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Cournot Competition, Informational Feedback, and Real Efficiency*

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Abstract

We revisit the relationship between firm competition and real efficiency in a novel setting with informational feedback from financial markets. While intensified competition can decrease market concentration in production, it reduces the value of proprietary information (on, e.g., market prospects) for speculators and discourages information production and price discovery in financial markets. Therefore, competition generates non-monotonic welfare effects through two competing channels: market concentration and information production. When information reflected in stock prices is sufficiently valuable for production decisions, competition can harm both consumer welfare and real efficiency. Our results are robust under cross-asset trading and learning, overall underscoring the importance of considering the interaction between product market and financial market in antitrust policy, e.g., concerning the regulation of horizontal mergers.

JEL Classification: D61. D83. G14. G34. L40.

Keywords: Antitrust, Feedback Effects, Horizontal Mergers, Information Production, Market Efficiency, Product Competition.

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1 Introduction

The interaction and alignment between financial market efficiency and real efficiency constitute a long-standing topic in financial economics, as recently highlighted in studies on feedback effects (Goldstein et al., 2013; Goldstein and Yang, 2019; Goldstein, 2023). Unlike traditional theories on price formation (Grossman and Stiglitz, 1980; Hellwig, 1980; Glosten and Milgrom, 1985; Kyle, 1985), here the information flow is bi-directional: stock prices not only aggregate information from firms, but also contain new information effectively aggregated from traders, which real decision makers (e.g., managers) learn and use to improve the efficacy of their decisions (e.g., investments and production decisions).

We revisit the link between firm competition and real efficiency in the presence of such feedback effects associated with stock prices. We show that the interaction between the financial market and the product market can enhance or undermine the effectiveness of competition. In particular, contrary to conventional wisdom, the positive relationship between product competition and real efficiency can be reversed due to incentives for learning and information production in the financial market. Through a parsimonious model in which firms' production decisions are endogenous to stock trading because of the informational feedback from stock prices, we provide new insights into competition and antitrust regulation.

Specifically, we consider a group of identical firms, each supervised by a manager, competing in a standard Cournot setting. The production decision of each firm depends on the assessment of uncertain market prospects, which managers can learn from stock prices. Meanwhile, stock prices aggregate the costly private information acquired by speculators who are incentivized by potential trading profits in financial markets. Firm managers then use the information extracted from stock prices to guide production decisions, which in turn affects firm valuation. The reliance of production decisions on stock prices establishes the feedback effect of the financial market on the real economy.

It is well known that firm competition increases total welfare by reducing market power concentration when firms engage in Cournot competition, which justifies the validity of antitrust regulations related to M&As, for example. However, when these firms are publicly traded, a countervailing force arises: intensified competition can reduce the information content of stock prices and decrease real efficiency. Therefore, intensified competition could generate a loss in total welfare rather than gains. Intuitively, with informational feedback, intensified competition generates both direct and indirect effects on total welfare. The direct effect entails the welfare gain as competition intensifies, reminiscent of that in conventional Cournot competition; the indirect effect comes from managerial learning from stock prices that aggregate individual speculators' information. Because intensified competition generally curbs the incentive for speculators to produce information, this translates into reduced information acquisition and incorporation into real decisions. A negative relationship between product competition and total welfare ensues when the indirect effect is dominant.

Surprisingly, this negative relationship occurs only for an intermediate level of parameters over information production cost, price sensitivity, market prospects, etc. The optimal market structure (i.e., the number of competing firms that maximizes total welfare), as a function of these parameters, is also non-monotonic. Note that the negative link between competition and welfare depends on the relative gap in information production, rather than the absolute intensity, as competition intensifies. For example, when the information acquisition cost is high or low, information production either ceases or is in full scale, leading to a minimal change in information production when competition intensifies. Therefore, the market concentration channel dominates, and thus competition always improves total welfare. In contrast, for an intermediate level of information cost, welfare-reducing competition always arises in the sense that any market structure with the total number of competing firms exceeding an exogenous threshold becomes sub-optimal due to welfare loss related to deteriorated managerial learning alone.

We identify product profitability and market uncertainty as two key determinants of the relative strength of the aforementioned competing forces. Both factors can contribute to the direct effect of product competition, although the positive effect of market uncertainty is more nuanced. With fixed information production for each stock, an increase in the number of stocks reduces the probability that all order flows are uninformative. However, intensified competition decreases information production, which indirectly leads to a large loss of welfare when amplified by the uncertainty of market prospects. Thus, one would expect the indirect effect to be dominant with low product profitability and high market uncertainty.

We extend the discussion in three important directions. First, we consider horizontal mergers by comparing the total welfare of a monopoly with that of a duopoly. Interestingly, a monopoly can dominate a duopoly in total welfare for an intermediate level of information production cost. When information production is too cheap or too costly, there is a small gap in the amount of information produced, and thus a monopoly is unlikely to be dominant.

Second, we consider cross-asset trading in which some traders with large investment opportunities (L-traders, including hedge funds, as introduced in Goldstein et al., 2014) can trade all stocks and the rest (S-traders such as individuals and some mutual funds) with small investment opportunities can only trade one stock. With cross-asset trading, the expected trading profits of L-traders, as competition intensifies, will first increase and then decrease, exhibiting an inverted U-shape pattern. Thus, the incentive for L-traders to acquire information will reach its maximum for a moderate level of competition. This differs sharply from S-traders, for whom the incentive of information production is always maximized in a monopoly. However, a negative relationship between competition and total welfare can still arise with L-traders, since the incentive of information production for L-traders will drop quickly after achieving its maximum level.

Third, we consider cross-asset learning in which market makers can observe the order flows of all stocks, rather than a single stock. This gives market makers more information advantages, reducing trading profits for both the S-traders and the L-traders. Actually, this makes S-traders more prone to competition compared to L-traders. Meanwhile, S-traders have a weaker incentive to acquire information compared to L-traders, implying that L-traders may "crowd out" S-traders due to cross-asset trading opportunities/abilities. Interestingly, we find that a negative relationship between product competition and total welfare can arise when S-traders are not fully crowded out by L-traders, which is more likely to occur if the cost of information production is relatively small.

Our results have immediate implications for antitrust regulations in practice, where efficiency and welfare are the primary considerations. For example, regulators worry that M&A deals may substantially reduce competition and generate significant welfare costs by giving firms excessive market power to exploit other market participants and consumers (Guesnerie and Hart, 1985; Farrell and Shapiro, 1990; Landes and Posner, 1997). Typically, the primary antitrust concern arises with proposed horizontal mergers between direct competitors. In particular, Section 7 of the Clayton Act, amended by the Celler-Kefauver Act later, prohibits mergers and acquisitions when the effect "may be substantially to lessen competition or to tend to create a monopoly." Consequently, the US Department of Justice (DOJ) and the Federal Trade Commission (FTC) have developed the Horizontal Merger Guidelines, delineating key factors and analytical frameworks, as well as many specific examples of how these principles can be applied in actual merger reviews.[1]

However, an important element largely missing from existing antitrust rules is due con-

[1] See, e.g., `https://www.justice.gov/atr/horizontal-merger-guidelines-0`.

sideration of the interaction between (financial) market efficiency and real efficiency. When stock prices contain a dimension of information that managers do not have, excessive competition can diminish the return of proprietary information for speculators, suppressing information production. The informational feedback from stock prices to real decisions then gives rise to a counter-intuitive result: reduced competition would create a social welfare gain rather than a welfare loss, when the feedback effect from the financial market is sufficiently large. Therefore, antitrust regulatory bodies must take into account the interaction between the financial market and the real economy when reviewing M&As.

Literature. Our study adds to the literature on the feedback effects of financial markets on real efficiency. Early studies include Fishman and Hagerty (1989), Leland (1992), Dow and Gorton (1997), and Subrahmanyam and Titman (1999). As reviewed by Bond et al. (2012), and recently by Goldstein (2023), real decision makers (e.g., firm managers) can collect new information from stock prices to improve investments and production decisions (Foucault and Frésard, 2014; Edmans et al., 2015; Lin et al., 2019; Goldstein et al., 2013; Edmans et al., 2017; Goldstein and Yang, 2019). Central to this strand of literature is the alignment of market efficiency (i.e., the prediction power of stock prices for future cash flows) and real efficiency (i.e., the usefulness of stock prices for investment and production decisions). These two notions of efficiency typically diverge under feedback effects (Dow and Gorton, 1997; Bond et al., 2012). Bai et al. (2016) derive a welfare-based measure of price informativeness and find a revelatory component has contributed significantly to the efficiency of capital allocation since 1960. Goldstein and Yang (2019) reveal a stark difference between market efficiency and real efficiency by considering multiple dimensions of information, generating interesting insights for optimal design of disclosure systems.[2]

Our paper differs by focusing on the welfare implications of intensified competition on real efficiency. In our model, product competition can increase real efficiency by reducing firms' market power and decrease real efficiency by reducing information production by speculators. The two competing forces of reducing market concentration and reducing information production jointly determine the impact of product competition on social welfare.

A closely related study is Xiong and Yang (2021), which emphasizes the strategic information disclosure of firms. Our paper differs from theirs in the following three aspects,

[2]More literature focusing on optimal disclosures include: Chen et al. (2021); Edmans et al. (2015); Boleslavsky et al. (2017); Gao and Liang (2013) and Jayaraman and Wu (2019).

including: First, in their model, competition reduces firms' voluntary disclosure, ultimately leading to a decrease in economic efficiency. In contrast, we stress the role of information production by speculators and show that this mechanism alone can generate a negative relationship between competition and total welfare. Second, their analysis mainly compares a monopoly product market with a perfect competition market, whereas we consider any arbitrary number of firms and characterize general conditions under which competition decreases total welfare. Third, speculators no longer exogenously possess private information, but instead endogenously choose whether to become informed in our model.[3] Huang and Xu (2023) also explore the secondary market and product market competition, but focus on how initial stock holdings affect arbitrageurs' buying and thus entry decisions of potential uninformed entrants through feedback effects. More broadly, our paper relates to the macroeconomics of information and production. For example, Angeletos et al. (2023) show that the two-way feedback between startup activity and investors beliefs can generate excessive and non-fundamental influences on firm activities and asset prices.

Our study is also related to the long-standing literature investigating the relationship between competition and economic efficiency and its implications for antitrust regulations. Dating back to Smith (1776) and Cournot (1838), the traditional wisdom — the existence of market power can generate market inefficiencies and reduce welfare by raising price and suppressing output — has greatly influenced the evolution of the Horizontal Merger Guidelines (Nocke and Whinston, 2022).[4] On the one hand, the unilateral effect analysis emphasizes the trade-off between post-merger market power and potential synergies (see, e.g., Williamson, 1968; Farrell and Shapiro, 1990; Nocke and Whinston, 2022).[5] On the other hand, the coordinated effect analysis concerns implicit anti-competitive coordination from mergers in the absence of explicit communication (see, e.g., Compte et al., 2002; Miller and Weinberg, 2017; Porter, 2020). Röller et al. (2001) and Asker and Nocke (2021) offer comprehensive surveys of this vast literature before 2001 and more recent developments, respectively. In addition, Peress (2010) analyzes how product market competition influences stock price informative-

[3]More precisely, Xiong and Yang (2021) also consider endogenous information acquisition by speculators in their Section 5.3. A key difference is that when the number of firms increases, information acquisition decreases in the extensive margin in our paper, while Xiong and Yang (2021) document a different pattern in which the extensive margin of information acquisition increases while the intensive margin decreases. This further suggests that this insight is robust to different ways of modeling information acquisition.

[4]The Horizontal Merger Guidelines feature two key considerations: unilateral price effects and coordinated effects. Other concerns include pro-competitive forces such as market entry and dynamic considerations (see, e.g., Mermelstein et al., 2020; Nocke and Whinston, 2010).

[5]Recently, a growing literature evaluates"merger simulations" to quantify unilateral price effects and welfare impacts (Werden and Froeb, 1994; Weinberg, 2011; Björnerstedt and Verboven, 2016; Nevo, 2000).

ness, which in turn affects capital allocation. We examine not only the potential negative impact of firm competition on price informativeness but also the informational feedback from stock prices to production decisions, with novel welfare and policy implications.

Several recent studies explore direct evidence for merger-specific efficiency (Ashenfelter et al., 2015; Braguinsky et al., 2015), and characterize what counts as an efficiency (Hemphill and Rose, 2017; Geurts and Van Biesebroeck, 2019). Covarrubias et al. (2020) identify good and bad concentrations at the aggregate and industry level in the United States over the past three decades. Our paper contributes to the discussion of positive merger-specific efficiencies by exploring a new channel through feedback effects between the product market and the financial market. Two other related papers, Edmans et al. (2012) and Luo (2005), similarly explore the feedback effect in mergers and acquisitions. Both emphasize how learning by insiders from outsiders' information affects the decision for M&As but do not focus on the link between competition and efficiency as we do.

The remainder of the paper is organized as follows: Section 2 sets up the model. Section 3 characterizes the equilibrium. Section 4 revisits the relationship between production competition and real efficiency in the presence of feedback effects. Section 5 extends the baseline model to consider the implications for M&As and discuss model robustness. Finally, Section 6 concludes. All proofs are relegated to the appendix.

2 Model Setup

We embed the feedback effect of stock prices on product decisions under market competition into an otherwise standard Cournot model. Consider $n \geq 2$ identical firms competing in production quantity, and each firm's equity is traded on a public stock exchange. Time is discrete and indexed by $t \in \{0, 1, 2\}$. At $t = 0$, a group of speculators decide whether to acquire private information on the market prospects of the product and subsequently decide how to trade stocks.[6] At $t = 1$, the manager of each firm makes a production decision, taking into account the production strategies of other firms and the trading on the stock exchange at $t = 0$. Finally, at $t = 2$, the cash flows for all firms are realized. The key departure from the Cournot model is that managers in our setting can learn and use information contained in stock prices for their production decisions.

[6] We follow the literature by assuming that speculators only acquire information once (See, e.g., Gao and Liang, 2013; Goldstein et al., 2014; Dow et al., 2017; Xiong and Yang, 2021).

The product market. Let q_i denote the output level of the ith firm at time $t = 1$, where $i \in \{1, \ldots, n\}$.[7] Denote the total supply of the product by $Q = \sum_{i=1}^{n} q_i = q_i + q_{-i}$, where $-i$ denotes all other firms. As in Xiong and Yang (2021), the market clearing price P is given by: $P = A - bQ$. Here, $b > 0$ indicates the sensitivity of demand to price and $A > 0$ captures the possible market prospect of the product. Depending on a relevant economic state $\omega \in \{H, L\}$, the realization of the market prospect is given by $A(\omega) = A_\omega$, where $A_H > A_L > 0$. Both states are equally likely ex ante, i.e., $\Pr(\omega = H) = \Pr(\omega = L) = 1/2$. Given the production decisions $\{q_i\}_{1 \le i \le n}$, the ith firm receives an operating profit given by:

$$TP_i(q_i) = q_i(A - bQ - MC), \tag{1}$$

where MC is a constant marginal production cost. Without loss of generality, we assume that $A_H > A_L \geq MC$. To highlight the core mechanism, we leave out financing constraints.

All firms decide simultaneously on the production level q_i at time $t = 1$. Each firm manager maximizes the expected value of the firm after the stock prices are observed. In other words, conditional on the information observed, $\mathcal{F}_m$, at $t = 1$, the firm manager chooses the output level q_i to maximize:

$$V_i(q_i) = \mathbb{E}[TP_i(q_i) \mid \mathcal{F}_m]. \tag{2}$$

The stock market. All firms are publicly traded by three types of investors: (i) a continuum of risk-neutral speculators who can choose to acquire costly information; (ii) a group of liquidity traders for each firm $i \in \{1, \cdots, n\}$, who jointly submit an aggregate order $z_i \sim U([-1, 1])$, independently and uniformly distributed over $[-1, 1]$ across the firm identity i; and (iii) a set of risk-neutral market makers. The free entry of market makers implies that each makes zero profit in equilibrium.

For each firm i, let $\alpha_i \in [0, 1]$ denote the size of speculators acquiring costly information at $t = 0$ as in Foucault and Frésard (2014). To endogenously determine the amount α_i of

[7] We focus on Cournot competition (i.e., quantity competition), rather than Bertrand price competition, for the following two reasons. First, in canonical Bertrand competition, the total welfare is independent of the total number of competing firms. Second, as shown in Kreps and Scheinkman (1983), the quantity (capacity) pre-commitment and the Bertrand price competition yield Cournot outcomes. In addition, we anticipate that Bertrand competition can weaken our result even with differentiated products. For example, Vives (1985) shows that prices and profits are generally higher and quantities are lower in Cournot competition than in Bertrand competition. Therefore, Bertrand competition can enhance the effect of market concentration, reducing the relative importance of information feedback.

informed speculators, we assume that each speculator k must pay a cost $c > 0$ to become informed, i.e., receiving an informative signal $m_k^i \in \{H, L\}$.[8] With precision $\theta > \frac{1}{2}$, the signal structure is given by:

$$\Pr\left(m_k^i = H | \omega = H\right) = \Pr\left(m_k^i = L | \omega = L\right) = \theta. \tag{3}$$

Conditional on the realization of ω, m_k^i is independently and identically distributed across speculators (as in Goldstein et al., 2013; Dow et al., 2017). Upon observing the signal m_k^i, the kth informed speculator can choose to trade x_k^i shares of the ith firm, where $x_k^i \in [-1, 1]$ as in Dow et al. (2017). Thus, the aggregate demand for the ith stock from speculators is given by: $x_i = \int_0^{\alpha_i} x_k^i dk$. Recall that all liquidity traders submit an aggregate order z_i that is uniformly distributed. The total order flow f_i for the ith stock is: $f_i = z_i + x_i$.

As in Kyle (1985), the order flow f_i in each stock i is absorbed by market makers, and the stock price s_i reflects the expected value of the firm conditional on the total order flow:

$$s_i(f_i) = \mathbb{E}\left[V_i \mid f_i\right]. \tag{4}$$

Equilibrium definition. The equilibrium concept that we use is perfect Bayesian equilibrium, which consists of: (i) a production strategy for each manager that maximizes the expected firm value given the information conveyed in stock prices; (ii) an information production strategy and a trading strategy for speculators that maximize the expected trading profit given all others' strategies; (iii) a price-setting strategy for market makers that allows them to break even in expectation given all others' strategies; (iv) managers and market makers update their beliefs about the economic state according to the Bayes rule; and (v) each player's belief about other players' strategies is correct in equilibrium.

3 Equilibrium Characterization

We solve the model backward. We first derive the equilibrium strategy at $t = 1$, taking as a given the amount α_i of informed speculators for each firm i, and then we endogenize α_i. As shown later, an informed speculator k with a private signal m_k^i always buys one share of the stock of the ith firm when $m_k^i = H$, and sells one share when $m_k^i = L$. Given this

[8] The superscript "i" in m_k^i is used to indicate that the kth speculator is trading the ith stock.

observation, we can now investigate the production strategies of firms and the pricing rules for stocks in equilibrium.

Let us first consider the limit where the information acquisition cost c is sufficiently high that all speculators abstain from acquiring information. When this occurs, the stock price is uninformative and the market outcome reduces to the standard Cournot competition outcome with n identical firms. Therefore, each firm produces an identical output:

$$q_M = \frac{\bar{A} - MC}{(n+1)b}, \tag{5}$$

where $\bar{A} = \frac{1}{2}(A_H + A_L)$.

This can be compared with the market outcome when the actual market prospect $A(\omega)$ is publicly known to all market participants. Specifically, when $A(\omega) = A_H$, each firm produces a quantity of $q_H = \frac{A_H - MC}{(n+1)b}$, making a profit of $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$. Similarly, when $A(\omega) = A_L$, each firm produces $q_L = \frac{A_L - MC}{(n+1)b}$, making a profit of $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$. In contrast, in the absence of information produced by speculators, the equilibrium output q_M under uncertainty is just the expectation of outputs in both states, i.e., $q_M = \frac{1}{2}(q_H + q_L)$.

