Persuading Multiple Audiences: Strategic Complementarities and (Robust) Regulatory Disclosures

by

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Abstract

How much information about financial institutions' balance sheets should regulators pass on to the market? To minimize the probability of inefficient default, the regulator optimally designs a disclosure regime that imposes transparency when the firm has weak fundamentals and opacity, otherwise. Intuitively, strategic complementarities among the investors, which are exacerbated by financial constraints, induce a preference for granular disclosures. The optimal policy is robust to investors' adversarial coordination and to the firm's agency, and remains optimal even if the firm can circumvent regulation and signal residual private information to the market. My results shed light on the optimal design of regulatory disclosures in environments with strategic complementarities, and provide a foundation for many empirical regularities found in practice.

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Keywords: Regulatory Disclosures, Robust Information Design, Strategic Complementarities, Financial Constraints, Transparency.

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Nicolas A. Inostroza

I have nothing to disclose.

1 Introduction

Information plays a key role in financial markets. Market participants routinely gather information from private and public sources and make investments decisions based on their findings. Information is especially critical for investors in institutions facing financial constraints, as the stakes are typically high and the survival of these institutions directly depends on the investors' decisions whether to continue pledging funds. However, how much information about these financial institutions' balance sheets should a regulator pass on to the market? Timely disclosures have the potential to restore market confidence about troubled institutions;[1] however, when not carefully designed, they risk unintentionally catalyzing a crisis.

When a regulator discloses information about a financial institution, she speaks to multiple *audiences* who care about different aspects of the institution's private information. Consider, for example, long-term investors interested in the profitability of the firm's assets (e.g., equity holders), short-term creditors (e.g., money market mutual funds) concerned about the firm's liquidity position, speculators interested in the fate of the firm, or counterparties exposed to a potential default. An optimally designed regulatory disclosure must necessarily account for the strategic reactions it induces in these multiple audiences.

Despite the recent attention that regulatory disclosures have received from the theoretical literature, the natural question concerning the *optimal* degree of transparency of such policies is not clear. One of the reasons behind this observation is the assumption, often encountered in the literature, of a single audience for the regulator's disclosure, which to a large extent oversimplifies the problem. When this is the case, the optimal policy is *opaque* and consists of a recommendation to the market whether to keep pledging funds to the firm.[2] In most cases this takes the form of a pass/fail test. With multiple audiences, however, disclosures intended for a particular audience are simultaneously *observed* by the rest of the market participants, generating an endogenous market reaction. As a result, the optimal degree of transparency of such disclosures is no longer clear.

Stated differently, a crucial ingredient determining regulatory disclosures' optimal degree of transparency is the strategic interaction among the multiple types of market participants concerned about the institution's (multidimensional) private information. This paper aims to shed light on this question and inform the debate on the optimal design of such disclosures.

I argue that the optimal level of transparency of regulatory disclosures is directly linked to the degree of strategic complementarities among the market participants directly concerned with the institution's fundamentals. When investors' incentives to pledge funds to the firm comove with other investors' decisions to provide financial support, then optimal regulatory disclosures aimed at maximizing efficiency (e.g., the flow of funds to solvent but potentially illiquid institutions) become transparent with respect to the institution's fundamentals. Intuitively, with strategic

[1] In the context of the global financial crisis, many scholars and regulators alike have argued that disclosing information about the health of systemically important banks, was a critical inflection point because it restored market confidence by providing investors with credible information about potential losses (Bernanke (2013), Hirtle and Lehnert (2015)).

[2] This is a manifestation of the revelation principle (Myerson (1982), Myerson (1986)).

complementarities, there exists an endogenous *amplification effect* associated with increasing the market's perception about the firm's financial health. Improving the investors' assessment of the firm's fundamentals induces investors to pledge more funds to the firm. The additional funds lead other investors to pledge more funds, which feeds back and induces yet more market participants to provide larger financial support. Thus, the complementarities between the investors induce an amplification mechanism that translates into a convex market response in the perception of the firm's fundamentals. These convexities imply that a regulator concerned with maximizing efficiency strictly benefits from finer disclosure policies. More granular disclosures increase the regulator's (ex ante) expected payoff in the same manner as a risk-loving decision maker benefits from swapping deterministic outcomes for lotteries with the same expected outcome.

Financial constraints exacerbate the complementarities among the financial institution's multiple audiences. When the difference between the amount of funds the firm can raise on short notice (e.g., by selling assets or pledging them as collateral) grows small with respect to the size of liabilities that may suddenly dry up (e.g., commercial paper, certificates of deposit), investors become concerned about whether the firm will be able to meet its short-term obligations. Investors' incentives to pledge funds then comove with other investors' funding decisions. Indeed, observing other market participants pledge funds, e.g., by purchasing the firm's assets, by lending short-term funds, or by refraining from speculating against the firm, increases each market participant's own incentives to provide financial support to the firm.

To fix ideas, consider the following minimal model that preserves the richness of the problem. The economy consists of a firm, a regulator, and two audiences: asset market investors and short-term creditors (henceforth, AM investors and ST creditors). The firm has private information about two dimensions, namely, (i) the long-term profitability of its assets and (ii) its liquidity position. I refer to these two variables as the firm's fundamentals. Uncertainty about the fundamentals is gradually resolved. While the profitability of the firm's assets is determined early, the firm's liquidity is determined at a later stage after a shock (potentially) materializes. The timing is meant to reflect the idea that the profitability of the firm's assets depends on investment decisions made in the past, whereas the firm's liquidity is subject to shocks and may suddenly dry up. The regulator's technology allows her to design regulatory disclosures about the firm's fundamentals.[3]

The first audience, AM investors, is primarily interested in learning about the profitability of the firm's assets (e.g., the amount of nonperforming loans). The second audience, ST creditors, on the other hand, is concerned with the firm's liquidity and its ability to repay short-term debt. Nevertheless, AM investors also care about disclosures concerning the firm's liquidity, as such information affects ST creditors' decisions of whether to roll over the firm's short-term debt. Given that ST creditors' claims are senior to those of AM investors, the latter may be wiped out if ST creditors choose to *run*. Therefore, AM investors are indirectly affected by disclosures about the firm's liquidity. In turn, ST creditors indirectly care about the profitability of the firm's assets.

[3] As is standard in the *information design* literature, I assume that the regulator has commitment power and chooses the information disclosure policy before observing the true realization of the firm's fundamentals.

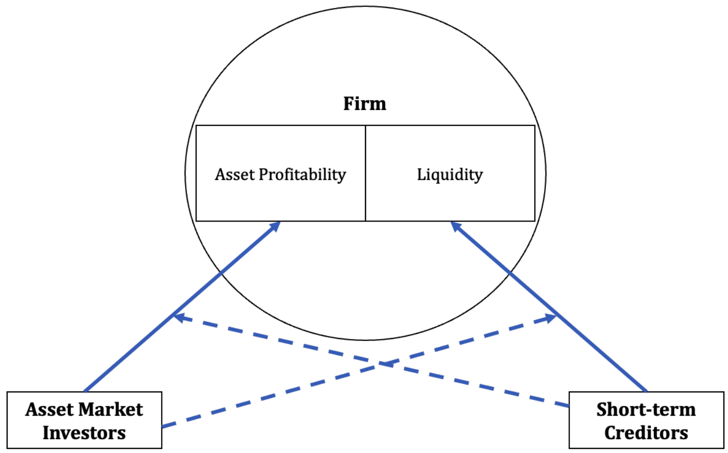

Figure 1: Persuading multiple Audiences.

Disclosures about this dimension determine the funds the firm can raise from AM investors either via asset sales or with collateralized borrowing.[4] The optimal regulatory disclosure thus has a fixed-point structure in that disclosures about one of the dimensions (e.g., asset profitability) account not only for the reaction of the audience who directly cares about that dimension (AM investors) but *also* for the endogenous reaction of the audiences who *indirectly* care about that dimension (ST creditors).

Using tools from the information design literature, I characterize the optimal public disclosure policy among *all* possible signal structures. I show that when the profitability of the firm's assets exceeds a threshold, the optimal policy is opaque and minimizes the information passed on to the market. By contrast, when the profitability of its assets falls below such a threshold, the optimal policy is transparent and provides granular information to all market participants.

The asymmetric structure of the optimal policy stems from the strategic interaction of the two audiences. When the profitability of the assets is low, the amplification mechanism described above gains traction. Improving the perception about the profitability of the firm's assets induces AM investors to pay larger prices for them. The additional funds increase the probability that the firm survives an eventual run by ST creditors. The higher resilience then induces AM investors to offer even higher prices for the firm's assets, and so forth. Thus, when the firm's financial constraints are stringent, the complementarities between the audiences gives rise to the amplification mechanism that translates into a convex survival probability in the perceived profitability of the firm's assets. The regulator thus prefers transparent disclosures over coarser rules.

In contrast, when the profitability of the firm's assets is high, the strategic complementarities weaken, and the amplification mechanism fades. The firm may prevent default altogether by raising sufficient funds to persuade ST creditors that it has enough liquidity buffers. Doing so dissipates the complementarities because AM investors are no longer concerned about ST creditors' behavior. Using a transparent policy in this case does not help and, in fact, may reduce risk-sharing

[4]Bolton et al. (2011) refer to the funds the firm is able to raise via asset sales or with collateralized borrowing as *outside* liquidity and to the firm's cash reserves as *inside* liquidity.

among firms with heterogeneous asset qualities. Thus, when the profitability of the firm's assets is sufficiently high, optimally designed disclosures are opaque.

I show that the predictions of the baseline model extend beyond the case of a single financial institution with two types of investors. Indeed, the same economic phenomenon manifests in a fairly large class of economies wherein the complementarities between market participants' actions are sufficiently strong. I show that as long as the audiences' behavior is vulnerable to the behavior of the other audiences, e.g., because of financial constraints, the optimal disclosure policy features a dichotomy between transparency and opacity, for poor and favorable fundamentals, respectively.

Furthermore, I show that the optimal disclosure policy is *robust* to both (a) adversarial coordination among the investors and (b) the financial institution's agency. On the first point, I take a conservative approach and assume that when multiple outcomes are consistent with equilibrium play, the audiences coordinate on the most adversarial (equilibrium) market response from the perspective of the regulator. This assumption captures the idea that when the regulator designs the disclosure policy, she does not trust her ability to coordinate the market on her most preferred outcome. Instead, the regulator is conservative and assumes that after disclosing the firm's information, the audiences will coordinate on the worst equilibrium profile. The optimal policy is thus conservative and accounts for the worst-case scenario.

Second, I assume that the financial institution is strategic and reacts to the regulator's disclosures. After the regulator reveals some of the firm's information to the market, the firm optimally chooses its funding strategy to maximize its profit. Optimal disclosures thus need to anticipate the firm's behavior and incorporate it into the design of the disclosure policy. Moreover, a financial institution with private information (arguably the more relevant case) may attempt to signal its residual private information (i.e., information not disclosed by the regulator) by strategically choosing its funding strategy, e.g., by choosing the security it sells to investors. Indeed, in many applications of interest, the firm's private information may be an important concern. In the case of banking, e.g., regulators and market participants alike pay close attention to the bank's superior information with respect to its opaque balance sheet (e.g., the amount of nonperforming loans). The firm's actions are then usually scrutinized and used as signals of its residual private information. I show that the optimal policy is robust to these signaling incentives. The optimal policy has the interesting feature that it induces no further revelation of the firm's private information to the market.

The theory in this paper predicts that when an institution faces strong financial constraints (e.g., a bank rolling over a large amount of short-term debt, an investment fund facing frequent redemptions), it should be subject to regulatory disclosures displaying a negative relationship between the degree of transparency and the institution's financial condition. The empirical literature on regulatory disclosures has found regularities consistent with this prediction. In the context of banks' stress tests, there is evidence that institutions with weaker fundamentals (e.g., riskier assets, larger quantities of nonperforming loans), are subject to more transparency than institutions with stronger fundamentals (Morgan et al. (2014), Flannery et al. (2017), and Ahnert et al. (2018)). Chen et al. (2022) find, in a recent paper, that Call Reports for US-based banks are more informative for banks with worse-performing assets. In the context of investment funds, Agarwal et al. (2013) find that hedge funds that request confidential treatment of their holdings in their Form 13F (i.e., those that delay disclosing their holdings for 45 or more days), exhibit significantly higher performance.

Furthermore, the optimal policy's asymmetric treatment between bad and good news is broadly consistent with the *conservatism principle* usually promoted by accounting standard-setters. According to the dictum, financial institutions should record losses as soon as they learn about them, whereas potential gains are to be recognized only after they materialize. A financial institution adhering to this accounting standard is prone to disclose more granular information when its assets perform poorly and to disclose coarser information otherwise, consistent with our insight. Thus, the theory can provide a microfoundation for the widespread accounting practice.

The remainder of this paper is organized as follows. Below I complete the introduction with a brief review of the pertinent literature. Section 2 presents the model. Section 3 describes the equilibrium concept and its properties taking the information disclosed by the regulator as given. Section 4 studies the optimal design or regulatory disclosures. Section 5.1 studies the robustness of the optimal disclosure policy. Finally, Section 6 extends the insights of the baseline model to a large class of economies. Omitted proofs are provided in the Appendix or Online Appendix.

Related literature. This paper is related to several strands of the literature. The first strand is the literature on *regulatory disclosures*. Faria-e Castro et al. (2016) study information disclosure in an environment with runnable liabilities and asymmetric information. The paper finds a monotonic relationship between the government's fiscal capacity and the regulatory disclosure's level of transparency. Goldstein and Leitner (2018) consider the problem of a regulator who seeks to facilitate risk-sharing among firms with assets of heterogeneous qualities. Inostroza and Pavan (2019) follow an adversarial approach and explore optimal disclosure policies with heterogeneously informed receivers. They find that optimal policies delete strategic uncertainty but preserve structural uncertainty.[5] Bouvard et al. (2015) study disclosures when firms are subject to rollover risk. The regulator cannot ex ante commit to her disclosures and hence is subject to a policy trap in that disclosing information signals negative information to the market. In contrast, I assume the regulator can commit to her disclosure policy before examining the firm's balance sheet.[6] Orlov et al. (2017) consider the joint design of stress tests and capital requirements. They study macroprudential stress tests for firms with correlated exposures. Also related is Quigley and Walther (2020), who study how firms react to regulatory disclosures by voluntarily disclosing private information. In my model, firms cannot disclose hard information but may signal information through their funding strategy.

My paper also contributes to the growing literature of optimal disclosures with multiple audiences. Malenko et al. (2021) study proxy advisors' recommendations to two type of investors, subscribers and nonsubsribers. Li et al. (2021) study how to induce heterogeneous responses from

[5] The literature on regulatory disclosures has grown at a swift pace in the last few years. Some recent contributions include Basak and Zhou (2020), Dai et al. (2021b), Huang (2020) Leitner and Williams (2021), Parlasca (2021), Parlatore and Philippon (2020).

[6] Regulators usually publish scenarios and the statistics that will be disclosed as part of their disclosure exercises prior to the assessment.

homogeneously informed audiences in the context of an entry game. Bond and Zeng (2019) study verifiable disclosures when the receiver's preferences are uncertain. Alonso and Camara (2016a) and Bardhi and Guo (2018) consider disclosures to a jury in a voting context. Li et al. (2020) and Morris et al. (2020) study persuasion with multiple receivers in binary action, supermodular games.

The paper also contributes to the extensive literature on security design with adverse selection, as in Myers and Majluf (1984), DeMarzo and Duffie (1999), and DeMarzo and Fishman (2007), among others.[7] I adopt the framework of Nachman and Noe (1994), who consider the problem of a seller with private information about the profitability of her assets and who issues claims on them in exchange for funds that help her meet a former liability. I modify their setting by introducing an endogenous probability of default, which is determined in equilibrium. In contrast to their celebrated result, which shows the existence of a unique equilibrium where all types of sellers pool over the same debt contract, I show that introducing the endogenous probability of default brings back multiplicity, but many of the qualitative properties in their paper remain true in this more general environment.

Finally, this paper relates more broadly to the literature on *information design*. This literature can be traced back to Myerson (1986). Recent developments include Kamenica and Gentzkow (2011), Kamenica and Gentzkow (2016), and Ely (2017). Bergemann and Morris (2016a) and Bergemann and Morris (2016b) characterize the set of outcome distributions that can be sustained as Bayes-Nash equilibria under arbitrary information structures consistent with a given common prior. Alonso and Camara (2016b) study public persuasion in a setting with multiple receivers with heterogeneous priors. Basak and Zhou (2017) and Doval and Ely (2017) study dynamic games in which the regulator can control both the agents' information and the timing of their actions.

2 Baseline Model

Players and Actions. The economy consists of a firm, a regulator, and two audiences: Short-term (ST) creditors and asset market (AM) investors. The firm may represent a financial intermediary (e.g., a bank, an investment fund), a corporation, etc. ST creditors represent market participants who have already pledged funds to the firm so that the latter invests the pool of funds and purchases assets. ST creditors may represent depositors of a bank, investors in mutual funds, limited partners in a VC fund, etc. AM investors, on the other hand, are agents who can purchase the firm's assets (or claims on them).

There are 3 periods, $T \equiv \{0, 1, 2\}$. The firm has two assets: (i) a safe and liquid asset (e.g., treasuries) and (ii) a risky and illiquid asset (e.g., a portfolio of loans, a venture project).[8] Both assets mature in period 3. The safe asset and the risky asset deliver observable stochastic cashflows

[7]Recent developments study the interplay between information and security design. Some recent papers include Daley et al. (2016), who consider the effect of ratings; Yang (2015), who studies security design with information acquisition; Szydlowski (2018) and Inostroza and Tsoy (2022), who consider when the joint design of information and securities; and Azarmsa and Cong (2020) who study the role of information in relationship finance.

[8]The illiquidity assumption captures the idea that the firm has technology to monitor the asset that cannot be easily transferred to external investors.

$\boldsymbol{\theta}_s$ and $\boldsymbol{\theta}_r$, respectively.

In period 0, to increase the amount of liquid funds available in period 1, the firm can sell claims on its risky assets (i.e., securities) to the asset market composed of a competitive and risk-neutral continuum of AM investors on $[0, 1]$. These are investors interested in the long-term profitability of the firm's assets. For each claim on the firm's future cash flows s (described below), each AM investor $j \in [0, 1]$, proposes a price $p_j \in \mathbb{R}_+$.

In period 1, the firm potentially suffers a temporary liquidity shock that impairs the safe asset, turning a fraction $\boldsymbol{\omega}$ of the asset illiquid. Specifically, $\boldsymbol{\omega} \in \Omega \equiv [0, 1]$ represents the largest fraction of the safe asset that the firm can liquidate during period 1 to repay early ST creditors. If the firm liquidates a fraction $\nu < \boldsymbol{\omega}$ of its safe asset in period 1, it obtains $\nu\theta_s/R$ units of funds. A value of $\boldsymbol{\omega} < 1$ can be interpreted as an unexpected liquidity shock that reduces the amount of liquid funds available at $t = 1$ (e.g., haircuts imposed in the repo market). I assume that the fraction of the safe asset that is not liquidated in period 1 becomes available in period 2; thus, $\boldsymbol{\omega}$ represents a *temporary* liquidity shock.[9] ,[10]

Finally, on the liability side of the balance sheet, a mass one of ST creditors, uniformly distributed over $[0, 1]$, is endowed with a contract (d_1, d_2). Each ST creditor has a claim promising a payoff d_1 if the ST creditor redeems *early* in period 1 or equal to d_2 if the ST creditor waits and redeems *late* in period 2. In the case of banking, e.g., these claims can be interpreted as uninsured deposits (e.g., repo, certificates of deposit), and the decision to wait can be regarded as rolling over the bank's debt. In the case of an investment fund, on the other hand, these claims can be interpreted as shares of the fund, and the decision to redeem early as the choice of selling the funds' shares. In that case, d_1 represents the *net asset value* (NAV) of the fund. ST creditors can reinvest the withdrawn funds elsewhere and guarantee a return normalized to 1. I assume that ST creditors' contract (d_1, d_2) is exogenous.

Let $a_i \in \{0, 1\}$ denote the action chosen by ST creditor i, where $a_i = 1$ represents the action of withdrawing late, and $a_i = 0$ represents the decision of withdrawing early. I denote by $A_{\text{ST}} \in [0, 1]$ the measure of ST creditors who pledge funds. Henceforth, I refer to the decision of redeeming early (resp., late) as *running* (resp., *pledging*), and as early (resp., late) ST creditors to those ST creditors who decide to run (resp., pledge). I assume that at most a fraction $1 - A_0 \in [0, 1]$ of ST creditors can run on the firm (i.e., a fraction A_0 of ST creditors always pledges). In the case of banking, the fraction A_0 represents the bank's long-term debt, which is less susceptible to runs. In the case of open-end funds, the fraction A_0 may represent an exogenous inflow of funds (as in Chen et al, 2010). In the case of VC or PE funds, on the other hand, the fraction A_0 may capture lock-up periods that prevent a fraction of ST creditors from running.

Fundamentals. The fundamentals of the firm's balance sheet are captured by the random

[9] The banking literature usually assumes a penalty for liquidating assets early. I assume instead that a fraction of the safe asset $\boldsymbol{\omega}$ remains perfectly liquid, whereas the remaining $1 - \boldsymbol{\omega}$ fraction becomes completely illiquid (e.g., MBSs during the global financial crisis). This assumption is consistent with the evidence documenting a large amount of heterogeneity in safe assets' liquidity.

[10] This assumption is made to facilitate the exposition. A model where a $1 - \omega$ fraction of the safe asset is destroyed during the interim period (i.e., a permanent liquidity shock) can be accommodated by assuming that $\inf X_R \geq d_1$.

vector $\vec{\boldsymbol{\vartheta}} \equiv (\boldsymbol{\theta}_r, \boldsymbol{\theta}_s, \boldsymbol{\omega})$. The variable $\boldsymbol{\theta}_r$ represents the risky asset's future cashflow, drawn from the absolutely continuous cdf F_r with support $X_r = [0, \bar{x}] \subseteq \mathbb{R}_+$. The variable $\boldsymbol{\theta}_s$ represents the safe asset's cashflow if held until period 2. I assume that $\boldsymbol{\theta}_s$ is *deterministic* and satisfies $\boldsymbol{\theta}_s = R$. The variable $\boldsymbol{\omega}$ represents the liquidity of the safe asset and is drawn from $F_\omega \in \Delta[0,1]$. I assume that F_ω is absolutely continuous over $[0,1)$ and that it has a mass point at $\omega = 1$ of size $\lambda \in [0,1)$. In other words, with probability λ the firm is perfectly liquid.[11]

Fund-raising Stage. In period 0, the firm sells a security s to AM investors, which corresponds to a claim on the risky asset's future cashflows. The market observes the security s issued by the firm and prices it according to the available public information. Let $\bar{P}(x)$ be the market value of a security promising to pay expected cashflows $x = \mathbb{E}(s(\boldsymbol{\theta}_r))$. This pricing function is endogenously determined in Section 3 and we treat it as given for the remainder of this section. If the firm raises $\bar{P}(\mathbb{E}(s(\boldsymbol{\theta}_r)))$ units of funds in period 0, then the amount of cash available to repay early withdrawals in period 1 is given by $\boldsymbol{\omega} + \bar{P}(\mathbb{E}(s(\boldsymbol{\theta}_r)))$.

Exogenous Information. There is *gradual resolution of uncertainty.* At $t = 0$, the risky asset's cashflow, $\boldsymbol{\theta}_r$, is drawn from F_r. The cashflow realization cannot be observed by any market participant.[12] The liquidity shock $\boldsymbol{\omega}$ is drawn from $F_\omega \in \Delta[0,1]$ at the beginning of period 1 and is only observed by the firm. The assumption of gradual resolution of uncertainty reflects the idea that the profitability of the firm's assets depends on investment decisions made in the past, whereas the firm's liquidity is subject to unexpected shocks and may suddenly change.

Firm's Payoff. If the firm raises $P_{\text{AM}} = \bar{P}(\mathbb{E}(s(\boldsymbol{\theta}_r)))$ from AM investors, it survives as long as the available funds are greater than its obligations, i.e., $P_{\text{AM}} + \boldsymbol{\omega} \geq d_1 \cdot (1 - A_{\text{ST}})$. In such a case, the firm reinvests the remaining cash and obtains a payoff of $R(P + \boldsymbol{\omega} - d_1 \cdot (1 - A_{\text{ST}}))$ at $t = 2$. Additionally, the firm must also repay late ST creditors in period 2, each of whom has been promised an amount d_2. I assume that (d_1, d_2) satisfy (a) $d_2 = Rd_1$,[13] and (b) $d_2 \in \left[\boldsymbol{\theta}_s, \boldsymbol{\theta}_s + \bar{P}(\mathbb{E}\{\boldsymbol{\theta}_r\}/R)\right]$. That is, ST creditors are promised a payment in period 2 at least equal to the return on the safe asset and at most equal to the expected value of the firm's assets. These assumptions imply that if the firm does not default in period 1, it does not default in period 2 either.[14] Thus, the firm's period 2 payoff is given by

[11] The assumption that $\boldsymbol{\theta}_r$ and $\boldsymbol{\omega}$ are independent does not mean that the firm's liquidity and asset profitability are uncorrelated. In fact, the amount of funds raised P correlates with its underlying quality; in the absence of information frictions, firms with better assets are able to secure more liquid funds at short notice.

[12] I consider departures from this assumption in Section 5.1

[13] This assumption is made on the grounds arbitrage. If $d_2 < Rd_1$, then ST creditors find it dominant to withdraw early. If in turn $d_2 > Rd_1$, then if ST creditors are subject to preference shocks as in Diamond and Dybvig (1983) (which I do not model here), then there are arbitrage opportunities.

[14] Indeed, the firm survives in period 1 if $P_{\text{AM}} + \omega \geq d_1(1 - A_{\text{ST}})$. This implies that in period 2 the firm has the reinvested funds $R(P_{\text{AM}} + \omega - d_1(1 - A_{\text{ST}}))$ plus the fraction of the safe asset that becomes available at that point $R(1 - \omega)$. Together, the two sources of liquid funds are enough to cover the liabilities in period 2, $d_2 A_{\text{ST}}$.

$$
\begin{aligned}
U\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}, s\right) &\equiv \{R\left(P_{\mathrm{AM}}+\boldsymbol{\omega}-d_1\left(1-A_{\mathrm{ST}}\right)\right)+\theta_s\left(1-\boldsymbol{\omega}\right) \\
&\quad -d_2 A_{\mathrm{ST}}+\boldsymbol{\theta_r}-s\left(\boldsymbol{\theta_r}\right)\}\cdot 1\left\{P_{\mathrm{AM}}+\boldsymbol{\omega}\geq d_1\left(1-A_{\mathrm{ST}}\right)\right\} \\
&= \left\{R\left(P_{\mathrm{AM}}-d_1\right)+\boldsymbol{\theta_s}+\boldsymbol{\theta_r}-s\left(\boldsymbol{\theta_r}\right)\right\}\cdot 1\left\{P_{\mathrm{AM}}+\boldsymbol{\omega}\geq d_1\left(1-A_{\mathrm{ST}}\right)\right\}. \quad (1)
\end{aligned}
$$

ST Creditors' Payoffs. ST creditors choose between running and pledging. It is without loss to focus on the differential payoff between the two options. Let $\Delta u_{\mathrm{ST}}(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}})$ represent the differential payoff between pledging and running. Then,

$$
\begin{aligned}
\Delta u_{\mathrm{ST}}(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}) &= g\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right)\cdot 1\left\{P_{\mathrm{AM}}+\boldsymbol{\omega}\geq d_1\left(1-A_{\mathrm{ST}}\right)\right\} \\
&\quad +b\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) 1\left\{P_{\mathrm{AM}}+\boldsymbol{\omega}< d_1\left(1-A_{\mathrm{ST}}\right)\right\}
\end{aligned}
$$

where $g\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) \in \left[\underline{g}, \bar{g}\right]$, $b\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) \in \left[\underline{b}, \bar{b}\right]$ for all $\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right)$, with g and b nondecreasing in $\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right)$, and $\underline{g} > 0 > \bar{b}$.[15]

AM Investors' Payoffs. I assume that the claims promised to AM investors are subordinated to those of ST creditors.[16] Hence, AM investors' claims are repaid *only if* the firm avoids default.[17] AM investors would therefore like to price the security to match its true value while accounting for the fact that the firm may default. I capture this with standard quadratic preferences. That is, the period 2 payoff of an arbitrary AM investor $j \in [0,1]$ who offers a price p_j is given by[18]

$$
u^j_{\mathrm{AM}}\left(p_j, \vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) = -\frac{1}{2}\left(p_j - \frac{s\left(\theta_r\right)}{R}\cdot 1\left\{P_{\mathrm{AM}}+\boldsymbol{\omega}\geq d_1\left(1-A_{\mathrm{ST}}\right)\right\}\right)^2. \quad (2)
$$

Regulator's Payoff. The regulator is concerned with economic efficiency and would like the firm to survive only if the latter is solvent.

Definition 1. We say that the firm is *ex ante solvent* if, at $t=0$, the market value of its assets is larger than the value of its liabilities. Formally,

$$
\bar{P}\left(\mathbb{E}\left(\boldsymbol{\theta}_r\right)\right)+\frac{\boldsymbol{\theta_s}}{R} > d_1. \quad (3)
$$

When inequality (3) holds, then from an ex ante perspective, the value of the firm's assets exceeds that of its liabilities. However, because of the liquidity shock, the firm may be ex ante solvent but still become illiquid in period 1.

Let $\theta^{\#}$ represent the expected cashflow threshold above which the firm becomes ex ante solvent.

