega\left(\omega - \theta\min\{\omega, z\} - (1-\theta)z\omega\right)}{1-z} \\
&= \begin{cases} \omega^2(1-\theta) & \text{if } \omega < z \\ \frac{\omega(\omega-(1-\theta)z\omega-\theta z)}{1-z} & \text{if } \omega \geq z \end{cases}
\end{aligned}$$

This then implies that, for any $\omega \neq z$,

$$f^{\theta}_{\boldsymbol{\omega}}\left(\omega|\boldsymbol{y}^{\theta} > z\right) = \begin{cases} 2\omega(1-\theta) & \text{if } \omega < z \\ \frac{2\omega(1-(1-\theta)z)-\theta z}{1-z} & \text{if } \omega > z. \end{cases}$$

We further note that $F_{\boldsymbol{\omega}}^{\theta}\left(\omega|\boldsymbol{y}^{\theta}>z\right)$ is continuous at $\omega=z$ and hence absolutely continuous over $[0,1]$.

Finally, we note that

$$\frac{\frac{\mathrm{d}}{\mathrm{d}\omega}f_{\boldsymbol{\omega}}^{\theta}\left(\omega|\boldsymbol{y}^{\theta}>z\right)}{f_{\boldsymbol{\omega}}^{\theta}\left(\omega|\boldsymbol{y}^{\theta}>z\right)}=\begin{cases}\frac{1}{\omega} & \text{if } \omega<z\\ \frac{2(1-(1-\theta)z)}{2\omega(1-(1-\theta)z)-\theta z} & \text{if } \omega>z\end{cases}$$

is nondecreasing in θ for any $\omega\in[0,1)$. This implies that the density $f_{\boldsymbol{\omega}}^{\theta}\left(\omega|\boldsymbol{y}^{\theta}>z\right)$ is log-supermodular in (θ,ω). Lemma 1 then implies that for any $u:[0,1]\to\mathbb{R}$ satisfying SCFB, (22) is satisfied. This proves the claim. *q.e.d.*

Claim 2. Suppose the issuer is restricted to use linear securities,

$$s\in\mathcal{S}^{E}\equiv\left\{s\in\mathcal{S}:\exists\alpha>0,s\left(y\right)=\alpha y,\forall y\in[0,1]\right\}.$$

Then, the (restricted) optimal mechanism is characterized by

$$\alpha_{\theta}^{*}\left(\omega\right)=1\left\{\omega\geq\omega_{\theta}^{*}\equiv\max\left\{\frac{3\theta-1}{4\theta},0\right\}\right\}.$$

Proof. We consider mechanisms of the form $\mathcal{M}_{E}^{\theta}=\left\{s_{E}^{\theta}\left[\omega\right],p_{E}^{\theta}\left(\omega\right)\right\}_{\omega\in[0,1]}$, where for all $\omega\in[0,1]$, $s_{E}^{\theta}\left(y|\omega\right)=\alpha^{\theta}\left(\omega\right)y$ and $p_{E}^{\theta}\left(\omega\right)\in\mathbb{R}$. Note that for any $\omega\in[0,1]$,

$$\mathbb{E}\left\{\boldsymbol{y}^{\theta}|\boldsymbol{\omega}=\omega\right\}=\underbrace{\theta\omega+\frac{1-\theta}{2}}_{\equiv h^{\theta}(\omega)}.$$

Furthermore, $h^{\theta}\left(\boldsymbol{\omega}\right)=\mathbb{E}\left\{\boldsymbol{y}^{\theta}|\boldsymbol{\omega}\right\}\sim U\left[\frac{1-\theta}{2},\frac{1+\theta}{2}\right]$.