Next, we consider the case of informative stock trading. Intuitively, due to information-based speculative trading, stock prices contain useful information for managers to guide production decisions. Thus, to solve for the production strategy with informational feedback effects, we need to analyze stock pricing rules in equilibrium. Following Kyle (1985), market makers set stock prices based on the updated belief about the value of firms, given the total order flow observed. Given the information structure in Equation (3), by the law of large numbers (Dow et al., 2017), the aggregate order of informed speculators is $x_i = \alpha_i(2\theta - 1)$ when $\omega = H$, generating a total order flow of $f_i = \alpha_i(2\theta - 1) + z_i$. Similarly, if $\omega = L$, then: $f_i = -\alpha_i(2\theta - 1) + z_i$.

In summary, market makers condition the stock price on the observed total order flow, which aggregates the information from the trading activities of informed speculators. Therefore, the stock price contains valuable information for managers, which establishes an information feedback channel to the real economy. As shown in Lemma 1, the optimal production strategies of firms explicitly depend on stock prices.

Lemma 1. *Given the measures of informed speculators $\{\alpha_i\}_{1 \le i \le n}$, the equilibrium stock price* *for the ith firm is given by:*

$$s_i\left(f_i\right)=\begin{cases} s_H, & \text{if } f_i>\gamma_i \\ s_M^i, & \text{if } -\gamma_i\le f_i\le\gamma_i \\ s_L, & \text{if } f_i<-\gamma_i \end{cases}, \tag{6}$$

where $s_H=\frac{(A_H-MC)^2}{(n+1)^2b}$, $s_M^i=\frac{1}{4(n+1)^2b}\left\{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\beta_i\left(A_H-A_L\right)^2\right\}$, $s_L=\frac{(A_L-MC)^2}{(n+1)^2b}$, $\gamma_i=1-\alpha_i(2\theta-1)$, *and* $\beta_i=\prod_{j\neq i}\gamma_j$.

Furthermore, given all stock prices $\{s_i\}_{1\le i\le n}$, *the ith firm produces an output of:*

$$q_i^*=\begin{cases} q_H, & \text{if } s_j=s_H \text{ for some } j \\ q_M, & \text{if } s_j=s_M^j \text{ for all } j \\ q_L, & \text{if } s_j=s_L \text{ for some } j \end{cases}, \tag{7}$$

where $q_H=\frac{A_H-MC}{(n+1)b}$, $q_L=\frac{A_L-MC}{(n+1)b}$, *and* q_M *is given by Equation (5).*

We make three comments on Lemma 1. First, the three conditions in Equation (6), as well as those in Equation (7), are mutually exclusive, which rules out the possibility of observing both $s_i=s_H$ and $s_j=s_L$ for some $i\neq j$.[9] Thus, the optimal production strategy q_i^* is well defined. Second, we can directly verify that $s_H>s_M^i>s_L$, which implies that the equilibrium stock price s_i increases weakly in the total order flow f_i. This result is consistent with those of the existing literature on feedback effects (Foucault and Frésard, 2014; Dow et al., 2017; Lin et al., 2019). Third, managers choose equilibrium output levels based on observed stock prices. Obviously, $q_H>q_M>q_L$, which implies that q_i^* generally tends to increase with stock prices.

We now proceed to analyze the optimal behavior of speculators in equilibrium. Specifically, we first derive the optimal trading strategy of an informed speculator and then calculate the resulting expected trading profits, which are summarized in Lemma 2 below.

Lemma 2. *For speculators focusing on the ith stock, the optimal trading strategy is to long one share (i.e.,* $x_k^i=+1$*) when* $m_k^i=H$ *and short one share (i.e.,* $x_k^i=-1$*) when* $m_k^i=L$*. The resulting expected trading profit is:*

$$\Pi_i(\boldsymbol{\alpha})=\frac{\gamma_i(2\theta-1)\left(2+(n-1)\beta_i\right)}{2(n+1)^2b}\left(\bar{A}-MC\right)\left(A_H-A_L\right).$$

[9] To see this, given that $s_i=s_H$, the state consistent with the order flow of noise trading can only admit $\omega=H$, contradicting $s_j=s_L$ which fully reveals that $\omega=L$.

Lemma 2 verifies the intuition that an informed speculator always follows his own signal, i.e., he longs the stock after receiving good news and shorts it after bad news. Also note that $\Pi_i(\boldsymbol{\alpha})$ depends on all $\{\alpha_i\}_{1\leq i\leq n}$ through γ_i and β_i. Furthermore, the expected trading profit $\Pi_i(\boldsymbol{\alpha})$ strictly increases both in the average profitability, as measured by $\left(\bar{A}-MC\right)$, and in the uncertainty about the market prospects, as measured by $(A_H - A_L)$.

Finally, Lemma 2 is an important intermediate step in understanding the incentive for information production. Specifically, when acquiring costly information on market prospects, an uninformed speculator balances between the cost of information production $c > 0$ and the value of proprietary information $\Pi_i(\boldsymbol{\alpha})$. Since all firms are identical in the Cournot competition, we hereafter focus on the symmetric case $\alpha_i = \alpha$ ($\forall\ 1 \leq i \leq n$) and define:

$$\Pi(\alpha) := \Pi_i(\boldsymbol{\alpha}) = \frac{\gamma(2\theta-1)\left(2+(n-1)\gamma^{n-1}\right)}{2(n+1)^2 b}\left(\bar{A}-MC\right)(A_H - A_L)\,, \tag{8}$$

where $\gamma = 1-\alpha(2\theta-1)$.

Note that $\Pi(\alpha)$ in Equation (8) strictly decreases in α, i.e., $\frac{\partial\Pi(\alpha)}{\partial\alpha} < 0$. Thus, the value of private information decreases when more agents choose to do so, implying that information acquisition is a strategic substitute among speculators.

Intuitively, when the cost of information acquisition is large enough such that $\Pi(0) \leq c$, no speculator has an incentive to acquire education. However, when the cost parameter is sufficiently small such that $c \leq \Pi(1)$, all speculators choose to acquire information. Together, these two conditions establish two cut-off points, including an upper bound $\overline{c} = \Pi(0)$ and a lower bound $\underline{c} = \Pi(1)$. Specifically, we define:

$$\overline{c}_n = \frac{(2\theta-1)}{2(n+1)b}\left(\bar{A}-MC\right)(A_H - A_L) \tag{9}$$

and

$$\underline{c}_n = \frac{(2\theta-1)(1-\theta)\left(2+(n-1)(2-2\theta)^{n-1}\right)}{(n+1)^2 b}\left(\bar{A}-MC\right)(A_H - A_L) \tag{10}$$

Let $\widehat{\alpha}$ denote the optimal intensity of information acquisition.

Lemma 3 (Information Acquisition Intensity)**.**

(i) When $c \in [\overline{c}_n, \infty)$ (i.e., a high cost for information acquisition), $\widehat{\alpha} = 0$;

(ii) When $c \in [0, \underline{c}_n]$ (i.e., a small cost for information acquisition), $\widehat{\alpha} = 1$; and

(iii) When $c \in (\underline{c}_n, \overline{c}_n)$, a unique interior solution $\widehat{\alpha} \in (0,1)$ exists and satisfies $\Pi(\widehat{\alpha}) = c$.

Two comments are in order. When $\Pi'(\widehat{\alpha}) < 0$, an interior solution $\widehat{\alpha}$ is said to be locally stable because when we start with $\alpha < \widehat{\alpha}$, more speculators find it optimal to acquire information, increasing the intensity of information acquisition and vice versa. Moreover, the incentive to acquire and trade on private information is negatively associated with the cost of information production.[10] A sufficiently large cost preempts the incentive to acquire information, and thus the informational feedback effect disappears. In general, the information content of stock prices depends on the amount of informed speculators in the stock market, which is pinned down uniquely by the information cost and other model parameters.

4 Competition and Efficiency Under Feedback Effects

We now establish that product competition can decrease the incentive for speculators to produce information, and then analyze the efficiency implications of firm competition with informational feedback from stock prices. Interestingly, under certain conditions, Cournot competition can generate negative welfare effects in the presence of feedback effects.

4.1 Information Production

We first analyze how information production, measured by the equilibrium size of informed speculators $\widehat{\alpha}_n := \widehat{\alpha}(n)$, varies with the number of firms n in the product market. For simplicity, we focus on the interior solution case; otherwise, we expect that $\partial\widehat{\alpha}_n/\partial n = 0$ under corner solutions. Then, we rewrite the equilibrium condition as:

$$\Pi(\widehat{\alpha}) = \Pi(n, \widehat{\alpha}_n) = c. \tag{11}$$

A direct application of the implicit function theorem implies the following:

Proposition 1 (Information Production). *When an interior solution $\widehat{\alpha}_n \in (0, 1)$ exists $c \in (\underline{c}, \overline{c})$, $\widehat{\alpha}_n$ strictly decreases in n, i.e., $\frac{\partial \widehat{\alpha}_n}{\partial n} < 0$.*

Proposition 1 verifies that the amount $\widehat{\alpha}_n$ of informed speculators decreases as competition intensifies due to reduced incentives to acquire information. This result is consistent with empirical evidence in Farboodi et al. (2022) in which investors have relatively more data on large firms than on small ones because the incentive for speculators to produce information decreases with fiercer competition, reducing both firm profitability and size.

[10] The equilibrium on information acquisition is reminiscent of that in Grossman and Stiglitz (1980).

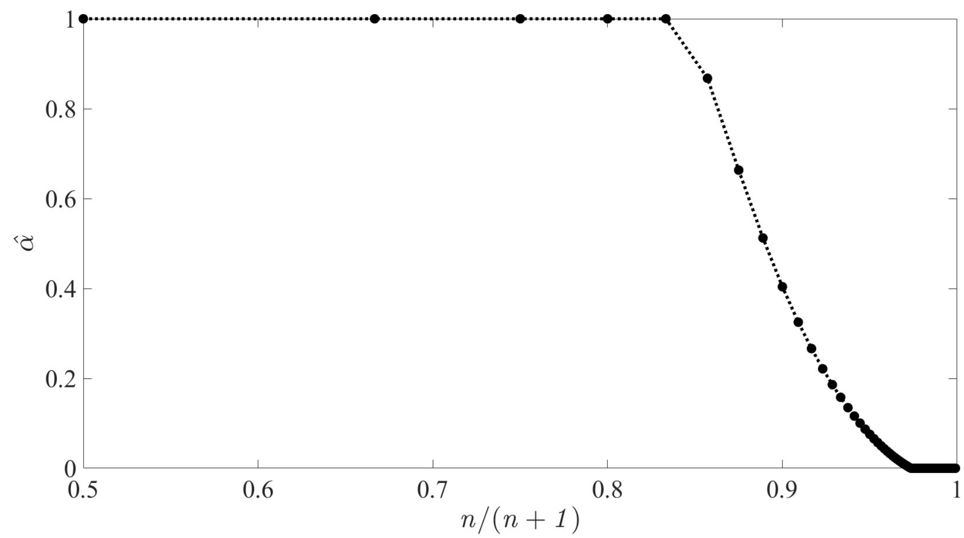

Figure 1: Product Competition and Information Production
Parameters: $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $c = 1.5$, $MC = 3$.

Proposition 1 is illustrated in Figure 1. When competition intensifies (i.e., $n \uparrow$), it features full information production (i.e., $\widehat{\alpha} = 1$ for $n \leq 5$), followed by partial information (i.e., $\widehat{\alpha} \in (0, 1)$ for $6 \leq n \leq 36$), and finally, no information (i.e., $\widehat{\alpha} = 0$ for $n \geq 37$).

Furthermore, it is also worth examining how information production is affected by changes in other model parameters related to the product market, including the unit production cost MC, the price sensitivity of demand b and market prospect parameters A_H and A_L. Again, we can apply the implicit function theorem to the equilibrium condition (11) to derive:

Corollary 1. *Suppose that $c \in (\underline{c}, \overline{c})$ so that an interior solution $\widehat{\alpha}_n \in (0, 1)$ exists. Then: $\frac{\partial \widehat{\alpha}_n}{\partial MC} < 0$, $\frac{\partial \widehat{\alpha}_n}{\partial b} < 0$, $\frac{\partial \widehat{\alpha}_n}{\partial A_H} > 0$, and $\frac{\partial \widehat{\alpha}_n}{\partial A_L} < 0$.*

Information production, measured by the amount $\widehat{\alpha}_n$ of informed speculators, decreases with the production cost MC. This result can be understood by analyzing the expected trading profit $\Pi(\alpha)$, which is lower for a higher MC. Obviously, a lower expected trading profit will reduce the incentive for speculators to produce information, decreasing the equilibrium amount of information production. Similarly, when demand becomes relatively more sensitive to price (i.e., $b \uparrow$), the amount $\widehat{\alpha}_n$ of informed speculators will also decrease, since the expected trading profit Π is lower for a higher b. Furthermore, $\widehat{\alpha}_n$ increases in A_H and decreases in A_L. To understand these, note that the expected trading profit Π increases in the market uncertainty that is proportional to $(A_H - A_L)^2$. Therefore, a larger gap of $(A_H - A_L)$ increases the expected trading profit of informed speculators, inducing them to acquire more information.

4.2 Real Efficiency

We now proceed to analyze the efficiency implications of product competition with feedback effects. Traditional wisdom claims that Cournot competition always improves economic efficiency and that imperfect/insufficient competition, such as oligopolies and monopolies, often leads to dead weight loss (Willner, 1989). However, all existing theoretical analyses that obtain a positive relationship between product competition and economic efficiency have commonly ignored the feedback effects of the financial market. Proposition 1 explains why this argument may fail by highlighting the information production mechanism through feedback effects, i.e., product competition reduces the incentive for speculators to acquire information, leading to inefficient production decisions in the real economy. To formalize this, we can use the inverse demand function $P = A - bQ$ to calculate total welfare when $n \geq 2$ firms engage in Cournot competition in the product market,

$$W = \frac{1}{2}(A(\omega) - P)Q + \sum_{i=1}^{n} TP_i, \tag{12}$$

where the first term captures the consumer surplus, while the second one captures the surplus for all producers. Since A is a random variable, the expected value of total welfare is given by: $\overline{W} = \mathbb{E}_\omega[W]$.

From Lemma 1, the equilibrium production strategy of each firm can be uniquely determined, given the amount $\widehat{\alpha}_n$ of informed speculators. Then, both the consumer and producer surpluses can be calculated. Thus, the expected total welfare can be written as a function of the size of informed agents $\widehat{\alpha}_n$ and the number of firms n, i.e., $\overline{W} = \overline{W}(\widehat{\alpha}_n, n)$. Specifically, the expected total welfare in the presence of feedback effects is given by:

$$\overline{W}(\widehat{\alpha}_n, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + (1 - \widehat{\gamma}_n{}^n)\left(A_H - A_L\right)^2\right), \tag{13}$$

where $\widehat{\gamma}_n = 1 - \widehat{\alpha}_n(2\theta - 1)$. Correspondingly, consumer welfare is given by:

$$\overline{CS}(\widehat{\alpha}_n, n) = \frac{n^2}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + (1 - \widehat{\gamma}_n{}^n)\left(A_H - A_L\right)^2\right). \tag{14}$$

Note that $\overline{W}(\widehat{\alpha}_n, n)$ strictly increases in average profitability $\left(\bar{A} - MC\right)$ and the uncertainty about market prospects $(A_H - A_L)$. Interestingly, a greater sensitivity of $\overline{W}(\widehat{\alpha}_n, n)$ to market uncertainty $(A_H - A_L)$ can be observed when there are more informed speculators in equilibrium (i.e., $\widehat{\alpha}_n \uparrow$). Clearly, this can arise only with the information feedback effect.

Next, we examine the relationship between total welfare and firm competition in the presence of feedback effects and investigate whether total welfare $\overline{W}(\widehat{\alpha}_n, n)$ can be negatively associated with the competition parameter n. To this end, we compute the total derivative of total welfare $\overline{W}(\widehat{\alpha}_n, n)$ with respect to n, the number of firms, as follows:

$$\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn} = \underbrace{\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial n}}_{\text{Competition Effects}} + \underbrace{\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n}\frac{\partial \widehat{\alpha}_n}{\partial n}}_{\text{Feedback Effects}}. \tag{15}$$

Equation (15) decomposes the total welfare effect into direct competition effects and feedback effects. Obviously, one can verify that $\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial n} > 0$, which is consistent with the conventional wisdom that product competition tends to increase total welfare (see, e.g., Willner, 1989). Meanwhile, since Proposition 1 establishes that $\frac{\partial \widehat{\alpha}_n}{\partial n} < 0$ (i.e., fierce product competition discourages information production), it might be possible for $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn}$ to be negative when $\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n}$ is positive and sufficiently large. Note that $\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n}$ measures the sensitivity of total welfare to the amount of information produced by speculators $\widehat{\alpha}_n$ in the stock market. Intuitively, as $\widehat{\alpha}_n$ increases, a higher level of informativeness of the stock market improves real efficiency in production, and thus a positive value of $\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n}$ follows.[11]

Define $G_1(A_H, A_L, MC) = 2 + 8\left(\bar{A} - MC\right)^2 / (A_H - A_L)^2$. The formulae for $g_1(\cdot, \cdot)$ and $g_2(\cdot, \cdot)$ can be found in the proof of Proposition 2 in Appendix A.

Proposition 2 (Competition and Real Efficiency)**.** *Product competition decreases total welfare, i.e.,* $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn} < 0$*, if and only if* $g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$*. Furthermore,* $\frac{d\overline{CS}(\widehat{\alpha}_n, n)}{dn} < 0$ *if and only if* $g_2(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$*.*

First, note that the auxiliary function $g_1(\widehat{\alpha}_n, n)$ depends only on the number of firms n and the equilibrium measure of informed speculators $\widehat{\alpha}_n$, while the function $G_1(A_H, A_L, MC)$ depends only on parameters related to market profitability, including A_H, A_L and MC. Second, $G_1(A_H, A_L, MC)$ increases strictly in the average profitability $\left(\bar{A} - MC\right)$ and decreases strictly in the market uncertainty $(A_H - A_L)$. Therefore, $g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$ is more likely to hold when the market uncertainty $(A_H - A_L)$ is high and the average profitability $\left(\bar{A} - MC\right)$ is low. Third, the condition in Proposition 2 is non-empty. For example, this occurs when the price sensitivity b of demand is sufficiently high that there is only a

[11]Using Equation (13), we can directly compute: $\frac{\partial \overline{W}(\widehat{\alpha}_n, n)}{\partial \widehat{\alpha}_n} = \frac{n^2(n+2)(2\theta-1)\widehat{\gamma}_n{}^{n-1}}{8b(n+1)^2}(A_H - A_L)^2 > 0$.

small fraction of informed speculators in equilibrium.[12]

Although Proposition 2 provides a tight characterization when competition can decrease total welfare, it involves an endogenous variable of information acquisition. To avoid this logic flaw, we provide a neat result using constructive derivations. Define $\mu := \frac{(A_H - A_L)^2}{4(\bar{A} - MC)^2}$ and

$$\Phi(l) := \left(1 - 1/(l+1)^2\right) * \left(1 + \mu * \left(1 - (2 - 2\theta)^l\right)\right). \tag{16}$$

Define $l_0 := \inf\{l \in \mathbb{N} : \Phi(l) \geq 1\}$. Note that l_0 always exists since $\Phi(l)$ increases strictly in l and $\lim_{l \to \infty} \Phi(l) = (1 + \mu) > 1$. Furthermore, we define:[13]

$$N(l) = \left[\frac{(l+1)^2}{(2-2\theta)\left(2 + (l-1)(2-2\theta)^{l-1}\right)}\right].$$

Theorem 1 (Informational Feedback & Over-Competition). *Fix any $l \geq l_0$. Then, for any $n \geq N(l) > l$, $\overline{W}(\widehat{\alpha}_l, l) > \overline{W}(\widehat{\alpha}_n, n)$ holds for any $c \in [\bar{c}_n, \underline{c}_l)$ with $\bar{c}_n < \underline{c}_l$.*

Theorem 1 highlights the welfare-reducing role of competition through informational feedback. Indeed, when information production $\hat{\alpha}$ is kept fixed, Equation (13) implies that an increase in the number of firms always increases total welfare. Therefore, the negative relationship between competition and total welfare in Theorem 1 is driven only by the information production channel. Furthermore, it quantifies over-competition in the presence of informational feedback. Specifically, there is a range of cost parameters c such that any market structure with $n \geq N(l_0)$ is suboptimal. Meanwhile, given the cost parameter c, a market structure with $n \geq N(l)$ is always dominated by that of l firms, where $N(l)$ is the smallest number satisfying $\bar{c}_{N(l)} < c < \underline{c}_l$.