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[15] For example, $g\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) = d_2 - d_1 = (R-1)\, d_1$ and $b\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}\right) = -\frac{P_{\mathrm{AM}}+\omega}{1-A_{\mathrm{ST}}} > -d_1$.

[16] I explore departures from this assumption in Subsection 4.5.

[17] In the case of banking, e.g., this captures the idea that subordinated debt and equity (AM investors' claims) are junior to uninsured deposits (ST creditors' claims).

[18] Equations (1) and (2) assume that the proceeds obtained by selling security s are used first to repay ST creditors withdrawing early. Any unused amount is distributed among the firm's *former* shareholders.

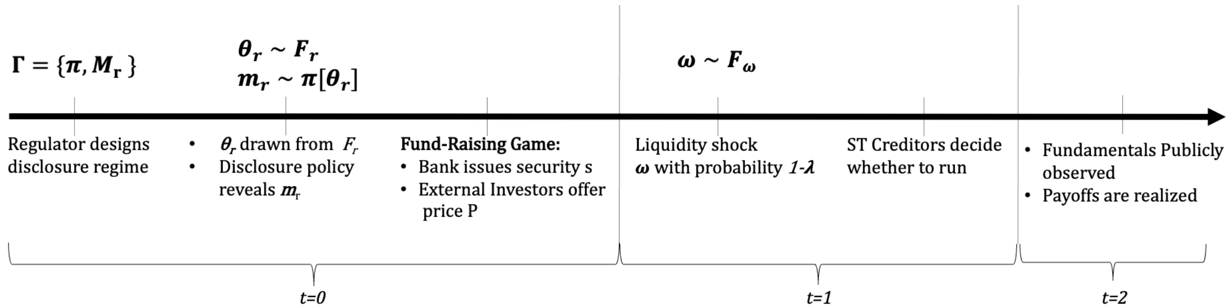

Figure 2: Timing.

That is, $\theta^{\#}$ is implicitly defined by $\bar{P}\left(\theta^{\#}\right) = d_1 - 1$. I denote by $\mathcal{R} = 0$ the event that the firm defaults and by $\mathcal{R} = 1$ the complementary event in which the firm survives. The regulator's ex ante payoff is measurable with respect to the firm's fate $\mathcal{R}$ and expected profitability of the firm's assets and is given by

$$U^R(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}, \mathcal{R}) \equiv L_0\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)(1-\mathcal{R}) + W_0\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)\mathcal{R}.$$

where $L_0\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right) \equiv \tau_L \max\left\{\theta^{\#} - \mathbb{E}\left\{\boldsymbol{\theta}_r\right\}, 0\right\}$ and $W_0\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right) \equiv \tau_W \max\left\{\mathbb{E}\left\{\boldsymbol{\theta}_r\right\} - \theta^{\#}, 0\right\}$, with $\tau_L \geq 0$ and $\tau_W > 0$. This specification captures the idea that, consistent with efficiency, the regulator obtains a weakly positive payoff when an insolvent firm defaults and that this magnitude decreases as the profitability of the firm's assets increases. In turn, the regulator's payoff is positive and increasing in the value of the firm's assets if the latter is solvent and avoids default.[19]

Regulatory Disclosures. The regulator has the technology and authority to implement regulatory disclosures that publicly disclose information about the firm's balance sheet to all market participants. The disclosure policy may represent stress testing exercises designed and conducted by a central bank (e.g., CCAR and DFAST), a report required by a financial supervisor (e.g., call reports filled by banks for the FDIC, or form 13F filled by institutional investment managers for the SEC). Finally, it may represent an accounting standard designed by an accounting system (e.g., GAAP). The assumption of gradual resolution of uncertainty implies that the regulator can disclose information about cashflows of the risky asset at $t = 0$, but not about $\boldsymbol{\omega}$ which materializes in period 1. I denote by Γ the regulatory disclosure about the profitability of the firm's risky asset $\boldsymbol{\theta}_r$. A regulatory disclosure $\Gamma = \{M_r, \pi\}$, consists of an arbitrary set of possible announcements M_r (e.g., scores, report) and a disclosure rule $\pi : X_r \to \Delta M_r$, which maps the realization of $\boldsymbol{\theta}_r$ into a (potentially stochastic) announcement $\boldsymbol{m}_r \in M_r$. I assume that the regulator cannot choose to learn information about $\boldsymbol{\theta}_r$ and not share it with market participants.[20] In Section 5, I explore the possibility of further disclosing information about $\boldsymbol{\omega}$ in period 1 and how this interacts with Γ.

Timing. The sequence of events is as follows:

[19] An alternative specification assumes that there exist large externalities from the firm's default and therefore the regulator maximizes the unrestricted probability of survival. A prior version of this manuscript, Inostroza (2019), studies that case and obtains the same qualitative results.

[20] This assumption is consistent with the idea that any information produced by the regulator leaks; therefore, if the regulator does not want the rest of the market participants to learn some specific information, she does not produce it in the first place. A similar assumption is made by Faria-e Castro et al. (2016).

Period 0. (a) The regulator designs the regulatory disclosure Γ and publicly announces it; (b) $\boldsymbol{\theta}_r$ is drawn from F_r; (c) the regulator publicly discloses information $\boldsymbol{m}_r$; and (d) The firm sells security $s \in S$ to AM investors at price P_{AM}.

Period 1. (a) $\boldsymbol{\omega}$ is drawn from F_ω; (b) ST creditors observe P_{AM} and the information disclosed by the regulator and decide whether to withdraw early; and (c) the firm liquidates a fraction of its safe asset, and its fate is determined according to whether $\boldsymbol{\omega} + P_{\mathrm{AM}} \geq d_1 (1 - A_{\mathrm{ST}})$.

Period 2. Conditional on the firm's survival, (a) ST creditors that pledged funds are paid back; (b) $\boldsymbol{\theta}_r$ is realized and $s(\boldsymbol{\theta}_r)$ is paid to AM investors, and the firm's shareholders obtain $\boldsymbol{\theta}_r - s(\boldsymbol{\theta}_r)$.

3 Equilibrium

3.1 Robust Approach

I assume that renegotiation between ST creditors and the firm is not feasible. Given the speed of events and the dispersion of ST creditors, renegotiation is, in most cases, unviable.[21] I follow a conservative approach and assume that when multiple action profiles are consistent with equilibrium play, ST creditors coordinate on the most aggressive outcome consistent with the rationality of both audiences (from the firm's perspective).

The adversarial approach implies that ST creditors run on the firm whenever running is the best response to everyone else running; that is, each ST creditor runs when

$$\mathbb{E}\left\{\Delta u_{\mathrm{ST}}(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}} = 1)\right\} \leq 0. \tag{4}$$

Define $K \geq 0$ as the minimum amount of funds needed to persuade ST creditors to pledge under adversarial coordination. That is,

$$K \equiv \inf\left\{P \geq 0 : \mathbb{E}\left\{\Delta u_{\mathrm{ST}}(\vec{\boldsymbol{\vartheta}}, P, 1)\right\} > 0\right\}.$$

In other words, the firm can make it dominant for ST creditors to pledge funds by raising K units of funds during the fund-raising stage. Let $A_{\mathrm{ST}}(P)$ be the smallest measure of ST creditors willing to pledge given a level of funds raised P. From the definition of K, under adversarial coordination, we have that $A_{\mathrm{ST}}(P) = A_0 + (1 - A_0)\, 1\{P > K\}$.

3.2 Fund-raising under Adverse Market Conditions

In period 1, the firm then enters the *fund-raising stage* by approaching AM investors and offering security s. We note that any securities with the same expected cashflows receive the same price P_{AM} from AM investors. Thus, henceforth, we refer to the price associated with any security inducing expected cashflows equal to $x = \mathbb{E}\left(s\left(\boldsymbol{\theta}_r\right)\right)$ as $\bar{P}\left(x\right)$.

[21] See, e.g., Landier and Ueda (2009) for a similar assumption.

For most of the paper, I assume that the distribution of the liquidity shock F_ω is severe in that if the firm does not raise additional funds, ST creditors find it optimal to run. Otherwise, the problem is uninteresting.

Assumption 1. $\mathbb{E}\left\{\Delta u_{ST}(\vec{\boldsymbol{\vartheta}}, P_{AM}=0, A_{ST}=1)\right\} \leq 0.$

By the end of period 0, ST creditors perfectly observe the amount of funds raised and decide whether to pledge. If the firm raises at least K, then no ST creditor runs, allowing the firm to survive with certainty. On the other hand, if the amount raised is less than K, then all ST creditors able to redeem early (i.e., a fraction $1-A_0$) run on the firm, in which case the survival of the latter depends on the amount raised and on the realization of the liquidity shock ω. Define the function $\bar{\omega}(P) \equiv d_1 \cdot (1 - A_{\mathrm{ST}}(P)) - P$, which identifies the cutoff for the liquidity shock below which the firm defaults when the capital raised is P. Note that, by definition, $\bar{\omega}(P) = 0$ for any $P \geq K$.

The price that AM investors are willing to pay for any security with expected cashflows $x = \mathbb{E}(s(\boldsymbol{\theta}_r))$ is then given by[22]

$$\bar{P}(x) \equiv \sup\left\{p \geq 0 : \frac{x}{R}\mathbb{P}\left\{\boldsymbol{\omega} \geq \bar{\omega}(p)\right\} \geq p\right\}. \tag{5}$$

3.3 Equilibrium Concept

Let $\mathbb{E}\left\{U\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}, s\right)\right\}$ be the firm's expected utility when it sells security s, raises P_{AM} from AM investors and faces a mass A_{ST} of pledging investors. Without the regulator's intervention, the firm's payoff can be written as

$$\begin{aligned}\mathbb{E}\left\{U\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, A_{\mathrm{ST}}, s\right)\right\} &= \mathbb{E}\left\{\left(R(P-d_1) + \theta_s + \boldsymbol{\theta}_r - s(\boldsymbol{\theta}_r)\right)\cdot 1\left\{\boldsymbol{\omega} + P \geq d_1 \cdot (1 - A_{\mathrm{ST}}(P))\right\}\right\} \\ &= \left(PR - R(d_1 - 1) + \mathbb{E}\left\{\boldsymbol{\theta}_r - s(\boldsymbol{\theta}_r)\right\}\right)\mathbb{P}\left\{\boldsymbol{\omega} \geq d_1 \cdot (1 - A_{\mathrm{ST}}(P)) - P\right\}\end{aligned} \tag{6}$$

I say that $\{s^\star, P^\star_{\mathrm{AM}}, A^\star_{\mathrm{ST}}\}$ is an equilibrium of the fund-raising game if:

$$\begin{aligned}\text{[Sequential Rationality]:} \quad & s^\star \in && \arg\max_s \mathbb{E}\left\{U\left(\vec{\boldsymbol{\vartheta}}, P^\star_{\mathrm{AM}}, A^\star_{\mathrm{ST}}, s\right)\right\} \\ \text{[Competitive Market]:} \quad & P^\star_{\mathrm{AM}}(s^\star) = && \bar{P}\left(\mathbb{E}\left\{s^\star\right\}\right) \\ \text{[Adversarial Coordination]:} \quad & A^\star_{\mathrm{ST}}(P) = && A_0 + (1 - A_0)\, 1\left\{P > K\right\}, \ \forall P \geq 0\end{aligned}$$

3.4 Strategic Complementarities and Convexity

Below, I introduce an assumption that guarantees the existence of strategic complementarities between the investors' actions.

Assumption 2. *The prior distribution of ω, F_ω, is concave over* $(\max\left\{d_1 \cdot (1 - A_0) - K, 0\right\}, 1)$.

[22]When inequality in (5) admits multiple solutions, $\bar{P}(x)$ corresponds to the largest solution. This selection can be microfounded by assuming that AM investors are competitive and the firm makes a take-it-or-leave-it (TIOLI) offer to them, $\left(s, \bar{P}\left(\mathbb{E}\left\{s\right\}\right)\right)$. The price is the maximal price accepted by rational investors concerned with default risk.

Assumption 2 reflects the idea that the firm's liquidity constraints are severe. Intuitively, when F_ω is concave, low realizations of $\boldsymbol{\omega}$ are more likely to occur, and hence stringent liquidity shocks become more probable. Severe liquidity constraints exacerbate the strategic complementarities between the two audiences. When assumption 2 holds, AM investors believe that it is plausible that the firm faces a massive run. The interaction of the two audiences then generates a negative feedback cycle as AM investors *price in* the firm's probability of default. This process depresses the price AM investors are willing to pay for s, which makes a run of ST creditors more likely. This further increases the probability of default, which translates into an even lower price, and so forth. Thus, when the firm is liquidity-constrained, the audiences' behavior reinforce each other and may amplify the probability of default.

Assumption 2 is a sufficient condition for financial constraints to induce and amplify the strategic complementarities between the audiences. When this assumption holds, the firm becomes vulnerable to the interaction of the two audiences, giving rise to an endogenous mechanism that makes the probability of survival *convex* in the market perception about the profitability of the firm's asset.

Define $\phi(x)$ as the probability that the firm survives conditional on selling a security with expected cashflows $x = \mathbb{E}\{s(\boldsymbol{\theta}_r)\}$. That is,

$$\phi(x) \equiv \mathbb{P}\left\{\boldsymbol{\omega} \geq \bar{\omega}\left(\bar{P}(x)\right)\right\} = 1 - F_\omega\left(\bar{\omega}\left(\bar{P}(x)\right)\right). \tag{7}$$

let $\underline{P} \equiv \max\{d_1(1-A_0) - 1, 0\}$ represent the wedge between the maximal amount of liabilities that can be redeemed in period 1, $d_1(1-A_0)$, and the maximal amount of liquid funds available at that period assuming no liquidity shock, i.e., $\boldsymbol{\omega} = 1$. Lemma 1 below shows that when assumption 2 holds, ϕ becomes convex over the critical region. Furthermore, $\underline{P}$ becomes the minimal amount that can be raised from AM investors.

Proposition 1. *Suppose that assumption 2 holds. Let $x \equiv \mathbb{E}(s(\boldsymbol{\theta}_r))$ represent the expected cashflows of the security offered by the firm. The function ϕ then satisfies the following properties*

(a) $\bar{P}(x) = \phi(x) = 0$ for any $x < \underline{P}R/\lambda$.

(b) $\phi(x) = 1$ for any $x \geq KR$.

(c) Suppose F_ω admits a continuously differentiable density f_ω over $[0,1)$. Then, ϕ is convex over $[\underline{P}R/\lambda, KR)$.[23]

The convexity of ϕ follows from the interaction of the two audiences. Adverse financial conditions, as captured by assumption 2, exacerbate the strategic complementarities between the different types of investors. When the firm is subject to liquidity constraints, the incentives of each audience to pledge funds increase when the other audience pledges more funds. Indeed, when the expected cashflows of the security sold to AM investors $\mathbb{E}\{s(\boldsymbol{\theta}_r)\}$ increase, the probability that the firm survives increases because the firm becomes resilient to more stringent liquidity shocks and hence

[23]The assumption that f_ω is continuously differentiable over $[0,1)$ simplifies the analysis. I can dispense with this assumption and show that, under a mild condition on f_ω, the functional operator implicit in the definition of the price satisfies the Blackwell conditions for a contraction. The convexity of $\bar{P}$ and ϕ is then naturally inherited from the concavity of $F_\omega(\cdot)$.

more resilient to a run by ST creditors. The larger probability of survival feeds back and increases the expected value of the security to AM investors and hence the price they are willing to offer. The higher price further increases the probability of survival, and so forth. As a result, in the absence of additional forces, this amplification mechanism induces a convex probability of survival as a function of the expected value of the firm's security.

Conversely, when F_ω is convex and therefore high levels of liquidity are more likely to occur, an improvement in the perceived profitability of the risky asset increases the probability of survival, $\mathbb{P}\{\boldsymbol{\omega} \geq \bar{\omega}(P)\}$, at a decreasing rate. If this effect is sufficiently strong, the amplification mechanism described above may dissipate. I discuss the role of the prior F_ω in detail in Section (4.5).

3.5 Firm's Optimal Funding Strategy

Financial institutions optimally respond to regulation. In the current framework, the firm chooses the security sold to the asset market and therefore has agency over its funding strategy. In this subsection, we characterize the firm's optimal funding strategy at any possible continuation game after the regulatory disclosure Γ has publicly revealed any public announcement $\boldsymbol{m}_r$.

For any $P \geq 0$, let

$$\varphi(P) \equiv \mathbb{P}\{\boldsymbol{\omega} \geq d_1 \cdot (1 - A_{\text{ST}}(P)) - P\}$$

be the probability of survival as a function of the price P. In particular, this means that $\phi(x) = \varphi\left(\bar{P}(x)\right)$. Let $V(x; \mathbb{E}\{\boldsymbol{\theta}_r\})$ be the firm's payoff from issuing a security with expected cashflows $x = \mathbb{E}\{s(\boldsymbol{\theta}_r)\}$, when the expected cashflows of the whole risky asset $\mathbb{E}\{\boldsymbol{\theta}_r\}$. That is,

$$\begin{aligned} V(x; \mathbb{E}\{\boldsymbol{\theta}_r\}) &\equiv \left(\bar{P}(x) R - R(d_1 - 1) + \mathbb{E}\{\boldsymbol{\theta}_r\} - x\right) \mathbb{P}\left\{\boldsymbol{\omega} \geq d_1 \cdot \left(1 - A_{\text{ST}}\left(\bar{P}(x)\right)\right) - \bar{P}(x)\right\}. \\ &= \left(\bar{P}(x) R - R(d_1 - 1) + \mathbb{E}\{\boldsymbol{\theta}_r\} - x\right) \varphi\left(\bar{P}(x)\right). \end{aligned}$$

The firm's problem reduces to issuing any security with a expected value

$$x^\star(\mathbb{E}\{\boldsymbol{\theta}_r\}) \equiv \underset{x \in [0, \mathbb{E}\{\boldsymbol{\theta}_r\}]}{\arg\max} \quad V(x; \mathbb{E}\{\boldsymbol{\theta}_r\}) \quad .$$

Let $h(x) \equiv \frac{x}{R}(1 - \phi(x))$ be the haircut associated with a security with expected cashflows $x = \mathbb{E}\{s(\boldsymbol{\theta}_r)\}$, that is, the difference between the safe value of the security and the equilibrium price that accounts for default risk. Next, recall that $\theta^\#$ represents the threshold above which the firm becomes (ex ante) solvent and is defined as the unique solution to the equation $\bar{P}\left(\theta^\#\right) = d_1 - 1$. The next result shows that when the firm is subject to adverse financial constraints, it optimally chooses to sell all the risky asset whenever the firm is ex-ante solvent, i.e., $\mathbb{E}\{\boldsymbol{\theta}_r\} \in \left[\theta^\#, KR\right)$, and on the contrary not to raise funds at all when the firm is insolvent, i.e., $\mathbb{E}\{\boldsymbol{\theta}_r\} < \theta^\#$.

Proposition 2. *Suppose that*

$$h(\underline{P}R/\lambda) \geq h(\theta^\#), \tag{8}$$

and that assumption (2) holds; then, the firm's optimal choice $x^{\star}\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)$ *takes the form:*

$$x^{\star}\left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)=\begin{cases}0 & \text{if } \mathbb{E}\left\{\boldsymbol{\theta}_r\right\}<\theta^{\#}\\ \mathbb{E}\left\{\boldsymbol{\theta}_r\right\} & \text{if } \mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\in\left[\theta^{\#},KR\right)\\ KR & \text{if } \mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\geq KR.\end{cases}$$

The assumption that $h\left(\underline{P}R/\lambda\right)\geq h\left(\theta^{\#}\right)$ in the proposition is a somewhat natural technical condition that provides tractability.[24] The assumption is closely related to the firm's financial constraints. Indeed, the inequality holds, e.g., when λ is small, that is, when the probability that the firm faces a liquidity shock (i.e., $1-\lambda$) is large, or when A_0 is small, meaning that the fraction of ST creditors who may run on the firm (i.e., mass $1-A_0$) is large.[25] The inequality guarantees that a firm at the verge of insolvency (i.e., $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}=\theta^{\#}$), prefers to maximize the amount of funds raised from AM investors by selling all of the risky asset rather than selling the fraction $\underline{P}R/\lambda$.

The proof of Proposition 2 shows that, when $h\left(\underline{P}R/\lambda\right)\geq h\left(\theta^{\#}\right)$ and, in addition, assumption (2) also holds, then for any $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\geq\underline{P}R/\lambda$, the firm's payoff $V\left(\cdot;\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)$ is v-shaped (and hence quasi-convex) over $\left[\underline{P}R/\lambda,KR\right]$ and attains a maximum at the corners of the interval. That is, conditional on the firm raising strictly positive funds, it either sells the whole risky asset or a security with expected cashflows $x=\underline{P}R/\lambda$. A key step in the proof is the observation that the firm's payoff $V\left(x;\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)$ has increasing differences in $\left(x;\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\right)$ (i.e., V is supermodular). This property implies that, if for some value $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\in\left[\underline{P}R/\lambda,KR\right]$ the firm prefers to sell the risky asset rather than issuing a security with value $x<\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}$, then any firm with expected cashflows larger than $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}$ prefers to sell the risky asset rather than a security with value x. The assumption that $h\left(\underline{P}R/\lambda\right)\geq h\left(\theta^{\#}\right)$ guarantees that a firm on the verge of solvency (i.e., $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}=\theta^{\#}$) prefers to maximize its probability of survival and sells all of the risky asset rather than the just a fraction $x=\underline{P}R/\lambda$. Because of the supermodularity property, this implies that all solvent but illiquid firms (i.e., those with $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}\in\left[\theta^{\#},KR\right)$) then raise the maximal amount of funds to minimize the probability of default.

4 Optimal Information Disclosure

The regulator can design mandatory disclosures that control the information that the firm passes on to the market. In period 0, the regulator designs a *regulatory disclosure* $\Gamma=\{M_r,\pi\}$, where M_r represents an arbitrary set of possible announcements (e.g., scores, report) and a disclosure policy

[24] When the condition is violated, the firm may be tempted to sell the smallest fraction of the asset that guarantees a nontrivial price, $x=\underline{P}R/\lambda$, if $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}$ is close to $\theta^{\#}$. The qualitative properties of optimal regulatory disclosures described below would not change, but the derivation of the optimal policy is substantially more intricate.

[25] A sufficient condition for $h\left(\underline{P}R/\lambda\right)\geq h\left(\theta^{\#}\right)$ is that

$$\lambda\leq\frac{d_1\left(1-A_0\right)-1}{\frac{d_1-1}{1-F_{\omega}\left(1-d_1A_0\right)}-d_1A_0},\tag{9}$$

which is satisfied for, e.g., when λ or A_0 is sufficiently small.

$\pi : X_r \to \Delta M_r$, which maps the realization of $\boldsymbol{\theta}_r$ into a (potentially stochastic) announcement $\boldsymbol{m}_r$. This formulation is general and encompasses all types of disclosures which are measurable with respect to the firm's assets.

I characterize below the optimal regulatory disclosure $\Gamma^\star$ accounting for the optimal responses of both audiences and the firm's funding strategy. We start from the observation that, any score m_r disclosed with positive probability induces a posterior expectation of $\boldsymbol{\theta}_r$, $\mathbb{E}(\boldsymbol{\theta}_r|\boldsymbol{m}_r = m_r)$. Let G^Γ be the distribution of posterior expectations induced by the policy Γ, i.e., the cdf of the random variable $\mathbb{E}(\boldsymbol{\theta}_r|\boldsymbol{m}_r)$. Strassen's theorem implies that, for any policy Γ, G^Γ must be a mean-preserving contraction of the prior F_r. Conversely, any mean-preserving contraction of the prior can be obtained with some disclosure policy Γ. Thus, the regulator's problem of maximizing over all possible disclosure policies is equivalent to the more tractable problem of optimizing over all mean-preserving contractions of the prior (Dworczak and Martini (2019), Gentzkow and Kamenica (2016)).[26]

4.1 The Regulator's Problem

For each announcement $\boldsymbol{m}_r = m_r$, let $\bar{\theta}_r = \mathbb{E}\left(\boldsymbol{\theta}_r|m_r\right)$ represent the induced posterior expectation of the risky asset's cashflows. Proposition 2 implies that the firm optimally chooses to sell a fraction $x^\star\left(\bar{\theta}_r\right)$ of the risky asset and secures $\bar{P}\left(x^\star\left(\bar{\theta}_r\right)\right)$ funds from AM investors. The regulator's payoff then becomes[27]

$$
\begin{aligned}
\mathcal{U}^R_\star(\bar{\theta}_r) &\equiv \mathbb{E}\left\{U^R(\bar{\theta}_r, \boldsymbol{\omega}, P^\star_{\text{AM}}, A^\star_{\text{ST}})\right\}, \\
&= \mathbb{E}\left\{L_0\left(\bar{\theta}_r\right) 1\left\{\boldsymbol{\omega} < \bar{\omega}\left(\bar{P}\left(x^\star\left(\bar{\theta}_r\right)\right)\right)\right\} + W_0\left(\bar{\theta}_r\right) 1\left\{\boldsymbol{\omega} \geq \bar{\omega}\left(\bar{P}\left(x^\star\left(\bar{\theta}_r\right)\right)\right)\right\}\right\} \\
&= L_0\left(\bar{\theta}_r\right)\left(1 - \phi\left(x^\star\left(\bar{\theta}_r\right)\right)\right) + W_0\left(\bar{\theta}_r\right)\phi\left(x^\star\left(\bar{\theta}_r\right)\right).
\end{aligned}
$$

Using the fact that $L_0\left(\bar{\theta}_r\right) = 0$ for all $\bar{\theta}_r \geq \theta^\#$, $W_0\left(\bar{\theta}_r\right) = 0$ for all $\bar{\theta}_r \leq \theta^\#$, and the characterization in propositions 1 and 2, we thus have

$$
\mathcal{U}^R_\star(\bar{\theta}_r) = \begin{cases} L_0\left(\bar{\theta}_r\right)\left(1 - \phi\left(0\right)\right) & \text{if } \bar{\theta}_r < \theta^\# \\ W_0\left(\bar{\theta}_r\right)\phi\left(\bar{\theta}_r\right), & \text{if } \bar{\theta}_r \in \left[\theta^\#, KR\right) \\ W_0\left(\bar{\theta}_r\right) & \text{if } \bar{\theta}_r \geq KR. \end{cases}
$$

[26] Let F and G be distribution functions with support in $X \subseteq \mathbb{R}$. We say that G is a mean-preserving contraction of F (alternatively, $F \succeq_{\text{MPS}} G$), if $\int_X \varphi(x)F(\text{d}x) \geq \int_X \varphi(x)G(\text{d}x)$, for any convex function φ in X.

[27] The regulator's payoffs L_0 and W_0 from the firm's default and survival depend on the ex ante value of the firm's assets $\bar{\theta}_r = \mathbb{E}\left(\boldsymbol{\theta}_r|m_r\right)$ (i.e., whether the firm is ex ante solvent) and not the amount of funds raised $x^\star\left(\bar{\theta}_r\right)$. The latter determines the fate of the firm.

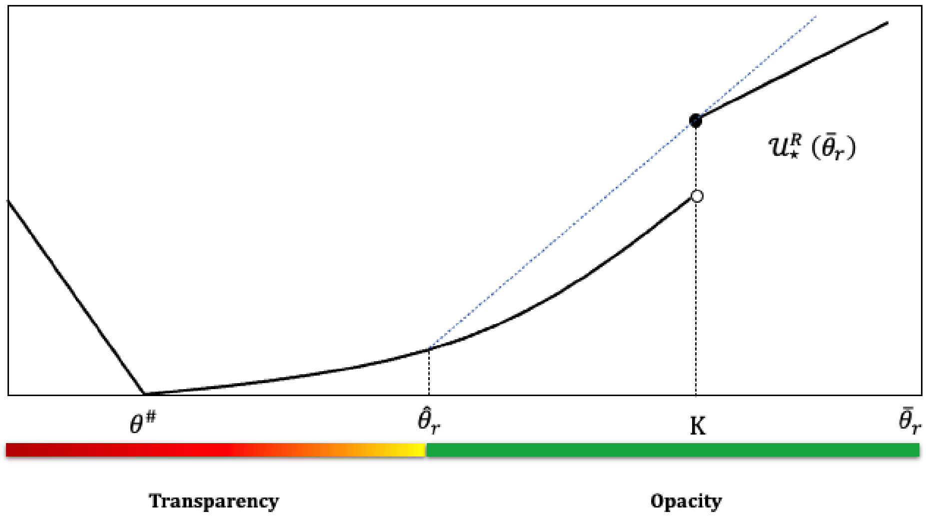

Figure 3: Regulator's payoff as a function of the induced posterior estimate $\bar{\theta}_r$.

The regulator's problem thus reduces to

$$\max_{G^\Gamma} \quad \int_0^\infty \left\{L_0\left(\bar{\theta}_r\right)\left(1-\phi\left(x^\star\left(\bar{\theta}_r\right)\right)\right)+W_0\left(\bar{\theta}_r\right)\phi\left(x^\star\left(\bar{\theta}_r\right)\right)\right\} G^{\Gamma_r}\left(\mathrm{d}\bar{\theta}_r\right)$$
$$\text{s.t:} \quad F_r \succeq_{\text{MPS}} G^\Gamma.$$

4.2 Transparency and Opacity

The next theorem shows that the optimal regulatory disclosure $\Gamma^\star$ is *transparent* for firms with nonperforming risky assets, and *opaque* for firms with highly profitable risky assets. Formally, there exists a cutoff $\hat{\theta}_r$ such that, any firm with a risky asset for which $\boldsymbol{\theta}_r<\hat{\theta}_r$, the regulator fully discloses the realization $\boldsymbol{\theta_r}$. In contrast, all cashflow realizations $\boldsymbol{\theta}_r \geq \hat{\theta}_r$ are pooled together under the same announcement, thereby minimizing the information passed on to the market. The cutoff $\hat{\theta}_r$ is chosen such that the posterior expectation induced by learning that $\boldsymbol{\theta}_r \geq \hat{\theta}_r$, satisfies $\mathbb{E}\left\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\} = KR$. Thus, $\hat{\theta}_r$ corresponds to the lowest cutoff that allows the firm to raise sufficient capital to persuade ST creditors to keep pledging funds to firm.