Next, for any $\omega,\tilde{\omega}\in[0,1]$ let

$$U_{\mathcal{M}_{E}^{\theta}}\left(\tilde{\omega};\omega\right)=\alpha^{\theta}\left(\tilde{\omega}\right)h^{\theta}\left(\omega\right)-p_{E}^{\theta}\left(\tilde{\omega}\right)$$

be the liquidity supplier's payoff when his true type is ω and he chooses to report $\tilde{\omega}$. The liquidity supplier's IC constraint is then given by

$$U_{\mathcal{M}_{E}^{\theta}}\left(\omega;\omega\right)=\max_{\tilde{\omega}}U_{\mathcal{M}_{E}^{\theta}}\left(\tilde{\omega};\omega\right).$$

Using the envelope theorem, we obtain that

$$\frac{\mathrm{d}}{\mathrm{d}\omega}U_{\mathcal{M}_E^\theta}(\omega;\omega)=\alpha^\theta(\omega)\frac{\mathrm{d}h^\theta(\omega)}{\mathrm{d}\omega},\ \forall\omega\in[0,1].$$

This further implies that

$$U_{\mathcal{M}_E^\theta}(\omega;\omega)=\int_0^\omega\alpha^\theta(\tilde{\omega})\frac{\mathrm{d}h^\theta(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega}.$$

We conclude that

$$p_E^\theta(\omega)=\alpha^\theta(\omega)h^\theta(\omega)-\int_0^\omega\alpha^\theta(\tilde{\omega})\frac{\mathrm{d}h^\theta(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega},$$

and therefore

$$\begin{aligned}\mathbb{E}\left\{p_E^\theta(\boldsymbol{\omega})\right\} &= \int_0^1\left\{\alpha^\theta(\omega)h^\theta(\omega)-\int_0^\omega\alpha^\theta(\tilde{\omega})\frac{\mathrm{d}h^\theta(\tilde{\omega})}{\mathrm{d}\omega}\mathrm{d}\tilde{\omega}\right\}\mathrm{d}\omega\\ &= \int_0^1\left\{\alpha^\theta(\omega)\left(h^\theta(\omega)-(1-\omega)\frac{\mathrm{d}h^\theta(\omega)}{\mathrm{d}\omega}\right)\right\}\mathrm{d}\omega\\ &= \int_0^1\left\{\alpha^\theta(\omega)\left(\theta\omega+\frac{1-\theta}{2}-(1-\omega)\theta\right)\right\}\mathrm{d}\omega\\ &= \int_0^1\left\{\alpha^\theta(\omega)\left(\theta\left(\frac{4\omega-3}{2}\right)+\frac{1}{2}\right)\right\}\mathrm{d}\omega.\end{aligned}$$

The issuer's can then be written as

$$\begin{aligned}\max_{\left\{\alpha^\theta(\omega)\right\}_{\omega\in[0,1]}}\quad & \mathbb{E}\left\{p_E^\theta(\boldsymbol{\omega})\right\}=\int_0^1\left\{\alpha^\theta(\omega)\left(\theta\left(\frac{4\omega-3}{2}\right)+\frac{1}{2}\right)\right\}\mathrm{d}\omega\\ \text{s.t.}\quad & \alpha^\theta(\cdot)\ \text{nondecreasing}.\end{aligned}$$

The issuer therefore optimally sets

$$\alpha_*^\theta(\omega)=1\left\{\omega\geq\omega_\theta^*\equiv\max\left\{\frac{3\theta-1}{4\theta},0\right\}\right\},$$

as claimed. *q.e.d.*

Claim 3. The optimal mechanism is characterized as follows. For any $\theta\in\left(\frac{1}{7},1\right)$, $D_\theta^*(\omega)=\omega$ for all $\omega\in[0,1]$. In turn, for any $\theta\in\left(0,\frac{1}{7}\right)$, $D_\theta^*(\omega)=1$ for all $\omega\in[0,1]$.

Proof. Fix any $\theta\in(0,1)$ and let $\mathcal{M}_*^\theta=\left\{s^\theta\equiv\min\left\{y,D_*^\theta(\omega)\right\},p_*^\theta(\omega)\right\}_{\omega\in\in[0,1]}$ represent the optimal menu of debt contracts (existence follows from the derivation below). Next, for any $\omega, \tilde{\omega} \in [0,1]$ let

$$\begin{aligned} U^{\theta}_{\mathcal{M}^{\theta}_{*}}(\tilde{\omega};\omega) &\equiv \mathbb{E}\left\{\min\left\{\boldsymbol{y}, D^{\theta}_{*}(\tilde{\omega})\right\}|\omega\right\} - p^{\theta}_{*}(\tilde{\omega}). \\ &= (1-\theta)\left(D^{\theta}_{*}(\tilde{\omega}) - \frac{D^{\theta}_{*}(\tilde{\omega})^{2}}{2}\right) + \theta\min\left\{\omega, D^{\theta}_{*}(\tilde{\omega})\right\} - p^{\theta}_{*}(\tilde{\omega}) \end{aligned}$$