Figure 2 illustrates a non-montonic welfare impact when competition intensifies, and the total welfare is maximized when the total number of firms $n = 6$. When the total number of firms increases, there are three cases: (i) when $n \leq 6$, total welfare increases in product competition because market power is reduced; (ii) when $6 \leq n \leq 37$, total welfare decreases in product competition because the feedback effect is dominant; and (iii) when $n \geq 37$, total welfare increases again because information production disappears and the feedback effect channel is shut down. Again, the market concentration effect dominates.

[12]Note that $\lim_{b \to \infty} \widehat{\alpha}_n = 0$. Then, we can get the approximation $g_1(\widehat{\alpha}_n, n) = \frac{n^2(n+1)(n+2)}{n(n-1)+2} + 2 + O(n\widehat{\alpha}_n)$, where $O(\cdot)$ means "big O". Now suppose that $g(0, n) > G_1$, or equivalently, $\frac{(\bar{A}-MC)^2}{(A_H - A_L)^2} < \frac{n^2(n+1)(n+2)}{8(n(n-1)+2)}$. By continuity, for any $\widehat{\alpha}_n > 0$ sufficiently small, $g_1(\widehat{\alpha}_n, n) > G_1(A_H, A_L, MC)$ holds.

[13]The floor function $[x]$ returns the largest integer no larger than x.

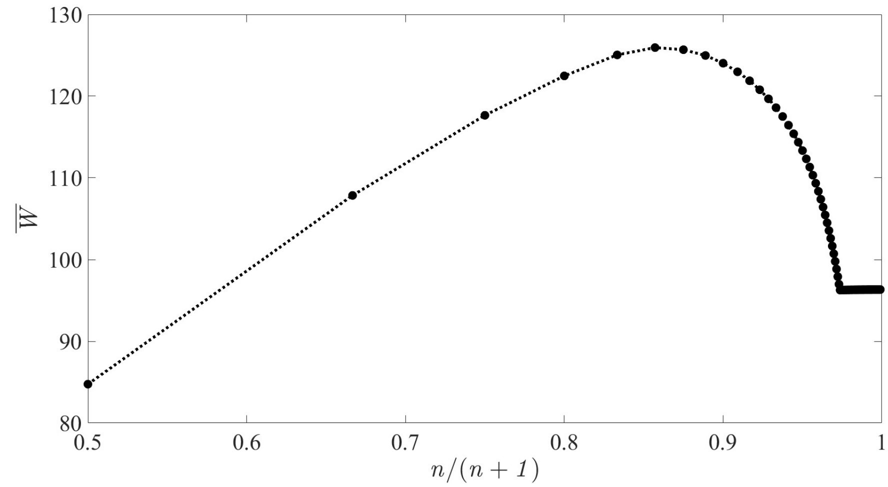

Figure 2: Product Competition and Total Welfare
Parameters: $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $c = 1.5$, $MC = 3$.

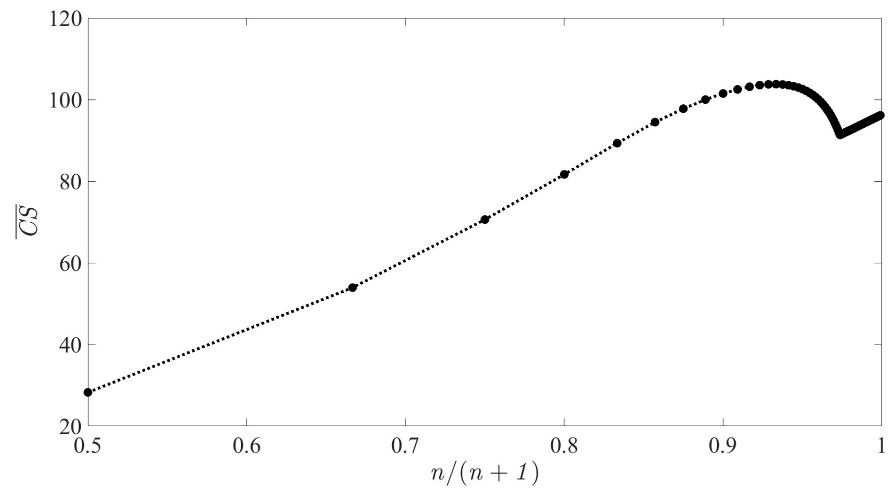

Figure 3: Product Competition and Consumer Surplus
Parameters: $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $c = 1.5$, $MC = 3$.

In addition, Figure 3 illustrates a similar non-monotonic pattern in consumer surplus when we vary the number of firms n. Specifically, the consumer surplus increases first for $n \leq 14$, then decreases for $14 \leq n \leq 37$, and finally increases again for $n \geq 37$. Note that the consumer surplus is maximized at $n = 14$, rather than at $n = 6$.

4.3 Comparative Statics

Let n^* denote the optimal market structure (i.e., the number of firms that maximize total welfare) in the presence of information feedback.

Proposition 3 (Non-Monotonicity)**.** *The negative relationship between competition and total welfare occurs only for an intermediate level of parameters, including information production cost c, price sensitivity b, and market prospect A_H. Furthermore, n^*, as a function of these parameters, is non-monotonic.*

The non-monotonicity is driven by the informational feedback effect. Specifically, when c is large, all speculators quit information production, and the information feedback channel is absent. Therefore, total welfare strictly increases as competition intensifies. Furthermore, in the limit where c is zero, all speculators choose to acquire information, implying that the information production gap between different market structures remains minimal for any c sufficiently small. Therefore, the market concentration channel dominates and thus the optimal market structure features perfect competition. Finally, for an intermediate level of information cost c correctly chosen, welfare-reducing competition arises in the sense of Theorem 1 such that a Cournot market with $n \geq N(\underline{l}_0)$ is strictly dominated in total welfare by a market with $n = \underline{l}_0$, due to the welfare loss related to deteriorated managerial learning from stock prices. In addition, the non-monotonicity of other parameters can be analyzed similarly and shown in Section B.1.

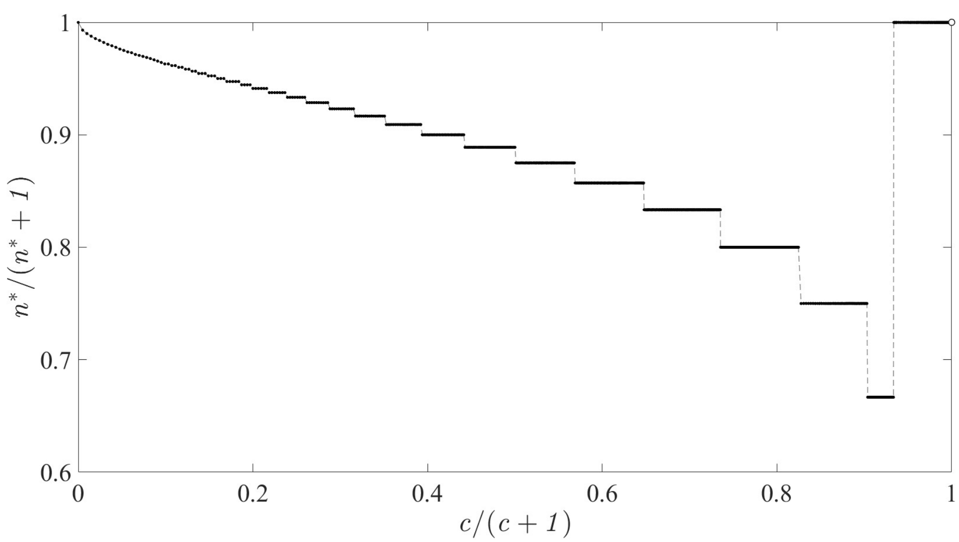

Figure 4: Optimal Market Structure n^*

Parameters: $\theta = 0.75$, $b = 1.5$, $MC = 3$, $A_H = 30$, $A_L = 10$.

Figure 4 depicts the optimal market structure index n^*. Initially, as the cost of information production c decreases, the optimal market structure n^* changes from perfect competition (i.e., $n^* \to \infty$) to a duopoly (i.e., $n^* = 2$) at the first cutoff $c = 14.0$. We can check that $\underline{c}_2 = 7.9$ and $\bar{c}_2 = 18.9$, which indicates that $\hat{\alpha}_2 \in (0, 1)$. Then, when c

decreases below the second cut-off point at $c = 9.4$, the optimal market structure admits three firms (i.e., $n^* = 3$). Note that $\underline{c}_3 = 4.4$ and $\bar{c}_3 = 14.2$. Therefore, when $c \in (9.4, 14.0)$, a duopoly dominates a triopoly, although under both market structures a positive fraction of speculators, not all, becomes informed. Finally, when c further decreases, the optimal market structure index n^* increases. Note that when c is small and the number of firms is relatively small, a high level of informed speculators ensues in equilibrium (i.e., $\hat{\alpha} \to 1$), implying that information production becomes insensitive to n when it is small. Therefore, total welfare improves when a larger n is chosen.

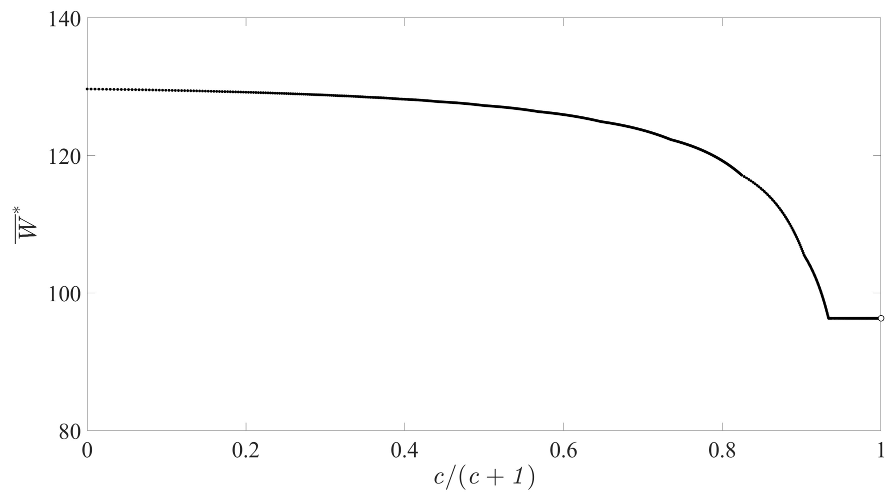

Figure 5: Total Welfare under Optimal Market Structure n^*

Parameters: $\theta = 0.75$, $b = 1.5$, $MC = 3$, $A_H = 30$, $A_L = 10$.

However, although the optimal market structure n^* is non-monotonic, total welfare under the optimal market structure is monotonic and increases when the information acquisition cost c decreases, as illustrated in Figure 5. Note that for any fixed market structure, the total welfare (weakly) decreases in the cost of information acquisition c. Then, the total welfare $\bar{W}(n^*, \widehat{\alpha}_{n^*})$ under the optimal market structure is just the supremum of a family of weakly increasing and continuous welfare functions $\bar{W}(n, \widehat{\alpha}_n)$ for all $n \in \mathbb{N}$, and therefore it is also weakly decreasing. In addition, a more detailed discussion about other parameters can be found in Section B.1.

Analysis for combined parameters when $n^* < \infty$. To better illustrate their economic intuition and implications, we discuss the role of average profitability and market uncertainty in shaping the link between competition and total welfare when $n^* < \infty$. Specifically, we use numerical methods to address the complexity of the auxiliary function $g_1(\alpha, n)$, complementing our earlier analytical results. Theoretical insights, including Proposition 2 and the following discussions in Section 4.2, provide guidance for the numerical analysis. We anticipate that a negative relationship between competition and total welfare is more likely to occur with high market uncertainty $(A_H - A_L)$ and low average profitability $\left(\bar{A} - MC\right)$. Meanwhile, by Equation (8) and Equation (11), these two factors also contribute to information production $\widehat{\alpha}$ in equilibrium.

We first define the incremental change in total welfare:

$$\Delta W_n := \overline{W}\left(\widehat{\alpha}_n, n\right) - \overline{W}\left(\widehat{\alpha}_{n-1}, n-1\right).$$

Obviously, a negative relationship between product competition and total welfare ensues when $\Delta W_n < 0$ is satisfied. We also focus on interior solutions of $\widehat{\alpha}_n$, and sensitivity analyses performed on a wide range of model parameter values have shown a similar pattern.

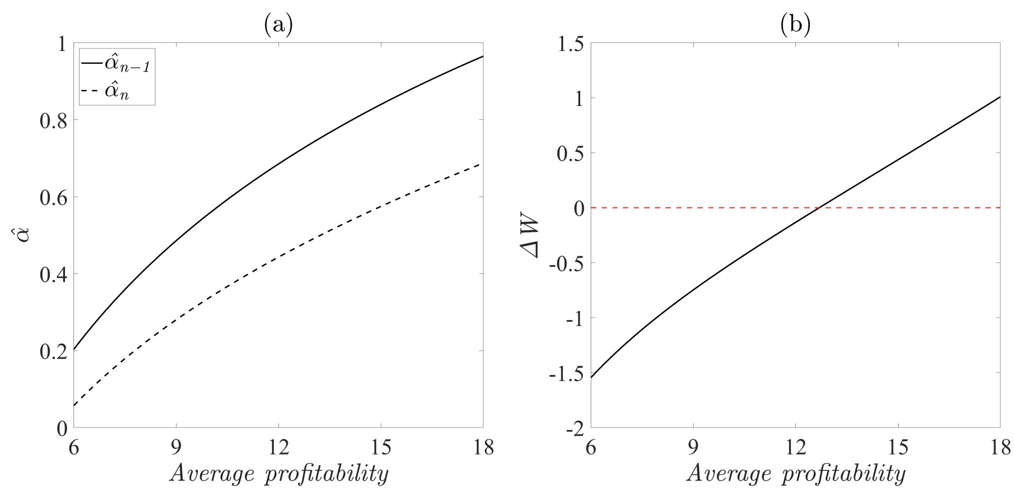

Figure 6: Average Profitability, Information Quality and Welfare.

Parameters: $A_H - A_L = 10$, $MC = 3$, $b = 1.5$, $n = 5$, $c = 1.5$ and $\theta = 0.75$.

Then we analyze the impact of average profitability $(\bar{A} - MC)$ on equilibrium information production $\widehat{\alpha}_n$ and total welfare ΔW_n. For this exercise, we fix the value of $(A_H - A_L)$ and other parameters. The results are plotted in Figure 6. We make three observations: First, Figure 6a shows that $\widehat{\alpha}_n$ is always lower than $\widehat{\alpha}_{n-1}$, which is consistent with the prediction of Proposition 1 that product competition dampens the incentive for speculators to produce information. Second, both $\widehat{\alpha}_n$ and $\widehat{\alpha}_{n-1}$ increase strictly in average profitability, implying that higher profitability improves information acquisition. Third, Figure 6b shows that the welfare gain ΔW_n is smaller for a lower level of average profitability. In particular, when the average profitability is sufficiently low, ΔW_n can be negative, indicating that intensified competition decreases the total welfare. Note that this result coincides with our discussion following Proposition 2.

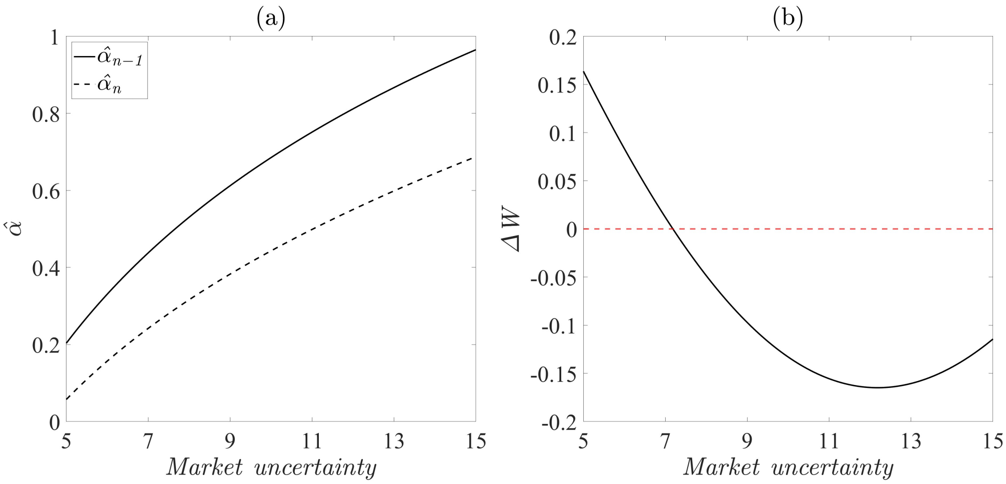

Figure 7: Market Uncertainty, Information Quality and Welfare.

Parameters: $\bar{A} = 30$, $MC = 3$, $b = 1.5$, $n = 5$, $c = 1.5$ and $\theta = 0.75$

Next, we investigate the effects of market uncertainty on $\widehat{\alpha}_n$ and ΔW_n by varying $(A_H - A_L)$ while keeping the average profitability $(\bar{A} - MC)$ and other parameters unchanged. These results are depicted in Figure 7. We make two observations: First, Figure 7a shows that both $\widehat{\alpha}_n$ and $\widehat{\alpha}_{n-1}$ increase as $(A_H - A_L)$ increases, which implies that increasing market uncertainty improves information production. Second, as shown in Figure 7b, competition decreases total welfare when market uncertainty is high, despite the high incentive of information production (i.e., $\widehat{\alpha}$ is high).

This illustrates a sharp difference between average profitability and market uncertainty. Although both exhibit similar effects on information production, the welfare implications of competition diverge. Specifically, a negative relationship between competition and total welfare is more likely to occur when: (i) the average profitability is low; or (ii) the market uncertainty is high. To understand this divergence, we highlight two observations: First, an increase in average profitability directly increases total welfare, which reduces the relative impact of information production, while an increase in market uncertainty amplifies that of information production (see Equation (13)). Second, the negative link between competition and welfare depends on the relative gap, rather than the absolute intensity, in information production when the level of competition varies.

5 Further Discussions

5.1 Implications for Horizontal Mergers

To better illustrate the empirical implications for horizontal mergers, we first compare a monopoly (i.e., $n = 1$) and a duopoly (i.e., $n = 2$) in perfectly symmetric Cournot competition. By Equation (13), the total welfare for a monopolist seller is given by:

$$\overline{W}\left(\widehat{\alpha}_1, 1\right) = \frac{3}{32b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \widehat{\gamma}_1\right)\left(A_H - A_L\right)^2\right) \tag{17}$$

and that for two duopoly sellers are given by

$$\overline{W}\left(\widehat{\alpha}_2, 2\right) = \frac{1}{9b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \left(\widehat{\gamma}_2\right)^2\right)\left(A_H - A_L\right)^2\right) \tag{18}$$

Obviously, if we fix the size of informed traders $\widehat{\alpha}_1 = \widehat{\alpha}_2$ (or equivalently $\widehat{\gamma}_1 = \widehat{\gamma}_2$) to shut down the information production channel, a duopoly market always outperforms a monopoly in total welfare. In other words, any regulatory action based on market concentration measures is well-founded. However, if we allow for endogenous information production, the above insight might not hold, as illustrated by Lemma 4 below.

Denote $\kappa = (2\theta - 1)(A_H - A_L)^2/b$.

Lemma 4 (Monopoly VS. Duopoly)**.** *Assume that $A_H > A_L = MC$.*

(i) When $\frac{\kappa}{12} \leq c < \frac{11}{108}\kappa$, then $\overline{W}\left(\widehat{\alpha}_1, 1\right) > \overline{W}\left(\widehat{\alpha}_2, 2\right)$; and

(ii) when $c \geq \frac{11}{108}\kappa$ or $c < \frac{(1-\theta)(2-\theta)\kappa}{9}$, then $\overline{W}\left(\widehat{\alpha}_1, 1\right) \leq \overline{W}\left(\widehat{\alpha}_2, 2\right)$.