Theorem 1. *Suppose that assumption 2 holds and that inequality (8) is satisfied. Then, the optimal policy $\Gamma^\star$ is fully transparent for any $\theta_r < \hat{\theta}_r$, and fully opaque $\theta_r \geq \hat{\theta}_r$, where $\hat{\theta}_r$ is implicitly defined by $\mathbb{E}\left\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\} = KR$.*

The optimal disclosure policy $\Gamma^\star$ pools all profitability levels above $\hat{\theta}_r$ so that the induced posterior expectation, $\mathbb{E}\left\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\} = KR$ and, hence, ST creditors are dissuaded from running. Using a more transparent disclosure policy for high values of $\boldsymbol{\theta}_r$ destroys risk-sharing opportunities among financial institutions with heterogeneous assets. In fact, under the opaque announcement, all firms whose risky assets' profitability is above $\hat{\theta}_r$ are spared an inefficient run by ST creditors. Enjoying risk-sharing opportunities by means of more opaque policies is usually referred to as the

Hirshleifer effect (see Hirshleifer (1971)) and has already been discussed in the context of regulatory disclosures (e.g., Goldstein and Leitner (2018)).

When $\boldsymbol{\theta}_r$ falls below $\hat{\theta}_r$, the optimal policy becomes fully transparent. This result is novel. The intuition is that, as explained in Subsection 3.4, there exists an endogenous amplification effect associated with increasing the market perception about the profitability of the firm's risky asset that originates from the strategic complementarities between the two audiences. Indeed, the interaction of both audiences generates a virtuous cycle that *convexifies* the probability of survival $\phi(\cdot)$ as a function of the profitability of the asset. When the profitability of the asset is low and $\phi(\cdot)$ is (locally) convex, the regulator prefers to separate different profitability levels $\boldsymbol{\theta}_r$ under different signals similar to a risk-loving agent who prefers to separate different states under different realizations rather than pooling them together. In other words, when ϕ is convex, a policy that induces dispersion of posterior beliefs dominates those inducing a contraction of posteriors. As a result, and perhaps surprisingly, the optimal policy to maximize the probability of survival of solvent institutions is *transparent* for financial institutions poor fundamentals.

4.3 Monotone Comparative Statics

Theorem 1 shows that the optimal policy is fully characterized by the threshold $\hat{\theta}_r$ below which the regulator perfectly discloses the risky asset's profitability. I show next that as the firm's financial condition deteriorates, the optimal policy becomes more informative. By virtue of Theorem 1, the informativeness of the optimal policy is formally captured by the magnitude of $\hat{\theta}_r$. That is, as the cutoff of the optimal policy $\hat{\theta}_r$ increases, the regulatory disclosure becomes more informative.[28]

Recall that $\hat{\theta}_r$ is implicitly defined as the unique solution to the equation $\mathbb{E}\left\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\} = KR$. This definition implies that $\hat{\theta}_r$ is determined by, among other things, $F_r, F_\omega, d_1(1-A_0)$. Indeed, the threshold K above which ST creditors are dissuaded from running depends on the underlying distribution of liquid funds F_ω and the size of liabilities, which may dry up on short notice $d_1(1-A_0)$. The expectation of $\boldsymbol{\theta}_r$ in turn is fully determined by F_r. Let $\hat{\theta}_r(F_r, F_\omega, d_1(1-A_0))$ be the threshold characterizing the regulator's optimal disclosure policy. Our next result provides monotone comparative statics of the optimal policy's informativeness, captured by variations in the threshold $\hat{\theta}_r(F_r, F_\omega, d_1(1-A_0))$.

Lemma 1. *Suppose that assumptions 8 and 2 hold, and that inequality (8) is satisfied. Then,*

(a) If $\tilde{F}_\omega \succeq_{MLRP} F_\omega$, *then* $\hat{\theta}_r\left(\tilde{F}_\omega, F_r, d_1(1-A_0)\right) \leq \hat{\theta}_r(F_\omega, F_r, d_1(1-A_0))$,

(b) If $\tilde{F}_r \succeq_{MLRP} F_r$, *then* $\hat{\theta}_r\left(F_\omega, \tilde{F}_r, d_1(1-A_0)\right) \leq \hat{\theta}_r(F_\omega, F_r, d_1(1-A_0))$,

(c) If $\tilde{d}_1(1-\tilde{A}_0) \geq d_1(1-A_0)$, *then* $\hat{\theta}_r\left(F_\omega, F_r, \tilde{d}_1(1-\tilde{A}_0)\right) \geq \hat{\theta}_r(F_\omega, F_r, d_1(1-A_0))$.

Lemma 1 1 implies that as the firm's financial condition deteriorates, either because of a depletion of its liquidity buffers, a deterioration of the performance of its assets, or an increase in the maturity mismatch between assets and liabilities, the regulator optimally responds by implementing more

[28]The distribution of posterior estimates induced by the optimal policy, $G^{\Gamma^\star}$, becomes larger under the MPS order.

transparent regulatory disclosures. We show in the next Subsection that these predictions resonate with empirical findings.

4.4 Empirical and Theoretical Predictions

Empirical. The theory in this paper predicts that when financial institutions face strong financial constraints (e.g., a bank rolling over a large amount of short-term debt, an investment fund facing frequent redemptions), it should be subject to regulatory disclosures displaying a negative relationship between the degree of transparency and the firm's financial condition. The empirical evidence on regulatory disclosure identifies regularities consistent with these predictions. In the context of stress tests in the banking sector, the literature has found evidence that institutions with weaker fundamentals (e.g., riskier assets, more leverage, larger quantities of nonperforming loans), are subject to more transparency than institutions with stronger fundamentals (Morgan et al. (2014), Flannery et al. (2017), and Ahnert et al. (2018)). In the context of Call Reports, Chen et al. (2022) find that for US-based banks, disclosures are more informative for banks with worse performing assets. In the context of investment funds, Agarwal et al. (2013) find that hedge funds that request confidential treatment of their holdings in their Form 13F (i.e., those that delay disclosing their holdings 45 or more days), exhibit significantly higher performance.

The underlying assumption for the regulatory disclosures described in the paper is the existence of commitment power so that after the regulator designs the disclosure policy, no agent can manipulate the information disclosed after the fundamentals have realized. This might be a strong assumption for some applications of interest wherein the firm has large degrees of freedom in the information it passes on to the market. The predictions of the model will most likely not fit the empirical patterns in that case.

A firm can nevertheless commit to disclose information by adhering to an accounting standard to report its financial information. The standard specifies how transactions and other events are to be recognized, measured, presented, and disclosed in financial statements to the rest of the market participants. Interestingly, the asymmetric treatment of the optimal policy between bad and good news is broadly consistent with the *conservatism principle* usually recommended by accounting standard-setters. Indeed, according to the dictum, financial institutions should record losses as soon as they learn about them, whereas potential gains are to be recognized only after they materialize. A financial institution adhering to this principle is prone to disclose more granular information when its assets perform poorly and to disclose coarser information otherwise.

Theory. As argued above, the combination of market participants' strategic complementarities and financial constraints induces an amplification mechanism that increases regulators' preferences for transparency. Consistent with these predictions, Dai et al. (2021a) find that a regulator concerned with financial stability prefers a transparent policy for systemic risk exposures, where arguably strategic complementarities are strong, and an opaque policy for financial institutions' idiosyncratic exposures, where the strategic complementarities disappear. Similarly, Huang (2020) shows that when disclosing information about institutions in a financial network, the optimal policy becomes more opaque as the aggregate level of the fundamentals improves, which is consistent with the idea that financial constraints relax.

4.5 Discussion: Strategic complementarities and Financial Constraints

In this subsection, I discuss the role of strategic complementarities and financial constraints in inducing the amplification mechanism and the consequent convexity of the regulator's payoff in the firm's perceived fundamentals. I argue that financial constraints are sufficient to generate the amplification mechanism but not necessary. I then argue that strategic complementarities are necessary for the key the economic mechanism in the paper.

4.5.1 Financial Constraints are sufficient but not necessary

First, consider the slightly more general version of the model where AM investors price the asset according to

$$P_{\text{AM}} = \mathbb{E}\left\{s\left(\boldsymbol{\theta}_r\right)\right\}\eta_i\left(P_{\text{AM}}, A_{\text{ST}}\right)\mathbb{P}\left\{\boldsymbol{\omega} + P_{\text{AM}} + A_{\text{ST}} \geq d_1\right\},$$

where the function $\eta_i\left(P_{\text{AM}}, A_{\text{ST}}\right)$ is increasing and convex. That is, each AM investor's valuation for the security depend on the financial support of both audiences, ST creditors and AM investors, beyond their effect through the firm's liquidity constraint. This parameterization can capture network externalities, productivity spillovers, scalability of the firm's projects, etc.

Further, assume that F_ω is uniform over $[0, 1]$, the limiting case where F_ω is both weakly concave and convex. Similar arguments to the one establishing property (c) in Proposition 1 (and, more generally, Proposition 6 in Section 6) imply that as long as

$$\varphi\left(P_{\text{AM}}, A_{\text{ST}}\right) \equiv \eta\left(P_{\text{AM}}, A_{\text{ST}}\right)\mathbb{P}\left\{\boldsymbol{\omega} + P_{\text{AM}} + A_{\text{ST}} \geq d_1\right\}$$

is increasing and (weakly) convex, the equilibrium price $P^\star_{\text{AM}}(x)$ is strictly increasing and *strictly* convex in $x = \mathbb{E}\left\{s\left(\boldsymbol{\theta}_r\right)\right\}$, and therefore so is

$$\tilde{\phi}(x) \equiv \mathbb{P}\left\{\boldsymbol{\omega} + P^\star_{\text{AM}}(x) + A_{\text{ST}}(x) \geq d_1\right\}.$$

This implies that the main results extend to the case where for cases where $F''_\omega(\omega)$ is sufficiently small (i.e., F_ω is not "too convex"). The role of the concavity of F_ω consists in guaranteeing that each AM investor's incentives to pay a larger price do not decrease when either ST creditors or the rest of AM investors pledge more funds to the firm. In this sense, assumption 2 ensures that the strategic complementarities are sufficiently strong.

Furthermore, the amplification mechanism manifests even in the case where $d_1 = 0$. This is the case where financial constraints are no longer relevant and the firm is *perfectly liquid*, regardless the distribution of F_ω. Indeed, in that case

$$P_{\text{AM}} = \mathbb{E}\left\{s\left(\boldsymbol{\theta}_r\right)\right\}\cdot\eta_i\left(P_{\text{AM}}, A_{\text{ST}}\right),$$

and the convexity of $P^{\star}_{\mathrm{AM}}(x)$ still prevails, inducing the regulator's preference for transparent disclosures.

This discussion suggests that stringent financial constraints, as implied by assumption 2, are *sufficient* to induce the amplification mechanism but are not *necessary*. The key economic property driving the result is the manifestation of strategic complementarities in the audiences' preferences.

4.5.2 Strategic Complementarities are necessary

Now, consider the case where strategic complementarities do not emerge. I slightly modify the model and assume that AM investors are protected against the firm's default, i.e., the firm ring-fences the risky asset (e.g., the firm securitizes the risky asset and sell it to AM investors). In this environment, AM investors price the asset according to

$$P_{\mathrm{AM}} = \mathbb{E}\left\{\boldsymbol{\theta}_r\right\}/R.$$

Suppose further that the regulator has a simple payoff structure and would like to maximize the firm's probability of survival. That is,

$$\mathcal{U}^R(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}) = \mathbb{P}\left\{P_{\mathrm{AM}} + \boldsymbol{\omega} \geq d_1\left(1 - A_{\mathrm{ST}}\right)\right\}.$$

Assume that F_ω is uniform over $[0,1]$. ST creditors' optimal action consists of running whenever $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\} < K$. Thus, the regulator's ex-ante payoff, when $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\} < K$, is given by

$$\begin{aligned} U^P(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}) &= \mathbb{P}\left\{\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}/R + \boldsymbol{\omega} \geq d_1\left(1 - A_0\right)\right\} \\ &= \left(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}/R + 1 - d_1\left(1 - A_0\right)\right), \end{aligned}$$

which is affine in $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}$. In contrast, when $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\} \geq K$, ST creditors are dissuaded from running and $\mathcal{U}^P(\mathbb{E}\left\{\boldsymbol{\theta}_r\right\}) = 1$.

The regulator's optimal policy consists of a binary rule which announces whether $\boldsymbol{\theta}_r \geq \hat{\theta}_r$ or $\boldsymbol{\theta}_r < \hat{\theta}_r$, where $\hat{\theta}_r = \hat{\theta}_r\left(F_\omega, F_r\right)$ is implicitly defined as the unique solution to $\mathbb{E}\left\{\boldsymbol{\theta}_r | \boldsymbol{\theta}_r \geq \hat{\theta}_r\right\} \geq K$. The regulator's optimal policy consists in pooling as many high states as possible as long as the posterior estimate induced by the knowledge that $\boldsymbol{\theta}_r$ belongs to this set is weakly higher than K (the Hirshleifer effect). In terms of informativeness, the optimal policy is opaque and has a monotone pass/fail structure. When the prior distributions are sufficiently favorable so that, in the absence of any announcement, $\mathbb{E}\left\{\boldsymbol{\theta}_r\right\} \geq K$, then the optimal policy is complete opacity and does not disclose any information to the investors.

Strategic complementarities provide an strict preference for transparency for low realizations of $\boldsymbol{\theta}_r$. Indeed, observe that starting from this model, one can add strategic complementarities by removing the ring-fencing assumption, thereby letting AM investors' payoff depend on the ST creditors' behavior. In that case,

$$P_{\text{AM}} \quad = \quad \frac{\mathbb{E}\{\boldsymbol{\theta}_r\}}{R}\mathbb{P}\{P_{\text{AM}} + \boldsymbol{\omega} \geq d_1(1-A_0)\}.$$

Thus, we obtain

$$P^{\star}_{\text{AM}}(\mathbb{E}\{\boldsymbol{\theta}_r\}) = \frac{\mathbb{E}\{\boldsymbol{\theta}_r\}(1-d_1(1-A_0))}{(R-\mathbb{E}\{\boldsymbol{\theta}_r\})}$$

which is strictly convex in $\mathbb{E}\{\boldsymbol{\theta}_r\}$ over the critical region $(0, KR)$. Thus, the regulator's ex-ante payoff, for $\mathbb{E}\{\boldsymbol{\theta}_r\} < K$, is given by

$$\mathcal{U}^P(\mathbb{E}\{\boldsymbol{\theta}_r\}) \quad = \quad P^{\star}_{\text{AM}}(\mathbb{E}\{\boldsymbol{\theta}_r\}) + 1 - d_1(1-A_0),$$

also strictly convex in $\mathbb{E}\{\boldsymbol{\theta}_r\}$ over the critical region. In turn, when $\mathbb{E}\{\boldsymbol{\theta}_r\} \geq K$, $\mathcal{U}^P(\mathbb{E}\{\boldsymbol{\theta}_r\}) = 1$. The optimal policy in this case is full transparency for any $\boldsymbol{\theta}_r < \hat{\theta}_r$, and opacity for all $\boldsymbol{\theta}_r \geq \hat{\theta}_r$. Furthermore, the binary policy described above for the case with ring-fencing is *strictly suboptimal*. Thus, the regulator strictly benefits from transparency when strategic complementarities are present, but does not otherwise.

5 Robust Information Disclosures

5.1 Private Information

A typical argument against increasing the transparency of financial markets is the idea that transparency may exacerbate agency conflicts. A firm that faces high disclosure requirements, the argument goes, might strategically find ways to act in a self-serving manner (Landier and Thesmar (2011), Leitner and Williams (2021)). I show that the optimal disclosure policy is generally *robust* to the firm's superior private information. In the current model, an informed firm may attempt to signal its private information by strategically choosing its funding strategy, that is, the security it offers to AM investors. In many applications of interest, the firm's private information may be an important concern when designing regulatory disclosures. In the case of banking, e.g., the regulator and market participants alike pay close attention to the bank's superior information with respect to its opaque balance sheets (e.g., the amount of nonperforming loans). The bank's actions are then usually scrutinized and used as signals of the firm's residual private information.

There is a vast theoretical literature arguing that the type of securities chosen by the issuer may signal her private information. I extend Nachman and Noe (1994)'s security design problem to the current environment characterized by an endogenous probability of default and show that, under the optimal policy described in the former section, the equilibrium outcome during the fund-raising stage features pooling among all firm types. That is, the optimal policy is robust to signaling incentives. This result is consistent with the findings of Quigley and Walther (2020) who show that when regulators account for financial institutions' voluntary disclosures, they optimally design policies that foster "private silence." In the current environment, firms' cannot make announcements to the market but still can provide useful information by signaling with their security choice. The result below shows that, under the optimal policy, firms do not signal their residual private information.

To incorporate the possibility of superior private information, I assume that, at the beginning of period 0, before the regulator discloses information about the risky asset, the firm learns a private signal about $\boldsymbol{\theta}_r$, $\boldsymbol{\xi} \in \Xi \equiv \{\xi_L, \xi_H\}$, with $\xi_L < \xi_H$, and updates beliefs about the realization of $\boldsymbol{\theta}_r$ according to the conditional cdf $F_r(\theta_r|\xi)$ (resp., pdf $f_r^{\xi}(\theta_r|\xi)$), $\xi \in \Xi$. I refer to ξ as the firm's type. Neither the investors nor the regulator observes the firm's signal. I assume that the conditional pdf $f_r^{\xi}(\theta_r|\xi)$ satisfies log-supermodularity in (θ_r, ξ) (or, equivalently, that cashflows are ordered according to MLRP).

After observing its private signal, the firm sells a security s to AM investors, which corresponds to a claim on the risky asset's future cashflow. Formally, any security s belongs to $S \equiv \{s : X_r \to \mathbb{R}_+ \text{ s.t: (LL),(M),(MR)}\}$ where (LL) $0 \leq s(\theta_r) \leq \theta_r,\ \forall \theta_r \in X_r$; (M) s is nondecreasing and (MR) $\theta_r - s(\theta_r)$ is nondecreasing.[29] The security s may represent an equity stake, a debt contract, or any arbitrary security.

The firm may try to signal its superior private information about its risky asset, through its security choice.[30] The signaling incentives, in turn, may compromise the regulator's desired outcome. Indeed, for any possible disclosure m_r, the fact that type ξ_H has a better risky asset than type ξ_L (a consequence of MLRP), implies that the former is relatively more willing to risk defaulting to signal its quality. In the next proposition, I show that when the interaction of both audiences substantially increases default risk, then despite the firm's private information, the regulator is able to implement the same outcome as in the absence of private information.

Proposition 3. *Suppose that*

$$\lim_{p \uparrow K} \frac{\mathbb{E}^{\xi_H}\left(\boldsymbol{\theta}_r | \boldsymbol{\theta}_r \geq \hat{\theta}_r\right)}{R} \varphi(p) < K, \tag{10}$$

then, the regulator's optimal policy coincides with $\Gamma^\star$. Furthermore, for each possible realization of $\boldsymbol{m}_r$, the firm's optimal strategy coincides with $x^\star(\cdot)$.

To understand inequality (10), first note that by definition, $\mathbb{E}\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\} = KR$. The fact that ξ_H is good news (Milgrom (1981)) then means that $\mathbb{E}^{\xi_H}\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\} > KR$. The assumption that inequality (10) holds, then implies that the probability of default is sufficiently important even when the firm has optimistic residual private information. The assumption captures the idea that, despite the firm's superior private information, the financial constraints are severe and all firm types are vulnerable to the interaction of the two audiences. When (10) holds, any firm type which raises

[29]The first constraint represents *limited liability* and states that a security is a sharing rule. The *monotonicity* condition requires that the security be nondecreasing in the asset's cashflows since the firm would otherwise have the option of requesting risk-free credit to boost its cashflows and decrease the amount owed to AM investors. The last constraint imposes that the share of cashflows retained by the firm is nondecreasing; otherwise, the firm would have incentives to *burn* part of them.

[30]There is vast theoretical literature arguing that the type of securities chosen by an issuer may signal her private information. See, for example, Leland and Pyle (1977) , Myers and Majluf (1984), Nachman and Noe (1994).

less than K – the amount needed to persuade ST creditors to keep pledging – experiences a discrete penalty (a haircut) when selling its asset to AM investors. This assumption means that even if the firm type was commonly known to be ξ_H, it would still have an incentive to raise enough funds to dissuade ST creditors from running.[31]

Proposition 3 establishes that, when default risk is sufficiently important, the firm refrains from signaling its private information. Consequently, the regulator can implement her optimal policy despite the underlying information frictions. The economic mechanism driving the result is reminiscent of the famous result in Nachman and Noe (1994), extended to the current environment with an endogenous probability of default. The fact that firm suffers a discrete penalty when not raising enough funds to dissuade ST creditors from running, serves as discipline device and leads all firm types to pool under the same (debt) security, thereby curbing their signaling incentives and implementing the regulator's most preferred outcome. The fact that adding residual private information information on the firm's end induces more constraints for the regulator, then implies that if the optimal solution in the less constrained enviornment (i.e., without the firm's private information), remains feasible under the new environment, then it must also be optimal under the additional constraints.

5.2 Disclosures about Firms' Liquidity

Thus far, we have restricted attention to the case where the only tools at the regulator's disposal are her ability to design and enforce regulatory disclosures with respect to the financial institution's assets. In practice, policy makers typically react when liquidity squeezes trigger financial distress at solvent but potentially illiquid large financial institutions. In this section, I explore the case in which the regulator can react to the liquidity shock by disclosing information about $\boldsymbol{\omega}$ to ST creditors before they make their rollover decision. This emergency response is inspired by the stress tests conducted both in the US (SCAP) and in Europe in the middle of the global financial crisis and more recently during the Covid crisis.

I assume that, in period 1, the regulator has the technology to conduct a liquidity disclosure $\Gamma_\omega[P] = \{M_\omega, \pi_\omega[P]\}$, which discloses information about the firm's liquidity according to the rule $\pi_\omega[P] : \Omega \to \Delta M_\omega$. The disclosure accounts for the amount of funds raised during the fund-raising stage, P. Importantly, the liquidity disclosure is *sequentially rational* and maximizes the regulator's period 2 payoff. This assumption captures the idea that the firm is *too big* or *too interconnected* to fail, and as a result, if the liquidity shock occurs, the regulator maximizes the probability that the firm survives, regardless of any promises made at $t = 0$.

I modify the period 1 sequence of events as follows: (a) the regulator observes P_{AM} and designs Γ_ω and publicly announces it; (b) $\boldsymbol{\omega}$ is drawn from F_ω; (c) the regulator discloses information

[31] Under inequality (10), there exist two equilibrium prices which are consistent with the rationality of the investors. One in which the AM investors pledge strictly less than K, and another one wherein they pledge the safe value of the expected cashflows, $\mathbb{E}^{\xi_H}\left\{\boldsymbol{\theta}_r|\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\}/R$. The assumption guarantees that even if the investors learned the firm has optimistic information about the asset, their incentives to pledge funds still comove with the rest of the investors' behavior, and therefore strategic complementarities manifest.

$\boldsymbol{m}_\omega$ according to Γ_ω; (d) ST creditors observe P_{AM} and $\boldsymbol{m}_\omega$ and decide whether to run; and (e) the firm liquidates a fraction of its safe asset, and its fate is determined according to whether $\boldsymbol{\omega} + P_{\text{AM}} \geq d_1 (1 - A_{\text{ST}})$.

The optimal liquidity disclosure in this environment can be interpreted as a pass-fail test, where given the level of funds raised P_{AM}, the regulator assigns a *passing* grade when the firm's liquidity is above the cutoff $\bar{\omega}^{\text{ST}}(P_{\text{AM}})$. Proposition 4 summarizes these findings.

Proposition 4. *Fix the amount of capital $P \geq 0$. Then, the liquidity disclosure, $\Gamma^\star_\omega [P]$, consists of a monotone pass-fail test with cutoff $\bar{\omega}^{ST}(P)$, such that $\Gamma^\star_\omega(P) = (\{G, B\}, \pi^\star_\omega [P])$, with $\pi^\star_\omega \{G|\omega; P\} = 1\left\{\omega \geq \bar{\omega}^{ST}(P)\right\}$. The cutoff $\bar{\omega}^{ST}(\cdot)$ is nonincreasing in P.*

The proof is in the Online Appendix. When the regulator announces that the firm's liquid funds exceed $\bar{\omega}^{\text{ST}}(P)$, all ST creditors rollover and survival occurs with certainty. By contrast, when the firm fails, all ST creditors withdraw early and the firm defaults. Indeed, $\bar{\omega}^{\text{ST}}(P) < d_1 (1 - A_0) - P$; therefore, announcing that $\boldsymbol{\omega} < \bar{\omega}^{\text{ST}}(P)$ induces firm failure with certainty.

In contrast to our findings in the previous section, the optimal disclosure policy about the firm's liquidity is thus coarse and minimizes the information passed on to the market. The regulator's announcement arrives after AM investors have made their investment decisions. At that point, the regulator maximizes the probability of survival by pooling as many states ω as possible while guaranteeing that it remains dominant for each ST creditor to pledge.

5.2.1 Self-defeating disclosures

A key feature of liquidity shocks is that they are, by definition, unexpected. We show that the promise of disclosing information about the firm's liquidity condition can be self-defeating and backfire. Indeed, if AM investors expect that the regulator will provide information about the firm's liquidity when a shock materializes, their assessment about ST creditors' response becomes more optimistic. This, in turn, exacerbates the firm's incentive to signal its private information during the fund-raising stage and to raise less funds than socially optimal. Without the regulator's disclosure, in turn, the threat of a run of ST creditors imposes *discipline* on the firm because it compels it to raise precautionary funds to prevent default, thereby dissipating the signaling incentive. The knowledge that the regulator can help in the case of an adversarial shock makes it easier for type ξ_H to separate from type ξ_L since default risk decreases. Signaling, however, increases the probability of default and destroys the benefits of disclosing information. The next proposition shows that, perhaps surprisingly, under some conditions, the regulator with the technology to conduct a liquidity disclosure may fare worse than a regulator who does not intervene at all.

Condition 1. The distribution of liquidity shocks F_ω and ST creditors' payoff functions g and b satisfy

(A) $(\exists \varepsilon > 0)$, $\Lambda(P) \equiv K\mathbb{P}\{\boldsymbol{\omega} \geq \bar{\omega}(P)\} - P > 0$ is strictly positive for all $P \in [K - \varepsilon, K)$.[32]

[32]This property is equivalent to requiring that

$$\lim_{\omega \to d_1(1-A_0)-K} (b(\omega, K, A_0) - g(\omega, K, A_0)) f_\omega(\omega, K, A_0) K < b(0, K, A_0).$$

(B) $\lim_{P \to K^-} 1 - F_\omega \left(d_1 \left(1 - A_0\right) - P \right) < \bar{\phi} \equiv \frac{\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)}{\mathbb{E}_H(y) - \mathbb{E}_L(y)}$, where $s_D \equiv \min\{y, D\}$ with $\mathbb{E}(s_D) = KR$.

Proposition 5. *Assume that condition 1 holds; then, under the optimal liquidity disclosure* $\Gamma_\omega^\star$*, default occurs with positive probability across all equilibria. In contrast, under the laissez-faire policy, the probability of default reduces to* 0.

As proved in Proposition 7 in the Appendix, at any equilibrium of the fund-raising stage, firms raise at most K when pooling. Furthermore, both firm types raise strictly less than K at any separating equilibrium. Assumption (A) in condition 1 implies that, under the optimal liquidity disclosure, $\Gamma_\omega^\star$, both firm types find it optimal to deviate from the pooling outcome where both raise K. Intuitively, under this assumption, a firm that raises slightly less than K faces a probability of default barely above 0. Such a deviation from the pooling equilibrium is always interpreted as coming from type ξ_H who has a better asset and therefore is relatively more willing to risk defaulting to signal its quality. Thus, small deviations are interpreted as coming from type ξ_H and priced accordingly. Both types then have the incentive to deviate from the situation where both raise K and raise strictly less funds, thus inducing ST creditors to run.

Assumption (B), on the other hand, implies that in the *absence* of liquidity disclosure, the probability of default is sufficiently large when the firm does not meet the cutoff K. This effect imposes discipline on the firm because it compels both types to raise sufficient funds to dissuade ST creditors from running. Under assumption (B), both firm types thus pool over the same debt contract $s_D = \min\{y, D\}$, with $\mathbb{E}(s_D) = KR$; as a result, they avoid default with certainty.

Surprisingly, under modest assumptions, the market may fare worse when the regulator who tries to maximize the probability of the firm's survival is equipped with a better technology.

6 General Model

We now generalize the results in the baseline model to a fairly large class of economies. Consider an economy composed of $N \geq 2$ audiences. These audiences may represent a financial institution's different types of investors (similar to the baseline model), a group of financial institutions (e.g., VC funds, private equity firms, mutual funds) financing one or multiple private companies, a group of institutional investors with a stake in one or more public firms, etc.