The fact that, downward incentive compatibility constraints bind implies that

$$\frac{\mathrm{d}}{\mathrm{d}\omega}U^{\theta}_{\mathcal{M}^{\theta}_{*}}(\omega;\omega) = \theta\cdot 1\left\{D(\omega) > \omega\right\},$$

and, therefore,

$$U^{\theta}_{\mathcal{M}^{\theta}_{*}}(\omega;\omega) = U^{\theta}_{\mathcal{M}^{\theta}_{*}}(0;0) + \theta\int_{0}^{\omega} 1\left\{D(\tilde{\omega}) > \tilde{\omega}\right\}\mathrm{d}\tilde{\omega}.$$

As a result, we obtain that the issuer's revenue is given by

$$\begin{aligned} \mathbb{E}\left\{p^{\theta}(\boldsymbol{\omega})\right\} &= \int_{0}^{1}\left\{(1-\theta)\left(D^{\theta}_{*}(\omega) - \frac{D^{\theta}_{*}(\omega)^{2}}{2}\right) + \theta\left(\min\{\omega, D^{\theta}_{*}(\omega)\} - \int_{0}^{\omega} 1\{D^{\theta}_{*}(\tilde{\omega}) > \tilde{\omega}\}\mathrm{d}\tilde{\omega}\right)\right\}\mathrm{d}\omega \\ &= \int_{0}^{1}\left\{(1-\theta)\left(D^{\theta}_{*}(\omega) - \frac{D^{\theta}_{*}(\omega)^{2}}{2}\right) + \theta\min\{\omega, D^{\theta}_{*}(\omega)\} - \theta(1-\omega)1\{D^{\theta}_{*}(\omega) > \omega\}\right\}\mathrm{d}\omega. \end{aligned}$$

Observe first that $D^{\theta}_{*}(\omega)\geq\omega$ for all ω. Indeed, for any ω, the first term is strictly increasing in $D^{\theta}_{*}(\omega)$, whereas the second term is also strictly increasing for any $D^{\theta}_{*}(\omega) < \omega$. The last term, in contrast, is constant in $D^{\theta}_{*}(\omega)$ everywhere except at ω where it suffers a discontinuous jump. Any menu for which $D^{\theta}_{*}(\omega) < \omega$ over a set with positive measure $\Omega^{-} \equiv \left\{\omega \in [0,1] : D^{\theta}_{*}(\omega) < \omega\right\}$ can then be strictly dominated by slightly increasing the value of $D^{\theta}_{*}(\cdot)$ for $\omega \in (\sup\Omega^{-} - \epsilon, \sup\Omega^{-})$, for $\epsilon > 0$ small.

Next, note that for any ω for which $D^{\theta}_{*}(\omega) > \omega$, it is (pointwise) optimal to set $D^{\theta}_{*}(\omega) = 1$, as both the second and third terms are invariants to increments in $D^{\theta}_{*}(\omega)$, whereas the first term is strictly increasing in $D^{\theta}_{*}(\omega)$.

We conclude that the optimal mechanism must take the form

$$D^{\theta}_{*}(\omega) = \begin{cases} \omega & \text{, for } \omega < x \\ 1 & \text{, for } \omega \geq x \end{cases}$$

for some $x \in [0,1]$. We thus optimize $\mathbb{E}\left\{p^{\theta}_{*}(\boldsymbol{\omega})\right\}$ by changing the value of x. For any such a menu, we have

$$\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\} = (1-\theta)\left(\int_0^x \left(\omega - \frac{\omega^2}{2}\right)\mathrm{d}\omega + \frac{1-x}{2}\right) + \theta\int_0^1 (\omega - (1-\omega)\,1\{\omega > x\})\,\mathrm{d}\omega.$$

This further implies that

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\} = \frac{(1-x)\left((1-\theta)\,x - (1-3\theta)\right)}{1-\theta}.$$