We briefly comment on Lemma 4. First, a monopoly dominates a duopoly for an intermediate level of information production cost c. In Statement (i), a lower bound $c \geq \frac{\kappa}{12}$ is imposed to completely remove information production in a duopoly market (i.e., $\widehat{\alpha}_2 = 0$), while an upper bound $c < \frac{11\kappa}{108}$ ensues that the incentive to produce information is strong enough in a monopoly market (i.e., $\widehat{\alpha}_1 \uparrow$). Second, when information production is too cheap or too costly, the relative gap in information production is small, and thus a duopoly market is more efficient due to lowered market concentration.

Obviously, our theory differs sharply from all the existing literature on merger analysis, which all features a monotonic relationship between competition and total welfare in perfectly symmetric Cournot competition when all firms are equally efficient (see, e.g., Farrell and Shapiro, 1990). Instead, even in the simplest example that compares a monopoly and a duopoly, merging two competing and equally efficient firms into a monopolist can improve social welfare for an intermediate level of information production cost when market concentration significantly increases information production. This naturally arises when managerial learning from the stock market benefits production decisions in a feedback loop. Therefore, our theory highlights the importance of considering the interaction between the product market and the financial market in M&As regulations.

Remark 1 (Beyond Monopoly & Duopoly). *When speaking to M&A, we also care about the comparison beyond monopoly and duopoly. Theorem 1 offers such a guide. Specifically, recall that* $l_0 := \inf\{l \in \mathbb{N} : \Phi(l) \geq 1\}$. *When* $n \geq N(l_0)$, *over-competition arises in the sense of total welfare for an intermediate range of information production cost, since it is strictly dominated by a market structure with* $n = l_0$. *Therefore, total welfare can be improved when the number of firms is reduced to* $n < N(l_0)$, *although the optimal number of firms* n^* *needs to be solved numerically.*[14]

Furthermore, our treatment of M&As closely follow the spirit of Cournot competition in the long-run sense, differing from that of Nocke and Whinston (2022), in which the post-merger Herfindahl index (HHI) simply combines the pre-merger market shares of firms in an M&A. Our analysis complements all existing forces in M&A by highlighting the importance of considering the interaction between the financial market and the product market, in addition to other channels well documented in the existing literature, including asymmetry in firms' production efficiency (Farrell and Shapiro, 1990), synergies (see, e.g., Maksimovic and Phillips, 2001), disclosure (Xiong and Yang, 2021), investment (Mermelstein et al., 2020; Motta and Tarantino, 2021), innovation (Yi, 1999; Aghion et al., 2005; Segal and Whinston, 2007; Spulber, 2013), etc.

Remark 2 (Feedback effects and allocative efficiency gains). *Informational feedback generates a new type of allocative efficiency gain in symmetric Cournot competition, which differs sharply from those related to synergies and marginal cost reduction in asymmetric Cournot games. Consider a horizontal merger between two of the* n *symmetric firms, resulting in a post-merger market structure with* $(n-1)$ *firms.*[15] *In the absence of feedback effects, such a horizontal merger is totally anti-competitive, leading to higher prices, lower quantities, and deteriorated total welfare. However, with informational feedback, managerial learning from*

[14] The dominated structures $n \geq N(l)$ can also be chosen conditional on the information cost c.

[15] This differs from the literature studying M&As in the short run (see, e.g., Nocke and Whinston, 2022).

stock prices leads to higher production decision-making efficiency. Therefore, a horizontal merger can increase real efficiency by improving information production. See Section B.2 for a detailed discussion.

5.2 Cross-Asset Trading

Although standard in the literature (see, e.g., Foucault and Frésard, 2014, 2019), bounded asset positions ($x_k^i \in [-1, 1]$) in our baseline model may not be as harmless as in other settings: If the total product market size is stable, with an increase in the number of firms, the size and, consequently, the equity value of each firm decrease. Therefore, the dollar value of the maximum trade size could decrease in n, and thus the incentive to acquire information might mechanically decrease. To address this concern and show robustness, we now allow cross-asset trading, in which a fraction of speculators can trade all stocks. All baseline findings continue to hold.

Specifically, we consider an economy with $n \geq 2$ identical firms competing in quantities and a stock exchange, which is populated with four types of investors, including: (i) a mass $\lambda \in [0, 1]$ of risk-neutral L-traders $k \in [0, \lambda]$, who choose whether to acquire a costly signal m_k at a cost $c_L > 0$, and trade all stock shares $y_k^i \in [-1, 1]$ for all i; (ii) a mass $1 - \lambda$ of risk-netural S-traders $k \in [0, 1 - \lambda]$ for each stock i, who choose whether to acquire a costly signal m_k^i at a cost $c_S > 0$ and only trade shares $x_k^i \in [-1, 1]$ for the ith stock. (iii) liquidity traders with aggregate demand z_i, uniformly distributed over $[-1, 1]$, for each firm i, and (iv) risk-neutral market makers who set prices to clear each stock.

Let $y_i = \int_0^{\alpha_L} y_k^i dk$ and $x_i = \int_0^{\alpha_i^S} x_k^i dk$ denote the aggregate demand for stock i by L- and S-traders. Recall that the aggregate order submitted by liquidity traders is z_i. Thus, the total order flow f_i for the ith stock is then given by: $f_i = x_i + y_i + z_i$. As in Goldstein et al. (2014), we assume that $c_L \leq c_S$, i.e., an L-trader has a relatively lower cost of information production.[16] For ease of reference, let α_L and $\alpha_{i,S}$ denote the measure of informed L-traders and that of informed S-traders for the ith firm. Define $\boldsymbol{\alpha} := (\alpha_L, \alpha_{1,S}, \cdots, \alpha_{n,S})$. All other features of the model are the same. Note that when $\lambda = 0$, it reduces to the baseline setup.

We briefly summarize the key novel insights, while the equilibrium analysis can be found in Section B.3. First, L-traders have a stronger incentive to acquire information than S-traders, given that $c_L \leq c_S$. Actually, the incentive for L-traders to acquire information can

[16] To be precise, Goldstein et al. (2014) sets $c_S > c_L = 0$, i.e., an L-trader costlessly observes a signal.

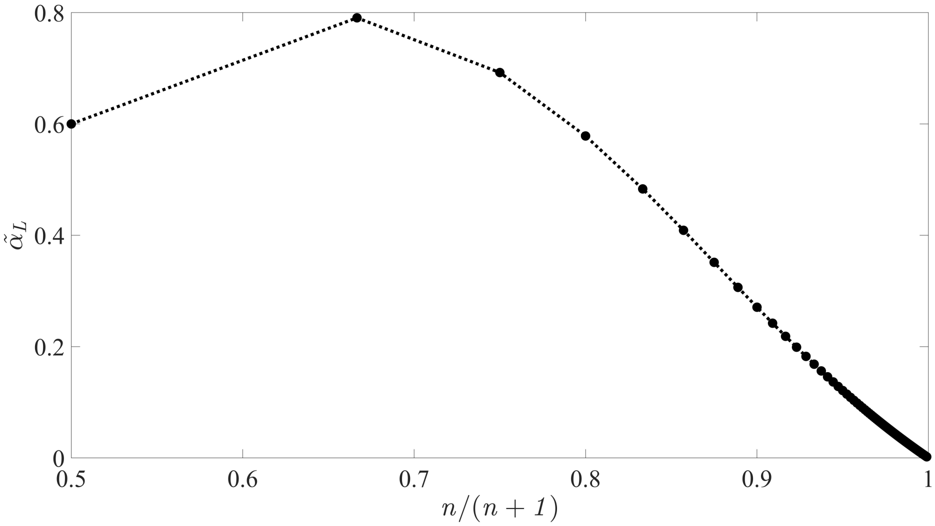

Figure 8: Trading Opportunities & (Non-monotonic) Information Production

Parameters: $\lambda = 0.8$, $\theta = 0.75$, $b = 3.5$, $A_H = 20$, $A_L = 10$, $MC = 9$, and $c_L = c_S = 1.5$. Note that $\widetilde{\alpha}_S = 0$.

even increase in the number of firms n, which differs sharply from S-traders for whom the incentive for information acquisition is always maximized in a monopoly. This complexity is illustrated in Figure 8. In particular, when we move from a monopoly ($n = 1$) to a duopoly ($n = 2$), the size of the informed L-traders $\widetilde{\alpha}_L$ first increases and then decreases.[17]

Second, our baseline result remains valid in the presence of L-traders, because the incentive for information production for L-traders will drop quickly after achieving its maximum level, and thus a negative relationship between competition and total welfare ensues.

5.3 Cross-Asset Learning

In the baseline model, we assume that the market maker of the ith firm does not observe the order flow of the other firms. Therefore, there may be arbitrage opportunities between competing firms. This section removes this restriction and considers cross-asset learning, which refers to the possibility that market makers observe the order flow in all stocks before setting the price (see, e.g., Pasquariello and Vega, 2015; Foucault and Frésard, 2019). Specifically, we modify the more general setup in Section 5.2 by allowing for cross-asset learning, i.e., the information set for market makers is $\Omega = \{f_1, \cdots, f_n\}$. Again, as in Kyle (1985), risk-neutral market makers absorb excess order flow and break even only in expectation.

[17] Vives (1985) shows that the profit of competing firms vanishes at a speed order of $1/n$. When multiplied by the number of firms n, the trading profits for L-traders can be non-monotonicity in n. We term this the "trading opportunity effect" in cross-asset trading.

Thus, the stock price of the ith firm at $t = 2$ is given by $s_i(\Omega) = \mathbb{E}[V_i|\Omega]$.

Here, we briefly discuss the main results with cross-asset learning, and delegate the formal analysis to Section B.4. First, the baseline result holds in the presence of cross-asset learning when there are only S-traders. Intuitively, cross-asset learning empowers market makers, reducing trading profits for speculators, except for the special case with a monopoly. This in turn makes the trading profits more sensitive to the change in the number of competing firms when it is small. Thus, the information feedback channel is strengthened.

Second, the non-monotonicity also holds when the cost of information production is small such that all L-traders choose to acquire information. Note that L-traders have a stronger incentive to acquire information compared to S-traders. Cross-asset trading makes S-traders more prone to competition compared to L-traders, and thus L-traders may crowd out S-traders due to their trading opportunities.

Third, when there are only L-traders in the presence of cross-asset learning, the total welfare increases strictly in the number of firms n. In other words, the baseline result holds with cross-asset trading or cross-asset learning, but not both. Intuitively, cross-asset trading endows L-traders with more trading opportunities, while cross-asset learning provides market makers with more information, decreasing speculators' trading profits and the relative importance of information production. Both forces reduce the impact of the information production channel. A more detailed discussion about the divergent impact of cross-asset learning on L-traders and S-traders can be found in Appendix B.4.

5.4 Investor Welfare

Investor welfare, especially that of liquidity traders, is largely missing from the total welfare defined in Equation (13), which essentially captures the welfare of the product market, including both the consumer surplus and the producers' surplus. We now show that our theoretical insights still hold when we include investor welfare in the calculation of total welfare. Recall that: (1) market makers always break even in expectation; (2) informed speculators incur acquisition costs but earn positive trading profits; (3) liquidity traders incur trading losses but enjoy liquidity benefits; and (4) informed speculators' trading profits equal liquidity traders' trading losses. Although liquidity benefits are conceptually endogenous, most papers treat them and liquidity trading as completely exogenous. The total cost of information acquisition varies with the size of informed speculators α, and given that we focus on the benefits of information, the cost of information acquisition should not be overlooked.

Specifically, let $B(n)$ denote the aggregate benefit of liquidity trading. Thus, total welfare $\overline{W}_{PF}$, including both product market welfare and investor welfare, can be measured as:

$$\overline{W}_{PF} = \overline{W} - n * \widehat{\alpha}_n * c + B(n) \tag{19}$$

where $\overline{W}(\widehat{\alpha}_n, n)$ is given by Equation (13).

When the aggregate benefits of liquidity trading are exogenously fixed (i.e., $B(n) = B_0$ for some non-negative constant B_0), a non-monotonic relationship between product competition and total welfare ensues, and the optimal market structure features a finite number of firms. This non-monotonicity also extends to other specifications when the aggregate benefits of liquidity trading are proportional to the number of stocks, although the optimal market structure might approach perfect competition when the benefits of liquidity trading become dominant. A formal analysis can be found in Appendix B.5.

6 Conclusion

Incorporating information production and learning into a standard Cournot competition game, we study the interaction between product market competition and informational feedback in financial markets. Although intensified competition can reduce market power and improve economic efficiency in the product market under exogenous information, it reduces the speculators' valuation of proprietary information on the market prospects of firms. When production decisions depend on the information conveyed through stock prices, a novel trade-off between economic efficiency and informational efficiency endogenously arises. Intensified product competition always discourages information production and reduces total welfare. A negative relationship between product competition and real efficiency emerges when the feedback effect of stock prices is large enough. Our tractable and transparent model provides insights for antitrust regulations in M&As and for further studies linking information economics and product market competition.

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Appendix

A Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

Proof. We first compute the beliefs of the market makers. Recall that the total order flow for the ith stock is $f_i = \alpha_i(2\theta - 1) * (\mathbb{1}(\{\omega = H\}) - \mathbb{1}(\{\omega = L\})) + z_i$.[18] Denote $\gamma_i = 1 - \alpha_i(2\theta - 1)$. Note that condition $f_i > \gamma_i$ contradicts the event that $\omega = L$ because: (1) $f_i = z_i + x_i$ by definition; (2) $x_i = -\alpha_i(2\theta - 1)$ if $\omega = L$ by the law of large numbers; and (3) $z_i \leq 1$. Conversely, when $z_i > \gamma_i - \alpha_i(2\theta - 1)$ and $\omega = H$, then $f_i > \gamma_i$. Therefore, the aggregate order flow f_i is a sufficient statistic to update the beliefs of the market makers. In summary, if the aggregate order flow satisfies $f_i > \gamma_i$, it can be inferred that $\omega = H$. Similarly, if the aggregate order flow of stock i is $f_i < -\gamma_i$, the market makers will infer that $\omega = L$. Furthermore, when the aggregate order flow satisfies $f_i \in (-\gamma_i, \gamma_i)$, an application of the Bayes rule implies that

$$\Pr\left(\omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{\Pr(\omega = H) \Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right)}{\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right)} = \frac{1}{2}$$

because $\Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) = \Pr\left(-\gamma_i - \alpha_i(2\theta - 1) \leq z_i \leq \gamma_i - \alpha_i(2\theta - 1)\right) = \gamma_i$ and $\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right) = \Pr\left(f_i \in (-\gamma_i, \gamma_i), \omega = H\right) + \Pr\left(f_i \in (-\gamma_i, \gamma_i), \omega = L\right) = \gamma_i$. This also means that an order flow such that $f_i \in [-\gamma_i, \gamma_i]$ is uninformative.

Second, we analyze the belief updating rule for the ith manager, given the equilibrium prices $\{s_i(f_i)\}_{1 \leq i \leq n}$. Specifically, when $s_i(f_i) = s_H$ is observed, the manager i infers that $f_i > \gamma_i$ and thus $\omega = H$, which is exactly the reason for the market makers. Similarly, when $s_i(f_i) = s_L$ is observed, it can be inferred that $f_i < -\gamma_i$ and thus $\omega = L$. Finally, when $s_i(f_i) = s_M^i$, it must be the case that $f_i \in (-\gamma_i, \gamma_i)$, implying that the ith firm stock price is not informative about the market prospects. The ith manager depends on all other firms' stock prices to infer about the state, and there are three cases, including: (i) there exists some $j \neq i$ such that $s_j = s_H$, then again $f_j > \gamma_j$ and thus $\omega = H$; (ii) if there exists some $j \neq i$ such that $s_j = s_L$, then $f_j < -\gamma_j$ and thus $\omega = L$; (iii) if for all $j \neq i$ such that $s_j = s_M^j$, then it can be inferred that all stock prices are uninformative.

Next, we analyze the ith firm's production strategy, given the manager's posterior belief on the state ω after observing stock prices. Let θ_m be the posterior probability of $\omega = H$. Then, the ith manager's problem is to choose the quantity q_i to maximize:

$$V_i(q_i) = \mathbb{E}\left[TP_i(q_i) \mid \theta_m\right] = q_i\left(A_m - b\left(q_i + q_{-i}\right) - MC\right) \tag{A.1}$$

where $A_m = \mathbb{E}[\tilde{A}|\mathcal{F}_m] = \theta_m A_H + (1 - \theta_m) A_L$ is the expected value of the market prospect A conditional on posterior belief. From Equation (A.1), we know that $V_i(q_i)$ is concave in q_i, and thus $q_i^*(q_{-i}) = \frac{1}{2b}(A_m - bq_{-i} - MC)$. Given a common posterior belief updating rule, we can invoke $q_i = q_j$ for any $i \neq j$. Therefore, $q_i^* = \frac{A_m - MC}{(n+1)b}$.

[18] $\mathbb{1}(\{x \in A\})$ is an indicator function that equals one only when $x \in A$ holds, and equals zero otherwise.

Denote $q_H = \frac{A_H - MC}{(n+1)b}, q_L = \frac{A_L - MC}{(n+1)b}$, and $\beta_i = \prod_{j \neq i} \gamma_j$. Then, combining the belief updating rule of the common posterior, we conclude: (1) if $s_j = s_H$ for some j, then $\theta_m = 1$, $A_m = A_H$ and $q_i^* = q_H$; (2) if $s_j = s_L$ for some j, then $\theta_m = 0$, $A_m = A_L$ and $q_i^* = q_L$; and (3) if $s_j = s_M^j$ for all $1 \leq j \leq n$, then $\theta_m = \frac{1}{2}$, $A_m = \bar{A}$ and $q_i^* = q_M$.

We now check that the stock price rule $s_i(f_i)$ in Equation (6) satisfies condition (4). First, when the total order flow of the ith stock satisfies $f_i > \gamma_i$, then $\omega = H$, and thus $q_i^* = q_H$. By Equations (1) and (2), $\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i\right] = \frac{(A_H - MC)^2}{(n+1)^2 b}$, which is equal to s_H. Thus, condition (4) is satisfied when $f_i > \gamma_i$. Second, when the total order flow satisfies $f_i < -\gamma_i$, the net demand for the ith stock reveals that $\omega = L$, and thus $q_i^* = q_L$. Hence, $\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i\right] = \frac{(A_L - MC)^2}{(n+1)^2 b}$ for $f_i < -\gamma_i$, which is equal to s_L. Thus, for $f_i < -\gamma_i$, condition (4) is satisfied.