Each audience consists of a mass 1 of atomistic investors. The fundamentals of the economy are captured by the random vector $\overrightarrow{\boldsymbol{\vartheta}} = (\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_N, \boldsymbol{\omega}) \in \Pi_{i=1}^N X_i \times \Omega$, where $X_i \equiv [\underline{x}_i, \bar{x}_i], \Omega \subseteq \mathbb{R}_+$. Each $\boldsymbol{\theta}_i \in X_i$ captures a dimension of the economy's fundamentals of direct interest to audience i. For example, when the audiences represent multiple investors interested in purchasing different types of assets from a given firm, $\boldsymbol{\theta}_i$ represents the returns associated with asset i. Alternatively, the audiences may represent multiple investment funds investing in a certain industry. In that case, each audience is interested in investing funds in a project or company whose fundamentals are parameterized by $\boldsymbol{\theta}_i$. From now on, I refer to $\boldsymbol{\theta}_i$ as the fundamentals' dimension i. Variable $\boldsymbol{\omega}$, in turn, captures the level of fragility of a reference entity and parameterizes the linkages between the audiences. We discuss its interpretation in detail below.

Information. Assume that all investors share the same prior beliefs about the economy's fundamentals, $F \in \Delta\left(\Pi_{i=1}^{N} X_i\right)$. For simplicity, I assume that, for any $i \neq j \in \{1, ..., N\}$, $\boldsymbol{\theta}_i \perp \boldsymbol{\theta}_j$ and $\boldsymbol{\theta}_i \perp \boldsymbol{\omega}$. I refer to the marginal distribution of dimension i as $F_i \in \Delta\Theta_i$ and use $F_\omega \in \Delta\Omega$ to denote the marginal distribution of $\boldsymbol{\omega}$.

Actions. Each investor $l \in [0, 1]$ in each audience i must choose an action $a_i^l \in X_i \equiv [\underline{x}_i, \bar{x}_i]$. For each $i \in \{1, ..., N\}$, we let $A_i \equiv \int_0^1 a_i^l \mathrm{d}l$ denote the *aggregate support* from audience $i \in \{1, ..., N\}$. We also let $A_{-i} \equiv \sum_{j \neq i} A_j$ denote the aggregate support from the rest of audiences $j \neq i$.

6.1 Strategic Complementarities

Preferences. The regulator is interested in maximizing the aggregate support of all the audiences. Her payoff is determined by a weighted average of the audiences' support. That is, she maximizes

$$U^R(\vec{A}) \equiv \sum_{i=1}^{N} \gamma_i A_i,$$

with $\gamma_i \geq 0$ for all i. The underlying assumption is that the projects or assets that the audiences invest in are welfare-improving and it is efficient to maximize their support.

In the case of the investors, on the other hand, I assume that each investor l in audience i cares about: (a) the fundamentals' i-th dimension, $\boldsymbol{\theta}_i$, (b) the aggregate support of all the audiences, (A_i, A_{-i}), and (c) the fragility of the reference entity, $\boldsymbol{\omega}$. Specifically, I assume that the preferences of audience i investors are captured by[33]

$$u_i\left(a_i^l, \theta_i, \omega, A_i, A_{-i}\right) \equiv -\frac{1}{2}\left(a_i^l - \theta_i \cdot 1\left\{\omega + A_i + A_{-i} \geq d\right\}\right)^2.$$

Define

$$U_i\left(a_i, A_i, A_{-i}\right) \equiv \mathbb{E}\left\{u_i\left(a_i, \boldsymbol{\theta}_i, \boldsymbol{\omega}, A_i, A_{-i}\right)\right\}.$$

The current specification implies that investors' marginal incentives to increase their action are captured by

$$\frac{\partial}{\partial a_i} U_i\left(a_i, A_i, A_{-i}\right) = \mathbb{E}\left\{\boldsymbol{\theta}_i\right\} \cdot \mathbb{P}\left\{\boldsymbol{\omega} + A_i + A_{-i} \geq d\right\} - a_i. \tag{11}$$

That is, taking the behavior of all the audiences (A_i, A_{-i}) as given, each investor in audience i

[33] The results below extend more generally to preferences of the form

$$u_i = -\frac{1}{2}\left(a_i^l - \eta_i\left(A_i, A_{-i}\right) 1\left\{\omega + A_i + A_{-i} \geq d_i\right\} - \kappa_i\right)^2,$$

for which the best responses take the form $\mathbb{E}\left\{\boldsymbol{\theta}_i\right\} \eta_i\left(A_i, A_{-i}\right)\left(1 - F_\omega\left(d_i - A_i - A_{-i}\right)\right) - \kappa_i$, as long as the functions $\eta_i\left(A_i, A_{-i}\right)$ are weakly positive, nondecreasing and weakly convex.

would like to match

$$\mathbb{E}\left\{\boldsymbol{\theta}_i\right\} \cdot \underbrace{\mathbb{P}\left\{\boldsymbol{\omega} + A_i + A_{-i} \geq d\right\}}_{\equiv \varphi(A_i, A_{-i})},$$

i.e., the expected value of the fundamentals' dimension in which they are interested, $\mathbb{E}\left\{\boldsymbol{\theta}_i\right\}$, scaled by the factor $\varphi_i\left(A_i, A_{-i}\right)$. This specification captures the idea that investors in audience i enjoy the fundamentals' dimension i, $\boldsymbol{\theta}_i$, as long as the level of support of all audiences is large enough as a function of fragility parameter $\boldsymbol{\omega}$. The random variable $\boldsymbol{\omega}$ thus captures the minimal amount of support from the audiences required to enjoy the future returns of the projects or assets. In the case where the audiences are investors from the same financial institution, $\boldsymbol{\omega}$ may represent the financial institution's liquidity (as in the baseline model). In turn, when the audiences are investors from different, interconnected firms, $\boldsymbol{\omega}$ may represent the liquidity of the most vulnerable firm in the network, or the critical mass of investment required for the industry to take off.

Note that the function $\varphi\left(A_i, A_{-i}\right)$ directly depends on the behavior of all the audiences and the distribution of $\boldsymbol{\omega}$. The probability $\varphi\left(A_i, A_{-i}\right)$ increases with the mass of investors in all audiences pledging support. This means that the audiences are exposed to the strategic behavior of the other audiences through the fragility of the reference entity (e.g., liquidity constraints). Indeed, each investor's marginal incentive to increase their action, as captured by equation 11, increases with $\mathbb{E}\left\{\theta_i\right\}$, A_i, and A_{-i}. In other words, investors' payoffs are supermodular with respect to (a) $\left(a_i, \mathbb{E}\left\{\boldsymbol{\theta}_i\right\}\right)$ and (b) $\left(a_i, A_i\right)$, and (c) $\left(a_i, A_{-i}\right)$. These properties are standard assumptions in games with strategic complementarities.[34] Property (a) implies that improving the perception of $\mathbb{E}\left\{\boldsymbol{\theta_i}\right\}$ increases the amount of support from investors in audience i. Properties (b) and (c), on the other hand, capture the idea that the investors' preferences display strategic complementarities among investors from the same audience but also among investors from different audiences.

Intuitively, $\boldsymbol{\theta}_i$ parameterizes the maximal profitability that the fundamentals' dimension i can potentially reach. Under the interpretation that each audience's support represents the amount of funds invested in assets with return $\boldsymbol{\theta}_i$, the specification captures the idea that the returns of asset i increase when audience i pledges more funds, but also increases when the other audiences pledge more funds to their respective projects.

When assumption (2) holds, implying a large degree of fragility, the marginal incentive to increase the level of support a_i, increases more when the audiences are providing a larger level of support, $\left(A_i, A_{-i}\right)$. To see this, note that the degree of strategic complementarities for investors in audience i, captured by

$$\frac{\partial^2 U_i\left(a_i, A_i, A_{-i}\right)}{\partial A_{-i} \partial a_i} \quad = \quad \mathbb{E}\left\{\boldsymbol{\theta}_i\right\} \frac{\partial \varphi}{\partial A_{-i}}\left(A_i, A_{-i}\right),$$

is larger for larger values of $\left(A_i, A_{-i}\right)$.

[34]They correspond, e.g., to assumptions A1 and A2 in Morris and Shin (2006), the canonical model of global games.

6.2 Financial Constraints and Strategic Complementarities

I formalize the idea that financial constraints exacerbate strategic complementarities. To see this, consider two (marginal) distributions $F_\omega^1, F_\omega^2 \in \Delta\Omega$ satisfying

$$\frac{f_\omega^1(x)}{1-F_\omega^1(x)} \geq \frac{f_\omega^2(x)}{1-F_\omega^2(x)},\ \forall x \in \Omega.$$

That is, F_ω^2 dominates F_ω^1 in the *hazard rate* (HR) order, which we write as $F_\omega^2 \succeq_{\text{HR}} F_\omega^1$. Intuitively, under F_ω^1 liquidity constraints are more stringent than under F_ω^2. Indeed, it is well-known that the HR order implies first order stochastic dominance (FOSD), which means that under F_ω^1 the level of liquidity is stochastically worse than under F_ω^2.[35] For example, it is easy to see that any distribution $\tilde{F}_\omega \in \Delta\Omega$ satisfying assumption 2 is dominated by the uniform distribution $F_\omega^{\text{Uniform}} \in \Delta\Omega$. Indeed,

$$\frac{1-\tilde{F}_\omega(x)}{\tilde{f}_\omega(x)} = \int_x^{\sup\Omega} \frac{\tilde{f}_\omega(z)}{\tilde{f}_\omega(x)}\mathrm{d}z \leq \int_x^{\sup\Omega} \mathrm{d}z = \frac{1-F_\omega^{\text{Uniform}}(x)}{f_\omega^{\text{Uniform}}(x)}.$$

Our next result shows that, when liquidity constraints are more stringent, each investor's marginal incentive to increase their own support increases proportionally more, and hence it is amplified, when other audiences increase their support.

Lemma 2. [Financial Constraints and Complementarities] *Consider $F_\omega^1, F_\omega^2 \in \Delta\Omega$ with $F_\omega^2 \succeq_{HR} F_\omega^1$; then, fixing (A_i, A_{-i}), the marginal incentives to increase the level of support increases proportionally more under F_ω^2 than under F_ω^1. Formally, for any (A_i, A_{-i}), let $\hat{a}_i(A_i, A_{-i}; F_\omega) \equiv \mathbb{E}\{\boldsymbol{\theta}_i\} \cdot (1 - F_\omega(d_i - A_i - A_{-i}))$ be the best response of audience i's investor to (A_i, A_{-i}). Then,*

$$\frac{\partial^2 U_i\left(\hat{a}_i, A_i, A_{-i}; F_\omega^2\right)}{\partial A_{-i}\partial a_i} \Big/ \hat{a}_i\left(A_i, A_{-i}; F_\omega^2\right) \geq \frac{\partial^2 U_i\left(\hat{a}_i, A_i, A_{-i}; F_\omega^1\right)}{\partial A_{-i}\partial a_i} \Big/ \hat{a}_i\left(A_i, A_{-i}; F_\omega^1\right).$$

6.3 Adversarial Equilibrium

Each investor conjectures that the aggregate support of the audiences is given by some profile (A_i, A_{-i}) and solves

$$\max_{a_i \in X_i} \mathbb{E}\left\{-\frac{1}{2}\left(a_i^l - \boldsymbol{\theta}_i \cdot 1\{\boldsymbol{\omega} + A_i + A_{-i} \geq d\}\right)^2\right\}.$$

The fact that all investors share the same prior beliefs about the fundamentals $\overrightarrow{\boldsymbol{\vartheta}}$, implies that the equilibrium aggregate support (A_i, A_{-i}) is measurable with respect to the prior F (and not with respect to the realization $\overrightarrow{\boldsymbol{\vartheta}}$). The fact that each investor is atomistic, then means that all investors in audience i choose the same action $a_i^*(F)$ given by

$$\begin{aligned} a_i^*(F) &= \mathbb{E}\left\{\boldsymbol{\theta}_i \cdot 1\left\{\boldsymbol{\omega} + A_i^*(F) + A_{-i}^*(F) \geq d\right\}\right\} \\ &= \mathbb{E}\{\boldsymbol{\theta}_i\} \cdot \varphi\left(A_i^*(F), A_{-i}^*(F)\right), \end{aligned}$$

[35] Further, the MLRP order implies the HR order. The MLRP order was used to perform comparative statics in Section 4 (see lemma 1).

where, for any $i \in \{1, ..., N\}$, $A_i^*(F) \equiv \int_0^1 a_i^*(F)\,\mathrm{d}l = a_i^*(F)$. This further means that, in equilibrium, investors' actions depend on the prior F only through the vector of prior expectations $\mathbb{E}\left\{\vec{\boldsymbol{\theta}}\right\}$ and are given by

$$a_i^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\}) = \mathbb{E}\{\boldsymbol{\theta}_i\} \cdot \varphi(a_i^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\}), a_{-i}^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\})),\ \forall i \in \{1, ..., N\} \tag{12}$$

where $a_{-i}^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\}) \equiv \sum_{j \neq i} a_j^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\})$.

The system in (12) may admit multiple solutions. Consistent with the idea of conservative regulatory disclosures, whenever there is multiplicity of equilibria, we focus on the most adversarial equilibrium, i.e., the smallest action profile $\left(a_i^*, a_{-i}^*\right)$ satisfying 12. Intuitively, this solution concept captures the idea that the regulator does not trust her ability to coordinate the market on her most preferred outcome when multiple action profiles are consistent with equilibrium play. The regulator is thus conservative and assumes that the audiences will coordinate on the worst equilibrium profile.

6.4 Convexity

I show next that, under adverse market conditions as captured by assumption (2), the strategic complementarities between the audiences lead to optimal actions that are first convex in the expected fundamentals of the economy and then comove in a linear manner with the fundamentals. To facilitate the exposition, I focus below on the case where $N = 2$. I extend the results to the case with arbitrary number of audiences in the Online Appendix.

Assumption 3. *Suppose that, for all i, $\bar{x}_i > \max\{d, 1/f_\omega(0)\}$ and that $\bar{x}_i(1 - F_\omega(d_i - A)) - A > 0$ for all $A \leq \bar{x}_i$.*

Our next result shows that, in equilibrium, investors' best responses are convex-then-linear in the fundamentals dimension of their interest.

Proposition 6. [Convex-then-Linear]*Suppose assumptions 2 and 3 hold. Then, for any $\bar{\theta}_j$, there exists $\bar{\theta}_i^{\#\#}(\bar{\theta}_j) \leq \bar{x}_i$, such that (a) for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}(\bar{\theta}_j)$, $a_i^*(\cdot, \bar{\theta}_j)$ and $a_j^*(\cdot, \bar{\theta}_j)$ are both strictly increasing and strictly convex in $\bar{\theta}_i$, whereas (b) for any $\bar{\theta}_i > \bar{\theta}_i^{\#\#}(\bar{\theta}_j)$, $a_i^*(\bar{\theta}_i, \bar{\theta}_j) = \bar{\theta}_i$ and $a_j^*(\bar{\theta}_i, \bar{\theta}_j) = \bar{\theta}_j$.*

Proposition 6 shows that the main qualitative features of the baseline model wherein the firm's probability of survival is convex for lower fundamentals and then linear for good fundamentals are a general insight that does not hinge on the specific institutional details assumed there. Roughly, the assumption that the audiences are fragile and vulnerable to the behavior of the rest of the audiences for low fundamentals, and that such fragility evaporates when the underlying fundamentals are strong implies that optimal market responses feature convexities for poor fundamentals that eventually fade away. As in the baseline model, I show below that these properties translate in optimal disclosures featuring transparency for weak fundamentals and opacity, otherwise.

6.5 Regulatory Disclosures

Assume that the regulator commits to a regulatory disclosure $\Gamma = \{\pi, M_1, ..., M_N\}$, which for each realization of the fundamentals $\vec{\theta}$, publicly discloses an announcement $\vec{m} = (m_1, ..., m_N) \in \Pi_{i=1}^N M_i$ with probability $\pi\left\{\vec{m}|\vec{\theta}\right\}$. Each announcement m_i represents information directly intended for audience i. However, the public nature of the disclosure implies that all audiences perfectly observe the whole vector $\vec{m}$.

I assume that, for each announcement $\vec{\boldsymbol{m}} = (\boldsymbol{m}_i)_{i=1}^N$ disclosed with positive probability, $\boldsymbol{m}_i = \mathbb{E}\{\boldsymbol{\theta}_i|\vec{\boldsymbol{m}}\} = \mathbb{E}\{\boldsymbol{\theta}_i|\boldsymbol{m}_i\}$. In other words, we identify each announcement with the posterior estimate about the fundamentals' i-th dimension after the information has been revealed to the market. Importantly, we assume that disclosures are orthogonal among themselves, meaning that along each dimension i, the information passed on to the investors, $\boldsymbol{m}_i$, reveals information exclusively about $\boldsymbol{\theta}_i$, that is, $\mathbb{E}\{\boldsymbol{\theta}_i|\vec{\boldsymbol{m}}\} = \mathbb{E}\{\boldsymbol{\theta}_i|\boldsymbol{m}_i\}$.[36]

The fact that disclosures are public and that investors do not have private information implies that, in equilibrium, strategies must be measurable with respect to the public announcement $\vec{\boldsymbol{m}}$. Thus, the equilibrium amount of support from audience i after announcement $\vec{m}$ is disclosed, is given by $A_i^*(\vec{m})$. In equilibrium, each investor l in each audience i thus conjectures a market response $\left(A_i^*(\vec{m}), A_{-i}^*(\vec{m})\right)$ and solves

$$\max_{a_i^l \in X_i} \mathbb{E}\left\{u_i\left(a_i^l, \boldsymbol{\theta}_i, A_i^*(\vec{\boldsymbol{m}}), A_{-i}^*(\vec{\boldsymbol{m}})\right)|\vec{\boldsymbol{m}} = \vec{m}\right\}.$$

The fact that

$$\begin{aligned}
\mathbb{E}\left\{\frac{\partial}{\partial a_i^l} u_i\left(a_i^l, \boldsymbol{\theta}_i, A_i^*(\vec{\boldsymbol{m}}), A_{-i}^*(\vec{\boldsymbol{m}})\right)|\vec{m}\right\} &= \mathbb{E}\left\{\boldsymbol{\theta}_i \varphi_i\left(A_i^*(\vec{\boldsymbol{m}}), A_{-i}^*(\vec{\boldsymbol{m}})\right) - a_i^l|\vec{\boldsymbol{m}} = \vec{m}\right\}. \\
&= \mathbb{E}\{\boldsymbol{\theta}_i|\vec{m}\}\varphi_i\left(A_i^*(\vec{m}), A_{-i}^*(\vec{m})\right) - a_i^l \\
&= m_i \varphi_i\left(A_i^*(\vec{m}), A_{-i}^*(\vec{m})\right) - a_i^l,
\end{aligned}$$

then implies that, investors' actions depend on the posterior beliefs distributions about $\vec{\boldsymbol{\theta}}$ only through the public announcement $\vec{m}$. Thus, we must have

$$a_i^*(\vec{m}) = m_i \cdot \varphi_i(a_i^*(\vec{m}), a_{-i}^*(\vec{m})), \ \forall i \in \{1, ..., N\}. \tag{13}$$

Next, we note that any regulatory disclosure Γ induces a distribution of posterior estimates $\{\mathbb{E}\{\boldsymbol{\theta}_i|\boldsymbol{m}_i\}\}_{i=1}^N$. Let G_i represent the cdf of posterior estimates of dimension i induced by regulatory disclosure Γ (i.e., G_i^Γ is the cdf of the random variable $\mathbb{E}\{\boldsymbol{\theta}_i|\boldsymbol{m}_i\}$). There exists a one-to-one mapping between (orthogonal) regulatory disclosures and distributions of posterior estimates $(G_1, ..., G_N)$ satisfying, for each dimension i, $F_i \succeq_{\text{MPS}} G_i$. Henceforth, we identify each regulatory disclosure with the distribution of posterior estimates that it generates and denote it by $\Gamma = \{G_i, G_j\}$.

[36] Provided that the regulator's disclosures are orthogonal, the assumption that $\boldsymbol{m}_i = \mathbb{E}\{\boldsymbol{\theta}_i|\boldsymbol{m}_i\}$ is without loss, as implied by Strassen's theorem.

The regulator's problem then reduces to

$$\begin{aligned} \max_{\{G_i\}_{i=1}^N} \quad & \sum_{i=1}^{N} \gamma_i \int_0^{\infty} a_i^* (m_i)\, G_i\,(\mathrm{d}m_i) \\ \text{s.t:} \quad & F_i \succeq_{\text{MPS}} G_i, \ \forall i. \end{aligned}$$

Our next result characterizes the optimal regulatory disclosure.

Theorem 2. *The optimal regulatory disclosure* $\Gamma^* = \left\{G_i^*, G_j^*\right\}$ *is characterized as follows. Fix an announcement on the j-th dimension,* $\boldsymbol{m}_j = m_j$*; then, there exists* $\hat{m}_i\,(m_j)$ *so that*

$$G_i^*\left(\bar{\theta}_i\right) = \begin{cases} F_i\left(\bar{\theta}_i\right) & \text{if } \bar{\theta}_i \leq \hat{m}_i\,(m_j) \\ F_i\left(\hat{m}_i\,(m_j)\right) & \text{if } \hat{m}_i\,(m_j) < \bar{\theta}_i < \bar{\theta}_i^{\#\#}\,(m_j) \\ 1 & \text{if } \bar{\theta}_i \geq \bar{\theta}_i^{\#\#}\,(m_j)\,, \end{cases}$$

where $\hat{m}_i\,(m_j)$ *is implicitly defined by* $\mathbb{E}\left\{\bar{\boldsymbol{\theta}}_i | \bar{\boldsymbol{\theta}}_i \geq \hat{m}_i\,(m_j)\right\} = \bar{\theta}_i^{\#\#}\,(m_j)$.

Theorem 2 shows that the main qualitative insight deduced in the Baseline Model extends more broadly to a rich class of economies where strategic complementarities manifest. In this Section, I have abstracted from many institutional details assumed in the Baseline Model to broaden the scope of the applications the model can be used for. Similar to the findings in Theorem 1, the optimal disclosure Γ^* imposes, along each dimension i, complete transparency for weak fundamentals, $\bar{\boldsymbol{\theta}}_i \leq \hat{m}_i\,(m_j)$, followed by complete opacity for strong fundamentals, $\bar{\boldsymbol{\theta}}_i > \hat{m}_i\,(m_j)$. I prove in the Online Appendix that the features of the optimal disclosures discussed in this Section further generalize for the case with $N > 2$ audiences and to all stable equilibria (Dixit (1986)) of the game.

7 Conclusions

This paper studies the optimal design of regulatory disclosures. I consider a rich environment that emphasizes the interaction among multiple audiences who care about different aspects of the firm's fundamentals. I show that the degree of transparency of the optimal policy is directly linked to the extent of strategic complementarities between the market participants. Poor financial conditions induce and amplify strategic complementarities among the firm's investors generating a regulator's preference for granular disclosures. As the firm's fundamentals improve, the strategic complementarities vanish, thereby dissipating the preference for transparency.

The optimal regulatory disclosure is robust to several practical concerns. Optimal disclosures are robust to (i) the adversarial coordination of the firm's investors, (ii) the firm's agency, and (iii) the introduction of asymmetric information. Interestingly, the main predictions of the model are consistent with recent empirical findings documenting the relationship between the informativeness of regulatory disclosures and the firms' financial conditions.

The above results are worth extending in several directions. The analysis assumes the regulator knows the distribution of the fundamentals in the economy when she designs the optimal her policy. Such knowledge may come from previous experience with similar firms. While this is a natural starting point, there are many environments in which it is more appropriate to assume that the regulator lacks information about the joint distribution of the underlying fundamentals. In future work, it would be interesting to investigate the optimal disclosure policy in such situations. One idea is to apply a robust approach to the regulator's problem, whereby the regulator expects nature to select the information structure that minimizes her payoff. The characterization of the optimal policy in this environment is highly relevant both from a theoretical standpoint and for the associated policy implications.

The analysis assumes that the only tool at the regulator's disposal is her ability to design regulatory disclosures. In many applications of interest, the regulator can complement disclosures with additional measures. For instance, she may impose further capital or liquidity restrictions, or react to liquidity squeezes by acting as a lender of last resort. In future work, it would be interesting to study the interplay between disclosures and other policy tools.

The model further assumes a one-shot interaction between the firm and its investors. However, firms' financial decisions are intrinsically dynamic phenomena. If the fundamentals are persistent over time, the optimal policy must also specify the timing of disclosures. In future work, it would also be interesting to extend the analysis in this direction.

Appendix A: Laissez Faire

Proof of Proposition *1.*

We start with claim (a). For any $P, z \in \mathbb{R}_+$, define the function

$$\zeta(P;z) \equiv P - \left(\frac{z}{R}\right) \cdot (1 - F_\omega(\bar{\omega}(P))).$$

Consider any $z \in [0, \underline{P}R/\lambda)$. Suppose first that $d_1(1-A_0) > 1$. This implies that $\zeta(0;z) = 0$ and that, for any $0 < P \leq \underline{P}$,

$$\begin{aligned} \zeta(P;z) &= P - \left(\frac{z}{R}\right) \cdot \mathbb{P}\{\boldsymbol{\omega} \geq \bar{\omega}(P)\} \\ &> 0. \end{aligned}$$

Next, suppose by contradiction that there exists $\hat{P} \in (\underline{P}, K)$ where the function $\zeta(\cdot;z)$ crosses 0, from positive to negative, that is, $\zeta\left(\hat{P};z\right) = 0$ and $\partial_- \zeta(P;z)|_{P=\hat{P}} = \lim_{h \downarrow 0} \frac{\zeta(\hat{P};z) - \zeta(\hat{P}-h;z)}{h} < 0$. Note that assumption 2 implies that $\zeta(\cdot;z)$ is concave and hence absolutely continuous. Thus,

$$\zeta'(P;z) = \underbrace{\zeta\left(\hat{P};z\right)}_{=0} + \int_{\hat{P}}^{P} \underbrace{\zeta'(p;z)}_{<0} \mathrm{d}p < 0, \ \forall P \in (\underline{P}, K),$$

where the fact that $\zeta'(p;z)<0$ for (almost) all $p>\hat{P}$ follows from the concavity of $\zeta(\cdot;z)$ and the construction of $\hat{P}$. The inequality above contradicts the fact that $\zeta(K;z)>0$. As a result, there is no positive price P satisfying $\zeta(P;z)=0$.

Assume now that $d_1(1-A_0)\leq 1$. Then, $[0,\underline{P}R/\lambda)=\emptyset$ and hence the claim is vacuously true. This proves claim a.

Claim (b) follows directly from the fact that $A_{\mathrm{ST}}=1\{P\geq K\}$ and the observation that $\bar{P}(z)=\frac{z}{R}$ for any $z\geq KR$.

Next, to see claim c, fix any $z\in[\underline{P}R/\lambda,KR)$. Assume that assumption 2 holds. We show that $\phi''(z)>0$ for any $z\in[\underline{P}R/\lambda,KR)$. Indeed, by differentiating (7) with respect to z, we obtain

$$\begin{aligned}\phi'(z) &= f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\bar{P}'(z)\\ &= f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\left(\frac{\phi(z)+z\phi'(z)}{R}\right),\end{aligned}\tag{14}$$

where the last equation follows from differentiating $\bar{P}(z)$ (recall the definition in 5). Differentiating (14) with respect to z, we get that

$$R\phi''(z)=-f_\omega'\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\bar{P}'(z)^2/R+f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\left(2\phi'(z)+z\phi''(z)\right),$$

and therefore we conclude that

$$\phi''(z)=\frac{-f_\omega'\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\bar{P}'(z)^2/R+2f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\phi'(z)}{R-zf_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)}.\tag{15}$$

Claim c. 1. $R-zf_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)>0$ for all $z\in[\underline{P}R/\lambda,KR)$.

Proof of claim c .1. We prove that the function

$$\zeta(P;z)\equiv P-\left(\frac{z}{R}\right)\cdot\left(1-F_\omega\left(\bar{\omega}(P)\right)\right)$$

crosses 0 exactly once from below over $P\in[\underline{P},K]$. Indeed,

$$\begin{aligned}\zeta(\underline{P};z) &= \underline{P}-\left(\frac{z}{R}\right)\cdot\mathbb{P}\left\{\boldsymbol{\omega}\geq\min\left\{1,d_1(1-A_0)\right\}\right\}\\ &\leq \underline{P}-\left(\frac{z}{R}\right)\lambda<0,\end{aligned}$$

where the last inequality follows from the assumption that $z\in[\underline{P}R/\lambda,KR)$. Next, note that $\zeta(K;z)=K-\frac{z}{R}>0$. The *intermediate value theorem* then implies that there exists $\tilde{P}\in(\underline{P},K)$, with $\zeta\left(\tilde{P};z\right)=0$.

We finally prove uniqueness. Note that the assumption that F_ω admits a continuously density over $[0,1)$ implies that $\zeta(\cdot;z)$ is concave and continuously differentiable. Suppose that the equation $\zeta(P;z)=0$ admits multiple solutions over $[\underline{P},K]$. Then, let $\tilde{P}_1$ and $\tilde{P}_2$ be two such solutions and assume that $\tilde{P}_1<\tilde{P}_2$ and that $\zeta(\cdot;z)$ crosses 0 from below at $\tilde{P}_1$ and from above at $\tilde{P}_2$. Then, it must be the case that $\zeta'\left(\tilde{P}_2; z\right) < 0$. The concavity of $\zeta(\cdot; z)$ then implies that $\zeta'(P; z) < 0$ for almost all $P \in \left[\tilde{P}_2, K\right]$. This means that

$$\zeta'(P; z) = \underbrace{\zeta\left(\tilde{P}_2; z\right)}_{=0} + \int_{\tilde{P}_2}^{P} \underbrace{\zeta'(p; z)}_{<0} \mathrm{d}p < 0, \ \forall P \in \left[\tilde{P}_2, K\right].$$

This contradicts that $\zeta(K; z) > 0$. Thus, there exists a unique $\tilde{P} \in (\max\{d_1(1 - A_0) - 1, 0\}, K)$, with $\zeta\left(\tilde{P}; z\right) = 0$. Moreover, $\zeta(\cdot; z)$ turns from negative to positve at this point. The definition of $\bar{P}(z)$ then implies that $\tilde{P} = \bar{P}(z)$. We must then have that

$$R\zeta'(P; z) = R - z f_\omega(\bar{\omega}(P)) > 0, \quad \text{all } P \in \left(\bar{P}(z) - \epsilon, \bar{P}(z) + \epsilon\right),$$

for some $\epsilon > 0$.■

Claim c. 2. $\phi'(z), \bar{P}'(z) > 0$ for almost all $z \in (\underline{P}R/\lambda, KR)$.