We finally note that, for any $\theta \in \left[\frac{1}{3}, 1\right)$, $\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\} \geq 0$ for all $x \in [0,1]$. We conclude that in that case, it is optimal to choose $x = 1$, and therefore $D_*^\theta(\omega) = \omega$ for all $\omega \in [0,1]$. In contrast, when $\theta < 1/3$, $\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\}$ is quasi-convex in x and the optimal choice is found at the corners. We thus need to compare the value of $\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\}$ at $x = 0$ and $x = 1$. We find that

$$\begin{aligned}
\mathbb{E}\left\{p_*^\theta(\boldsymbol{\omega})\right\}\Big|_{x=1} &= (1-\theta)\left(\int_0^1 \left(\omega - \frac{\omega^2}{2}\right)\mathrm{d}\omega\right) + \theta\int_0^1 \omega\mathrm{d}\omega \\
&= \frac{2+\theta}{6},
\end{aligned}$$

whereas

$$\begin{aligned}
\mathbb{E}\left\{p^\theta(\boldsymbol{\omega})\right\}\Big|_{x=0} &= \frac{1-\theta}{2} + \theta\int_0^1 (2\omega - 1)\,\mathrm{d}\omega, \\
&= \frac{1-\theta}{2}.
\end{aligned}$$

We conclude that for any $\theta \in \left(\frac{1}{7}, \frac{1}{3}\right)$, it is optimal to set $x = 1$, and then $D_*^\theta(\omega) = \omega$ for all $\omega \in [0,1]$, whereas, for any $\theta \in \left(0, \frac{1}{7}\right)$, it is optimal to set $x = 0$, and then $D_*^\theta(\omega) = 1$ for all $\omega \in [0,1]$. *q.e.d.*

Proof of Proposition 2. Claim (i) follows from Proposition 2 in Jewitt [2007] which states that, an experiment F'' Lehmann-dominates another experiment F', if and only if, for any arbitrary prior distribution Φ, the induced joint distributions $\mathbf{F}''_\Phi$ and $\mathbf{F}'_\Phi$ are ranked in the positive quadrant dependence (PQD) order.[24] For random vectors of dimension $N = 2$, the PQD order, in turn, is equivalent to the supermodular order (Tchen [1980]). Claim (ii) is standard (see, e.g., Müller and Stoyan [2002]) and follows from the fact that the domination in the supermodular order implies a higher degree of interdependence. Claim (iii)

[24] A distribution $P \in \Delta\mathbb{R}^N$ dominates $Q \in \Delta\mathbb{R}^N$ in the PQD order, if $P(z_1, ..., z_N) \leq Q(z_1, ..., z_N)$, $\forall (z_1, ..., z_N) \in \mathbb{R}^N$. See, e.g., Shaked and Shanthikumar [2007].

follows from noting that, for any nondecreasing function $u(\cdot)$, and any $z \in [0,1]$, the function $\mathbb{I}\{\Phi(\omega) \geq z\} u(y)$ is supermodular in (y, ω) and, therefore,

$$\int_{\mathbb{R}_+} \int_{\Omega} \mathbb{I}\{\Phi(\omega) \geq z\} u(y) \, \mathrm{d}\mathbf{F}''_{\Phi}(y, \omega) \geq \int_{\mathbb{R}_+} \int_{\Omega} \mathbb{I}\{\Phi(\omega) \geq z\} u(y) \, \mathrm{d}\mathbf{F}'_{\Phi}(y, \omega).$$

This means that, for any nondecreasing function $u(\cdot)$,

$$\mathbb{E}_{\mathbf{F}''_{\Phi}}\left(u(\boldsymbol{y}) \,|\Phi(\boldsymbol{\omega}) \geq z\right) \geq \mathbb{E}_{\mathbf{F}'_{\Phi}}\left(u(\boldsymbol{y}) \,|\Phi(\boldsymbol{\omega}) \geq z\right),$$

and, therefore ,

$$\mathbf{F}''_{\Phi}(y|\Phi(\boldsymbol{\omega}) \geq z) \succeq_{\mathrm{FOSD}} \mathbf{F}'_{\Phi}(y|\Phi(\boldsymbol{\omega}) \geq z), \ \forall z \in [0,1].$$