Third, when $f_i \in (-\gamma_i, \gamma_i)$, the investor demand for the ith stock is not informative about the state, i.e., $\Pr\left(\omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}$. Furthermore, by the argument of common posterior belief above, the manager i will produce q_H if $s_j = s_H$ for some $j \neq i$, produce q_L if $s_j = s_L$ for some $j \neq i$, and produce q_M if $s_j = s_M^j$ for all $j \neq i$. Thus, given that $f_i \in (-\gamma_i, \gamma_i)$ and $\exists j \neq i : s_j = s_H$, the ith firm's total profit at time $t = 2$ from producing q_H is

$$TP_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$$

When $f_i \in (-\gamma_i, \gamma_i)$ and $\exists j \neq i : s_j = s_L$, firm i 's total profit from producing q_L is

$$TP_L = \frac{(A_L - MC)^2}{(n+1)^2 b}.$$

When $f_i \in (-\gamma_i, \gamma_i)$ and $s_j = s_M^j$ for $\forall j \neq i$, we deduce that: (1) if $\omega = H$, firm i 's total profit in $t = 2$ from producing q_M is

$$TP_{MH} = \frac{(n+1)\left(\bar{A} - MC\right)\left(A_H - MC\right) - n\left(\bar{A} - MC\right)^2}{(n+1)^2 b};$$

and (2) if $\omega = L$, firm i 's total profit in $t = 2$ from producing q_M is

$$TP_{ML} = \frac{(n+1)\left(\bar{A} - MC\right)\left(A_L - MC\right) - n\left(\bar{A} - MC\right)^2}{(n+1)^2 b}.$$

Furthermore, by Equation (2), we obtain the following.

$$\begin{aligned}
\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i \in (-\gamma_i, \gamma_i)\right] \quad = \quad & \Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_H \\
+ \quad & \Pr\left(\exists j \neq i : s_j = s_L \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_L \\
+ \quad & \Pr\left(\forall j \neq i : s_j = s_M^j, \omega = H \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_{MH} \\
+ \quad & \Pr\left(\forall j \neq i : s_j = s_M^j, \omega = L \mid f_i \in (-\gamma_i, \gamma_i)\right) \times TP_{ML}.
\end{aligned}$$

To compute $\mathbb{E}\left[V_i\left(q_i^*\right) | f_i \in (-\gamma_i, \gamma_i)\right]$, we first calculate the conditional probabilities. Applying the Bayes rule, we get:

$$\Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i)\right)}{\Pr\left(f_i \in (-\gamma_i, \gamma_i)\right)}. \tag{A.2}$$

Using the law of total probability, we have

$$\begin{aligned}\Pr\left(\exists j \neq i : s_j = s^H, f_i \in (-\gamma_i, \gamma_i)\right) &= \Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = H\right) \\ &+ \Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = L\right)\end{aligned}$$

Note that $\Pr(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = L) = 0$ and that

$$\begin{aligned}\Pr\left(\exists j \neq i : s_j = s_H, f_i \in (-\gamma_i, \gamma_i), \omega = H\right) &= \Pr(\omega = H) \times \Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) \\ &\times \Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i), \omega = H\right) = \frac{1}{2}(1 - \beta_i)\gamma_i\end{aligned}$$

Thus, $\Pr\left(\exists j \neq i : s_j = s^H, f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1 - \beta_i)\gamma_i$.

Plugging this into Equation (A.2), we obtain: $\Pr\left(\exists j \neq i : s_j = s_H \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1 - \beta_i)$.

Analogously, we can show: $\Pr\left(\exists j \neq i : s_j = s^L \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}(1 - \beta_i)$ and

$$\begin{aligned}&\Pr(\forall j \neq i : s_j = s_j^M, \omega = H | f_i \in (-\gamma_i, \gamma_i)) \\ &\qquad = \Pr\left(\forall j \neq i : s_j = s_j^M, \omega = L \mid f_i \in (-\gamma_i, \gamma_i)\right) = \frac{1}{2}\beta_i\end{aligned}$$

Finally, plugging in these conditional probabilities, we have:

$$\mathbb{E}\left[V_i\left(q_i^*\right) \mid f_i \in (-\gamma_i, \gamma_i)\right] = \frac{2\left((A_H - MC)^2 + (A_L - MC)^2\right) - \beta_i\left(A_H - A_L\right)^2}{4(n+1)^2 b}$$

which is equal to S_M^i. Therefore, condition (4) is satisfied for $f_i \in [-\gamma_i, \gamma_i]$. The proof concludes. □

A.2 Proof of Lemma 2

Proof. Let $\Pi_i\left(x_k^i, m_k^i\right)$ be the expected profit of the speculator k who trades $x_k^i \in [-1, 1]$ shares of the ith firm when his signal is m_k^i, and let V_2^i be the market value of the ith firm at $t = 2$. Since each speculator is risk neutral and a price taker in the stock market, speculators will trade the maximum size possible if they acquire information, i.e., $x_k^i = \pm 1$.

First, consider an informed speculator who observes $m_k^i = H$. If he buys the asset, his expected profit is $\Pi_k^i(+1, H) = \mathbb{E}\left[V_2^i - s_i(f_i) \mid m_k^i = H, x_k^i = 1\right]$.

From the proof of Lemma 1, firm i 's value at $t = 2$ is

$$V_2^i = \begin{cases} TP_H & \text{if} \quad \exists j \in \{1, \ldots, n\} \text{ such that } s_j = s_H; \\ TP_{MH} & \text{if} \quad \omega = H \text{ \& } s_j = s_M^j, \forall j \in \{1, \ldots, n\}; \\ TP_L & \text{if} \quad \exists j \in \{1, \ldots, n\} \text{ such that } s_j = s_L; \\ TP_{ML} & \text{if} \quad \omega = L \text{ \& } s_j = s_M^j, \forall j \in \{1, \ldots, n\}. \end{cases} \tag{A.3}$$

Thus, using Equation (A.3), we can calculate $\Pi_i(+1, H)$ as follows:

$$\begin{aligned}
\Pi_i(+1, H) = {} & \Pr\left(\omega = H, f_i > \gamma_i \mid m_k^i = H\right) \times (TP_H - s_H) \\
& + \Pr\left(\omega = L, f_i < -\gamma_i \mid m_k^i = H\right) \times (TP_L - s_L) \\
& + \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right) \times \left(TP_H - s_M^i\right) \\
& + \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) \times \left(TP_{MH} - s_M^i\right) \\
& + \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) \times \left(TP_L - s_M^i\right) \\
& + \Pr(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\Big) \times \left(TP_{ML} - s_M^i\right).
\end{aligned}$$

Since $s^H = TP_H$ and $s^L = TP_L$, we can rewrite the expression of $\Pi_i(+1, H)$ as:

$$\begin{aligned}
\Pi_i(+1, H) & = & \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right) \times \left(TP_H - s_M^i\right) \\
& + & \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) \times \left(TP_{MH} - s_M^i\right) \\
& + & \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) \times \left(TP_L - s_M^i\right) \\
& + & \Pr(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\Big) \times \left(TP_{ML} - s_M^i\right).
\end{aligned}$$

Now, we use the Bayes rule to calculate $\Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H\right)$.

$$\begin{aligned}
\Pr(\omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H \mid m_k^i = H) = \frac{1}{\Pr\left(m_k^i = H\right)} \times \Pr(\omega = H) \\
\times \Pr\left(f_i \in (-\gamma_i, \gamma_i) \mid \omega = H\right) \times \Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i)\right) \\
\times \Pr\left(m_k^i = H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H)\right) = \theta\gamma_i (1 - \beta_i)
\end{aligned}$$

We have used the following facts in the last equation, including:

$$\begin{aligned}
& \Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i)\right) = \Pr\left(\exists j \neq i : s_j = s_H \mid \omega = H\right) = 1 - \beta_i; \\
& \Pr(m_k^i = H \mid \omega = H, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_H = \Pr(m_k^i = H \mid \omega = H) = \theta; \\
& \Pr(m_k^i = H) = \sum_{\omega \in \{H, L\}} \Pr(\omega) \Pr(m_k^i = H \mid \omega) = \frac{1}{2}.
\end{aligned}$$

Similarly, we have:

$$\begin{aligned}
& \Pr\left(\omega = H, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) = \theta\gamma_i\beta_i; \\
& \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \exists j \neq i : s_j = s_L \mid m_k^i = H\right) = \gamma_i(1 - \theta)(1 - \beta_i); \\
& \Pr\left(\omega = L, f_i \in (-\gamma_i, \gamma_i), \forall j \neq i : s_j = s_M^j \mid m_k^i = H\right) = \gamma_i\beta_i(1 - \theta).
\end{aligned}$$

Plugging these conditional probabilities back into the formula of $\Pi_i(+1, H)$, we have:

$$\Pi_i(+1, H) = \frac{(2\theta - 1)\gamma_i(2 + \beta_i(n - 1))\left((A_H - MC)^2 - (A_L - MC)^2\right)}{4(n + 1)^2 b} > 0$$

If instead the speculator sells, his expected profit is

$$\Pi_i(-1,H) = -\frac{(2\theta-1)\gamma_i(2+\beta_i(n-1))\left((A_H-MC)^2-(A_L-MC)^2\right)}{4(n+1)^2 b} < 0$$

Thus, the optimal trading strategy is to buy $x_{ki} = +1$ when $m_k^i = H$.

Symmetric reasoning shows that the speculator's optimal trading strategy is to sell $x_k^i = +1$ when $m_{ki} = L$. And in this case, his trading profit satisfies $\Pi_i(-1,L) = \Pi_i(+1,H)$. Furthermore, since $(A_H - MC)^2 - (A_L - MC)^2 = 2\left(\bar{A} - MC\right)(A_H - A_L)$, we conclude that

$$\Pi_i = \frac{(2\theta-1)\gamma_i\left(2+(n-1)\beta_i\right)\left(\bar{A}-MC\right)(A_H-A_L)}{2(n+1)^2 b}.$$

The proof concludes. □

A.3 Proof of Lemma 3

Proof. By Equation (8), $\frac{\partial \Pi(\alpha)}{\partial \alpha} < 0$. Thus, $\Pi(0) > \Pi(\alpha) > \Pi(1)$ for all $\alpha \in (0,1)$. Furthermore, by definition, we have: (i) when $c \geq \Pi(0) =: \bar{c}$, $\Pi(\alpha) < 0$ for any $\alpha > 0$, and thus $\widehat{\alpha} = 0$; (ii) when $c \leq \Pi(1) =: \underline{c}$, $\Pi(\alpha) < 0$ for any $\alpha < 1$, and thus $\widehat{\alpha} = 1$; and (iii) when $c \in (\underline{c}, \bar{c})$, by the intermediate value theorem and $\Pi(0) - c > 0 > \Pi(1) - c$, there exists a solution $\widehat{\alpha}$ such that $\Pi(\widehat{\alpha}) = c$, which is also unique since $\Pi'(\alpha) < 0$. □

A.4 Proof of Proposition 1

Proof. First, we can use Equation (8) to calculate the partial derivatives:

$$\begin{aligned}
\frac{\partial \Pi\left(n,\widehat{\alpha}_n\right)}{\partial \widehat{\alpha}_n} &= -\frac{(2\theta-1)^2\left(2+n(n-1)\widehat{\gamma}^{n-1}\right)\left(\bar{A}-MC\right)(A_H-A_L)}{2b(n+1)^2} \\
\frac{\partial \Pi\left(n,\widehat{\alpha}_n\right)}{\partial n} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(\bar{A}-MC\right)(A_H-A_L)}{2b(n+1)^3}\left\{4+\widehat{\gamma}_n^{n-1}\left(n-3+\left(n^2-1\right)\ln\frac{1}{\widehat{\gamma}_n}\right)\right\}
\end{aligned}$$

where $\widehat{\gamma}_n = 1 - \widehat{\alpha}_n(2\theta - 1)$.

By the implicit function theorem, we further have:

$$\begin{aligned}
\frac{\partial \widehat{\alpha}_n}{\partial n} &= -\left(\frac{\partial \Pi\left(n,\widehat{\alpha}_n\right)}{\partial n}\right) \Big/ \left(\frac{\partial \Pi\left(n,\widehat{\alpha}_n\right)}{\partial \widehat{\alpha}_n}\right) \\
&= -\frac{\widehat{\gamma}_n{}^n}{(2\theta-1)(n+1)\left(2+n(n-1)\widehat{\gamma}_n{}^{n-1}\right)}\left(4\widehat{\gamma}_n^{1-n}+n-3+(n+1)(n-1)\ln\frac{1}{\widehat{\gamma}_n}\right)
\end{aligned} \tag{A.4}$$

Obviously, when $n \geq 3$, it is easy to verify that $\frac{\partial \widehat{\alpha}_n}{\partial n} < 0$. Furthermore, we next show that $\frac{\partial \widehat{\alpha}_n}{\partial n} < 0$ holds when $n = 2$. Plugging in $n = 2$, it yields:

$$\frac{\partial \widehat{\alpha}_n}{\partial n}\bigg| n = 2 = -\frac{\widehat{\gamma}_2^2}{6(2\theta-1)\left(1+\widehat{\gamma}_2\right)}\left(4\widehat{\gamma}_2^{-1}+3\ln\frac{1}{\widehat{\gamma}_2}-1\right)$$

Since $0 \leq \widehat{\gamma}_n = 1 - \widehat{\alpha}_n(2\theta - 1) \leq 1$, the result follows. The proof concludes. □

A.5 Proof of Corollary 1

Proof. We first show that $\frac{\partial \widehat{\alpha}_n}{\partial A_H} > 0$. Applying the implicit function theorem implies:

$$\frac{\partial \widehat{\alpha}_n}{\partial A_H} = -\left(\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_H}\right) \Big/ \left(\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial \widehat{\alpha}_n}\right)$$

We have already shown in the proof of Proposition 1 that $\frac{\partial \Pi(\widehat{\alpha}_n)}{\partial \widehat{\alpha}_n} < 0$. Hence, it suffices to show that $\frac{\partial \Pi(\widehat{\alpha}_n)}{\partial A_H} > 0$. Again, Using Equation (8), we obtain:

$$\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_H} = \frac{2\widehat{\gamma}_n(2\theta-1)\left(A_H - MC\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{4b(n+1)^2} > 0$$

Similarly, we can show that:

$$\begin{aligned}
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial A_L} &= -\frac{2\widehat{\gamma}_n(2\theta-1)\left(A_L - MC\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{4b(n+1)^2} < 0,\\
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial MC} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(A_H - A_L\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{2b(n+1)^2} < 0,\\
\frac{\partial \Pi\left(\widehat{\alpha}_n\right)}{\partial b} &= -\frac{\widehat{\gamma}_n(2\theta-1)\left(\bar{A} - MC\right)\left(A_H - A_L\right)\left(2+(n-1)\widehat{\gamma}_n{}^{n-1}\right)}{2b^2(n+1)^2} < 0.
\end{aligned}$$

Hence, $\frac{\partial \widehat{\alpha}_n}{\partial A_L} < 0$, $\frac{\partial \widehat{\alpha}_n}{\partial MC} < 0$, and $\frac{\partial \widehat{\alpha}_n}{\partial b} < 0$. The proof concludes. □

A.6 Derivation of Equation (13) and (14)

From Lemma 1 and Equation (12), we can calculate total welfare at $t = 2$ as

$$W = \begin{cases} W_H & \text{if } s_i = s_H \text{ for some } i \in \{1, \ldots, n\}; \\ W_{MH} & \text{if } \ \omega = H \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \\ W_{ML} & \text{if } \omega = L \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \text{and} \\ W_L & \text{if } s_i = s_L \text{ for some } i \in \{1, \ldots, n\}. \end{cases}$$

where $W_H = \frac{n(n+2)(A_H-MC)^2}{2b(n+1)^2}$, $W_{MH} = \frac{n\left(\bar{A}-MC\right)((2n+4)(A_H-MC)+n(A_H-A_L))}{4b(n+1)^2}$, $W_L = \frac{n(n+2)(A_L-MC)^2}{2b(n+1)^2}$, and $W_{ML} = \frac{n\left(\bar{A}-MC\right)((2n+4)(A_L-MC)+n(A_L-A_H))}{4b(n+1)^2}$.

Then, the expected total welfare is given by:

$$\begin{aligned}
\overline{W} &= \Pr\left(\exists i : s_i = s_H\right) \times W_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times W_{MH}\\
&\quad + \Pr\left(\exists i : s_i = s_L\right) \times W_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times W_{ML}
\end{aligned}$$

From the proof of Lemma 1, we already know that $f_i > \widehat{\gamma}_n$ (i.e., $s_i = s_H$) is impossible when $\omega = L$ and $f_i < \widehat{\gamma}_n$ (i.e., $s_i = s_L$) is impossible when $\omega = H$. Hence, we have:

$$\begin{aligned}
\overline{W} &= \Pr\left(\exists i : s_i = s_H, \omega = H\right) \times W_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times W_{MH}\\
&\quad + \Pr\left(\exists i : s_i = s_L, \omega = L\right) \times W_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times W_{ML}
\end{aligned}$$

To compute $\overline{W}$, we use the Bayes rule to calculate $\Pr(\exists i : s_i = s_H, \omega = H)$.

$$\Pr\left(\exists i : s_i = s_H, \omega = H\right) = \Pr(\omega = H)\Pr\left(\exists i : s_i = s_H \mid \omega = H\right)$$

Using the expression of $s_i(f_i)$ in Equation (6), we know:

$$\Pr\left(s_i = s_M^i \mid \omega = H\right) = \Pr\left(-\widehat{\gamma}_n \le f_i \le \widehat{\gamma}_n \mid \omega = H\right) = \widehat{\gamma}_n$$
$$\Pr\left(s_i = s_H \mid \omega = H\right) = \Pr\left(f_i > \widehat{\gamma}_n \mid \omega = H\right) = 1 - \widehat{\gamma}_n$$

and thus: $\Pr\left(\exists i : s_i = s_H \mid \omega = H\right) = 1 - \Pr\left(\forall i : s_i = s_M^i \mid \omega = H\right) = 1 - (\widehat{\gamma}_n)^n$.

Since $\Pr(\omega = H) = 1/2$, we further have:

$$\Pr\left(\exists i : s_i = s_H, \omega = H\right) = \frac{1 - (\widehat{\gamma}_n)^n}{2}$$

Similarly, we have

$$\Pr\left(\exists i : s_i = s_L, \omega = L\right) = \frac{1 - (\widehat{\gamma}_n)^n}{2},$$
$$\Pr\left(\forall i : s_i = s_M^i, \omega = H\right) = \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) = \frac{(\widehat{\gamma}_n)^n}{2}$$

Therefore, $\overline{W}$ can be written as

$$\overline{W}(\widehat{\alpha}_n, n) = \frac{n(n+2)}{8(n+1)^2 b}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widehat{\gamma}_n)^n\right)(A_H - A_L)^2\right)$$

Obviously, $\overline{W}$ depends on n and $\widehat{\alpha}_n$, which implicitly depends on n, and we can explicitly write: $\overline{W}(\widehat{\alpha}_n, n)$. Given the monotone relationship between $\widehat{\alpha}_n$ and n, we know that the expected total welfare is uniquely determined for any fixed n.

Last, note that we can show for the formula of $\overline{CS}(\widehat{\alpha}_n, n)$ in a similar way. Again, from Lemma 1 and Equation (12), we can calculate total welfare at $t = 2$ as

$$CS = \begin{cases} CS_H & \text{if } s_i = s_H \text{ for some } i \in \{1, \ldots, n\}; \\ CS_{MH} & \text{if } \ \omega = H \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \\ CS_{ML} & \text{if } \omega = L \ \& \ s_i = s_M^i \ \forall i \in \{1, \ldots, n\}; \text{and} \\ CS_L & \text{if } s_i = s_L \text{ for some } i \in \{1, \ldots, n\}. \end{cases}$$

where $CS_H = \frac{n^2(A_H - MC)^2}{2b(n+1)^2}$, $CS_L = \frac{n(n+2)(A_L - MC)^2}{2b(n+1)^2}$, and $CS_{MH} = CS_{ML} = \frac{n^2\left(\bar{A} - MC\right)^2}{2b(n+1)^2}$

Furthermore, similar to $\bar{W}$, we have:

$$\overline{CS} = \Pr\left(\exists i : s_i = s_H, \omega = H\right) \times CS_H + \Pr\left(\forall i : s_i = s_M^i, \omega = H\right) \times CS_{MH}$$
$$+ \Pr\left(\exists i : s_i = s_L, \omega = L\right) \times CS_L + \Pr\left(\forall i : s_i = s_M^i, \omega = L\right) \times CS_{ML}$$

Thus, $\overline{CS}$ can be calculated as

$$\overline{CS} = \frac{1 - (\widehat{\gamma}_n)^n}{2} \times (CS_H + CS_L) + \frac{(\widehat{\gamma}_n)^n}{2} \times (CS_{MH} + CS_{ML})$$

From the expression of the consumer surplus at $t=2$, we further have:

$$\overline{CS}\left(\widehat{\alpha}_n, n\right) = \frac{n^2}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \left(\widehat{\gamma}_n\right)^n\right)\left(A_H - A_L\right)^2\right).$$

The derivation concludes.