Proof of Claim c.2. From equation (14), we know that, for all $z \in \left(\frac{\max\{d_1(1-A_0)-1,0\}}{\lambda} R, KR\right)$,

$$\phi'(z) = \frac{f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)\phi(z)}{R - z f_\omega\left(\bar{\omega}\left(\bar{P}(z)\right)\right)}.$$

Claim a then implies that $\phi'(z) > 0$ for almost all $z \in (\underline{P}R/\lambda, KR)$. The fact that $\bar{P}(z) = \frac{z}{R}\phi(z)$, together with the last result, then jointly imply that $\bar{P}'(z) > 0$ for almost all $z \in [\underline{P}R/\lambda, KR)$.■

The proof of claim (c) then follows from combining the results in Claims c. 1 and c. 2, and equation (15).□

Proof of Proposition 2. We first observe that $\bar{P}$ is nontrivial only for assets with expected cashflows $\bar{\theta}_r \geq \underline{P}R/\lambda$. Indeed, as proved in Claim I of Proposition 1, for any $x \leq \bar{\theta}_r < \bar{P}R/\lambda$, $\bar{P}(x) = 0 = \varphi\left(\bar{P}(x)\right)$, and therefore the firm optimally chooses $x^\star = 0$.

Next, assume that $\bar{\theta}_r \in [\underline{P}R/\lambda, KR)$. We show that the firm's expected utility is quasi-convex for any $x \in \left[\underline{P}R/\lambda, \bar{\theta}_r\right]$ and always attains its global maximum at one of the corners $x \in \left\{0, \bar{\theta}_r\right\}$. Indeed, note first that

$$\begin{aligned}
\frac{\partial}{\partial x} V\left(x; \bar{\theta}_r\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left\{\left(\bar{P}(x) R - R(d_1 - 1) + \bar{\theta}_r - x\right)\varphi\left(\bar{P}(x)\right)\right\} \\
&= \left\{R\varphi\left(\bar{P}\right) + \left(\bar{P}R - R(d_1 - 1) + \bar{\theta}_r - x\right)\varphi'\left(\bar{P}\right)\right\}\bar{P}'(x) - \varphi\left(\bar{P}\right) \\
&= \left\{R\varphi\left(\bar{P}\right) + \left(\bar{P}R - R(d_1 - 1) + \bar{\theta}_r - x\right)\varphi'\left(\bar{P}\right)\right\}\left(\frac{\varphi\left(\bar{P}\right)}{R - x\varphi'\left(\bar{P}\right)}\right) - \varphi\left(\bar{P}\right) \\
&= R\bar{P}'(x) \cdot \left\{\varphi\left(\bar{P}\right) + \left(\bar{P} - (d_1 - 1) + \bar{\theta}_r/R\right)\varphi'\left(\bar{P}\right) - 1\right\},
\end{aligned}$$

where the third and fourth equalities obtain from noting that implicit differentiation of the $\bar{P}$

function yields

$$\bar{P}'(x)=\frac{\varphi\left(\bar{P}\right)}{R-x\varphi'\left(\bar{P}\right)}.$$

Define

$$\chi\left(x;\bar{\theta}_r\right)\equiv\varphi\left(\bar{P}(x)\right)+\left(\bar{P}(x)-(d_1-1)+\bar{\theta}_r/R\right)\varphi'\left(\bar{P}(x)\right)-1.$$

The monotonicity of φ and $\bar{P}$ and the convexity of φ (implied by assumption 2) means that $\chi\left(\cdot;\bar{\theta}_r\right)$ is monotone. Let $x_0\left(\bar{\theta}_r\right)$ be implicitly defined as the solution to $\chi\left(x_0;\bar{\theta}_r\right)=0$ if such a solution exists for some $x\in\left[\underline{P}R/\lambda,\bar{\theta}_r\right]$. Otherwise, let $x_0\left(\bar{\theta}_r\right)=\bar{\theta}_r$ if $\chi\left(x;\bar{\theta}_r\right)<0$ for all $x\in\left[\underline{P}R/\lambda,\bar{\theta}_r\right]$, and $x_0\left(\bar{\theta}_r\right)=\underline{P}R/\lambda$ if $\chi\left(x;\bar{\theta}_r\right)>0$ for all $x\in\left[\underline{P}R/\lambda,\bar{\theta}_r\right]$.

Assume first that $x_0\left(\bar{\theta}_r\right)\in\left(\underline{P}R/\lambda,\bar{\theta}_r\right)$. The fact that $\chi\left(x;\bar{\theta}_r\right)<0$ for all $x<x_0\left(\bar{\theta}_r\right)$ and $\chi\left(x;\bar{\theta}_r\right)>0$ for all $x>x_0\left(\bar{\theta}_r\right)$ implies that $V\left(\cdot;\bar{\theta}_r\right)$ is decreasing over $x\in\left[\underline{P}R/\lambda,x_0\left(\bar{\theta}_r\right)\right]$ and increasing for any $x>x_0\left(\bar{\theta}_r\right)$. Thus, $V\left(\cdot;\bar{\theta}_r\right)$ attains a (local) minimum at $x=x_0\left(\bar{\theta}_r\right)$, and a (local) maximum at the corners $x\in\left\{\underline{P}R/\lambda,\bar{\theta}_r\right\}$. We note next that $V\left(x;\bar{\theta}_r\right)$ is supermodular in $\left(x,\bar{\theta}_r\right)$, which means that, if $V\left(x=\bar{\theta}_r;\bar{\theta}_r\right)>V\left(x=\underline{P}R/\lambda;\bar{\theta}_r\right)$ for some $\bar{\theta}_r$, then we must we have that, for any $\bar{\theta}_r'>\bar{\theta}_r$,

$$\begin{aligned}0<V\left(x=\bar{\theta}_r;\bar{\theta}_r\right)-V\left(x=\underline{P}R/\lambda;\bar{\theta}_r\right) &< V\left(x=\bar{\theta}_r;\bar{\theta}_r'\right)-V\left(x=\underline{P}R/\lambda;\bar{\theta}_r'\right)\\ &< V\left(x=\bar{\theta}_r';\bar{\theta}_r'\right)-V\left(x=\underline{P}R/\lambda;\bar{\theta}_r'\right),\end{aligned}$$

where the second inequality obtains from the fact that $V\left(\cdot;\bar{\theta}_r\right)$ is increasing for any $x>x_0\left(\bar{\theta}_r\right)$ and the fact that $\bar{\theta}_r'>\bar{\theta}_r>x_0\left(\bar{\theta}_r\right)$. Thus, if for some $\bar{\theta}_r$, the firm prefers to sell the whole asset $x=\bar{\theta}_r$ over selling the minimum amount leading to a nontrivial price, i.e., $x=\underline{P}R/\lambda$, then, any firm with an asset with $\bar{\theta}_r'>\bar{\theta}_r$ prefers to sell the whole asset as well.

Suppose now that $x_0\left(\bar{\theta}_r\right)=\bar{\theta}_r$. This means that $V\left(\cdot;\bar{\theta}_r\right)$ is decreasing over $\left[\underline{P}R/\lambda,\bar{\theta}_r\right]$ and therefore attains a local maximum at $x=\underline{P}R/\lambda$. Assume next, instead, that $x_0\left(\bar{\theta}_r\right)=\underline{P}R/\lambda$. This implies that $V\left(\cdot;\bar{\theta}_r\right)$ is increasing over $\left[\underline{P}R/\lambda,\bar{\theta}_r\right]$ and hence attains a local maximum at $x=\bar{\theta}_r$.

Finally, we need to check whether, for any $\bar{\theta}_r\in[\underline{P}R/\lambda,KR)$, the local maximum described above is, in fact, a global maximum. As argued above, provided that the firm chooses to sell a fraction $x<\underline{P}R/\lambda$, it is optimal to not sell at all (i.e., $x=0$), as any asset with expected cashflows below $\underline{P}R/\lambda$ leads to a null price. We conclude that for any $\bar{\theta}_r\in[\underline{P}R/\lambda,KR)$, $x^\star\left(\bar{\theta}_r\right)\in\left\{0,\underline{P}R/\lambda,\bar{\theta}_r\right\}$.

Next, for any $\bar{\theta}_r\in\left[\underline{P}R/\lambda,\theta^\#\right)$,

$$\begin{aligned}V\left(\underline{P}R/\lambda,\bar{\theta}_r\right) &= \left(\bar{P}\left(\underline{P}R/\lambda\right)R-R\left(d_1-1\right)+\bar{\theta}_r-\underline{P}R/\lambda\right)\cdot\varphi\left(\bar{P}\left(\underline{P}R/\lambda\right)\right)\\ &= \left(\underline{P}R-R\left(d_1-1\right)+\bar{\theta}_r-\underline{P}R/\lambda\right)\cdot\lambda.\end{aligned}$$

We show that $V\left(\underline{P}R/\lambda,\bar{\theta}_r\right)<0$ for all $\bar{\theta}_r\in\left[\underline{P}R/\lambda,\theta^\#\right)$. Indeed, the definition of $\theta^\#$ implies that

$$\bar{P}\left(\theta^\#\right)=\frac{\theta^\#}{R}\varphi\left(\bar{P}\left(\theta^\#\right)\right)=d_1-1,$$

and, therefore,

$$\theta^{\#}=\frac{R\left(d_{1}-1\right)}{\varphi\left(\bar{P}\left(\theta^{\#}\right)\right)}=\frac{R\left(d_{1}-1\right)}{1-F_{\omega}\left(1-d_{1}A_{0}\right)}.$$

This means that

$$\begin{aligned}
V\left(\underline{P}R/\lambda,\theta^{\#}\right) &= \lambda\left(\underline{P}R-R\left(d_{1}-1\right)+\theta^{\#}-\underline{P}R/\lambda\right)\\
&= \lambda R\left(d_{1}-1\right)\frac{F_{\omega}\left(1-d_{1}A_{0}\right)}{1-F_{\omega}\left(1-d_{1}A_{0}\right)}-\underline{P}R\left(1-\lambda\right)\\
&= \frac{R\left\{\lambda\left(d_{1}-1\right)F_{\omega}\left(1-d_{1}A_{0}\right)-\underline{P}\left(1-\lambda\right)\left(1-F_{\omega}\left(1-d_{1}A_{0}\right)\right)\right\}}{1-F_{\omega}\left(1-d_{1}A_{0}\right)}\\
&= \frac{R\lambda\left\{d_{1}-1-d_{1}A_{0}\left(1-F_{\omega}\left(1-d_{1}A_{0}\right)\right)\right\}}{1-F_{\omega}\left(1-d_{1}A_{0}\right)}\\
&\quad \frac{-\underline{P}R\left(1-F_{\omega}\left(1-d_{1}A_{0}\right)\right)}{1-F_{\omega}\left(1-d_{1}A_{0}\right)}\\
&< 0,
\end{aligned}$$

where the inequality directly follows from inequality (8). We have used the fact that $\underline{P}=d_{1}\left(1-A_{0}\right)-1>0$, which is also implied by inequality (8). The monotonicity of $V\left(\underline{P}R/\lambda,\cdot\right)$ then implies that $V\left(\underline{P}R/\lambda,\bar{\theta}_{r}\right)<0$ for all $\bar{\theta}_{r}\in\left[\underline{P}R/\lambda,\theta^{\#}\right)$.

Note next that

$$\begin{aligned}
V\left(\bar{\theta}_{r},\bar{\theta}_{r}\right) &= \left(\bar{P}\left(\bar{\theta}_{r}\right)R-R\left(d_{1}-1\right)\right)\cdot\varphi\left(\bar{P}\left(\bar{\theta}_{r}\right)\right)\\
&\leq 0,\ \forall\bar{\theta}_{r}\in\left[\underline{P}R/\lambda,\theta^{\#}\right),
\end{aligned}$$

with equality if, and only if, $\bar{\theta}_{r}=\theta^{\#}$. This means that, when $\bar{\theta}_{r}\in\left[\underline{P}R/\lambda,\theta^{\#}\right)$, both $x=\underline{P}R/\lambda$ and $x=\bar{\theta}_{r}$ are dominated by $x=0$. In other words, the firm does not sell any security to AM investors.

Next, the *mean value theorem* implies that, because $V\left(\theta^{\#},\theta^{\#}\right)-V\left(\underline{P}R/\lambda,\theta^{\#}\right)>0$, there must exist $\tilde{x}\in\left(\underline{P}R/\lambda,\theta^{\#}\right)$ such that

$$\frac{\partial}{\partial x}V\left(\tilde{x};\theta^{\#}\right)=\frac{V\left(\theta^{\#},\theta^{\#}\right)-V\left(\underline{P}R/\lambda,\theta^{\#}\right)}{\theta^{\#}-\underline{P}R/\lambda}>0.$$

This further implies that $\frac{\partial}{\partial x}\chi\left(\tilde{x};\theta^{\#}\right)>0$. The monotonicity of $\chi\left(x;\bar{\theta}_{r}\right)$ in $\left(x,\bar{\theta}_{r}\right)$, together with the monotonicity of $\bar{P}'\left(\cdot\right)$ (recall that under assumption (2) $\bar{P}$ is convex), then implies that

$$\frac{\partial}{\partial x}V\left(x;\bar{\theta}_{r}\right)\geq 0,\ \text{for all } x\geq\tilde{x},\bar{\theta}_{r}\geq\theta^{\#}. \tag{16}$$

In other words, $x_0\left(\theta^{\#}\right) \in [\underline{P}R/\lambda, \tilde{x})$. Therefore, for any $\bar{\theta}_r > \theta^{\#}$,

$$\begin{aligned}
V\left(\underline{P}R/\lambda;\bar{\theta}_r\right) &= \underbrace{V\left(\underline{P}R/\lambda;\theta^{\#}\right)}_{<0} + \int_{\theta^{\#}}^{\bar{\theta}_r} \frac{\partial}{\partial\bar{\theta}_r} V\left(\underline{P}R/\lambda;\theta\right) \mathrm{d}\theta \\
&< \int_{\theta^{\#}}^{\bar{\theta}_r} \frac{\partial}{\partial\bar{\theta}_r} V\left(\theta;\theta\right) \mathrm{d}\theta \\
&< \int_{\theta^{\#}}^{\bar{\theta}_r} \left(\frac{\partial}{\partial x} V\left(\theta;\theta\right) + \frac{\partial}{\partial\bar{\theta}_r} V\left(\theta;\theta\right)\right) \mathrm{d}\theta \\
&= \underbrace{V\left(\theta^{\#};\theta^{\#}\right)}_{=0} + \int_{\theta^{\#}}^{\bar{\theta}_r} \frac{\mathrm{d}}{\mathrm{d}\theta} V\left(\theta;\theta\right) \mathrm{d}\theta \\
&= V\left(\bar{\theta}_r;\bar{\theta}_r\right)
\end{aligned}$$

where the first inequality follows from the supermodularity of $V\left(x,\bar{\theta}_r\right)$ (implied by the monotonicity of φ and $\bar{P}$), which means that for any $\bar{\theta}_r > \underline{P}R/\lambda$,

$$\frac{\partial}{\partial\bar{\theta}_r} V\left(\bar{\theta}_r;\bar{\theta}_r\right) > \frac{\partial}{\partial\bar{\theta}_r} V\left(\underline{P}R/\lambda;\bar{\theta}_r\right).$$

The second inequality, in turn, obtains from (16). We conclude that, for any $\bar{\theta}_r \in \left[\theta^{\#}, KR\right)$,

$$V\left(\bar{\theta}_r;\bar{\theta}_r\right) \geq \max\left\{V\left(0;\bar{\theta}_r\right), V\left(\underline{P}R/\lambda;\bar{\theta}_r\right)\right\},$$

with strict inequality for any $\bar{\theta}_r > \theta^{\#}$, and therefore, for any $\bar{\theta}_r \in \left(\theta^{\#}, KR\right)$, $x^{\star}\left(\bar{\theta}_r\right) = \bar{\theta}_r$.

Finally, consider the case where $\bar{\theta}_r \geq KR$. Then, the firm can secure the maximal possible payoff by issuing any security with expected value $x = KR$. Indeed, for any $x \geq KR$,

$$V\left(x;\bar{\theta}_r\right) = \bar{\theta}_r - R\left(d_1 - 1\right).$$

This completes the proof of Proposition (2).□

Appendix B: Optimal Information Disclosure

Proof of Theorem 1.

Let $\mathcal{U}(z) \equiv L_0(z)(1-\phi(z)) + W_0(z)\phi(z)$. Under the assumptions in the theorem, the function $\mathcal{U}(\cdot)$ is convex for any $z < KR$ and hence differentiable almost everywhere. Using integration by parts (see Theorem VI.90 in Dellacherie and Meyer (1982)), we can rewrite the regulator's problem as

$$\begin{aligned} \min_{G} \quad & \int_0^{\bar{x}} G(z)\,\mathrm{d}\mathcal{U}(z) - \Delta\mathcal{U}(KR)\,\Delta G(KR) \\ \text{s.t:} \quad & F_r \succeq_{\text{MPS}} G, \end{aligned}$$

where $\Delta\mathcal{U}(KR) \equiv \mathcal{U}(KR^+) - \mathcal{U}(KR^-)$ and $\Delta G(KR) \equiv G(KR^+) - G(KR^-)$.

Using the definition of L_0 and W_0, the objective then becomes

$$-\tau_L \int_0^{\theta^\#} G(z)\,\mathrm{d}z + \int_{\theta^\#}^{KR} (W_0(z)\,\phi(z))'\,G(z)\,\mathrm{d}z + \tau_W \int_{KR}^{\bar{x}} G(z)\,\mathrm{d}z - \Delta\mathcal{U}(KR)\,\Delta G(KR).$$

Consider an arbitrary feasible distribution H satisfying $F_r \succeq_{\text{MPS}} H$. Let μ_{F_r} be the probability measure inducing F_r. Suppose that $\mu_{F_r}\{\theta_r : H(\theta_r) \neq G^\star(\theta_r)\} > 0$. We show that such a distribution is necessarily dominated.

Step 1. Assume that $\mu_{F_r}\left\{\theta_r \leq \hat{\theta}_r : H(\theta_r) < G^\star(\theta_r)\right\} > 0$. The definition of $\hat{\theta}_r$ implies that $\mu_{F_r}\{\theta_r < KR : H(\theta_r) > G^\star(\theta_r)\} > 0$. To see the last observation, note that, if this is not the case, then

$$\begin{aligned} \int_0^{\bar{x}} H(z)\,\mathrm{d}z &< \int_0^{KR} \min\left\{F_r(z), F_r(\hat{\theta}_r)\right\}\mathrm{d}z + \int_{KR}^{\bar{x}} 1\mathrm{d}z \\ &= \int_0^{\hat{\theta}_r} F_r(z)\,\mathrm{d}z + F_r(\hat{\theta}_r)(KR - \hat{\theta}_r) + (\bar{x} - KR). \\ &= \bar{x} - \mathbb{E}_0(\boldsymbol{\theta}_r) \\ &= \int_0^{\bar{x}} F_r(z)\,\mathrm{d}z, \end{aligned} \tag{17}$$

where the inequality follows from the definition of $G^\star$, the first equality is self-evident, and the last equality obtains from noting that, by definition of $\hat{\theta}_r$,

$$\int_0^{\hat{\theta}_r} zF_r(\mathrm{d}z) + KR(1 - F_r(\hat{\theta}_r)) = \mathbb{E}_0(\boldsymbol{\theta}_r),$$

and, therefore, using integration by parts,

$$\int_0^{\hat{\theta}_r} F_r(z)\,\mathrm{d}z = \hat{\theta}_r F_r(\hat{\theta}_r) + KR(1 - F_r(\hat{\theta}_r)) - \mathbb{E}_0(\boldsymbol{\theta}_r).$$

Inequality (17), however, contradicts the assumption that $F_r \succeq_{\text{MPS}} H$. We thus focus on policies H satisfying $\mu_{F_r}\{\theta_r < KR : H(\theta_r) > G^\star(\theta_r)\} > 0$.

Next, pick two adjacent sets $\Theta_- \subseteq \left[0, \hat{\theta}_r\right]$, $\Theta_+ \subseteq [0, KR)$, with $\sup\Theta_- = \inf\Theta_+$, satisfying (a) $\mu_{F_r}\{\Theta_-\}, \mu_{F_r}\{\Theta_+\} > 0$, (b) $H(\theta_r) > G^\star(\theta_r)$ μ_{F_r}-almost all $\theta_r \in \Theta_+$ and $H(\theta_r) \leq G^\star(\theta_r)$ for

μ_{F_r}-almost all $\theta_r \in \Theta_-$ with $\mu_{F_r}\{\theta_r \in \Theta_- : H(\theta_r) < G^\star(\theta_r)\} > 0$, and (c)[37]

$$\int_{\Theta_-} (F_r(z) - H(z))\,\mathrm{d}z = \int_{\Theta_+} (H(z) - G^\star(z))\,\mathrm{d}z. \tag{18}$$

Construct an alternative policy $\hat{H}$ defined as follows: $\hat{H}(\theta_r) = H(\theta_r)$ for all $\theta_r \notin \Theta_- \cup \Theta_+$, $\hat{H}(\theta_r) = F_r(\theta_r)$ for all $\theta_r \in \Theta_-$, and $\hat{H}(\theta_r) = G^\star(\theta_r) = \min\left\{F_r(\theta_r), F_r\left(\hat{\theta}_r\right)\right\}$ for all $\theta_r \in \Theta_+$. We note that the new policy is feasible as, by construction,

$$\int_0^{\theta_r} \hat{H}(z)\,\mathrm{d}z \leq \int_0^{\theta_r} F_r(z)\,\mathrm{d}z,\ \forall\theta_r,$$

and

$$\begin{aligned}\int_0^{\bar{x}} \hat{H}(z)\,\mathrm{d}z &= \int_{\Theta\setminus(\Theta_-\cup\Theta_+)} H(z)\,\mathrm{d}z + \int_{\Theta_-} F_r(z)\,\mathrm{d}z + \int_{\Theta_+} G^\star(z)\,\mathrm{d}z \\ &= \int_0^{\bar{x}} H(z)\,\mathrm{d}z = \int_0^{\bar{x}} F_r(z)\,\mathrm{d}z,\end{aligned}$$

where the second equality is a consequence of (18).

The new policy strictly improves upon H as $\mathcal{U}'(z)$ is nondecreasing over $[0, KR)$, and $\hat{H}$ is contructed from H by moving probability mass from high realizations of $\boldsymbol{\theta}_r$ to low realizations.

Step 2. By virtue of Step 1, assume without loss that $H(\theta_r) = F_r(\theta)$ for all $\theta_r \leq \hat{\theta}_r$. For any such a distribution H which also satisfies $F_r \succeq_{\text{MPS}} H$, let

$$V[H] \equiv \int_{\hat{\theta}_r}^{\bar{x}} \mathcal{U}'(z)\,H(z)\,\mathrm{d}z - \Delta H(KR)\,\Delta\mathcal{U}(KR).$$

Note that

$$V[G^\star] = F_r\left(\hat{\theta}_r\right)\int_{\hat{\theta}_r}^{KR} \mathcal{U}'(z)\,\mathrm{d}z + \tau_W(\bar{x} - KR) - (1 - F_r(\hat{\theta}_r))\Delta\mathcal{U}(KR).$$

[37] Existence of Θ_- and Θ_+ is guaranteed from the assumption that $\mu_{F_r}\left\{\theta_r \leq \hat{\theta}_r : H(\theta_r) < G^\star(\theta_r) = F_r(\theta_r)\right\} > 0$, the observation above that $\mu_{F_r}\{\theta_r < KR : H(\theta_r) > G^\star(\theta_r)\} > 0$, and the fact that $\int_0^{\theta_r} H(z)\,\mathrm{d}z \leq \int_0^{\theta_r} F_r(z)\,\mathrm{d}z$ for all θ_r.

Thus,

$$\begin{aligned} V[H]-V[G^{\star}] &= \int_{\hat{\theta}_r}^{KR} \mathcal{U}'(z)\left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z - \tau_W \int_{KR}^{\bar{x}} (1-H(z))\,\mathrm{d}z \\ &\quad + \left(1-F_r(\hat{\theta}_r)-\Delta H(KR)\right)\Delta\mathcal{U}(KR) \\ &= \int_{\hat{\theta}_r}^{KR} \mathcal{U}'(z)\left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z - \tau_W \int_{\hat{\theta}_r}^{KR} \left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z \\ &\quad + \left(1-F_r(\hat{\theta}_r)-\Delta H(KR)\right)\Delta\mathcal{U}(KR) \\ &= \int_{\hat{\theta}_r}^{KR} (\mathcal{U}'(z)-\tau_W)\left(H(z)-F_r(\hat{\theta}_r)\right))\mathrm{d}z + (1-F_r(\hat{\theta}_r)-\Delta H(KR))\Delta\mathcal{U}(KR), \end{aligned} \tag{19}$$

where the second equality follows from noting that

$$\int_{\hat{\theta}_r}^{\bar{x}} H(z)\,\mathrm{d}z = \int_{\hat{\theta}_r}^{\bar{x}} G^{\star}(z)\,\mathrm{d}z = F_r(\hat{\theta}_r)\left(KR-\hat{\theta}_r\right)+\bar{x}-KR,$$

which implies that

$$\int_{\hat{\theta}_r}^{KR} \left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z = \int_{KR}^{\bar{x}} (1-H(z))\,\mathrm{d}z.$$

We note that if $\int_{\hat{\theta}_r}^{KR} (\mathcal{U}'(z)-\tau_W)\left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z \geq 0$, then (19) directly implies the result in the theorem, since $1-F_r(\hat{\theta}_r) > \Delta H(KR)$. We assume therefore that,

$$\int_{\hat{\theta}_r}^{KR} (\tau_W - \mathcal{U}'(z))\left(H(z)-F_r(\hat{\theta}_r)\right)\mathrm{d}z > 0.$$

Note that (19) then implies that

$$\begin{aligned} V[H]-V[G^{\star}] &= (H(KR^-)-F_r(\hat{\theta}_r))\left\{\Delta\mathcal{U}(KR) - \int_{\hat{\theta}_r}^{KR} (\tau_W-\mathcal{U}'(z))\frac{H(z)-F_r(\hat{\theta}_r)}{H(KR^-)-F_r(\hat{\theta}_r)}\mathrm{d}z\right\} \\ &\quad + \left(1-H\left(KR^+\right)\right)\Delta\mathcal{U}(KR) \\ &> \left(H\left(KR^-\right)-F_r\left(\hat{\theta}_r\right)\right)\left\{\int_{\hat{\theta}_r}^{KR} (\tau_W-\mathcal{U}'(z))\left(1-\frac{H(z)-F_r(\hat{\theta}_r)}{H(KR^-)-F_r(\hat{\theta}_r)}\right)\mathrm{d}z\right\}, \end{aligned} \tag{20}$$

where the inequality obtains from noting that

$$\begin{aligned} \mathcal{U}(KR)-\mathcal{U}\left(\hat{\theta}_r\right) &= W_0(KR)-W_0\left(\hat{\theta}_r\right)\phi\left(\hat{\theta}_r\right) \\ &> W_0(KR)-W_0\left(\hat{\theta}_r\right) \\ &= \tau_W\left(KR-\hat{\theta}_r\right), \end{aligned}$$

and therefore

$$\mathcal{U}(KR)-\mathcal{U}\left(\hat{\theta}_r\right)=\int_{\hat{\theta}_r}^{KR}\mathcal{U}\left(z^-\right)\mathrm{d}z+\Delta\mathcal{U}(KR)>\tau_W\left(KR-\hat{\theta}_r\right),$$

implying that $\Delta\mathcal{U}(KR)>\int_{\hat{\theta}_r}^{KR}\left(\tau_W-\mathcal{U}\left(z^-\right)\right)\mathrm{d}z$.

The next claim is instrumental to prove that

$$\int_{\hat{\theta}_r}^{KR}\left(\tau-\mathcal{U}'(z)\right)\left(1-\frac{H(z)-F_r(\hat{\theta}_r)}{H\left(KR^-\right)-F_r(\hat{\theta}_r)}\right)\mathrm{d}z>0, \tag{21}$$

and, therefore, from (20), that $V[H]-V[G^\star]>0$.

Claim 3. Consider two functions $w,J:[a,b]\subseteq\mathbb{R}_+\to\mathbb{R}$, satisfying (a) J nondecreasing, continuous over $[a,b)$, with $J(a)=0$ and $J(b)=1$, (b) w nonincreasing, and (c) $\int_a^b w(x)J(x)\,\mathrm{d}x>0$. Then, we must necessarily have $\int_a^b w(x)(1-J(x))\,\mathrm{d}x>0$.