In other words, $\mathbf{F}''_{\Phi}$ dominates $\mathbf{F}'_{\Phi}$ in the Monotone Information Order for Nondecreasing objective functions (MIO-ND) sense (see Athey and Levin [2018]). Theorem 1 in Ganuza and Penalva [2010] then implies that $\mathbb{E}_{\mathrm{F}''_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega}) \succeq_{\mathrm{cvx}} \mathbb{E}_{\mathrm{F}'_{\Phi}}(\boldsymbol{y}|\boldsymbol{\omega})$. □

Proof of Lemma 6. Note that

$$\begin{aligned}
\mathbb{E}\left(p_*(\omega;F)\right) &= \int_{\Omega}\left\{\int_{\mathbb{R}_+} \min\{y, D_*(\omega;F)\}\left(1-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\left(\frac{\frac{\partial f(y|\omega)}{\partial \omega}}{f(y|\omega)}\right)\right)\mathrm{d}F(y|\omega)\right\}\mathrm{d}\Phi(\omega) \\
&= \int_{\Omega}\left\{\int_0^{D_*(\omega;F)} y\left(1-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\left(\frac{\frac{\partial f(y|\omega)}{\partial \omega}}{f(y|\omega)}\right)\right)\mathrm{d}F(y|\omega)\right\}\mathrm{d}\Phi(\omega) \\
&= \int_{\Omega}\left\{\int_0^{D_*(\omega;F)} \left(F(D_*(\omega;F)|\omega) - F(y|\omega)\right)\mathrm{d}y\right. \\
&\quad \left. -\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_0^{D_*(\omega;F)} \frac{\partial}{\partial\omega}\left(F(D_*(\omega;F)|\omega) - F(y|\omega)\right)\mathrm{d}y\right\}\mathrm{d}\Phi(\omega) \\
&= \int_{\Omega}\int_0^{D_*(\omega;F)} (1-F(y|\omega))\left\{1-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\frac{\partial}{\partial\omega}(1-F(y|\omega))}{1-F(y|\omega)}\right\}\mathrm{d}y\mathrm{d}\Phi(\omega),
\end{aligned}$$

where the second equality obtains from the definition of $D_*(\omega;F)$, the third equality follows from applying integration by parts, and the fourth equality obtains from rearranging terms and using the definition of $D_*(\omega;F)$, which implies that

$$1-F(D_*(\omega;F)|\omega) - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\partial}{\partial\omega}\left(1-F(D_*(\omega;F)|\omega)\right) = 0.$$

This completes the proof of the lemma. □

Appendix D: Proof of Theorem 3

Proof. Let $\tilde{F}$ be an arbitrary experiment. Define $\psi_{\tilde{F}}[D]$ as the issuer's revenue when she proposes a menu of incentive compatible debt contracts characterized by $\{D(\omega)\}_{\omega\in\Omega}$. By virtue of lemma 4, this means that

$$\psi_{\tilde{F}}[D] = \int_{\Omega}\left(\int_{\mathbb{R}_+} u\left(\min\{y, D(\omega)\}, \omega\right)\left(1 - \left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\left(\frac{\frac{\partial}{\partial\omega} f(y|\omega)}{f(y|\omega)}\right)\right) \mathrm{d}F(y|\omega)\right)\mathrm{d}\Phi(\omega).$$

We show that the function $\psi_{\tilde{F}}[D]$ has the *single crossing differences* property in $\left(D, \tilde{F}\right)$. That is, we show that, for any $D'' > D'$, and any $F'' \succeq_{\text{Lehmann}} F'$,

$$\psi_{F'}[D''] - \psi_{F'}[D'] \leq 0 \Rightarrow \psi_{F''}[D''] - \psi_{F''}[D'] \leq 0.$$

To see this, first note that

$$\begin{aligned}\frac{\partial}{\partial D(\omega)}\psi_F[D] &= \phi(\omega)\left(1 - F\left(D(\omega)|\omega\right)\right) - \left(1-\Phi(\omega)\right)\frac{\partial}{\partial\omega}\left(1 - F\left(D(\omega)|\omega\right)\right) \\ &= -\frac{\partial}{\partial\omega}\left(\left(1-\Phi(\omega)\right)\left(1 - F\left(D(\omega)|\omega\right)\right)\right). \qquad (23)\end{aligned}$$