A.7 Proof of Proposition 2

Proof. For ease of reference, define:

$$g_1(\alpha, n) = 2\gamma^n + \frac{n(n+2)\gamma^n}{2 + n(n-1)\gamma^{n-1}}\left(4n + n(n-3)\gamma^{n-1} - 2(n+1)\ln\frac{1}{\gamma}\right)$$

and

$$g_2(\alpha, n) = 2\gamma^n + \frac{n\gamma^n}{2 + n(n-1)\gamma^{n-1}}\left(4n + n(n-3)\gamma^{n-1} - 2(n+1)\ln\frac{1}{\gamma}\right)$$

where $\gamma = 1 - \alpha(2\theta - 1)$.

(i) Total welfare. Based on the expression for $\overline{W}(\widehat{\alpha}_n, n)$ in Equation (13), we know that

$$\frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn} = \frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial n} + \frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial\widehat{\alpha}_n}\frac{\partial\widehat{\alpha}_n}{\partial n}$$

First, the partial derivative of $\overline{W}\left(\widehat{\alpha}_n, n\right)$ with respect to n can be calculated as

$$\begin{aligned}\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial n} &= \frac{n(n+2)\left(A_H - A_L\right)^2\left(\widehat{\gamma}_n\right)^n\ln(1/\widehat{\gamma}_n)}{8b(n+1)^2} \\ &+ \frac{1}{4b(n+1)^3}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - \left(\widehat{\gamma}_n\right)^n\right)\left(A_H - A_L\right)^2\right)\end{aligned}$$

Second, we calculate the partial derivative of $\overline{W}\left(\widehat{\alpha}_n, n\right)$ with respect to $\widehat{\alpha}_n$ as follows:

$$\frac{\partial\overline{W}\left(\widehat{\alpha}_n, n\right)}{\partial\widehat{\alpha}_n} = \frac{\left(\widehat{\gamma}_n\right)^{n-1}n^2(n+2)(2\theta-1)\left(A_H - A_L\right)^2}{8b(n+1)^2}.$$

Using Equations (A.4) and the two partial derivatives above, we get:

$$\frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn} = \frac{\left(A_H - A_L\right)^2}{8b(n+1)^3}\left\{\frac{2\left(4\left(\bar{A} - MC\right)^2 + \left(A_H - A_L\right)^2\right)}{\left(A_H - A_L\right)^2} - g_1\left(\widehat{\alpha}_n, n\right)\right\}$$

Therefore, $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn} < 0$ holds if and only if: $g_1\left(\widehat{\alpha}_n, n\right) > \frac{8\left(\bar{A} - MC\right)^2}{\left(A_H - A_L\right)^2} + 2$.

(ii) Consumer surplus. Obviously, $\overline{CS}\left(\widehat{\alpha}_n, n\right) = \frac{n}{n+2}\overline{W}\left(\widehat{\alpha}_n, n\right)$. Thus, the total derivative of $\overline{CS}\left(\widehat{\alpha}_n, n\right)$ with respect to n can be written as follows:

$$\frac{d\overline{CS}\left(\widehat{\alpha}_n, n\right)}{dn} = \frac{n}{n+2} \times \frac{d\overline{W}\left(\widehat{\alpha}_n, n\right)}{dn} + \frac{2}{(n+2)^2} \times \overline{W}\left(\widehat{\alpha}_n, n\right)$$

Recall that $\overline{W}(\widehat{\alpha}_n, n) = \frac{n(n+2)}{8b(n+1)^2}\left\{4\left(\bar{A} - MC\right)^2 + (1 - (\widehat{\gamma}_n)^n)(A_H - A_L)^2\right\}$ and $\frac{d\overline{W}(\widehat{\alpha}_n, n)}{dn} = \frac{(A_H - A_L)^2}{8b(n+1)^3}(G_1 - g_1(\widehat{\alpha}_n, n))$. Then, we can calculate $d\overline{CS}(\widehat{\alpha}_n, n)/dn$ as follows:

$$\frac{d\overline{CS}(\widehat{\alpha}_n, n)}{dn} = \frac{n(A_H - A_L)^2}{8b(n+1)^3}(G_1 - g_2(\widehat{\alpha}_n, n))$$

Thus, $\frac{d\overline{CS}(\widehat{\alpha}_n, n)}{dn} < 0$ holds if and only if $g_2(\widehat{\alpha}_n, n) > G_1$ is true. The proof concludes. □

A.8 Proof of Theorem 1

Proof. The idea is to construct a set A of the information production cost such that for any $c \in A$, we have: (i) $\hat{\alpha}_l = 1$, $\hat{\alpha}_n = 0$; (ii) $n > l$; and (iii) $\overline{W}(\widehat{\alpha}_l, l) > \overline{W}(\widehat{\alpha}_n, n)$. It suffices to show that competition can decrease total welfare through informational feedback when $A \neq \varnothing$, because whenever information production is fixed, an increase in the number of firms always improves total welfare in Cournot competition.

Now, we come to construct A. First, given condition (i),

$$\frac{\overline{W}(\widehat{\alpha}_l, l)}{\overline{W}(\widehat{\alpha}_n, n)} = \frac{\left(1 - \frac{1}{(l+1)^2}\right) * \left(1 + \mu * (1 - (2-2\theta)^l)\right)}{\left(1 - \frac{1}{(n+1)^2}\right)}$$

Thus, $\overline{W}(\widehat{\alpha}_l, l) > \overline{W}(\widehat{\alpha}_n, n)$ holds whenever $\Phi(l) \geq 1$ is true, since the denominator is always smaller than 1 for any $n \in \mathbb{N}$.

Second, since $\Phi(l)$ is continuous and strictly increasing in m and that $\lim_{l\to\infty} \Phi(l) = (1+\mu) > 1$, there exists some l_0 sufficiently large such that $\Phi(l) \geq 1$ for all $l \geq l_0$. Fix any l such that $\Phi(l) \geq 1$, and we can define $\underline{c}_l$ by Equation (10).

Third, we can use the floor function $[x] = \{z \in \mathbb{Z} : z \leq x\}$ to define:

$$N(l) = \left[\frac{(l+1)^2}{(2-2\theta)\left(2 + (l-1)(2-2\theta)^{l-1}\right)}\right]$$

By construction, we have $\underline{c}_l > \bar{c}_N$. Therefore, we can define $A = [\bar{c}_n, \underline{c}_l)$ for any $n \geq N$ because $\bar{c}_n$ is strictly decreasing in n. By construction, $A = [\bar{c}_n, \underline{c}_l)$ is the desired set that satifies conditions (i)-(iii). The proof concludes. □

A.9 Proof of Proposition 3

Proof. We prove this result for all parameters one by one.

Case (i): Information production cost c. First, when $c = 0$, $\hat{\alpha}_n = 1$ for all $n \in \mathbb{N}$. Therefore, $n^* \to \infty$. Second, when $c > \bar{c}_1$, then $\hat{\alpha}_n = 0$, and thus $n^* \to \infty$. Then, the non-monotonicity of $n^*(c)$ follows from Corollary A.1 below.

Corollary A.1. *Consider n_1 such that $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. Then:*

(1) When $c < \underline{c}_{n_2}$ or $c > \bar{c}_{n_1}$, $\overline{W}(\widehat{\alpha}_{n_2}, n_2) > \overline{W}(\widehat{\alpha}_{n_1}, n_1)$; and

(2) When $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, $\overline{W}(\widehat{\alpha}_{n_2}, n_2) < \overline{W}(\widehat{\alpha}_{n_1}, n_1)$.

Note that Corollary A.1 follows directly from Theorem 1.

Case (ii): Price sensitivity b. First, when $b \to \infty$, we have $\Pi(\alpha) \to 0$, which implies that $\hat{\alpha}_n = 0$ for all $n \in \mathbb{N}$ and thus $n^* \to \infty$. Second, when $b \to 0$, then $\hat{\alpha}_n = 1$, and thus $n^* \to \infty$. Then, the non-monotonicity of $n^*(b)$ follows from Corollary A.1. To see it, select positive integers n_1 and n_2 such that: $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. By Corollary A.1, $n^* < n_2$ when $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, which translates into:

$$\frac{(2\theta-1)(A_H - A_L)(\bar{A} - MC)}{2(n_2+1)c} < b < \frac{(2\theta-1)(1-\theta)(2+(n_1-1)(2-2\theta)^{n_1-1})(A_H-A_L)(\bar{A}-MC)}{(n_1+1)^2 c}$$

Therefore, n^* is non-monotonic in b.

Case (iii): Market prospect in good state A_H. First, when $A_H \to \infty$, we have $\Pi(\alpha) \to \infty$, which implies that $\hat{\alpha}_n = 1$ for all $n \in \mathbb{N}$ and thus $n^* \to \infty$. Second, when $(A_H - A_L) \to 0$, then $\hat{\alpha}_n = 0$, and thus $n^* \to \infty$. Then, the non-monotonicity of n^* follows from Corollary A.1. To see it, select positive integers n_1 and n_2 such that: $\Phi(n_1) \geq 1$ and $n_2 \geq N(n_1)$. By Corollary A.1, $n^* < n_2$ when $\bar{c}_{n_2} < c < \underline{c}_{n_1}$, which translates into:

$$A_L + \frac{2(n_2+1)bc}{(2\theta-1)(\bar{A}-MC)} > A_H > A_L + \frac{(n_1+1)^2 bc}{(2\theta-1)(1-\theta)(2+(n_1-1)(2-2\theta)^{n_1-1})(\bar{A}-MC)}$$

Thus, $n^* < \infty$ can be finite. Therefore, n^* is non-monotonic in $(A_H - A_L)$. The proof concludes. □

A.10 Proof of Lemma 4

Proof. First, note that by the assumed condition $A_L = MC$, $4(\bar{A} - MC)^2 = (A_H - A_L)^2$. Thus, $\overline{W}(\widehat{\alpha}_1, 1) > \overline{W}(\widehat{\alpha}_2, 2)$ reduces to:

$$\frac{3}{32}(2 - \widehat{\gamma}_1) \geq \frac{1}{9}(2 - (\widehat{\gamma}_2)^2)$$

Second, when $c \geq \frac{(2\theta-1)(A_H-A_L)^2}{6b}$, by Equation (9), we have: $\widehat{\alpha}_2 = 0$ and thus $\widehat{\gamma}_2 = 1$. This further implies that $\overline{W}(\widehat{\alpha}_1, 1) > \overline{W}(\widehat{\alpha}_2, 2)$ if and only if $\widehat{\gamma}_1 \leq \frac{22}{27}$.

Finally, note that $\widehat{\gamma}_1$ is governed by Equation (8). Simple algebra yields the bound $c \leq \frac{11}{108}\kappa$. The other condition $c < \frac{(1-\theta)(2-\theta)\kappa}{9}$ follows from the definition of $\underline{c}$ for $n = 1$ and $n = 2$. Indeed, if $c < \min\{c_l(1), c_l(2)\}$, then $\widehat{\gamma}_1 = \widehat{\gamma}_2 = 1$, and thus $\overline{W}(\widehat{\alpha}_1, 1) \leq \overline{W}(\widehat{\alpha}_2, 2)$. The proof concludes. □

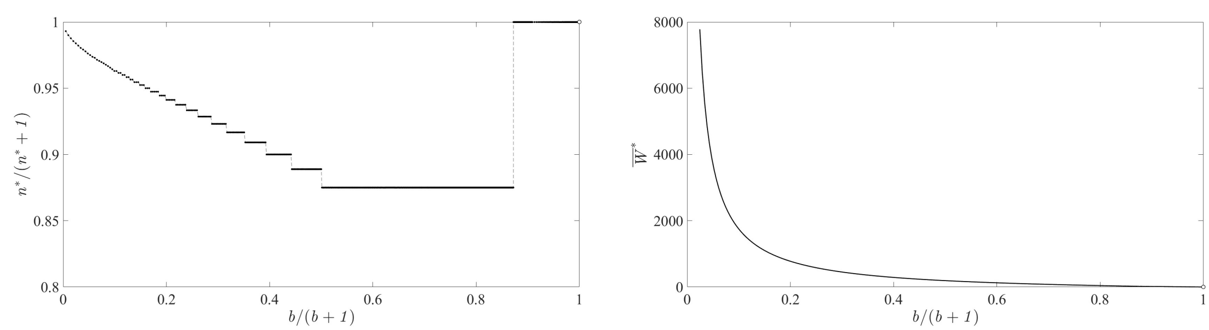
Online Appendix

B Extended Discussions

B.1 An Extended Discussion for Section 4.3

This section provides an additional discussion of comparative statics skipped in Section 4.3.

Price sensitivity b**.** Figure 17b depicts the optimal market structure $n^*/(n^*+1)$ and the corresponding total welfare $W(n^*)$ under the optimal market structure n^*. When b is high, the market price is very sensitive to the quantity of production, reducing profits for the firms and thus discouraging the production of information. Therefore, the information production gap disappears when we vary n, leading to a dominant role of market concentration. Similarly, when b is low, the market price is insensitive, increasing profits for all firms and thus enhancing information production. Again, the information production gap disappears when we vary n, and the market concentration channel becomes dominant. For an intermediate level of price sensitivity b, the information production gap can be relatively large when changing the number of firms in the market, and the information production channel can dominate that of market concentration. This pattern is illustrated in Figure 9a. However, note that a decrease in b always improves total welfare, because it directly increases firms' profits and consumer welfare and indirectly improves total welfare by enhancing information production.

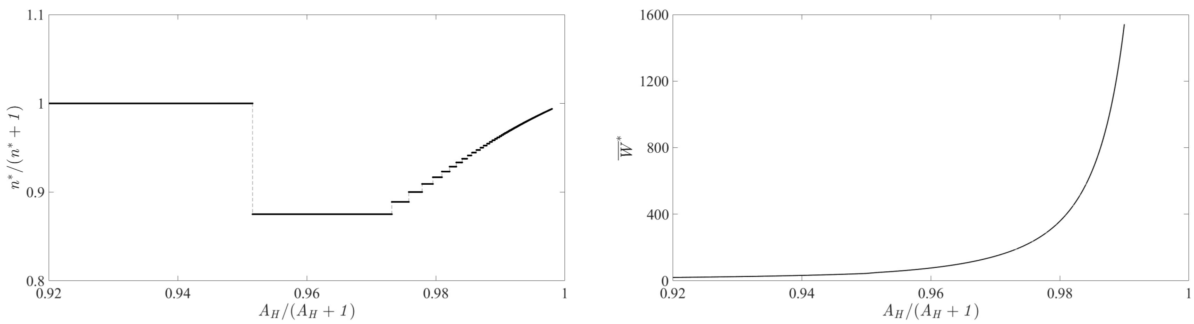


(a) Optimal Market Structure n^* (b) Total Welfare $W(n^*)$

Figure 9: Price Sensitivity b

Parameters: $\theta = 0.75$, $c = 1.5$, $MC = 3$, $A_H = 30$, $A_L = 10$.

Market prospect parameters A_H**.** Figure 10 depicts n^* and $W(n^*)$ when we vary the market prospect A_H in the good state $\omega = H$. Specifically, when A_H increases from zero to ∞, the optimal market structure n^* first decreases and then increases. Similar to other parameters, the total welfare under the optimal market structure always increases in A_H. Unlike other parameters, A_H affects the equilibrium through two forces, including market uncertainty $(A_H - A_L)$ and average profitability. These two forces can both increase information production (see, e.g., Equation (8)). However, their impacts on the optimal market structure can diverge, as illustrated in the discussion below, i.e., the negative relationship between competition and total welfare is more likely to occur when average profitability is relatively small (but not too tiny, otherwise the information production gap disappears) or the uncertainty is relatively large (but not too large). In other words, an increase in average profitability weakens, while an increase in market uncertainty reinforces the importance of the information production channel in the negative relationship between competition and total welfare.



(a) Optimal Market Structure n^* (b) Total Welfare $W(n^*)$

Figure 10: Market Prospect Paramter A_H

Parameters: $\theta = 0.75$, $b = 1.5$, $c = 1.5$, $MC = 3$, $A_L = 10$.

B.2 Feedback effects and allocative efficiency gains.

This section discusses allocative efficiency gains due to feedback effects. To this end, we first abstract away the cost reduction by focusing on the symmetric Cournot game. Specifically, we consider the cases of $\omega = H$ and $\omega = L$ separately.

(i) The case of $\omega = H$. From Lemma 1, we can derive that

$$\Pr(\forall i : q_i^* = q_M \mid \omega = H) = (\widehat{\gamma}_n)^n \quad \text{and} \quad \Pr(\forall i : q_i^* = q_H \mid \omega = H) = 1 - (\widehat{\gamma}_n)^n$$

Thus, with probability $1 - (\widehat{\gamma}_n)^n$, the true state $\omega = H$ is revealed, leading to a total output of $Q_H = \frac{n(A_H - MC)}{b(n+1)}$ and a price of $P_H = \frac{A_H + nMC}{(n+1)}$. However, with complementary probability $(\widehat{\gamma}_n)^n$, the prices of stocks are uninformative, leading to a lower total output $Q_M = \frac{n(\bar{A} - MC)}{b(n+1)} < Q_H$, and a higher price $P_{MH} = P_H + \frac{n(A_H - A_L)}{2(n+1)}$. Thus, $(\widehat{\gamma}_n)^n$ measures the probability of misallocation when $\omega = H$, and that of $\omega = L$ by symmetry.

Furthermore, since $\widehat{\gamma}_n = 1 - \widehat{\alpha}_n(2\theta - 1)$, a higher level of information production $\widehat{\alpha}_n$ reduces the probability of misallocation $(\widehat{\gamma}_n)^n$. Then, conditional on $\omega = H$, an increase in $\widehat{\alpha}_n$ leads to a lower (expected) price $\bar{P}_H(n)$ decreases, a larger consumer surplus $\overline{CS}_H(n)$, and ambiguous impact on $\Gamma_H(n)$, i.e.,

$$\bar{P}_H(n) = \frac{(A_H + nMC)}{(n+1)} + \frac{n\,(A_H - A_L)}{2(n+1)}\,(\widehat{\gamma}_n)^n$$

$$\overline{CS}_H(n) = \frac{n^2}{2b(n+1)^2}\left\{(A_H - MC)^2 - \left((A_H - MC)^2 - (\bar{A} - MC)^2\right)(\widehat{\gamma}_n)^n\right\}$$

$$\Gamma_H(n) = \frac{n}{2b(n+1)^2} \left\{ 2\left(A_H - MC\right)^2 + \left(A_H - A_L\right) \left[n\left(\bar{A} - MC\right) - \left(A_H - MC\right) \right] \left(\widehat{\gamma}_n\right)^n \right\}$$

We now discuss the implications of a horizontal merger. Specifically, by Proposition 1, it increases post-merger information production and decreases the probability of misallocation (i.e., $\left(\widehat{\gamma}_{n-1}\right)^{n-1} < \left(\widehat{\gamma}_n\right)^n$). The price effect of the merger in $\omega = H$ then is

$$\Delta \bar{P}_H(n) = \bar{P}_H(n-1) - \bar{P}_H(n) = \frac{A_H - MC}{n(n+1)} + \frac{A_H - A_L}{2n(n+1)} \left[\left(n^2 - 1\right) \left(\widehat{\gamma}_{n-1}\right)^{n-1} - n^2 \left(\widehat{\gamma}_n\right)^n \right]$$

Interestingly, a lower post-merger price can occur (i.e., $\Delta \bar{P}_H(n) < 0$) when the informational feedback effect is sufficiently strong such that

$$\left(\widehat{\gamma}_{n-1}\right)^{n-1} < \frac{n^2}{\left(n^2 - 1\right)} \left(\widehat{\gamma}_n\right)^n - \frac{2\left(A_H - MC\right)}{\left(n^2 - 1\right)\left(A_H - A_L\right)}. \tag{B.1}$$

Similarly, the change in consumer surplus $\Delta \overline{CS}_H(n)$ due to the merger is given by:

$$\begin{aligned} \Delta \overline{CS}_H(n) = \overline{CS}_H(n-1) - \overline{CS}_H(n) = & \frac{\left(A_H + \bar{A} - 2MC\right)\left(A_H - A_L\right)}{2bn^2(n+1)^2} \\ & \times \left\{ n^4 * \left(\widehat{\gamma}_n\right)^n - \left(n^2 - 1\right)^2 \left(\widehat{\gamma}_{n-1}\right)^{n-1} - \frac{\left(2n^2 - 1\right)\left(A_H - MC\right)^2}{\left(A_H + \bar{A} - 2MC\right)\left(A_H - A_L\right)} \right\} \end{aligned}$$

When Equation (B.1) holds, consumer surplus also increases, i.e., $\Delta \overline{CS}_H(n) > 0$.[19]

(ii) The case of $\omega = L$. Similarly, with probability $1 - \left(\widehat{\gamma}_n\right)^n$, the true state $\omega = L$ is revealed, leading to a total output of $Q_L = \frac{n(A_L - MC)}{b(n+1)}$ and a price of $P_L = \frac{A_L + nMC}{(n+1)}$. With complementary probability $\left(\widehat{\gamma}_n\right)^n$, stock prices are uninformative, leading to a total output of $Q_M = \frac{n\left(\bar{A} - MC\right)}{b(n+1)}$ and a price of $P_{ML} = \frac{A_L + nMC}{(n+1)} - \frac{n(A_H - A_L)}{2(n+1)}$.