Proof. Using integration by parts, we have

$$\int_a^b w(x)J(x)\,\mathrm{d}x=\int_a^b w(x)\,\mathrm{d}x-\int_a^b\underbrace{\left(\int_a^x w(z)\,\mathrm{d}z\right)}_{\equiv q(x)}\mathrm{d}J(x), \tag{22}$$

where we have used the assumption that $J(a)=0$ and $J(b)=1$. This equation implies that

$$\int_a^b w(x)(1-J(x))\,\mathrm{d}x=\int_a^b q(x)\,\mathrm{d}J(x).$$

Next, we note that part (a) implies that $J(\cdot)$ is a probability measure over $[a,b]$. Part (b) further implies that $q(\cdot)$ is globally concave. Construct an alternative measure

$$\bar{J}(x)\equiv\left(\frac{b-\int_a^b x\mathrm{d}J(x)}{b-a}\right)+\left(\frac{\int_a^b x\mathrm{d}J(x)-a}{b-a}\right)\cdot 1\{x=b\},\ \forall x\in[a,b].$$

That is, $\bar{J}$ allocates all probability mass to either $x=a$ or $x=b$, and satisfies $\int_a^b x\mathrm{d}\bar{J}(x)=\int_a^b x\mathrm{d}J(x)$. We observe that, by construction, $\bar{J}\succ_{\mathrm{MPS}}J$. The concavity of $q(\cdot)$ then implies that

$$\int_a^b q(x)\,\mathrm{d}J(x)>\int_a^b q(x)\,\mathrm{d}\bar{J}(x)=q(b)\cdot\left(\frac{\int_a^b x\mathrm{d}J(x)-a}{b-a}\right).$$

Next, property (c) and equation (22) jointly imply that

$$q(b)=\int_a^b w(x)\,\mathrm{d}x>\int_a^b q(x)\,\mathrm{d}J(x).$$

The last two inequalities, together with the fact that $\frac{\int_a^b x\mathrm{d}J(x)-a}{b-a}<1$, then imply that $q(b)>0$, and therefore

$$\int_a^b w(x)(1-J(x))\,\mathrm{d}x = \int_a^b q(x)\,\mathrm{d}J(x) > 0,$$

as claimed. ■

By letting $w(z) \equiv \tau - \mathcal{U}'(z)$ and $J(z) \equiv \frac{H(z)-F_r(\hat{\theta}_r)}{H(KR^-)-F_r(\hat{\theta}_r)}$ in claim 3, we conclude that (21) holds and therefore $V[H] > V[G^\star]$. This, in turn, implies that G^* solves the regulator's problem.

Appendix C: Robust Information Disclosures

D1 Refinement. Define first the set of *best responses* to an arbitrary security s, $BR(s)$, as the set of prices which are consistent with rationality of the investors under some belief about the type of the firm:[38]

$$BR(s) \equiv \left\{ P \geq 0 : \frac{\mathbb{E}_H(s)}{R}\mathbb{P}\left\{\boldsymbol{\omega} + P \geq A^\star(P)\right\} \geq P \right\}.$$

Define then,

$$\mathcal{D}(\theta|s) \equiv \left\{P \in BR(s) : V(P,s,\theta) > V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}$$

$$\mathcal{D}^0(\theta|s) \equiv \left\{P \in BR(s) : V(P,s,\theta) = V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}.$$

The profile $\left\{\{s^\star_\theta\}_{\theta\in\Theta}, \mu^\star, P^\star, A^\star\right\}$ satisfies the D1 criterion if for any security $s \in S$ with $s \neq s_*(\theta)$ all $\theta \in \Theta$, $\mu_*(s)$ is such that $\forall \theta, \theta'$ $\left(\mathcal{D}(\theta|s) \cup \mathcal{D}^0(\theta|s)\right) \subset \mathcal{D}(\theta'|s)\right) \Rightarrow \mu_*(\theta|s) = 0$.

Definition 2. We say a function $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from above* (SCFA), if there exists some $y \in Y$ such that $g(y) < 0$, then $\forall \tilde{y} > y$, $g(\tilde{y}) \leq 0$. Similarly, we say that $h : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from below* (SCFB), if the following holds true: if there exists some $y \in Y$ such that $h(y) > 0$, then $\forall \tilde{y} > y$, $h(\tilde{y}) \geq 0$.

Lemma 3. *Suppose that* $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ *satisfies SCFA and that* $f(y,t)$ *is log-supermodular for all* $(y,t) \in Y \times T \subseteq \mathbb{R}^2$. *Define* $\phi(t) \equiv \int_Y g(y) f(y,t) dy$ *and let* $y_0 \equiv \inf\{y \in Y : g(y) < 0\}$. *Then,* $\forall \tilde{t} > t \in T$:

$$\phi\left(\tilde{t}\right) = 0 \Rightarrow \phi(t) > 0.$$

Proof. That $f(y,t)$ is log-SM implies that $\frac{f(\cdot,t)}{f(\cdot,\tilde{t})}$ is non-increasing. Then,

$$\begin{aligned}
\phi(t) &= \int_Y 1\{y \leq y_0\}\, g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t})\mathrm{d}y + \int_Y 1\{y > y_0\}\, g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t})\mathrm{d}y \\
&\geq \left(\frac{f(y_0,t)}{f(y_0,\tilde{t})}\right)\phi\left(\tilde{t}\right)
\end{aligned}$$

[38] First-order stochastic dominance (which is implied by MLRP) means that

$$\left\{P > 0 : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\} = \bigcup_{\mu\in\Delta\Theta} \left\{P > 0 : \frac{\mathbb{E}(s;\mu)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\}$$

which implies the result. $\square$

Proofs Subsection 5.1.

As an intermediate step, we first characterize the set of equilibrium outcomes that arise in the fund-raising game in the absence of regulatory disclosures. Proposition7 below extends the results in Nachman and Noe (1994) to the current environment, where the probability of default is endogenously determined by the interaction between the two audiences.[39] The proposition below shows that, although the celebrated uniqueness result of Nachman and Noe (1994) may not hold in the current environment, some qualitative properties remain true.

The next proposition shows that when the firm faces severe liquidity constraints, i.e., when assumption 2 holds, the existence of a firm type with sufficiently poor assets may completely freeze the asset market, preventing both firm types from raising funds. AM investors' ability to foresee the possibility of a run and to price assets accordingly, together with the incentives of type ξ_H to separate from type ξ_L, induces a *contagion* effect so severe that no firm is able to raise funds. Furthermore, if the aggregate expected profitability of the firms' assets is lower than K, market freezing becomes the unique equilibrium of the fund-raising stage.

Proposition 7. Suppose that assumption (2) holds. Then,

1. *All pooling equilibria are in debt contracts* ($s_{pool} = \min\{y, d\}$, $d \geq 0$). *Moreover,* $P(s_{pool}) \leq K$.

2. *Suppose that* $\mathbb{E}^{\xi_H}\{\boldsymbol{\theta}_r\} > KR > \mathbb{E}^{\xi_L}\{\boldsymbol{\theta}_r\}$; *then, in any separating equilibrium, any security issued by type* ξ_H *satisfies* $P\left(s_H^{sep}\right) \leq \mathbb{E}_L(y) < KR$.

Proof. The proof below applies regardless of whether the regulator has disclosed information about the fundamentals $\vec{\boldsymbol{\vartheta}} = (\boldsymbol{\theta}_r, \boldsymbol{\omega})$. Assume that the survival probability can be written as $\mathbb{P}\left\{\boldsymbol{\omega} \geq \omega^\sharp(z)\right\}$, where $\omega^\sharp(\cdot)$ represents a decreasing function, continuously differentiable for almost all $z < K$, and with $\omega^\sharp(\tau) = 0$, for all $z \geq K$. In the context of Section 3 and 4, $\omega^\sharp(P) = \bar{\omega}(P)$, while in the context of section 5, $\omega^\sharp = \bar{\omega}^{\text{LST}}$. Define $\Pi(z)$ as the set of prices which induce a nonnegative profit to AM investors when a security of expected value z is purchased. That is

$$\Pi(z) \equiv \left\{P \geq 0 : (z/R)\,\mathbb{P}\left\{\omega \geq \omega^\sharp(P)\right\} \geq P\right\}.$$

Part 1.a. We first rule pooling in securities other than debt contracts. Suppose that there exists an equilibrium of the fund-raising game, $\left\{\{\sigma_\xi\}_{\xi\in\Xi}, \mu, P, A\right\}$, and any nontrivial security $\hat{s} \in S$ with $\sigma_\xi(\hat{s}) > 0$, for all $\xi \in \Xi$. Suppose by contradiction that $\hat{s}$ is not a debt contract. Define the debt security $s_D \equiv \min\{y, D\}$ where D is such that $\mathbb{E}_H(s_D - \hat{s}) = 0$. Note that $s_D - \hat{s}$ satisfies *single*

[39] The model in Nachman and Noe (1994) assumes that the seller of the asset (i.e., the firm in our environment) survives with probability 1 if the latter raises an exogenous amount K and defaults, also with certainty, if the firm does not.

crossing from above (SCFA) and hence lemma 3 implies that $\mathbb{E}_L(s_D - \hat{s}) > 0 = \mathbb{E}_H(s_D - \hat{s})$. Thus,

$$\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D) > \mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s}). \tag{23}$$

Next, let $P^\sharp(z) \equiv \sup \Pi(z)$ and define $\Delta V_\xi(P)$ as the difference in payoffs for type ξ obtained by switching to security s_D, and sell it at price P, instead of issuing security $\hat{s}$ at price $P(\hat{s}) \equiv P^\sharp(\mathbb{E}_{\hat{\mu}}(\hat{s}))$, with $\hat{\mu} = \sigma_H(\hat{s}) / (\sigma_L(\hat{s}) + \sigma_H(\hat{s})) \in (0,1)$. That is,

$$\begin{aligned} \Delta V_\xi\left(\tilde{P}\right) &= V\left(\tilde{P}, s_D, \xi\right) - V\left(P(\hat{s}), \hat{s}, \xi\right) \\ &= \left(\tilde{P}R - R(d_1 - 1) + \mathbb{E}_\xi(\boldsymbol{\theta}_r - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^\sharp\left(\tilde{P}\right)\right\} \\ &\quad - \left(P(\hat{s})R - R(d_1 - 1) + \mathbb{E}_\xi(\boldsymbol{\theta}_r - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^\sharp\left(P(\hat{s})\right)\right\}, \ \xi \in \Xi. \end{aligned}$$

Inequality (23) together with the fact that $\theta_r - s_D$ and $\theta_r - \hat{s}$ are monotone then imply that

$$\begin{aligned} \Delta V_H\left(\tilde{P}\right) - \Delta V_L\left(\tilde{P}\right) &= \left(\mathbb{E}_H(\boldsymbol{\theta}_r - s_D) - \mathbb{E}_L(\boldsymbol{\theta}_r - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^\sharp\left(\tilde{P}\right)\right\} \\ &\quad - \left(\mathbb{E}_H(\boldsymbol{\theta}_r - \hat{s}) - \mathbb{E}_L(\boldsymbol{\theta}_r - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^\sharp\left(P(\hat{s})\right)\right\} \\ &> 0, \quad \forall \tilde{P} \geq P(\hat{s}). \end{aligned} \tag{24}$$

Next, the fact that F_ω is nondecreasing and right-continuous implies that $\Pi(\cdot)$ is compact and strictly increasing for any $\tau \geq 0$.[40] Thus,

$$P(\hat{s}) = \max\ \Pi\left(\mathbb{E}_{\hat{\mu}}(\hat{s})\right) < \max\ \Pi\left(\mathbb{E}_H(\hat{s})\right) = \max\ BR(s_D),$$

where the first equality follows from the compactness of Π and the definition of $P(\hat{s})$. The inequality arises from the strict monotonicity of Π and the MLRP ordering. The second equality is by definition of $BR(\cdot)$ and the construction of s_D.

Finally, note that $\mathbb{E}_L(\boldsymbol{\theta}_r - \hat{s}) > \mathbb{E}_L(\boldsymbol{\theta}_r - s_D)$ implies that $\Delta V_L(P(\hat{s})) < 0$. By construction, we also have that $\Delta V_H(P(\hat{s})) = 0$, which together with the fact that $P(\hat{s}) \in \Pi(\mathbb{E}_{\hat{\mu}}(\hat{s})) \subset BR(s_D)$ and the result in (24) imply that

$$\mathcal{D}(\theta_L | s_D) \cup \mathcal{D}^0(\theta_L | s_D) \subset \mathcal{D}(\theta_H | s_D).$$

As a consequence, market beliefs consistent with D1 must necessarily assign $\mu(\xi_H | s_D) = 1$. This implies that the market prices the security s_D at $P^\sharp(\mathbb{E}_H(s_D)) > P(\hat{s})$ and therefore both types have incentives to deviate and issue s_D instead, which contradicts the assumption that $\left\{\{\sigma_\xi\}_{\xi\in\Xi}, \mu, P, A\right\}$ is an equilibrium.

Part 1.b. Next, we show that under pooling no type raises more than K. Suppose, by contradiction, that there exists an equilibrium of the fund-raising game, $\left\{\{\sigma_\xi\}_{\xi\in\Xi}, \mu, P, A\right\}$, and any

[40]We say that a correspondence $\varphi : \mathbb{R}_+ \to 2^{\mathbb{R}_+}$ is strictly increasing if, for any $z, z' \in \mathbb{R}_+$, with $z < z'$, $\varphi(z) \subsetneq \varphi(z')$.

nontrivial security $s_d \equiv \min\{y,d\}$ with $\sigma_\xi(s_d) > 0$, for all $\xi \in \Xi$. Define $P^\sharp(z) \equiv \sup \Pi(z)$ and let $\mu_d \equiv \sigma_H(s_d)/(\sigma_L(s_d)+\sigma_H(s_d))$. We must then have that $P(s_d) \equiv P^\sharp(\mathbb{E}_{\mu_d}(s_d))$. Assume by contradiction that $P(s_d) = P^\sharp(\mathbb{E}_{\mu_d}(s_d)) > K$. Consider the alternative debt contract $s_\epsilon = \min\{y, d-\epsilon\}$ with $\epsilon > 0$ small so that (a) $\mathbb{E}_H(s_\epsilon) > \mathbb{E}_{\mu_d}(s_d)$, and (b) $\mathbb{E}_{\mu_d}(s_d - s_\epsilon) < R(P(s_d) - K)$. We show that type θ_H can profitably deviate to s_ϵ. Observe that $s_d - s_\epsilon$ is an increasing function. FOSD (which is implied by MLRP) then means that $\mathbb{E}_H(s_d - s_\epsilon) > \mathbb{E}_L(s_d - s_\epsilon)$, or equivalently,

$$\mathbb{E}_H(y - s_\epsilon) - \mathbb{E}_L(y - s_\epsilon) > \mathbb{E}_H(y - s_d) - \mathbb{E}_L(y - s_d). \tag{25}$$

Define $\Delta V_\xi\left(\tilde{P}; s_\epsilon, s_d\right) \equiv V\left(\tilde{P}, s_\epsilon, \xi\right) - V(P(s_d), s_d, \xi)$ as the difference in payoffs for firm ξ obtained by switching to security s_ϵ, and sell it at price $\tilde{P}$, instead of issuing security s_d at price $P(s_d)$. That is,

$$\begin{aligned}\Delta V_\xi\left(\tilde{P}; s_\epsilon, s_d\right) &= V\left(\tilde{P}, s_\epsilon, \xi\right) - V(P(s_d), s_d, \xi)\\ &= \left(\tilde{P}R + \mathbb{E}_\xi(\theta_r - s_\epsilon)\right)\mathbb{P}\left\{\boldsymbol{\omega} \geq \omega^\sharp\left(\tilde{P}\right)\right\}\\ &\quad - (P(s_d)R + \mathbb{E}_\xi(\theta_r - s_d))\,\mathbb{P}\left\{\boldsymbol{\omega} \geq \omega^\sharp(P(s_d))\right\},\ \xi \in \Xi.\end{aligned}$$

Next, the fact that $\mathbb{E}_H(s_\epsilon) > \mathbb{E}_{\mu_d}(s_d)$ implies that

$$\Pi(\mathbb{E}_{\mu_d}(s_d)) \subsetneq \Pi(\mathbb{E}_H(s_\epsilon)) = BR(s_\epsilon),$$

and hence $P(s_d) \in BR(s_\epsilon)$. Moreover, given that s_ϵ is smaller than s_d, we must have that $\Delta V_\xi(P(s_d)) > 0$ for both $\xi \in \Xi$, and therefore that $\mathcal{D}(\xi_L|s_\epsilon), \mathcal{D}(\xi_H|s_\epsilon) \neq \emptyset$. Next, inequality 25 implies that

$$\begin{aligned}\Delta V_H\left(\tilde{P}; s_\epsilon, s_d\right) - \Delta V_L\left(\tilde{P}; s_\epsilon, s_d\right) &= (\mathbb{E}_H(\theta_r - s_\epsilon) - \mathbb{E}_L(\theta_r - s_\epsilon)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(\tilde{P}\right)\right\}\\ &\quad - (\mathbb{E}_H(\theta_r - s_d) - \mathbb{E}_L(\theta_r - s_d)) \times \underbrace{\mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_d))\right\}}_{=1}\\ &> 0, \quad \forall \tilde{P} \geq K,\end{aligned} \tag{26}$$

Finally, let

$$\tilde{P}_\epsilon \equiv P(s_d) - \mathbb{E}_{\mu_d}(s_d - s_\epsilon)/R.$$

Condition (b) above implies that $\tilde{P}_\epsilon \in [K, P(s_d))$. This means that

$$\frac{\mathbb{E}_H(s_\epsilon)}{R}\mathbb{P}\left\{\boldsymbol{\omega} \geq \omega^\sharp\left(\tilde{P}_\epsilon\right)\right\} = \frac{\mathbb{E}_H(s_\epsilon)}{R} > \frac{\mathbb{E}_{\mu_d}(s_d)}{R} = P(s_d) > \tilde{P}_\epsilon,$$

and therefore $\tilde{P}_\epsilon \in BR(s_\epsilon)$. Moreover, by construction, we have that $\Delta V_H\left(\tilde{P}_\epsilon\right) > 0 > V_L\left(\tilde{P}_\epsilon\right)$.

Indeed,

$$\begin{aligned}
\Delta V_L\left(\tilde{P}_\epsilon\right) &= V\left(\tilde{P}_\epsilon, s_\epsilon, \xi_L\right) - V\left(P\left(s_d\right), s_d, \xi_L\right) \\
&= \left(\tilde{P}_\epsilon R + \mathbb{E}_L\left(\theta_r - s_\epsilon\right)\right) - \left(P\left(s_d\right) R + \mathbb{E}_L\left(\theta_r - s_d\right)\right) \\
&= \mathbb{E}_L\left(s_d - s_\epsilon\right) - \mathbb{E}_{\mu_d}\left(s_d - s_\epsilon\right) < 0 \\
&< \mathbb{E}_H\left(s_d - s_\epsilon\right) - \mathbb{E}_{\mu_d}\left(s_d - s_\epsilon\right) \\
&= \left(\tilde{P}_\epsilon R + \mathbb{E}_H\left(\theta_r - s_\epsilon\right)\right) - \left(P\left(s_d\right) R + \mathbb{E}_H\left(\theta_r - s_d\right)\right) \\
&= \Delta V_H\left(\tilde{P}_\epsilon\right)
\end{aligned}$$

where the second and fifth equalities follow from the fact $\tilde{P}_\epsilon, P(s_d) > K$, the third and fourth equalities obtain by definition of $\tilde{P}_\epsilon$, and the two inequalities follow from FOSD. Thus, $\mathcal{D}\left(\xi_L|s_\epsilon\right) \cup \mathcal{D}^0\left(\xi_L|s_\epsilon\right) \subset \mathcal{D}\left(\xi_H|s_\epsilon\right)$, and consequently market beliefs consistent with D1 must assign $\mu\left(\xi_H|s_\epsilon\right) = 1$. Together with condition (a), this implies that both types can profitably deviate to s_ϵ. This is a contradiction and therefore $P(s_d) \leq K$.

Part 2. Consider any security s_H issued only by type ξ_H. Assume by contradiction that $P\left(s_H\right) R > \mathbb{E}_L\left(\boldsymbol{\theta}_r\right)$. Denote by s_L any security issued with positive probability by type θ_L. That this is separating equilibrium, together with assumption **??**, means that

$$P\left(s_L\right) = \max \Pi\left(\mathbb{E}_L\left(s_L\right)\right) < \mathbb{E}_L\left(s_L\right)/R,$$

Hence,

$$P\left(s_H\right) R > \mathbb{E}_L\left(\boldsymbol{\theta}_r\right) > P\left(s_L\right) R + \mathbb{E}_L\left(\boldsymbol{\theta}_r - s_L\right) \tag{27}$$

As a result, type ξ_L has incentives to mimic type ξ_H. To see this, note that

$$\begin{aligned}
& V\left(P\left(s_H\right), s_H, \xi_L\right) - V\left(P\left(s_L\right), s_L, \xi_L\right) \\
= \; & \left(P\left(s_H\right) R - R(d_1 - 1) + \mathbb{E}_L\left(\boldsymbol{\theta}_r - s_H\right)\right) \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(P\left(s_H\right)\right)\right\} \\
& - \left(P\left(s_L\right) R - R(d_1 - 1) + \mathbb{E}_L\left(\boldsymbol{\theta}_r - s_L\right)\right) \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(P\left(s_L\right)\right)\right\} \\
> \; & \left(P\left(s_H\right) R - R(d_1 - 1) + \mathbb{E}_L\left(\boldsymbol{\theta}_r - s_H\right) - \left(P\left(s_L\right) R - R(d_1 - 1) + \mathbb{E}_L\left(\boldsymbol{\theta}_r - s_L\right)\right)\right) \times \\
& \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(P\left(s_L\right)\right)\right\} \\
> \; & 0,
\end{aligned}$$

where the first inequality arises from the fact that $P\left(s_H\right) > P\left(s_L\right)$ and the monotonicity of $\omega^\sharp$. The second inequality, in turn, is a consequence of equation (27). This is a contradiction and hence $P\left(s_H\right) \leq \frac{1}{R}\mathbb{E}_L\left(\boldsymbol{\theta}_r|m^y\right) < K$. $\square$

Proof of Proposition 3.

We prove that, when the disclosure policy announces that $m_r = \left\{\boldsymbol{\theta}_r \geq \hat{\theta}_r\right\}$, then the (best) equilibrium of the fund-raising stage consists of both firm types selling the whole risky asset at a price $P = K$. Recall that the definition of $\hat{\theta}_r$ is $\mathbb{E}\left(\boldsymbol{\theta}_r | \boldsymbol{\theta}_r \geq \hat{\theta}_r\right) = KR$.

To see that this is an equilibrium, fix an arbitrary security $\tilde{s}$ and define $\Delta V_\xi (P, \tilde{s})$ as the differential payoff obtained by type ξ by switching from pure equity, that is, $s(\cdot) = \mathrm{Id}(\cdot)$, to an alternate security $\tilde{s}$ and receiving a price P for the latter. That is,

$$\begin{aligned} \Delta V_\xi (P, \tilde{s}) &\equiv V_\xi (P, \tilde{s}, m_r) - V_\xi (K, \mathrm{Id}, m_r). \\ &= R\left\{\left(P - (d_1 - 1) + \mathbb{E}^\xi (\boldsymbol{\theta}_r - \tilde{s}(\boldsymbol{\theta}_r) | m_r) / R\right) \varphi(P) - (K - (d_1 - 1))\right\} \end{aligned}$$

We show that beliefs that assign probability 1 to the type being θ_L are consistent with D1. Clearly, under such beliefs no firm type has incentive to deviate.

The next claim reduces the set of deviations that need to be considered.

Claim 1. Fix an arbitrary security $s \in \mathcal{S}$, let $s_d \equiv \min\{y, d\}$ be the equivalent debt contract from type ξ_H's perspective, that is, s_d is such that $\mathbb{E}^{\xi_H}(s - s_d | m_r) = 0$. Then, $\Delta V_{\xi_L}(P, s_d) \leq \Delta V_{\xi_L}(P, s)$.

Proof. By virtue of Lemma 3 (which applies as MLRP is robust to bayesian updating) and the definition of s_d, we have that $\mathbb{E}^{\xi_L}(s - s_d | m_r) < 0$. The result follows from noting that

$$\Delta V_{\xi_L}(P, s_d) - \Delta V_{\xi_L}(P, s) = \mathbb{E}^{\xi_L}(s - s_d | m_r) \varphi(P) \leq 0. \square$$

Claim 1 implies that the only deviations that need to be considered are those to debt contracts. Indeed, for any security $s \in \mathcal{S}$, the *equivalent debt* security s_d minimizes the set of prices that would induce type ξ_L to deviate while keeping the set of prices for type ξ_H unchanged (since by construction, $\Delta V_{\xi_H}(P, s_d) = \Delta V_{\xi_H}(P, s)$). Under the D1 criterion, off-path beliefs at any security s, must assign all weight to the firm type with the largest set of prices consistent with a profitable deviation.[41] Claim 1 thus proves that, to show that all possible deviations can be attributed to type ξ_L, it is enough to restrict attention to debt contracts.

Consider an arbitrary debt contract $\tilde{s} = \min\left\{\theta_r, \tilde{d}\right\}$ with $\tilde{d} > 0$. For any $P \geq K$, we have that

$$\Delta V_\xi (P, \tilde{s}) = (P - K) R + \mathbb{E}^\xi (\boldsymbol{\theta}_r - \tilde{s}(\boldsymbol{\theta}_r) | m_r) > 0, \ \xi \in \Xi.$$

Next, we prove that $\Delta V_{\xi_H}(P, \tilde{s}) < 0$ for any $P < K$ satisfying $P \in \mathrm{BR}(\tilde{s})$. Define

$$\hat{P}^-(z) \equiv \min\left\{P \geq 0 : \frac{z}{R}\varphi(P) = P\right\}$$

to be the smallest price consistent with selling a security with expected cashflows z.[42] Note that

[41]To be precise, the set of relevant prices are those in $BR(s) = \left\{P \geq 0 : \frac{\mathbb{E}_H(s)}{R}\varphi(P) \geq P\right\}$. This set remains unchanged when considering the equivalent debt security s_d, by construction.

[42]Under assumption (2), for any $z > KR$, there exist exactly two solutions to the equation $\frac{z}{R}\varphi(P) = P$, $\bar{P}(z)$ and

$\Delta V_{\xi}(P,\tilde{s})$ is strictly increasing in P. This means that, to show that $\Delta V_{\xi_H}(P,\tilde{s})<0$ for any $P\in \mathrm{BR}(\tilde{s})\cap[0,K)$, it is enough to prove that $\Delta V_{\xi_H}(\sup \mathrm{BR}(\tilde{s})\cap[0,K),\tilde{s})<0$. Let $x\equiv \mathbb{E}^{\xi_H}(\tilde{s}(\boldsymbol{\theta}_r)|m_r)$ and observe that, under assumption (10), $\hat{P}^{-}(x)=\sup \mathrm{BR}(\tilde{s})\cap[0,K)$. Then, for any $P\leq \hat{P}^{-}(x)$,

$$\begin{aligned}
\frac{\Delta V_{\xi_H}(P,\tilde{s})}{R} &= \left(P-(d_1-1)+\left(\mathbb{E}^{\xi_H}(\boldsymbol{\theta}_r|m_r)-x\right)/R\right)\varphi(P)-(K-(d_1-1)) \\
&< \left(\hat{P}^{-}(x)-(d_1-1)+\left(\mathbb{E}^{\xi_H}(\boldsymbol{\theta}_r|m_r)-x\right)/R\right)\varphi\left(\hat{P}^{-}(x)\right)-(K-d_1+1) \\
&< (K-(d_1-1))\,\varphi(K)-(K-d_1+1) \\
&< 0,
\end{aligned}$$

where the first inequality follows from the monotonicity of $\Delta V_{\xi_H}(P,\tilde{s})$. The second inequality follows from the fact that, by definition,

$$x\varphi\left(\hat{P}^{-}(x)\right)/R=\hat{P}^{-}(x)<\hat{P}^{-}(x)\,\varphi\left(\hat{P}^{-}(x)\right),$$

and the assumption in (10). As a result, $[K,\infty)=\mathcal{D}(\theta_L|\tilde{s})=\mathcal{D}(\theta_H|\tilde{s})$ and, therefore, beliefs satisfying $\mu(\tilde{s})=1\{\theta=\theta_L\}$ are consistent with D1. This completes the proof that $s(\cdot)=\mathrm{Id}(\cdot)$ is an equilibrium of the fund-raising stage.□

Proof of Proposition 5.

Step 1. First, we prove that under the laissez-faire policy there exists an equilibrium of the fund-raising stage where both firm types pool over the debt contact $s_D\equiv\min\{y,D\}$, with D chosen so that $\mathbb{E}(s_D)/R=K$. At this equilibrium, the market keeps its prior belief about θ, μ_0, when observing security s_D and thus offers a payoff equal to K for s_D.

To see that this is an equilibrium, fix an arbitrary security $\tilde{s}$ and define $\Delta V_{\theta}(P|\tilde{s})$ as the differential payoff obtained by type θ by switching from security s_D to $\tilde{s}$ and receiving a price P for the latter. That is,

$$\Delta V_{\theta}(P|\tilde{s})\equiv(PR+\mathbb{E}_{\theta}(y-\tilde{s}(y)))\,\phi(P)-(KR+\mathbb{E}_{\theta}(y-s_D(y))),$$

where $\phi(P)=1-F^{\omega}(1-P)$. We show that beliefs that assign probability 1 to the type being θ_L are consistent with D1. Clearly, under such beliefs no firm type has incentive to deviate. The next claim reduces the set of deviations that need to be considered.