Next, let $\boldsymbol{y}''$ (resp. $\boldsymbol{y}'$) be the cashflows obtained from drawing $\boldsymbol{\omega}$ from Φ and then applying the experiment F'' (resp. F'). Claim (1) in proposition 2 implies that the induced marginal distributions of $\boldsymbol{y}''$ and $\boldsymbol{y}'$ coincide and equals Ψ_Φ. The fact that $F'' \succeq_{\text{Lehmann}} F'$ implies that, for all $z \in [0,1]$, [25]

$$\frac{\frac{\partial}{\partial\omega} f''\left(\omega|\Psi_\Phi(\boldsymbol{y}'') \geq z\right)}{f''\left(\omega|\Psi_\Phi(\boldsymbol{y}'') \geq z\right)} \geq \frac{\frac{\partial}{\partial\omega} f'\left(\omega|\Psi_\Phi(\boldsymbol{y}') \geq z\right)}{f'\left(\omega|\Psi_\Phi(\boldsymbol{y}') \geq z\right)},$$

or, equivalently,

$$\frac{\frac{\partial}{\partial\omega}\left\{\Pr\left\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)\right\}}{\Pr\left\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)} \geq \frac{\frac{\partial}{\partial\omega}\left\{\Pr\left\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)\right\}}{\Pr\left\{\Psi_\Phi(\boldsymbol{y}') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)}.$$

Next, note that

$$\frac{\frac{\partial}{\partial\omega}\left\{\Pr\left\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)\right\}}{\Pr\left\{\Psi_\Phi(\boldsymbol{y}'') \geq z|\boldsymbol{\omega}=\omega\right\}\phi(\omega)} = \frac{\frac{\partial}{\partial\omega}\left(1 - F''\left(\Psi_\Phi^{-1}(z)|\omega\right)\right)}{1 - F''\left(\Psi_\Phi^{-1}(z)|\omega\right)} + \frac{\frac{\mathrm{d}}{\mathrm{d}\omega}\phi(\omega)}{\phi(\omega)},$$

[25] See corollary 1 in Athey and Levin [2018].

and similarly,

$$\frac{\frac{\partial}{\partial\omega}\left\{\Pr\left\{\Psi_{\Phi}\left(\boldsymbol{y}'\right)\geq z|\boldsymbol{\omega}=\omega\right\}\phi\left(\omega\right)\right\}}{\Pr\left\{\Psi_{\Phi}\left(\boldsymbol{y}'\right)\geq z|\boldsymbol{\omega}=\omega\right\}\phi\left(\omega\right)}=\frac{\frac{\partial}{\partial\omega}\left(1-F'\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)\right)}{1-F'\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)}+\frac{\frac{\mathrm{d}}{\mathrm{d}\omega}\phi\left(\omega\right)}{\phi\left(\omega\right)},$$

Thus, we must have that, for all $z\in[0,1]$,

$$\frac{\frac{\partial}{\partial\omega}\left(1-F''\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)\right)}{1-F''\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)}\geq\frac{\frac{\partial}{\partial\omega}\left(1-F'\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)\right)}{1-F'\left(\Psi_{\Phi}^{-1}\left(z\right)|\omega\right)}. \tag{24}$$

Finally, suppose that for some mechanism characterized by $D\left(\cdot\right)$, $\frac{\partial}{\partial D(\omega)}\psi_{F'}\left[D\right]\leq 0$. From (23), this is equivalent to having

$$\frac{\phi\left(\omega\right)}{1-\Phi\left(\omega\right)}\quad\leq\quad\frac{\frac{\partial}{\partial\omega}\left(1-F'\left(D\left(\omega\right)|\omega\right)\right)}{1-F'\left(D\left(\omega\right)|\omega\right)}.$$

Inequality (24) then implies that necessarily $\frac{\partial}{\partial D(\omega)}\psi_{F''}\left[D\right]\leq 0$. Further, note that, under assumptions 1 and 2, for any experiment $\tilde{F}$, the optimal mechanism $D_{*}\left(\cdot;\tilde{F}\right)$ is determined by pointwise maximization. The result then follows from Milgrom and Shannon [1994]. □

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