Conditional on $\omega = L$, an increase in information production and thus a lower proability misallocation $\left(\widehat{\gamma}_n\right)^n$ leads to a higher price $\bar{P}_L(n) = E[P \mid \omega = L] = \frac{A_L + nMC}{(n+1)} - \frac{n(A_H - A_L)}{2(n+1)} \left(\widehat{\gamma}_n\right)^n$, a lower post-merger consumer surplus, and an ambiguous impact on firms' profits, which can be positive when the change in information production is large. The economic efficiency is therefore mainly achieved through improving firm profits in this case. In particular, define $\Gamma_L(n) := \mathbb{E}\left[\sum_{i=1}^n TP_i \mid \omega = L\right]$ and $\Delta \Gamma_L(n) := \Gamma_L(n-1) - \Gamma_L(n)$. Then, the net change in firms' profits is given by:

$$\begin{aligned} \Delta \Gamma_L(n) = & \frac{\left(A_L - MC\right)^2 \left(n^2 - n - 1\right)}{bn^2(n+1)^2} + \frac{n\left(A_H - A_L\right)\left(\widehat{\gamma}_n\right)^n}{2b(n+1)^2} * \left(n\left(\bar{A} - MC\right) - \left(A_L - MC\right)\right) \\ & - \frac{(n-1)\left(A_H - A_L\right) * \left(\widehat{\gamma}_{n-1}\right)^{n-1}}{2bn^2} * \left((n-1)\left(\bar{A} - MC\right) - \left(A_L - MC\right)\right) \end{aligned}$$

which can be positive when $\widehat{\gamma}_{n-1}$ is sufficiently small.

[19] Note that $\Delta \overline{CS}_H(n) > \frac{\left(A_H + \bar{A} - 2MC\right)\left(A_H - A_L\right)}{2bn^2(n+1)^2} \times \left\{ n^2 \left(\widehat{\gamma}_n\right)^n + \frac{2\left(n^2 - 1\right)\left(A_H - MC\right)}{\left(A_H - A_L\right)} - \frac{2\left(2n^2 - 1\right)\left(A_H - MC\right)}{\left(3A_H + A_L - 4MC\right)} \frac{\left(A_H - MC\right)}{\left(A_H - A_L\right)} \right\}$ $> \frac{\left(A_H + \bar{A} - 2MC\right)\left(A_H - A_L\right)}{2bn^2(n+1)^2} * \left\{ n^2 * (\hat{\gamma})_n^n + \frac{2(n^2 - 2)(A_H - MC)}{3(A_H - A_L)} \right\} > 0.$

In summary, a new type of allocative efficiency gain associated with a horizontal merger ensues with informational feedback in a perfectly symmetric Cournot game.

B.3 Equilibrium Analysis in Section 5.2

This section analyzes the equilibrium for the cross-asset trading setup in Section 5.2. We first solve the equilibrium, taking as given the measures of informed speculators $\boldsymbol{\alpha}$, which is then determined by investigating the incentive for information acquisition. Analogous to Lemma 1, given $\boldsymbol{\alpha}$, the stock price $s_i(f_i)$ is determined as:

$$s_i(f_i) = \begin{cases} s_H & \text{if } f_i \in (\gamma_i^{LS}, \infty); \\ s_M^j & \text{if } f_i \in [-\gamma_i^{LS}, \gamma_i^{LS}]; \\ s_L & \text{if } f_i \in (-\infty, -\gamma_i^{LS}). \end{cases}$$

where $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$, $s_M^j = \frac{1}{4(n+1)^2 b} * \left(2(A_H - MC)^2 + 2(A_L - MC)^2 - \beta_i^{LS}(A_H - A_L)^2\right)$, $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$, $\gamma_i^{LS} = 1 - (2\theta - 1)(\alpha_L + \alpha_{i,S})$ and $\beta_i^{LS} = \prod_{j \neq i} \gamma_i^{LS}$.

Furthermore, the ith firm's optimal production strategy, conditional on the stock prices observed, is given by:

$$q_i^*(\boldsymbol{s}) = \begin{cases} q_H & \text{if } \exists j \in \{1, \ldots, n\} : s_j = s_H; \\ q_M & \text{if } \forall j \in \{1, \ldots, n\} : s_j = s_M^j; \\ q_L & \text{if } \exists j \in \{1, \ldots, n\} : s_j = s_L. \end{cases}$$

where $q_H = \frac{A_H - MC}{(n+1)b}$, $\bar{A} = \frac{1}{2}(A_H + A_L)$, $q_M = \frac{\bar{A} - MC}{(n+1)b}$, and $q_L = \frac{A_L - MC}{(n+1)b}$.

Next, we endogenize the measure of informed traders $\boldsymbol{\alpha}$. Specifically, for an informed L-trader k with a private signal m_k, the optimal trading strategy is to hold $y_k^j = +1$ ($y_k^j = -1$) share of each firm $j \in \{1, \ldots, n\}$ when $m_k = H$ ($m_k = L$), leading to an expected trading profit given by:

$$\Pi_L(\boldsymbol{\alpha}) = \frac{\left(\bar{A} - MC\right)(A_H - A_L)(2\theta - 1)\sum_{j=1}^n \gamma_j^{LS}\left(2 + (n-1)\beta_j^{LS}\right)}{2b(n+1)^2}$$

Similarly, for an informed S-trader k with a private signal m_k^i, the optimal trading strategy is to buy $x_k^i = +1$ shares of the ith stock when $m_k^i = H$, and sell $x_k^i = -1$ shares of the ith stock when $m_k^i = L$. This leads to an expected trading profit:

$$\Pi_S^i(\boldsymbol{\alpha}) = \frac{\left(\bar{A} - MC\right)(A_H - A_L)(2\theta - 1)\gamma_i^{LS}\left(2 + (n-1)\beta_i^{LS}\right)}{2b(n+1)^2}$$

Since all firms in the Cournot competition are identical, we can focus on the symmetric equilibrium in which $\alpha_{i,S} = \alpha_S$. Then, with information acquisition, the expected profits for the L- and S-traders can be further written as: $\Pi_L(\boldsymbol{\alpha}) = n\Pi_S(\boldsymbol{\alpha})$ and

$$\Pi_S(\boldsymbol{\alpha}) = \Pi_S(\alpha_L, \alpha_S) = \frac{\left(\bar{A} - MC\right)(A_H - A_L)(2\theta - 1)\gamma^{LS}\left(2 + (n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2} \tag{B.2}$$

where $\gamma^{LS} = 1 - (2\theta - 1)(\alpha_L + \alpha_S)$.

By comparing $\Pi_L(\boldsymbol{\alpha})$ and $\Pi_S(\boldsymbol{\alpha})$, we can observe that L-traders have a stronger incentive to acquire information than S-traders, given that $c_L \leq c_S$.This further implies: (1) if $\alpha_S > 0$, then $\alpha_L = \lambda$; and (2) if $\alpha_L < \lambda$, then $\alpha_S = 0$. Using this property, we can derive the optimal strategies for information production as follows.

Lemma B.1 (Information Production)**.** *The equilibrium intensity of information production* $(\widetilde{\alpha}_L, \widetilde{\alpha}_S)$ *satisfies the following:*

(i) when $c_L \geq \Pi_L(0,0)$*, then* $\widetilde{\alpha}_L = \widetilde{\alpha}_S = 0$*;*

(ii) when $\Pi_L(\lambda, 0) < c_L < \Pi_L(0,0)$*, then* $\widetilde{\alpha}_S = 0$ *and* $\widetilde{\alpha}_L \in (0, \lambda)$*, where* $\Pi_L(\widetilde{\alpha}_L, 0) = c_L$*;*

(iii) when $c_L < \Pi_L(\lambda, 0)$ *and* $c_S \geq \Pi_S(\lambda, 0)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S = 0$*;*

(iv) when $c_L < \Pi_L(\lambda, 0)$ *and* $\Pi_S(\lambda, 1-\lambda) < c_S < \Pi_S(\lambda, 0)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S \in (0, 1-\lambda)$*, where* $\Pi_S(\lambda, \widetilde{\alpha}_S) = c_S$*; and*

(v) when $c_L < \Pi_L(\lambda, 0)$ *and* $c_S \leq \Pi_S(\lambda, 1-\lambda)$*, then* $\widetilde{\alpha}_L = \lambda$ *and* $\widetilde{\alpha}_S = 1 - \lambda$*.*

Define $\widetilde{\boldsymbol{\alpha}}_n := \widetilde{\boldsymbol{\alpha}}(n)$. Finally, following the derivation of Equation (13), we can compute the expected total welfare $\widetilde{W}(\widetilde{\boldsymbol{\alpha}}_n, n)$ as follows:

$$\widetilde{W}(\widetilde{\boldsymbol{\alpha}}_n, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widetilde{\gamma}^{LS})^n\right)(A_H - A_L)^2\right) \tag{B.3}$$

where $\widetilde{\gamma}^{LS} = 1 - (\widetilde{\alpha}_L + \widehat{\alpha}_S) \times (2\theta - 1)$.

Furthermore, define $\gamma_S = 1 - (2\theta - 1)(\lambda + \widetilde{\alpha}_S)$, $\gamma_L = 1 - \widetilde{\alpha}_L(2\theta - 1)$,

$$g_S(\widetilde{\alpha}_S, n) = 2\gamma_S^n + \frac{n(n+2)\gamma_S^n}{2 + n(n-1)\gamma_S^{n-1}}\left(4n + n(n-3)\gamma_S^{n-1} - 2(n+1)\ln\frac{1}{\gamma_S}\right)$$

and

$$g_L(\widetilde{\alpha}_L, n) = \frac{(\gamma_L)^n \times \left(2n(n-1)(n+2) + 4 - 3n^2(n+1)\gamma_L^{n-1} - 2n(n+1)(n+2)\ln\frac{1}{\gamma_L}\right)}{2 + n(n-1)\gamma_L^{n-1}}$$

With the aid of Equation (B.3), we can check the relationship between competition and total welfare when an interior solution arises for information production.

Lemma B.2 (Competition and Welfare with Cross-Asset Trading)**.** *Product competition decreases total welfare* $\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)$*, i.e.,* $\frac{d\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)}{dn} < 0$*, when:*

(i) $g_S(\widetilde{\alpha}_S, n) > G_1(A_H, A_L, MC)$ *in Case 1 such that* $\widetilde{\alpha}_L = \lambda$*; and*

(ii) $g_L(\widetilde{\alpha}_L, n) > G_1(A_H, A_L, MC)$ *in Case 2 so that* $\widetilde{\alpha}_S = 0$.

We make two comments. First, Lemma B.2 verifies the validity of our key result on the non-monotonic relationship between competition and total welfare in the presence of L-traders. The numerical insights are similar and are shown in Section B.3.

Second, the incentive for information production can increase with the number of firms for L-traders (i.e., $\frac{d\widetilde{\alpha}_L}{dn} > 0$ for a certain range of n when $\widetilde{\alpha}_S = 0$), which differs significantly from the case for S-traders when $\lambda = 0$ (i.e., $\frac{d\widetilde{\alpha}_S}{dn} < 0$ by Proposition 1). This complexity is illustrated in

Figure 8. In particular, when we move from a monopoly ($n = 1$) to a duopoly ($n = 2$), the size of the informed L-traders $\widetilde{\alpha}_L$ first increases and then decreases when n increases. To understand this non-monotonicity, we plug in $\widetilde{\alpha}_S = 0$ and use Equation (B.2) to obtain:

$$\Pi_L(\boldsymbol{\alpha}) = \Pi_S(\alpha_L, \alpha_S) = \frac{n\widetilde{\gamma}\left(\bar{A} - MC\right)\left(A_H - A_L\right)(2\theta - 1)\left(2 + (n-1)\widetilde{\gamma}^{n-1}\right)}{2b(n+1)^2}$$

where $\widetilde{\gamma} = 1 - (2\theta - 1)\widetilde{\alpha}_L$. We can further compute:

$$\frac{\partial \Pi_L}{\partial n} = \frac{(2\theta - 1)\left(A_H - A_L\right)\left(\bar{A} - MC\right)}{2b(n+1)^3} \times \left\{\widetilde{\gamma}^n(3n-1) - 2\widetilde{\gamma}(n-1) - \left(\log\frac{1}{\widetilde{\gamma}}\right)\widetilde{\gamma}^n n(n-1)(n+1)\right\}$$

Therefore, it is possible that $\frac{\partial \Pi_L}{\partial n} > 0$. For example, when α_L is sufficiently small,

$$\frac{\partial \Pi_L}{\partial n} = \frac{(2\theta - 1)\left(A_H - A_L\right)\left(\bar{A} - MC\right)}{2b(n+1)^2} + \frac{n(n-1)\widetilde{\alpha}_L}{(n+1)^2} \times O(1) > 0$$

Note that $\frac{\partial \Pi_L}{\partial n} > 0$ implies that increased competition in the product market can strengthen the incentive for L-traders to acquire and trade on private information. Intuitively, as shown in Vives (1985), the profit of firms converges to zero at a speed of $1/n$. When multiplied by the number of firms n, the trading profits for L-traders can be non-monotonicity in n. We term this the "trading opportunity effect" in cross-asset trading.

Numerical Analysis Here, we use numerical methods to verify that the basic insights still hold when there are both L-traders and S-traders in the stock market. Again, let $\Delta\widetilde{W}_n$ denote the incremental change in total welfare when the number of firms increases from $(n-1)$ to n, i.e., $\Delta\widetilde{W}_n = \widetilde{W}\left(\widetilde{\boldsymbol{\alpha}}_n, n\right) - \widetilde{W}\left(\widetilde{\boldsymbol{\alpha}}_{n-1}, n-1\right)$.

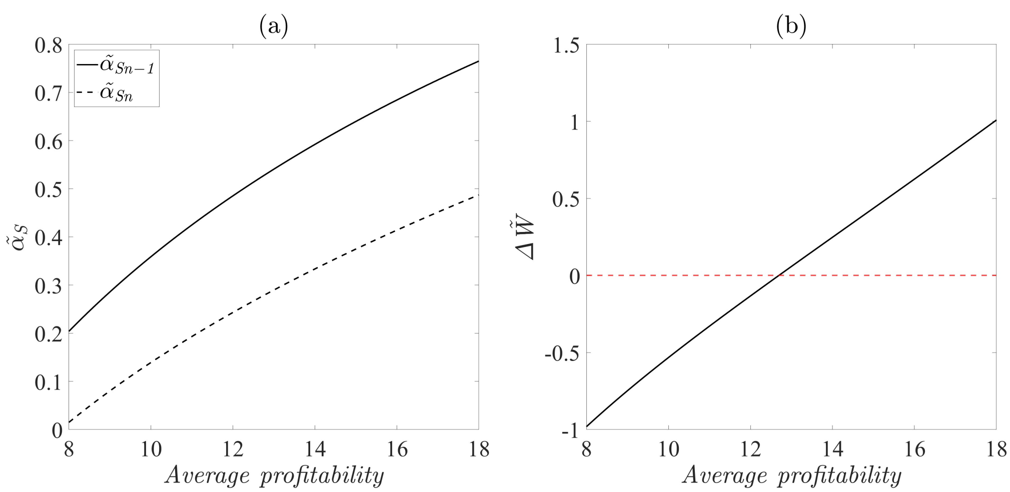


Figure 11: Average Profitability, Information Quality and Welfare.

Parameters: $A_H - A_L = 10, b = 1.5, \theta = 0.75, n = 5, MC = 3, c_L = c_S = 1.5, \lambda = 0.2.$

Remark: (Case 1) the intensity of information production for L-traders satisfies: $\widetilde{\alpha}_L = \lambda$.

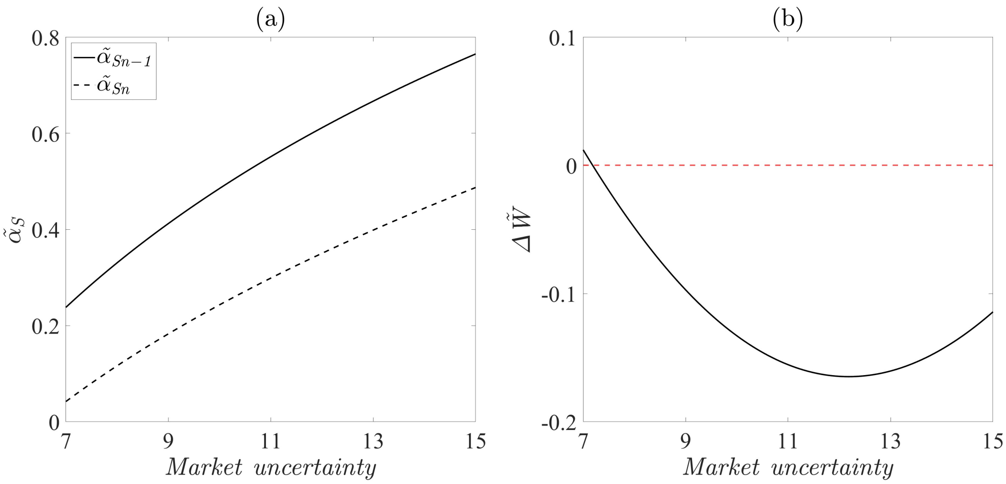


Figure 12: Uncertainty, Information Quality and Welfare.

Parameters: $\bar{A} = 15, b = 1.5, \theta = 0.75, n = 5, MC = 3, c_L = c_S = 1.5, \lambda = 0.2$.

Remark: (Case 1) the intensity of information production for L-traders satisfies: $\widetilde{\alpha}_L = \lambda$.

First, Figure 11 illustrates how average profitability $\left(\bar{A} - MC\right)$ affects information production $\widetilde{\alpha}$ and total welfare $\Delta\widetilde{W}_n$ when all L-traders choose to acquire information. Specifically, similar to Figure 6, it delivers three messages, including: (1) the intensity of information production $\widetilde{\boldsymbol{\alpha}}_n$ decreases in the number of firms n; (2) both $\widetilde{\boldsymbol{\alpha}}_n$ and $\widetilde{\boldsymbol{\alpha}}_{n-1}$ increase the average profitability $\left(\bar{A} - MC\right)$; and (3) the welfare gain $\Delta\widetilde{W}_n$ is smaller for a lower average profitability, which can even be negative when the average profitability is sufficiently low.

Furthermore, Figure 12 shows the impact of uncertainty, measured by $(A_H - A_L)$, on information production and total welfare. Specifically, it delivers three messages, including: (1) the intensity of information production $\widetilde{\boldsymbol{\alpha}}_n$ decreases in the number of firms n; (2) both $\widetilde{\boldsymbol{\alpha}}_n$ and $\widetilde{\boldsymbol{\alpha}}_{n-1}$ increase in market uncertainty $(A_H - A_L)$; and (3) the incremental welfare change can be negative when market uncertainty $(A_H - A_L)$ is high. Finally, a similar pattern ensues when all S-traders abstain from acquiring information and only a fraction of L-traders choose to produce information.