Claim 2. Fix an arbitrary security $s\in\mathcal{S}$, let $s_d\equiv\min\{y,d\}$ be such that $\mathbb{E}_H(s-s_d)=0$. Then, $\Delta V_L(P|s_d)\leq\Delta V_L(P|s)$.

Proof. By virtue of Lemma 3 and the definition of s_d, we have that $\mathbb{E}_L(s-s_d)<0$. The result follows from noting that

$$\Delta V_L(P|s_d)-\Delta V_L(P|s)=\mathbb{E}_L(s-s_d)\,\phi(P)\leq 0.\square$$

$\hat{P}^{-}(z)$, whereas for any $z\leq KR$, there exists only one solution at $P=\bar{P}(z)=\hat{P}^{-}(z)$.

Claim 2 implies that the only deviations that need to be considered are those to debt contracts. Indeed, for any security $s \in \mathcal{S}$, the *equivalent debt* security s_d minimizes the set of prices that would induce type θ_L to deviate while keeping the set of prices for type θ_H unchanged (since by construction, $\Delta V_H(P|s_d) = \Delta V_H(P|s)$). Under the D1 criterion, off-path beliefs at any security s, must assign all weight to the firm type with the largest set of prices consistent with a profitable deviation.[43] Claim 2 can then be used to show that, if for a given debt contract s_d we have that $\mathcal{D}(\theta_L|s_d) \cup \mathcal{D}^0(\theta_L|s_d) \supseteq \mathcal{D}(\theta_H|s_d)$, then we must necessarily have that

$$\mathcal{D}(\theta_L|s) \cup \mathcal{D}^0(\theta_L|s) \supseteq \mathcal{D}(\theta_L|s_d) \cup \mathcal{D}^0(\theta_L|s_d) \supseteq \mathcal{D}(\theta_H|s_d) = \mathcal{D}(\theta_H|s).$$

Claim 2 thus proves that, to show that all possible deviations can be attributed to type θ_L, it is enough to restrict attention to debt contracts.

Consider first deviations to debt contracts $\tilde{s} = \min\left\{y, \tilde{d}\right\}$ with $\tilde{d} > D$. In this case, for any $P \geq K$,

$$\Delta V_\theta(P|\tilde{s}) = (P - K)R - \mathbb{E}_\theta(\tilde{s} - s_D), \; \theta \in \{L, H\}.$$

The fact that $\tilde{s}$ is a debt contract implies that $\tilde{s} - s_D$ is nondecreasing and therefore FOSD (implied by MLRP) means that $\mathbb{E}_H(\tilde{s} - s_D) > \mathbb{E}_L(\tilde{s} - s_D) > 0$. As a result, there exists a price $\hat{P} > K$ for which

$$\Delta V_L\left(\hat{P}|\tilde{s}\right) > 0 > \Delta V_H\left(\hat{P}|\tilde{s}\right).$$

This implies that beliefs satisfying $\mu(\tilde{s}) = 1\{\theta = \theta_L\}$ are consistent with D1.

Now consider the case where $\tilde{s}$ is a debt contract with $\tilde{d} < K$. For any $P \geq K$, we have that

$$\Delta V_\theta(P|\tilde{s}) = (P - K)R + \mathbb{E}_\theta(s_D - \tilde{s}), \; \theta \in \{\theta_L, \theta_H\}.$$

That $\tilde{s}$ is a debt contract implies that $s_D - \tilde{s}$ is positive and nondecreasing. Thus, $\Delta V_\theta(P|\tilde{s}) > 0$ for all θ, and all $P \geq K$. Next, for any $P < K$,

$$\begin{aligned}
\Delta V_H(P|\tilde{s}) - \Delta V_L(P|\tilde{s}) &= (\mathbb{E}_H(y - \tilde{s}) - \mathbb{E}_L(y - \tilde{s}))\varphi(P) - (\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)) \\
&< (\mathbb{E}_H(y - \tilde{s}) - \mathbb{E}_L(y - \tilde{s}))\bar{\varphi} - (\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)) \\
&= \left(\frac{\mathbb{E}_H(y - \tilde{s}) - \mathbb{E}_L(y - \tilde{s})}{\mathbb{E}_H(y) - \mathbb{E}_L(y)} - 1\right)(\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)) \\
&< 0,
\end{aligned}$$

where the first inequality follows from assumption (c) in Condition 1. The second equality is by definition of $\bar{\varphi}$. The last inequality follows from noting that $\mathbb{E}_H(\tilde{s}) - \mathbb{E}_L(\tilde{s}) > 0$ since $\tilde{s}$ is monotone and signals are ordered according to MLRP. As a result, $\mathcal{D}(\theta_L|\tilde{s}) \supseteq \mathcal{D}(\theta_H|\tilde{s})$ and, therefore, beliefs satisfying $\mu(\tilde{s}) = 1\{\theta = \theta_L\}$ are consistent with D1. This completes the proof that s_D is an

[43] To be precise, the set of relevant prices are those in $BR(s) = \left\{P \geq 0 : \frac{\mathbb{E}_H(s)}{R}\phi(P) \geq P\right\}$ (see the equilibrium definition in the Appendix). This set remains unchanged when considering the equivalent debt security s_d, by construction.

equilibrium of the fund-raising stage.

Step 2. Next, we prove that, under the sequentially optimal LST Γ^{ω}, having both firm types pooling over the security s_D cannot be an equilibrium outcome. To show this, we prove that there exists a profitable deviation. In fact, consider the security $s_\epsilon = \min\{y, D-\epsilon\}$ with $\epsilon > 0$ small. Similarly to the analysis above, define $\Delta V_\theta^{\Gamma^\omega}(P|\tilde{s})$ as the differential payoff obtained by type θ when switching from security s_D to s_ϵ and receiving a price P, when the regulator runs the sequentially optimal LST Γ^ω. That is,

$$\Delta V_\theta^{\Gamma^\omega}(P|s_\epsilon) \equiv (PR + \mathbb{E}_\theta(y - s_\epsilon))\,\hat{\phi}(P) - (KR + \mathbb{E}_\theta(y - s_D)),$$

where $\hat{\phi}(P) = \mathbb{P}\{\omega \geq \bar{\omega}(P)\} = 1 - F^\omega(\bar{\omega}(P))$. For any $P \geq K$, we have that

$$\Delta V_\theta^{\Gamma^\omega}(P|s_\epsilon) = (P-K)\,R + \mathbb{E}_\theta(s_D - s_\epsilon),\ \theta \in \{\theta_L, \theta_H\}.$$

Thus, $\Delta V_\theta^{\Gamma^\omega}(P|s_\epsilon) > 0$ for any $P \geq K$, and any θ. Next, note that

$$\Delta V_H^{\Gamma^\omega}(K|s_\epsilon) - \Delta V_L^{\Gamma^\omega}(K|s_\epsilon) = \mathbb{E}_H(s_D - s_\epsilon) - \mathbb{E}_L(s_D - s_\epsilon) > 0,$$

as $s_D - s_\epsilon$ is nondecreasing. We prove that, under the assumptions in Condition 1, there exists a price $P_\epsilon < K$ satisfying that $\Delta V_H^{\Gamma^\omega}(P_\epsilon|s_\epsilon) > 0 > \Delta V_L^{\Gamma^\omega}(P_\epsilon|s_\epsilon)$.

To see this, let $\tilde{P}_\epsilon < K$ be defined as the unique solution to $\Delta V_H^{\Gamma^\omega}(P|s_\epsilon) = 0$. Note that the definition of $\bar{\omega}(\cdot)$ implies that $\lim_{P\to K^-} \hat{\phi}(P) = 1$ and, therefore, $\lim_{\epsilon\to 0^+} \tilde{P}_\epsilon = K$. Next, we rewrite $\Delta V_\theta^{\Gamma^\omega}\left(\tilde{P}_\epsilon|s_\epsilon\right)$ using the first-order Taylor expansion as

$$\Delta V_\theta^{\Gamma^\omega}\left(\tilde{P}_\epsilon|s_\epsilon\right) = \Delta V_\theta^{\Gamma^\omega}(K|s_\epsilon) + \partial_P^- \Delta V_\theta^{\Gamma^\omega}(K|s_\epsilon)\left(\tilde{P}_\epsilon - K\right) + o\left(\tilde{P}_\epsilon - K\right),$$

where $\partial_P^- \Delta V_\theta^{\Gamma^\omega}(K|s_\epsilon) \equiv \lim_{P\to K^-}\lim_{\delta\to 0^+} \frac{\Delta V_\theta^{\Gamma^\omega}(P|s_\epsilon) - \Delta V_\theta^{\Gamma^\omega}(P-\delta|s_\epsilon)}{\delta}$ represents the *left* derivative of $\Delta V_\theta^{\Gamma^\omega}(P|s_\epsilon)$ at K^-. Thus, we can express

$$\Delta V_L^{\Gamma^\omega}\left(\tilde{P}_\epsilon|s_\epsilon\right) = \Delta V_L^{\Gamma^\omega}(K|s_\epsilon) - \partial_P^- \Delta V_L^{\Gamma^\omega}(K|s_\epsilon)\underbrace{\frac{\Delta V_H^{\Gamma^\omega}(K|s_\epsilon) + o\left(\tilde{P}_\epsilon - K\right)}{\partial_P^- \Delta V_H^{\Gamma^\omega}(K|s_\epsilon)}}_{=K-\tilde{P}_\epsilon} + o\left(\tilde{P}_\epsilon - K\right). \quad (28)$$

Next, assumption (b) in Condition 1, together with the fact $\lim_{P\to K^-} \bar{\omega}(P) = 0$, imply that

$$\lim_{P\to K^-} \hat{\phi}'(P) = \lim_{P\to K^-} f^\omega(\bar{\omega}(P))\,\bar{\omega}'(P) = 0,$$

which in turn implies that

$$\frac{\partial_P^- \Delta V_L^{\Gamma^\omega}(K|s_\epsilon)}{\partial_P^- \Delta V_H^{\Gamma^\omega}(K|s_\epsilon)} = \lim_{P\to K^-} \frac{R\phi(P) + (PR + \mathbb{E}_L(y - s_\epsilon))\,\hat{\phi}'(P)}{R\phi(P) + (PR + \mathbb{E}_H(y - s_\epsilon))\,\hat{\phi}'(P)} = 1.$$

Thus, by choosing $\tilde{\epsilon}$ sufficiently close to 0, we obtain that $\Delta V_L^{\Gamma^\omega}\left(\tilde{P}_{\tilde{\epsilon}}|s_{\tilde{\epsilon}}\right) < 0 = \Delta V_H^{\Gamma^\omega}\left(\tilde{P}_{\tilde{\epsilon}}|s_{\tilde{\epsilon}}\right)$, which can be seen by taking the limit $\epsilon \downarrow 0$ in equation (28).

Finally, consider $\tilde{\tilde{\epsilon}}$ sufficiently small so that $\frac{\mathbb{E}_H\left(s_{\tilde{\tilde{\epsilon}}}\right)}{R} > K$. Note that assumption (a) in Condition 1 implies that $BR\left(s_{\tilde{\tilde{\epsilon}}}\right) = \left[0, \frac{\mathbb{E}_H\left(s_{\tilde{\tilde{\epsilon}}}\right)}{R}\right]$. By picking $\epsilon = \min\left\{\tilde{\epsilon}, \tilde{\tilde{\epsilon}}\right\}$ we then have that $\mathcal{D}\left(\theta_H|s_\epsilon\right) \supsetneq \mathcal{D}\left(\theta_L|\epsilon\right)$. As a consequence, beliefs consistent with D1 necessarily assign $\mu\left(s_\epsilon\right) = 1\left\{\theta = \theta_H\right\}$ and therefore such a deviation receives a price $P = \mathbb{E}_H\left(s_{\tilde{\tilde{\epsilon}}}\right)/R > K$ which leads both types to choose s_ϵ over s_D. This proves that s_D cannot be an equilibrium. The rest of the proof follows from results (1) and (2) in Proposition 7 which show that (i) any pooling contract always delivers a price weakly smaller than K, and that (ii) in any separating equilibrium, type H always raises less than K.[44] This concludes the proof of the proposition. □

Appendix D: General Model

Proof of Proposition 6.

The main difficulty of the proof is the fact that (12) may admit multiple solutions. We characterize the properties of the smallest of such solutions. Fix $\bar{\theta}_j > 0$ and define

$$a_j^{\#}\left(a_i;\bar{\theta}_j\right) \equiv \inf\left\{a_j : \bar{\theta}_j\varphi\left(a_i, a_j\right) - a_j \leq 0\right\},$$

whenever $\left\{a_j : \bar{\theta}_j\varphi_j\left(a_i, a_j\right) - a_j \leq 0\right\} \neq \emptyset$, and let $a_j^{\#}\left(a_i;\bar{\theta}_j\right)\equiv\bar{\theta}_j$ otherwise. In other words, $a_j^{\#}\left(a_i;\bar{\theta}_j\right)$ represents audience j investors' (smallest) best response to a_i and corresponds to the smallest solution to the equation $a_j = \bar{\theta}_j\mathbb{P}\left\{\boldsymbol{\omega} \geq d - a_i - a_j\right\}$ whenever it exists. I omit henceforth the dependence of $a_j^{\#}\left(a_i;\bar{\theta}_j\right)$ on $\bar{\theta}_j$ for brevity.

Claim 3. $a_j^{\#}\left(\cdot;\bar{\theta}_j\right)$ is strictly monotone and strictly convex for any $a_i \leq \hat{a}_i\left(\bar{\theta}_j\right)$, whereas $a_j^{\#}\left(a_i;\bar{\theta}_j\right) = \bar{\theta}_j$ for any $a_i > \hat{a}_i\left(\bar{\theta}_j\right)$.

Proof of Claim 3. Let

$$\Psi_j\left(a_i;\bar{\theta}_j\right) \equiv \min_{0\leq a_j\leq\bar{\theta}_j} \bar{\theta}_j\left(1 - F_\omega\left(d - a_i - a_j\right)\right) - a_j.$$

By assumption (3), we have that, for any $a_i > 0$, $\Psi_j\left(a_i;\bar{x}_j\right) > 0$ and $\lim_{\bar{\theta}_j\to0^+}\Psi_j\left(a_i;\bar{\theta}_j\right) < 0$. Further, the envelope theorem implies that Ψ_j is a monotone function. Let $\bar{\bar{\theta}}_j < \bar{x}_j$ be the highest value of $\bar{\theta}_j$ for which there exists a_i so that $\Psi_j\left(a_i;\bar{\theta}_j\right) \leq 0$. Consider the case where $\bar{\theta}_j < \bar{\bar{\theta}}_j$ and let $\hat{a}_i\left(\bar{\theta}_j\right)$ be implicitly defined by the equation $\Psi_j\left(\hat{a}_i\left(\bar{\theta}_j\right);\bar{\theta}_j\right) = 0$. Intuitively, for any $a_i \leq \hat{a}_i\left(\bar{\theta}_j\right)$, the set $\left\{a_j : \bar{\theta}_j\varphi_j\left(a_i, a_j\right) - a_j \leq 0\right\} \neq \emptyset$ (and therefore there exists at least one solution to the equation $a_j = \bar{\theta}_j\varphi\left(a_i, a_j\right)$). In turn, when $a_i > \hat{a}_i\left(\bar{\theta}_j\right)$, $\bar{\theta}_j\varphi\left(a_i, a_j\right) > a_j$ for all $a_j \leq \bar{\theta}_j$ and hence audience

[44] Note that the proof of Proposition 1 is general and works not only for the laissez faire policy but also under the sequentially rational ERP Γ^ω.

j investors' best response is given by $a_j^{\#}(a_i) = \bar{\theta}_j$. For the case where $\bar{\theta}_j \geq \bar{\bar{\theta}}_j$, we let $\hat{a}_i(\bar{\theta}_j) = 0$, and therefore for any $a_i > 0 = \hat{a}_i(\bar{\theta}_j)$, $a_j^{\#}(a_i) = \bar{\theta}_j$.

Suppose that $a_i < \hat{a}_i(\bar{\theta}_j)$ and therefore that $a_j^{\#}(a_i) < \bar{\theta}_j$. I first show that $a_j^{\#}(\cdot)$ is strictly monotone and strictly convex over this region. Indeed, for any $a_i \leq \hat{a}_i(\bar{\theta}_j)$, $a_j^{\#}(a_i)$ is the smallest solution to the equation $a_j = \bar{\theta}_j \varphi_j(a_i, a_j)$. Under assumption (2), $\varphi(\cdot,\cdot)$ is a convex function, and hence it is differentiable almost everywhere. The fact that F_ω admits a monotone density (by assumption (2)), further implies that $\varphi(\cdot,\cdot)$ is twice differentiable almost everywhere. We must then have that

$$\mathrm{d}^2_{a_i} a_j^{\#}(a_i) = \frac{-\bar{\theta}_j f'_\omega\left(d - a_i - a_j^{\#}\right)\left(1 + \mathrm{d}_{a_i} a_j^{\#}(a_i)\right)^2}{1 - \bar{\theta}_j f_\omega\left(d - a_i - a_j^{\#}\right)}, \tag{29}$$

where d_{a_i} and $\mathrm{d}^2_{a_i}$ represent the first and second derivative with respect to a_i, respectively. The convexity of $\bar{\theta}_j \varphi(a_i, a_j) - a_j$ in a_j, coupled with the facts that $\left(\bar{\theta}_j \varphi(a_i, a_j) - a_j\right)\big|_{a_j=0} > 0$ and that $a_i < \hat{a}_i(\bar{\theta}_j)$, jointly imply that the function $\bar{\theta}_j \varphi(a_i, a_j) - a_j$ crosses 0, for the first time, from positive to negative at $a_j^{\#}(a_i)$, and therefore must have a nonpositive slope (except for the case wherein $a_i = \hat{a}_i(\bar{\theta}_j)$ in which case $\bar{\theta}_j \varphi(\hat{a}_i(\bar{\theta}_j), a_j) - a_j$ is tangent at 0). Thus, $\bar{\theta}_j f_\omega\left(d - a_i - a_j^{\#}(a_i)\right) \leq 1$ with equality only for $a_i = \hat{a}_i(\bar{\theta}_j)$. From (29), we conclude that $a_j^{\#}(\cdot)$ is a convex function for any $a_i < \hat{a}_i(\bar{\theta}_j)$. This completes the proof of the claim. ■

Next, define

$$\Lambda_i\left(a_i; \vec{\theta}\right) \equiv \bar{\theta}_i \varphi\left(a_i, a_j^{\#}(a_i; \bar{\theta}_j)\right) - a_i.$$

Note, in particular, that $\Lambda\left(a_i; \vec{\theta}\right) = \bar{\theta}_i \varphi(a_i, \bar{\theta}_j) - a_i$ for any $a_i > \hat{a}_i(\bar{\theta}_j)$. We are interested in characterizing

$$a_i^\star\left(\vec{\theta}\right) = \inf\left\{a_i \geq 0 : \Lambda\left(a_i; \vec{\theta}\right) \leq 0\right\}.$$

Define

$$\bar{\theta}_i^{\#\#}(\bar{\theta}_j) \equiv \sup\left\{\bar{\theta}_i \geq 0 : \exists a_i \leq \hat{a}_i(\bar{\theta}_j) \text{ s.t.} \Lambda\left(a_i; \vec{\theta}\right) \leq 0\right\}.$$

The monotonicity of Λ_i in $\bar{\theta}_i$ implies that $\bar{\theta}_i^{\#\#}(\bar{\theta}_j)$ is well-defined.

Claim 4. $a_i^\star(\cdot, \bar{\theta}_j)$ is strictly monotone and strictly convex for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}(\bar{\theta}_j)$, whereas $a_i^\star(\bar{\theta}_i, \bar{\theta}_j) = \bar{\theta}_i$ for any $\bar{\theta}_i > \bar{\theta}_i^{\#\#}(\bar{\theta}_j)$.

Proof of Claim 4. Consider any $\bar{\theta}_i < \bar{\theta}_i^{\#\#}(\bar{\theta}_j)$. By definition of $\bar{\theta}_i^{\#\#}(\bar{\theta}_j)$, we must have that $a_i^\star\left(\vec{\theta}\right) < \hat{a}_i(\bar{\theta}_j)$. Furthermore, $a_i^\star\left(\vec{\theta}\right)$ is the point at which $\Lambda_i\left(a_i; \vec{\theta}\right)$ crosses 0 for the first time, and it does it from positive to negative. Thus, we have that

$$a_i^\star\left(\vec{\theta}\right) = \bar{\theta}_i \varphi\left(a_i^\star\left(\vec{\theta}\right), a_j^{\#}\left(a_i^\star\left(\vec{\theta}\right)\right)\right), \tag{30}$$

and, at the same time,

$$\bar{\theta}_i f_\omega\left(d - a_i^\star\left(\vec{\theta}\right) - a_j^{\#}\left(a_i^\star\left(\vec{\theta}\right)\right)\right)\left(1 + \mathrm{d}_{a_i} a_j^{\#}(a_i^\star)\right) \leq 1.$$

The monotonicity of $a_j^{\#}(a_i)$ then implies that

$$\bar{\theta}_i f_\omega \left(d - a_i^\star\left(\vec{\theta}\right) - a_j^{\#}\left(a_i^\star\left(\vec{\theta}\right)\right)\right) < 1. \tag{31}$$

Further, the monotonicity of Λ_i in $\bar{\theta}_i$, implies that $a_i^\star\left(\bar{\theta}_i, \bar{\theta}_j\right)$ is monotone in $\bar{\theta}_i$.

Next, we prove that for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$, $a_i^\star\left(\vec{\theta}\right)$ is strictly convex in $\bar{\theta}_i$. To see this, we differentiate (30) twice to obtain

$$\begin{aligned} \mathrm{d}^2_{\bar{\theta}_i} a_i^\star &= 2 f_\omega \left(d - a_i^\star - a_j^{\#}\left(a_i^\star\right)\right)\left(1 + \mathrm{d}_{a_i} a_j^{\#}\left(a_i^\star\right)\right)\mathrm{d}_{\bar{\theta}_i} a_i^\star\left(\vec{\theta}\right) \\ &\quad + \bar{\theta}_i f_\omega\left(d - a_i^\star - a_j^{\#}\left(a_i^\star\right)\right)\left(\mathrm{d}^2_{\bar{\theta}_i} a_i^\star\left(\vec{\theta}\right) + \mathrm{d}_{a_i} a_j^{\#}\left(a_i^\star\right)\cdot\left(\mathrm{d}_{\bar{\theta}_i} a_i^\star\left(\vec{\theta}\right)\right)^2\right) \\ &\quad - \bar{\theta}_i f'_\omega\left(d - a_i^\star - a_j^{\#}\left(a_i^\star\right)\right)\left(1 + \mathrm{d}_{a_i} a_j^{\#}\left(a_i^\star\right)\right)^2\left(\mathrm{d}_{\bar{\theta}_i} a_i^\star\left(\vec{\theta}\right)\right)^2, \end{aligned}$$

where $\mathrm{d}_{\bar{\theta}_i}$ and $\mathrm{d}^2_{\bar{\theta}_i}$ represent the first and second derivative with respect to $\bar{\theta}_i$, respectively. Using inequality (31) and assumption (2), we conclude that, for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$, $\mathrm{d}^2_{\bar{\theta}_i} a_i^\star > 0$.

Finally, we argue that for any $\bar{\theta}_i > \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$, $a_i^\star\left(\vec{\theta}\right) = \bar{\theta}_i$. Consider any $\bar{\theta}_i > \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$. By definition, $\hat{a}_i\left(\bar{\theta}_j\right)$ is *not* a function of $\bar{\theta}_i$ and further satisfies $\frac{\partial}{\partial a_i}\Psi_j\left(a_i; \bar{\theta}_j\right)\Big|_{a_i = \hat{a}_i\left(\bar{\theta}_j\right)} = 0$. This implies that $\bar{\theta}_j f_\omega\left(d - \hat{a}_i - a_j^{\#}\left(\hat{a}_i\right)\right) = 1$. Assumption (2) then implies that, if we define

$$\psi_j\left(a_i, a_j; \bar{\theta}_j\right) \equiv \bar{\theta}_j\left(1 - F_\omega\left(d - a_i - a_j\right)\right) - a_j,$$

then

$$\begin{aligned} \bar{\theta}_j - \left(d - \hat{a}_i\right) - \psi_j\left(\hat{a}_i, a_j^{\#}\left(\hat{a}_i\right); \bar{\theta}_j\right) &= \psi_j\left(\hat{a}_i, d - \hat{a}_i; \bar{\theta}_j\right) - \Psi_j\left(\hat{a}_i; \bar{\theta}_j\right) \\ &= \int_{a_j^{\#}\left(\hat{a}_i\right)}^{d - \hat{a}_i} \frac{\partial}{\partial a_j}\psi_j\left(\hat{a}_i, x; \bar{\theta}_j\right)\mathrm{d}x \\ &= \int_{a_j^{\#}\left(\hat{a}_i\right)}^{d - \hat{a}_i} \underbrace{\left(\bar{\theta}_j f_\omega\left(d - \hat{a}_i - x\right) - 1\right)}_{>0}\mathrm{d}x \\ &> 0. \end{aligned}$$

We conclude that, for any $\bar{\theta}_j$,

$$\hat{a}_i\left(\bar{\theta}_j\right) > d - \bar{\theta}_j + \Psi_j\left(\hat{a}_i; \bar{\theta}_j\right) \geq d - \bar{\theta}_j. \tag{32}$$

That $\bar{\theta}_i > \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$ implies that $\Lambda\left(\hat{a}_i; \vec{\theta}\right) > 0$. This means that $\bar{\theta}_i \geq \bar{\theta}_i \varphi\left(\hat{a}_i, a_j^{\#}\left(\hat{a}_i\right)\right) > \hat{a}_i$. Now, recall that, for any $a_i > \hat{a}_i\left(\bar{\theta}_j\right)$, $a_j^{\#}\left(a_i\right) = \bar{\theta}_j$. This fact coupled with inequality (32) jointly imply that, for any $a_i > \hat{a}_i\left(\bar{\theta}_j\right)$, $\Lambda_i\left(a_i; \vec{\theta}\right) = \bar{\theta}_i - a_i$. We conclude that for any $a_i \in \left(\hat{a}_i\left(\bar{\theta}_j\right), \bar{\theta}_i\right)$, $\Lambda_i\left(a_i; \vec{\theta}\right) > 0$. Thus, the first and only point at which Λ_i reaches 0 is at $a_i = \bar{\theta}_i$. We conclude that

$a_i^\star\left(\vec{\theta}\right) = \bar{\theta}_i$ for any $\vec{\theta}$ where $\bar{\theta}_i > \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$. This completes the proof of the claim. ■

Finally, we argue that, for any $\vec{\theta}$, $\left(a_i^\star, a_j^\#\left(a_i^\star\right)\right)$ corresponds to the smallest solution of (12) and therefore corresponds to our notion of equilibrium. That is, $\left(a_i^*(\vec{\theta}), a_j^*(\vec{\theta})\right) = \left(a_i^\star(\vec{\theta}), a_j^\#(a_i^\star(\vec{\theta}))\right)$. Indeed, the definition of $a_j^\#$ implies that, taking $a_i^\star \leq \hat{a}_i$ as given, $a_j^\#\left(a_i^\star\right)$ is the smallest solution to $\bar{\theta}_j\varphi\left(a_i^\star, a_j^\#\left(a_i^\star\right)\right) = a_j^\#\left(a_i^\star\right)$, implying both the optimality of audience j investors' action and the adversarial selection. Similarly, whenever $a_i^\star \leq \hat{a}_i$, we have $\bar{\theta}_i\varphi\left(a_i^\star, a_j^\#\left(a_i^\star\right)\right) = a_i^\star$. The convexity of $\bar{\theta}_i\varphi\left(\cdot, a_j^\#\left(a_i^\star\right)\right) - \cdot$, coupled with inequality (31) implies that $a_i^\star$ is the first crossing and hence also corresponds to the adversarial selection.

That $a_i^*(\vec{\theta})$ has the properties stated in the proposition follows directly from claim 4. The definition of $\bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$ implies that, for any $\bar{\theta}_i \geq \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$, $a_j^\#\left(a_i^\star\left(\bar{\theta}_i, \bar{\theta}_j\right)\right) = \bar{\theta}_j$. That $a_j^*(\vec{\theta}) = a_j^\#(a_i^\star(\vec{\theta}))$ is strictly monotone and strictly convex for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$ follows from combining claims 3 and 4. . This concludes the proof of the proposition. □

Proof of Theorem 2.

The proof is analogous to the proof of Theorem 1, and hence omitted.

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Internet Appendix (Not for Publication)

This document contains proofs and additional results for the manuscript "Persuading Multiple Audiences: An Information Design Approach to Banking Regulation". All numbered items (i.e., sections, subsections, lemmas, conditions, propositions, and equations) in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text. Please refer to the main text for notation and definitions. The notation and definitions are the same as in the main text.

Section S1: Omitted Proofs for Section 5.1

The solution to the regulator's problem is characterized by the binary-monotone policy $\Gamma_\star^\omega = (\{G,B\},\pi_\star^\omega)$, which satisfies $\pi_\star^\omega\{G|\omega\} = 1\{\omega > \bar{\omega}^{\mathrm{LST}}(P)\}$, where $\bar{\omega}^{\mathrm{LST}}(P)$ is the smallest liquidity cutoff such that when ST creditors learn that the liquidity is above the cutoff, it becomes dominant to rollover.[45] That is,

$$\bar{\omega}^{\mathrm{LST}}(P) \equiv \inf\left\{\tilde{\omega} \geq 0 : \mathbb{E}\left\{\Delta u\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, 1\right)|\boldsymbol{\omega} > \tilde{\omega}\right\} > 0\right\}. \tag{33}$$

Assume the firm raises P_{AM} during the fund-raising stage. For any announcement $m_\omega \in M_\omega$, let $F^{\omega|m}(\cdot|m_\omega)$ be the posterior cdf characterizing the beliefs about $\boldsymbol{\omega}$. Denote by $\mathbb{E}\left\{\Delta u\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, 1\right)|m_\omega\right\}$ the expected posterior utility of an ST creditor who observes the announcement m_ω and believes that all ST creditors will run on the firm. Under adversarial coordination, when ST creditors have homogeneous beliefs, the regulator's task reduces to convincing ST creditors that rolling over is a dominant strategy. That is, that their expected payoff from rolling over is positive, even if all other ST creditors run.[46]

Every score $\boldsymbol{m}^\omega = m^\omega$ generates a posterior expected adversarial utility (PEAU), $\mathbb{E}\left\{\Delta u\left(\vec{\boldsymbol{\vartheta}}, P_{\mathrm{AM}}, 1\right)|m_\omega\right\}$. Denote by G^{Γ^ω} the distribution of PEAU induced by stress test Γ^ω, and let $G_{\mathrm{FD}}^\omega(\cdot;P)$ be the distribution induced by the full-disclosure policy (i.e., the policy that follows the rule $\Gamma_{\mathrm{FD}}^\omega \equiv \{M^\omega = \Omega, \pi_{\mathrm{FD}}^\omega\}$, with $\pi_{\mathrm{FD}}^\omega(m^\omega|\omega) = 1\{m^\omega = \omega\}$).

The next proposition shows that the problem of finding the optimal stress test is equivalent to finding the distribution of posterior expected adversarial utilities that maximizes the mass assigned to the event $\{\omega : \mathbb{E}(u(\omega + P, 1)|m^\omega) > 0\}$. Intuitively, under adversarial coordination, when ST creditors have homogenous beliefs, the regulator's task reduces to convincing ST creditors that it is dominant to rollover. That is, that their expected payoff if they rollover the firm's debt is positive,

[45] Rigorously, the problem does not admit an optimal policy. If the regulator announces that $\omega > \bar{\omega}(P)$, then under adversarial coordination, all ST creditors run on the firm because $\mathbb{E}(u(\omega + P, 1)|\omega > \bar{\omega}(P)) = 0$. Nonetheless, the regulator can guarantee herself a payoff arbitrarily close to that induced by $\Gamma_\star^\omega$. With abuse of notation, I refer to $\Gamma_\star^\omega$ as the optimal policy.

[46] Inostroza and Pavan (2019) show, in an environment with heterogeneous beliefs, that the optimal disclosure perfectly coordinates ST creditors' actions. The current specifications capture the *perfect coordination property* while simplifying the intricacies of characterizing the optimal policy in the richer environment.

even if the rest of ST creditors choose to run on the firm.[47]

Proposition S1. *Fix $P \geq 0$. The stress test that maximizes the regulator's payoff which solves*

$$\max_{\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}} \quad \mathbb{E}\left(W_0\left(\bar{A}\left(P,m^{\omega}\right)\right)1_{\{\omega+P\geq\bar{A}(P,m^{\omega})\}}\right)$$
$$s.t: \quad \bar{A}\left(P,m^{\omega}\right)=1\left\{\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right)\leq 0\right\},$$

is characterized by the distribution of posterior expected adversarial utility, $G^{\Gamma^{\omega}}$, which among all mean preserving contractions of the full-disclosure distribution, G^{ω}_{FD}, maximizes $1-G^{\Gamma^{\omega}}(0)$. That is,

$$\max_{G^{\Gamma^{\omega}}} \quad 1-G^{\Gamma^{\omega}}(0)$$
$$s.t: \quad G^{\omega}_{FD}\succeq_{MPS} G^{\Gamma^{\omega}}.$$

Proof. Below I prove a sequence of lemmas that induce the result.

Lemma S1. *Fix the amount raised by the firm, $P \geq 0$. The problem of designing a stress test that maximizes the regulator's payoff :*

$$\max_{\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}} \quad \mathbb{E}\left(W_0\left(\bar{A}\left(P,m^{\omega}\right)\right)1_{\{\omega+P\geq\bar{A}(P,m^{\omega})\}}\right)$$
$$s.t: \quad \bar{A}\left(P,m^{\omega}\right)=1\left\{\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right)\leq 0\right\},$$

is equivalent to maximizing the probability that ST creditors find it dominant to rollover (i.e., maximizing $\mathbb{P}\left\{\mathbb{E}\left(u\left(\omega+P,1\right);\Gamma^{\omega}\right)>0\right\}$). The regulator's problem can thus be written as

$$\max_{\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}}\int_{\Omega\times M^{\omega}} 1\left\{\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right)>0\right\}\pi^{\omega}\left(dm^{\omega}|\omega\right)F^{\omega}(d\omega). \tag{34}$$

Proof. Consider an arbitrary stress test $\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}$. Assume that there exists some score $\bar{m}$ for which (i) $\bar{A}\left(P,\bar{m}\right)=1$, and (ii)

$$\mathbb{P}\left\{\omega:\omega+P\geq 1 \text{ and } \pi^{\omega}\left(\bar{m}|\omega\right)>0\right\}>0.$$

That is, score $\bar{m}$ induces all ST creditors to withdraw early and satisfies that the set of realizations of

[47] Inostroza and Pavan (2019) show, in an environment with heterogeneous beliefs, that the optimal disclosure policy perfectly coordinates ST creditors actions. The current model can be modified to accommodate heterogeneous beliefs. The specification in the current model retains the same qualitative properties as the former paper (e.g., perfect coordination of ST creditors) while simplifying the intrincancies of characterizing the optimal policy in the richer environment.

ω for which the firm *survives* even if all ST creditors withdraw early, has positive measure. Consider then the aAMernative policy $\hat{\Gamma}^\omega = \left\{\hat{\pi}^\omega, \hat{M}^\omega = M^\omega \cup \{\bar{m}_0, \bar{m}_1\}\right\}$ constructed as follows. For any $m \in M^\omega$ different from $\bar{m}$, $\hat{\pi}^\omega (m|\cdot) = \pi^\omega (m|\cdot)$. Additionally, $\hat{\pi}^\omega (\bar{m}_0|\omega) = \pi^\omega (\bar{m}|\omega) 1_{\{\omega+P\geq 1\}}$ and $\hat{\pi}^\omega (\bar{m}_1|\omega) = \pi^\omega (\bar{m}|\omega) 1_{\{\omega+P<1\}}$ for all $\omega \in \Omega$. Policy $\hat{\Gamma}^\omega$ improves the probability that the firm survives and decreases the number of ST creditors who run when observing message $\bar{m}_0$. Therefore, $\hat{\Gamma}^\omega$ weakly dominates Γ^ω. As a result, assuming that the optimal policy maximizes the probability that ST creditors refrain from attacking is without loss. $\square$

The next lemma shows that the distribution of posterior expected adversarial utilities induced by stress test Γ^ω, G^{Γ^ω}, corresponds to a mean-preserving contraction of the distribution associated with the full-disclosure policy $\Gamma^\omega_{\text{FD}}$, G^ω_{FD}, and a mean-preserving spread of the no-disclosure policy, $G^\omega_\emptyset$. That is, $G^\omega_{\text{FD}} \succeq_{\text{MPS}} G^{\Gamma^\omega} \succeq_{\text{MPS}} G^\omega_\emptyset$, where the partial order $\succeq_{\text{MPS}}$ is defined as follows.

Definition 3. Let F and G be distribution functions with support in $X \subseteq \mathbb{R}$. We say that F dominates G in the MPS order, $F \succeq_{\text{MPS}} G$, if $\int_X \varphi(x)F(dx) \geq \int_X \varphi(x)G(dx)$ for any convex function φ in X.

Lemma S2. [Blackwell] Let $\Gamma^\omega_1 = (M^\omega_1, \pi^\omega_1)$ and $\Gamma^\omega_2 = (M^\omega_2, \pi^\omega_2)$ be two stress tests. Assume that there exists $z : M^\omega_1 \times M^\omega_2 \to [0,1]$ such that:

(i) $\pi^\omega_2 (m_2|\omega) = \sum_{M^\omega_1} z(m_1, m_2)\, \pi^\omega_1 (m_1|\omega)\,, \quad \forall \omega \in [0,1], \forall m_2 \in M^\omega_2$

(ii) $\sum_{M^\omega_2} z(m_1, m_2) = 1, \quad \forall m_1 \in M^\omega_1$.

Then the distributions of posterior expected adversarial utility induced by Γ^ω_1 and Γ^ω_2 are such that $G^{\Gamma^\omega_1} \succeq_{\text{MPS}} G^{\Gamma^\omega_2}$.

Proof. Let $f^{m_i} \in \Delta[0,1]$ be the posterior pdf after observing message $m_i \in M^\omega_i$, and $\pi^\omega_i (m_i) = \int \pi^\omega_i (m_i|\omega)\, f^\omega(\omega) d\omega$ the total probability of observing disclosure m_i, under policy Γ^ω_i, $i \in \{1,2\}$. Observe that *bayesian updating* together with property (i) imply that, for any message $m_2 \in M^\omega_2$ with $\pi^\omega_2(m_2) > 0$, we have

$$f^{m^2}(\omega) = \sum_{m_1 \in M^\omega_1} \left(\frac{\pi^\omega_1 (m_1)\, z(m_1, m_2)}{\pi^\omega_2 (m_2)} \right) f^{m^1}(\omega).$$

This implies that, for any convex function φ,

$$\begin{aligned}
\sum_{m_2 \in M^\omega_2} \pi^\omega_2 (m_2)\, \varphi \left(\int_0^1 \omega f^{m_2}(\omega) d\omega \right) &= \sum_{m_2 \in M^\omega_2} \pi^\omega_2 (m_2)\, \varphi \left(\sum_{m_1 \in M^\omega_1} \left(\frac{\pi^\omega_1 (m_1)\, z(m_1, m_2)}{\pi^\omega_2 (m_2)} \right) \int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&\leq \sum_{m_2 \in M^\omega_2} \sum_{m_1 \in M^\omega_1} \pi^\omega_1 (m_1)\, z(m_1, m_2)\, \varphi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&= \sum_{m_1 \in M^\omega_1} \pi^\omega_1 (m_1)\, \varphi \left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right),
\end{aligned}$$

where the inequality obtains from Jensen's inequality and the last equality from property (ii). As a result, $G^{\Gamma_1^\omega} \succeq_{\mathrm{MPS}} G^{\Gamma_2^\omega}$. □

Lemma S2 shows that stress tests that are more informative in the Blackwell sense induce distributions of posterior expected adversarial utilities that dominate in the MPS order. As a result, $G^\omega_{\mathrm{FD}} \succeq_{MPS} G^{\Gamma^\omega} \succeq_{MPS} G^\omega_\emptyset$.

Consider then the problem of maximizing the likelihood that ST creditors keep pledging to the firm. Using lemmas S1 and S2, the policy-maker's problem can be reformulated as maximizing

$$\mathbb{P}\left\{\mathbb{E}\left(u\left(\omega+P,1\right);\Gamma^\omega\right)>0\right\} = 1 - G^{\Gamma^\omega}(0;P)$$

among all possible disclosure policies over ω. That is,

$$\begin{aligned} \max_{G^{\Gamma^\omega}} \quad & 1 - G^{\Gamma^\omega}(0) \\ \text{s.t:} \quad & G^\omega_{\mathrm{FD}} \succeq_{\mathrm{MPS}} G^{\Gamma^\omega}. \end{aligned}$$

This concludes the proof of Proposition S1. □

Next, for any stress test Γ^ω, and any amount $P \geq 0$ raised by the firm in period 1, define the *integral function* $\mathcal{G}^{\Gamma^\omega}(t;P) \equiv \int_{\tilde{u}=u(0,P,1)}^{t} G^{\Gamma^\omega}(\tilde{u};P)\,d\tilde{u}$. Let $\mathcal{G}^\omega_{\mathrm{FD}}$ and $\mathcal{G}^\omega_\emptyset$ be the integral functions associated with the full-disclosure policy, $\Gamma^\omega_{\mathrm{FD}}$, and no-disclosure policy, $\Gamma^\omega_\emptyset$, respectively. The set of feasible stress tests Γ^ω, coincides with the set of convex functions that lie between $\mathcal{G}^\omega_{\mathrm{FD}}$ and $\mathcal{G}^\omega_\emptyset$.

Lemma S3. *Consider an arbitrary stress test Γ^ω. Then, $\mathcal{G}^{\Gamma^\omega}(t;P)$ is convex and satisfies $\mathcal{G}^\omega_{FD}(t) \geq \mathcal{G}^{\Gamma^\omega}(t) \geq \mathcal{G}^\omega_\emptyset(t)$ for all $t \in [u(P,1), u(1+P,1)]$. Conversely, any convex function $h(\cdot)$, satisfying $\mathcal{G}^\omega_{FD}(t) \geq h(t) \geq \mathcal{G}^\omega_\emptyset(t)$ for all $t \in [u(0,P,1), u(1,P,1)]$ corresponds to the integral function of some disclosure policy Γ^ω.*

Proof. Under full-disclosure, each disclosure $m^\omega = \omega$ generates a degenerate posterior distribution with a mass of 1 at $u(\omega,P,1)$, which also coincides with the posterior expected adversarial utility induced by m^ω. As a result, $\mathcal{G}^\omega_{\mathrm{FD}}(t;P) = \int_{u(0,P,1)}^{t} G^\omega_{\mathrm{FD}}(\tilde{u};P)\,d\tilde{u}$, where

$$G^\omega_{\mathrm{FD}}(\tilde{u};P) = \int_{u(0,P,A=1)}^{\tilde{u}} \frac{f_\omega\left(u^{-1}(z;P,1)\right)}{\partial_\omega u\left(u^{-1}(z;P,1),\tau,1\right)}dz$$

corresponds to the distribution of $u(\omega,P,1)$ under full-disclosure. Next, notice that under no-disclosure, the posterior mean remains unchanged and equal to $\mathbb{E}\left(u\left(\omega+P,1\right)|\emptyset\right)$. Thus, $\mathcal{G}^\omega_\emptyset(t;P) = \int_{u(0,P,1)}^{t} 1\left\{\tilde{u} \geq \mathbb{E}\left(u\left(\omega,P,1\right)|\emptyset\right)\right\} d\tilde{u}$. To save on notation, hereafter we will omit the dependence on P of all disclosure policies and associated distributions.

Any disclosure policy Γ^ω induces an integral function $\mathcal{G}^{\Gamma^\omega}(t) \equiv \int_{u(0,P,1)}^{t} G^{\Gamma^\omega}(\tilde{u})\,d\tilde{u}$. That $G^\omega_{\mathrm{FD}} \succeq_{\mathrm{MPS}} G^{\Gamma^\omega} \succeq_{\mathrm{MPS}} G^\omega_\emptyset$ implies that $\mathcal{G}^\omega_{\mathrm{FD}}(t) \geq \mathcal{G}^{\Gamma^\omega}(t) \geq \mathcal{G}^\omega_\emptyset(t)$ for all $t \in [u(P,1), u(1+P,1)]$, which can be seen from applying the definition of $\succeq_{\mathrm{MPS}}$ to the convex function $\max\{\omega - t, 0\}$. Moreover, $\mathcal{G}^{\Gamma^\omega}$ is convex since G^{Γ^ω} is non-decreasing. Conversely, any non-decreasing, convex func-

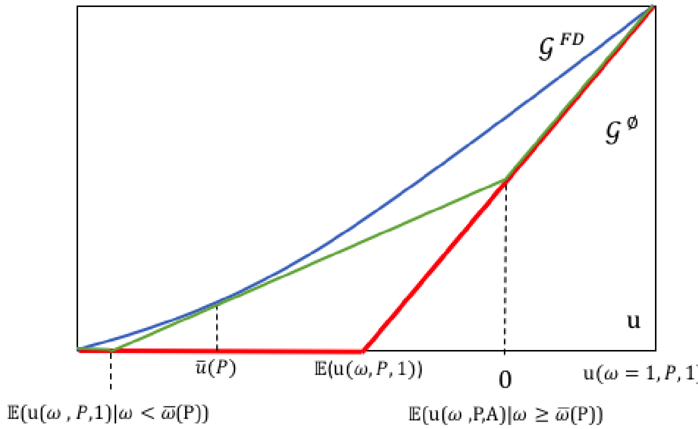

Figure 4: Optimal Stress Test.

tion h in $[u(P,1), u(1+P,1)]$, which satisfies that $\mathcal{G}^{\omega}_{\text{FD}}(t) \geq h(t) \geq \mathcal{G}^{\omega}_{\emptyset}(t)$ can be induced by some policy Γ^{ω}. To see this note that h is differentiable almost everywhere and its right derivative is always well-defined since it is convex. Let $G(\tilde{u}) \equiv h'(\tilde{u}^{+})$ be the right-derivative of h at $\tilde{u}$. Observe next that $\lim_{\tilde{u}\to\infty} G(\tilde{u}) = 1$, and thus G is a distribution. Finally, note that G^{ω}_{FD} is a mean-preserving spread of G and therefore there must exist a policy that induces it by Strassen's Theorem (See Theorem 1.5.20 in Müller and Stoyan (2002)). □

The regulator's problem is thus equivalent to finding the policy Γ^{ω} which generates the convex function $\mathcal{G}^{\Gamma^{\omega}}$, between $\mathcal{G}^{\omega}_{\emptyset}$ and $\mathcal{G}^{\omega}_{\text{FD}}$, with minimal slope at 0. As can be seen from Figure 4, the solution to the regulator's problem is thus given by the monotone-binary policy $\Gamma^{\omega}_{\star} = (\{0,1\}, \pi^{\omega}_{\star})$ that satisfies

$$\pi^{\omega}_{\star}(0|\omega) = 1\left\{u(\omega + P, 1) \geq \bar{u}(P)\right\} = 1\{\omega \geq \bar{\omega}\left(P\right)\},$$

where $\bar{u}(P)$ corresponds to the point at which $\mathcal{G}^{\omega}_{\text{FD}}$ is tangent to the the (convex) integral function with minimal slope to the left of 0. The value of $\bar{u}(P)$ can also be characterized by $\bar{u}(P) = u\left(\bar{\omega}\left(P\right) + P, 1\right)$, where $\bar{\omega}(P)$ represents the liquidity cutoff defined as[48]

$$\bar{\omega}(P) \equiv \inf\left\{\tilde{\omega} \geq 0 : \mathbb{E}\left(u\left(\omega + P, 1\right)|\omega \geq \tilde{\omega}\right) > 0\right\}. \tag{35}$$

Proposition S2 below summarizes these findings.

Proposition S2. *Fix the amount of capital $P \geq 0$. Then, the optimal stress test $\Gamma^{\omega}_{\star}[P]$ consists of a monotone pass-fail test with cutoff $\bar{\omega}(P)$, such that $\Gamma^{\omega}_{\star}(P) = (\{0,1\}, \pi^{\omega}_{\star}[P])$, with $\pi^{\omega}_{\star}(0|\omega; P) = 1_{\{\omega \geq \bar{\omega}(P)\}}$. The cutoff $\bar{\omega}(\cdot)$ is non-increasing with P.*

[48] To see this, note that the policy $\Gamma^{\omega}_{\star}$ induces a distribution of posterior means $G^{\Gamma^{\omega}_{\star}}$ which assigns positive probability to only two points, which coincide with the points at which $\mathcal{G}^{\Gamma^{\omega}_{\star}}$ changes slope. To see that the first point at which $\mathcal{G}^{\Gamma^{\omega}_{\star}}$ changes slope coincides with

$$\mathbb{E}\left(u\left(\omega + P, 1\right)|\omega < \bar{\omega}(P)\right),$$

note that the tangency condition implies that $G^{\Gamma^{\omega}_{\star}}(\bar{u}(P)) = G^{\omega}_{\text{FD}}(\bar{u}(P))$, where the RHS equals $F^{\omega}(\bar{\omega}(P))$.

Section S2: The case with $N > 2$ Audiences.

Consider the case with $N > 2$ audiences. The analysis in the main text imply that, at any equilibrium, investors' action depend on the prior F only through the vector of prior expectations $\mathbb{E}\left\{\vec{\theta}\right\}$ and are given by

$$\begin{aligned} a_i^*\left(\mathbb{E}\left\{\vec{\boldsymbol{\theta}}\right\}\right) &= \mathbb{E}\{\boldsymbol{\theta}_i\}\varphi\left(a_i^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\}), a_j^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\})\right) \\ &= \mathbb{E}\{\boldsymbol{\theta}_i\}\left(1 - F_\omega\left(d - a_i^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\}) - a_{-i}^*(\mathbb{E}\{\vec{\boldsymbol{\theta}}\})\right)\right), \ \forall i \in \{1, ..., N\} \end{aligned} \tag{36}$$

where $a_{-i}^*\left(\mathbb{E}\left\{\vec{\boldsymbol{\theta}}\right\}\right) \equiv \sum_{j \neq i} a_j^*\left(\mathbb{E}\left\{\vec{\boldsymbol{\theta}}\right\}\right)$.

7.1 Convexity and Stability

We show that, under adverse market conditions as captured by assumption (2), the complementarities between audiences lead to optimal actions which are convex in the expected fundamentals of the economy. Fix an audience $i \in \{1, ..., N\}$ and, to ease notation, let $\bar{\theta}_i \equiv \mathbb{E}\{\boldsymbol{\theta}_i\}$ and $\bar{\theta}_{-i} \equiv \mathbb{E}\{\boldsymbol{\theta}_{-i}\}$, $i \in \{1, ..., N\}$.

As in the baseline model, there may be multiple outcome profiles consistent with equilibrium play. Indeed, the system (36) may have multiple solutions. We restrict attention henceforth to stable equilibria (Dixit (1986)).

Definition 4. [Stability] The outcome profile $\vec{a} = (a_i, a_{-i})$ is a stable equilibrium of the game if it solves (36) and, in addition, satisfies[49]

$$\left(\sum_{i=1}^N \bar{\theta}_i\right) f_\omega\left(d - a_i - a_{-i}\right) < 1. \tag{37}$$

Proposition 8. *Suppose assumption 2) holds and that f_ω is continuous. Then, in any stable equilibrium, for any $i \in \{1, ..., N\}$, and any $\bar{\theta}_{-i}$, there exists $\bar{\theta}_i^{\#\#}\left(\bar{\theta}_{-i}\right) \leq \bar{x}_i$, such that (a) for any $\bar{\theta}_i \leq \bar{\theta}_i^{\#\#}\left(\bar{\theta}_{-i}\right)$, and any $j \neq i$, $a_j^*\left(\cdot, \bar{\theta}_{-i}\right)$ is both strictly increasing and strictly convex in $\bar{\theta}_i$, whereas (b) for any $\bar{\theta}_i > \bar{\theta}_i^{\#\#}\left(\bar{\theta}_j\right)$, $a_j^*\left(\bar{\theta}_i, \bar{\theta}_j\right) = \bar{\theta}_j$.*

[49] The assumption that $\vec{a}$ satisfies (37) implies inequality (36-v) in (Dixit (1986)). Indeed, define $\alpha_i \equiv \frac{\partial^2 U_i}{\partial a_i^2} = \bar{\theta}_i f_\omega\left(d - a_i - a_{-i}\right) - 1$ and $\beta_i \equiv \frac{\partial^2 U_i}{\partial a_{-i}\partial a_i} = \bar{\theta}_i f_\omega\left(d - a_i - a_{-i}\right)$. Then,

$$\begin{aligned} 0 < 1 + \sum_{i=1}^N \frac{\beta_i}{\alpha_i - \beta_i} &= \frac{\Pi_{i=1}^N\left(\alpha_i - \beta_i\right) + \sum_{i=1}^N \beta_i \Pi_{j \neq i}\left(\alpha_j - \beta_j\right)}{\Pi_{i=1}^N\left(\alpha_i - \beta_i\right)} \\ &= \frac{(-1)^N + (-1)^{N-1}\left(\sum_{i=1}^N \bar{\theta}_i\right) f_\omega\left(d - a_i - a_{-i}\right)}{(-1)^N} \\ &= 1 - \left(\sum_{i=1}^N \bar{\theta}_i\right) f_\omega\left(d - a_i - a_{-i}\right). \end{aligned}$$

Proof. Under assumptions (2), $\varphi(\cdot)$ is a convex function, and hence it is differentiable almost everywhere, for all $i \in \{1, ..., N\}$. We must then have

$$\begin{aligned}
\mathrm{d}_{\bar{\theta}_i} a_i^* &= \varphi\left(a_i^*, a_{-i}^*\right) + \bar{\theta}_i \left(\partial_i \varphi\left(a_i^*, a_{-i}^*\right) \mathrm{d}_{\bar{\theta}_i} a_i^* + \left\langle \partial_{-i} \varphi\left(a_i^*, a_{-i}^*\right), \mathrm{d}_{\bar{\theta}_i} a_{-i}^* \right\rangle\right), \\
&= \varphi\left(a_i^*, a_{-i}^*\right) + \bar{\theta}_i f_\omega\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right), \forall i \in \{1, ..., N\} \qquad (38)
\end{aligned}$$

where $\mathrm{d}_{\bar{\theta}_i}$ represents the derivative with respect to $\bar{\theta}_i$ (i.e., $\frac{\mathrm{d}}{\mathrm{d}\bar{\theta}_i}$), ∂_i the partial derivative against A_i (i.e., $\frac{\partial}{\partial A_i}$), ∇_{-i} is the vector of partial derivatives against A_{-i}, and $\langle \cdot, \cdot \rangle$ represents the inner product in $\mathbb{R}^{N-1}$. Similarly,

$$\begin{aligned}
\mathrm{d}_{\bar{\theta}_j} a_i^* &= \bar{\theta}_i \left(\partial_i \varphi\left(a_i^*, a_{-i}^*\right) \mathrm{d}_{\bar{\theta}_j} a_i^* + \left\langle \nabla_{-i} \varphi\left(a_i^*, a_{-i}^*\right), \mathrm{d}_{\bar{\theta}_j} a_{-i}^* \right\rangle\right), \\
&= \bar{\theta}_i f_\omega\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{k=1}^{N} \mathrm{d}_{\bar{\theta}_j} a_k^*\right) \qquad (39)
\end{aligned}$$

Using (38) and (39), we conclude that

$$\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^* = \frac{\varphi\left(a_i^*, a_{-i}^*\right)}{1 - \left(\sum_{i=1}^{N} \bar{\theta}_i\right) f_\omega\left(d - a_i - a_{-i}\right)} \geq 0,$$

with strict inequality whenever $a_i^* + a_{-i}^* < d$. The inequality follows from the fact that $\left(a_i^*, a_{-i}^*\right)$ is stable. Equalities (38) and (39) then imply that $a_i^*\left(\bar{\theta}_i, \bar{\theta}_{-i}\right)$ is nondecreasing in $\left(\bar{\theta}_i, \bar{\theta}_{-i}\right)$.

Next, differentiang (38) once more with respect to $\bar{\theta}_i$, we obtain that, for all $i \in \{1, ..., N\}$,

$$\begin{aligned}
\mathrm{d}_{\bar{\theta}_i}^2 a_i^* &= 2\left\langle \nabla\varphi, \mathrm{d}_{\bar{\theta}_i} \vec{a}^* \right\rangle + \bar{\theta}_i \left(\left\langle \nabla\varphi\left(a_i^*, a_j^*\right), \mathrm{d}_{\bar{\theta}_i}^2 \vec{a}^* \right\rangle + \left(\mathrm{d}_{\bar{\theta}_i} \vec{a}^*\right)^T \left(\mathrm{H}\varphi_i\right) \left(\mathrm{d}_{\bar{\theta}_i} \vec{a}^*\right)\right) \\
&= 2 f_\omega\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right) \\
&\quad + \bar{\theta}_i \left(f_\omega\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i}^2 a_j^*\right) - f_\omega'\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right)^2\right). \qquad (40)
\end{aligned}$$

where $\mathrm{d}_{\bar{\theta}_i}^2$ represents the secord-order derivative with respect to $\bar{\theta}_i$ (i.e., $\frac{\mathrm{d}^2}{\mathrm{d}\bar{\theta}_i^2}$). Similarly, for any $j \neq i$, we can show that,

$$\begin{aligned}
\mathrm{d}_{\bar{\theta}_i}^2 a_j^* &= \bar{\theta}_j \left(\left\langle \nabla\varphi\left(a_j^*, a_{-j}^*\right), \mathrm{d}_{\bar{\theta}_i}^2 a^* \right\rangle + \left(\mathrm{d}_{\bar{\theta}_i} \vec{a}^*\right)^T \left(\mathrm{H}\varphi\right) \left(\mathrm{d}_{\bar{\theta}_i} \vec{a}^*\right)\right). \\
&= \bar{\theta}_j \left(f_\omega\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i}^2 a_j^*\right) - f_\omega'\left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right)^2\right). \qquad (41)
\end{aligned}$$

Using (40) and (41), we obtain that

$$\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^* = \frac{2 f_\omega \left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right) - \left(\sum_{i=1}^{N} \bar{\theta}_i\right) f_\omega' \left(d - a_i^* - a_{-i}^*\right) \left(\sum_{j=1}^{N} \mathrm{d}_{\bar{\theta}_i} a_j^*\right)^2}{1 - \left(\sum_{i=1}^{N} \bar{\theta}_i\right) f_\omega \left(d - a_i - a_{-i}\right)} \geq 0,$$

with strict inequality whenever $a_i^* + a_{-i}^* < d$. The inequality obtains from assumption (2) and the fact that $\left(a_i^*, a_{-i}^*\right)$ is stable. The result then follows from the continuity of f_ω and the monotonicity of the optimal strategies. □