B.4 Equilibrium Analysis in Section 5.3

Equilibrium Analysis. Recall that we let α_L and $\alpha_{i,S}$ denote the measure of informed L-traders and that of informed S-traders for the ith firm, and the size of L-traders is $\lambda = 0$. We first solve the equilibrium for a fixed $\boldsymbol{\alpha}$. Specifically:

$$s_i(\Omega) = \begin{cases} s_H & \text{if } \exists j : f_j \in (\gamma_j^{LS}, \infty); \\ s_M & \text{if } \forall j : f_j \in [-\gamma_j^{LS}, \gamma_j^{LS}]; \\ s_L & \text{if } \exists j : f_j \in (-\infty, -\gamma_j^{LS}). \end{cases} \tag{B.4}$$

where $s_H = \frac{(A_H - MC)^2}{(n+1)^2 b}$, $s_M = \frac{\left(\bar{A} - MC\right)^2}{(n+1)^2 b}$, $s_L = \frac{(A_L - MC)^2}{(n+1)^2 b}$, and $\gamma_i^{LS} = 1 - (2\theta - 1)(\alpha_L + \alpha_{i,S})$.

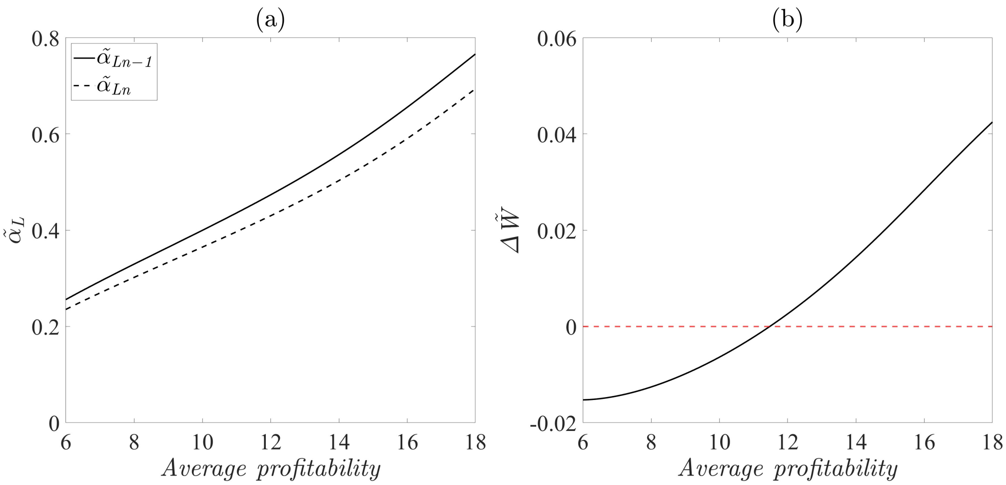


Figure 13: Average Profitability, Information Quality and Welfare.

Parameters: $A_H - A_L = 10, b = 2.5, \theta = 0.75, n = 14, MC = 6.5, c_L = c_S = 1.5, \lambda = 0.8.$

Remark: (Case 2) the intensity of information production for S-traders satisfies: $\widetilde{\alpha}_S = 0$.

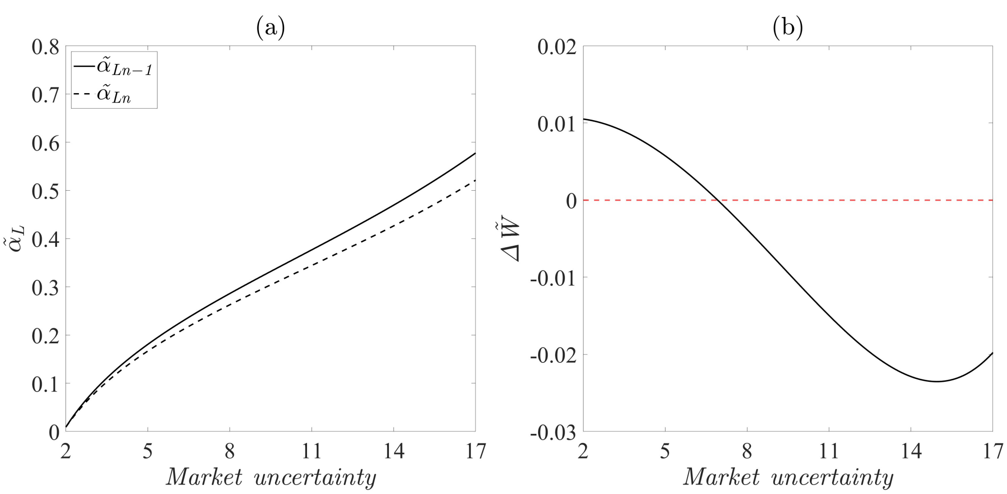


Figure 14: Uncertainty, Information Quality and Welfare.

Parameters: $A_H = 20, A_L = 10, b = 2.5, \theta = 0.75, n = 14, MC = 6.5, c_L = c_S = 1.5, \lambda = 0.8.$

Remark: (Case 2) the intensity of information production for S-traders satisfies: $\widetilde{\alpha}_S = 0$.

Furthermore, the ith firm optimally chooses production based on observed stock prices:

$$q_i^*(\boldsymbol{s}) = \begin{cases} q_H & \text{if } \exists j : s_j = s_H; \\ q_M & \text{if } \forall j : s_j = s_M; \\ q_L & \text{if } \exists j : s_j = s_L. \end{cases}$$

where $q_H = \frac{A_H - MC}{(n+1)b}$, $q_M = \frac{\bar{A} - MC}{(n+1)b}$ and $q_L = \frac{A_L - MC}{(n+1)b}$.

Again, for an informed L-trader k with a private signal m_k, the optimal trading strategy is to buy $y_k^j = +1$ ($y_k^j = -1$) share of each firm j when $m_k = H$ ($m_k = L$), leading to an expected trading profit given by:

$$\Pi_{L,C}(\boldsymbol{\alpha}) = \frac{n(2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)\left(\prod_{j=1}^n \gamma_j^{LS}\right)}{2b(n+1)^2}$$

Similarly, for an informed S-trader k with a private signal m_k^i, the optimal trading strategy is to buy $x_k^i = +1$ shares of the ith stock when $m_k^i = H$, and sell $x_k^i = -1$ shares of the ith stock when $m_k^i = L$, leading to an expected trading profit of:

$$\Pi_{S,C}(\boldsymbol{\alpha}) = \frac{(2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)\left(\prod_{j=1}^n \gamma_j^{LS}\right)}{2b(n+1)^2}$$

Here, the symbol "C" in the subscript means "cross-asset learning".

By focusing on the symmetric equilibrium (i.e., $\alpha_{i,S} = \alpha_S$), the expected profits for the L- and S-traders can be further written as: $\Pi_L(\boldsymbol{\alpha}) = n\Pi_S(\boldsymbol{\alpha})$ and

$$\Pi_{S,C}(\boldsymbol{\alpha}) = \frac{(2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)\gamma_{LS}^n}{2b(n+1)^2} \tag{B.5}$$

where $\gamma^{LS} = 1 - (2\theta - 1)(\alpha_L + \alpha_S)$.

Now, we turn to equilibrium information production. Define

$$\nu = \frac{1}{(2\theta-1)} - \frac{1}{(2\theta-1)}\left(\frac{2bc_L(n+1)}{n(2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)}\right)^{1/n}, \quad \text{and}$$

$$\xi = \frac{1}{(2\theta-1)} - \frac{1}{(2\theta-1)}\left(\frac{2bc_S(n+1)}{n(2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)}\right)^{1/n} - \lambda$$

Lemma B.3 (Information Production). *The equilibrium intensity of information production $(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C})$ satisfies the following:*

(i) when $c_L \geq \Pi_{L,C}(0,0)$, then $\widetilde{\alpha}_{L,C} = \widetilde{\alpha}_{S,C} = 0$;

(ii) when $\Pi_{L,C}(\lambda, 0) < c_L < \Pi_{L,C}(0,0)$, then $\widetilde{\alpha}_{S,C} = 0$ and $\widetilde{\alpha}_{L,C} = \nu \in (0, \lambda)$;

(iii) when $c_L < \Pi_{L,C}(\lambda, 0)$ and $c_S \geq \Pi_{S,C}(\lambda, 0)$, then $\widetilde{\alpha}_{L,C} = \lambda$ and $\widetilde{\alpha}_{S,C} = 0$;

(iv) when $c_L < \Pi_{L,C}(\lambda, 0)$ and $\Pi_{S,C}(\lambda, 1-\lambda) < c_S < \Pi_{S,C}(\lambda, 0)$, then $\widetilde{\alpha}_{L,C} = \lambda$ and $\widetilde{\alpha}_{S,C} = \xi \in (0, 1-\lambda)$; and

(v) when $c_L < \Pi_{L,C}(\lambda, 0)$ and $c_S \leq \Pi_{S,C}(\lambda, 1-\lambda)$, then $\widetilde{\alpha}_{L,C} = \lambda$ and $\widetilde{\alpha}_{S,C} = 1 - \lambda$.

Define $\widetilde{\boldsymbol{\alpha}}_n := \widetilde{\boldsymbol{\alpha}}(n)$. Finally, following the derivation of Equation (13), we can compute the expected total welfare $\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n, n)$ as follows:

$$\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4\left(\bar{A} - MC\right)^2 + \left(1 - (\widetilde{\gamma}^{LS})^n\right)(A_H - A_L)^2\right) \tag{B.6}$$

where $\widetilde{\gamma}^{LS} = 1 - (2\theta - 1) * (\widetilde{\alpha}_L + \widetilde{\alpha}_S)$.

Recall that $\gamma_S = 1 - (2\theta - 1)(\lambda + \widetilde{\alpha}_S)$, $\gamma_L = 1 - \widetilde{\alpha}_L(2\theta - 1)$. Define

$$g_{S,C}(\gamma_S, n) = (\gamma_S)^n(2 + n(n+1)(n+2)).$$

Lemma B.4 (Competition and Welfare with Cross-Asset Learning)**.**

(i) Case 1: $\widetilde{\alpha}_{L,C} = \lambda$. Then, the total welfare decreases in the number of firms n (i.e., $\frac{d\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n,n)}{dn} < 0$) if and only if $g_{S,C}(\xi, n) > G_1(A_H, A_L, MC)$; and

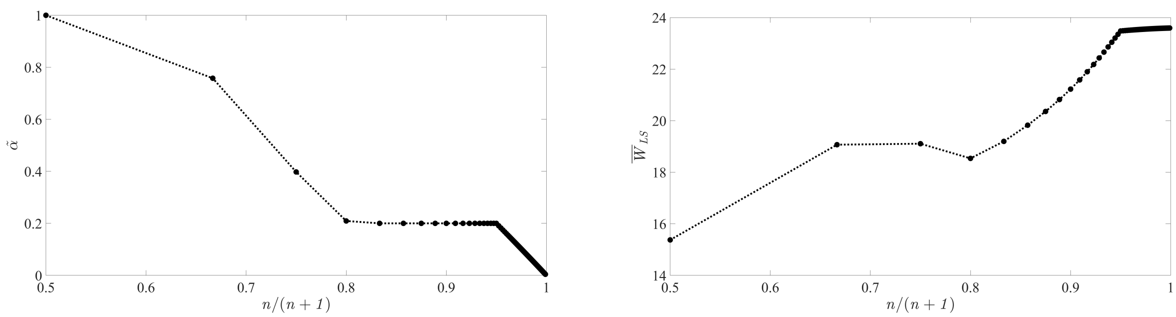
(ii) Case 2: $\widetilde{\alpha}_{S,C} = 0$. Then, the total welfare increases strictly in the number of firms n, i.e., $\frac{d\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n,n)}{dn} > 0$.

Lemma B.4 requires several additional clarifications, given that market makers can observe the flow of orders in all stocks. First, when there are only S-traders in the stock market (i.e., $\lambda = 0$ and thus $\widetilde{\alpha}_{L,C} = 0 = \lambda$ always holds), the nonmonotonic relationship between competition and total welfare still holds. Second, the non-monotonicity also holds when the cost of information production is small such that $\widetilde{\alpha}_{L,C} = \lambda$. Note that L-traders have a stronger incentive to acquire information, compared to S-traders. Third, when there are only L-traders (i.e., $\lambda = 1$ and thus $\widetilde{\alpha}_{S,C} = 0$ always holds), the total welfare increases strictly in the number of firms n. In other words, the non-monotonic relationship between competition and total welfare holds when we allow cross-asset trading by L-traders or cross-asset learning by market makers, but not both. Intuitively, there are two economic forces behind this. On the one hand, as discussed in Section 5.2, intensified competition can improve trading profits for L-traders by granting them more trading opportunities. On the other hand, cross-asset learning provides market makers with more information, decreasing speculators' trading profits, and information production in equilibrium. In summary, both the trading opportunity effect and the cross-asset learning effect reduce the impact of the information production channel. A more detailed discussion about the divergent impact of cross-asset learning on L-traders and S-traders can be found in Appendix B.4.

We first illustrate how competition shapes information production and total welfare when market makers can observe the order flow of all stocks.

Numerical analysis. With intensified Cournot competition ($n \uparrow$), the incentive to acquire information weakly decreases. This is illustrated in Figure 15a. First, when $n \leq 4$, an increase in n reduces the measure of informed S-traders, who have a relatively smaller incentive to acquire information. Second, when $4 < n \leq 18$, S-traders quit from acquiring information and trading on private information, while all L-traders choose to acquire information. Third, when $n \geq 18$, an increase in n further reduces the incentive for L-traders to acquire information.

Correspondingly, Figure 15b depicts total welfare when the number of firms n increases. When $n \leq 4$, total welfare first increases and then decreases and reaches a local minimum when all S-



(a) Information Production (b) Total Welfare

Figure 15: Price Sensitivity b

Parameters: $\lambda = 0.2$, $\theta = 0.75$, $b = 1.5$, $A_H = 20$, $A_L = 10$, $MC = 8$, and $c_L = c_S = 1.5$.

traders abstain from information production. However, when $n \geq 4$, total welfare increases strictly in the number of firms, indicating a dominant role of the market concentration channel.

Understanding the impact of cross-asset learning. By Lemma B.4, cross-asset learning affects L-traders differently from S-traders. Here, we show that this complexity is primarily caused by the combination of the trading opportunity effect and the cross-asset learning effect.

(i) Cross-asset learning effect.

Specifically, with cross-asset learning, market makers can observe the order flow of all stocks, enabling more efficient pricing against informed speculators. Thus, trading profits decrease for both L-traders and S-traders and are lower than those without cross-asset learning. Indeed, given $\widetilde{\gamma}^{LS}$ (or equivalently, $\widetilde{\alpha}_{L,C} + \widetilde{\alpha}_{S,C}$), we have:

$$\frac{\Pi_{L,C}}{\Pi_L} = \frac{\Pi_{S,C}}{\Pi_S} = f_C(n) \tag{B.7}$$

where $f_C(n) = \frac{(n+1)}{2(\widetilde{\gamma}^{LS})^{1-n}+(n-1)}$. Obviously, $f_C(n) \in (0,1)$ and $f'_C(n) < 0$. Therefore, the trading profits of an informed L-trader and an informed S-trader will shrink proportionally by a ratio of $f_C(n)$ when market makers can observe the order flow of all stocks, and this effect is more pronounced when n is large.

(ii) Trading opportunity effect.

This effect arises from the opportunity to access all stock, and thus only exists for L-traders. Unlike an S-trader with small trading opportunities, an L-trader can earn a higher trading profit by acquiring costly information, i.e., $\Pi_L = n\Pi_S$ and $\Pi_{L,C} = n\Pi_{S,C}$. Therefore, the expected trading profit of an L-trader can increase with n, especially when n is small. For example, we can verify that $\frac{\partial \Pi_L}{\partial n} > 0$ for $n = 1$, which differs from the case with an S-trader whose expected trading profit always decreases in n. However, note that $\frac{\partial \Pi_L}{\partial n} < 0$ when n is large enough. Figure 16 illustrates the pattern of trading profits with (blue dashed line) and without (red solid line) cross-asset learning by market makers.

We now examine how cross-asset learning affects the incentive for information production. We first consider S-traders, whose expected trading profits Π_S strictly decrease in n and are further

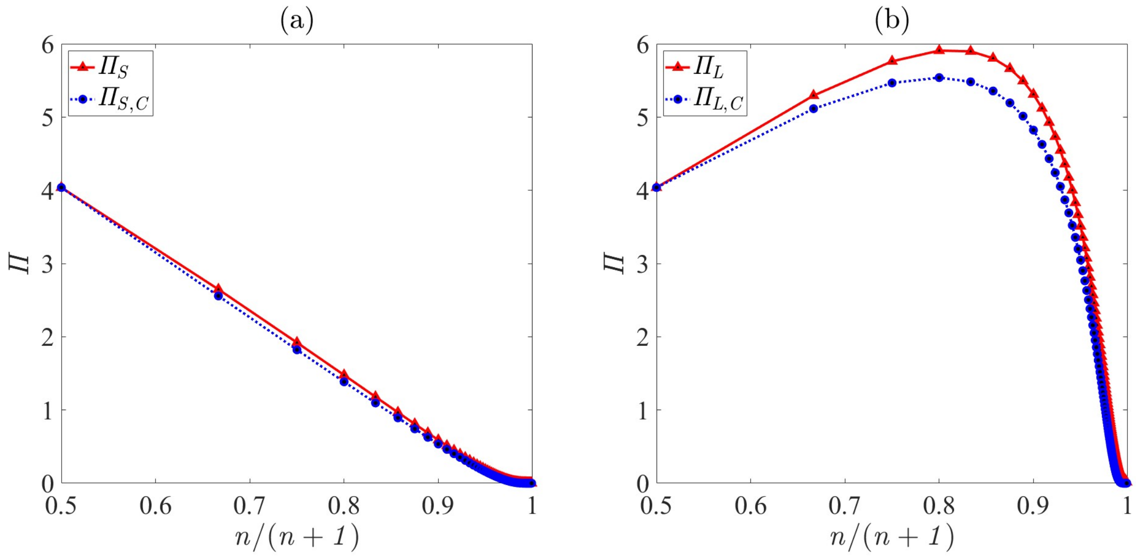


Figure 16: Trading profits with/witout cross-asset learning

Parameters: $\theta = 0.75$, $b = 2.5$, $A_H = 20$, $A_L = 10$, $MC = 6.5$, and $\widetilde{\alpha}_{L,C} + \widetilde{\alpha}_{S,C} = 0.1$.

reduced by cross-asset learning (i.e., $\frac{d\Pi_{S,C}}{dn} < 0$). Note that $\Pi_S = \Pi_{S,C}$ when $n = 1$ or $n \to \infty$. Then, one would expect that when n is relatively small, $\Pi_{S,C}$ decreases relatively faster than $\Pi_{S,C}$ as n increases. This is illustrated in panel (a) of Figure 16. Therefore, with cross-asset learning, the expected trading profit of an informed S-trader exhibits a higher level of sensitivity in the number of firms (n), which implies that intensified market competition can further reduce the incentive for S-traders to trade on proprietary information compared to the case without cross-asset learning. In other words, it reinforces the informational feedback channel, leading to a stronger (negative) effect of competition on real efficiency.

Next, we consider L-traders, whose expected trading profits Π_L are non-monotonic in n. Specifically, due to the trading opportunity effect, Π_L first increases and then decreases, generating an inverted U-shape pattern when n increases. Similarly, cross-asset learning also decreases the expected trading profit $\Pi_{L,C}$ for L-traders and flattens the inverted U-shape pattern, as shown in panel (b) of Figure 16. Thus, with cross-asset learning by market makers, the expected trading profit of an informed L-trader becomes less sensitive to the number of firms (n) when n is relatively small, leading to weaker informational feedback effects. Therefore, the non-monotonic link between competition and total welfare fails because the trading opportunity effect and cross-asset learning reinforce each other.

As a final remark, Figure 16 appears to indicate that the expected trading profits Π_L and $\Pi_{L,C}$ for L-traders are relatively more sensitive to changes in n when n is large, compared to those of S-traders Π_S and $\Pi_{S,C}$. However, this does not mean that a change in n affects L-traders more than S-traders when it comes to information production. More formally, recall that $\Pi_L = n\Pi_S$ and $\Pi_{L,C} = n\Pi_{S,C}$, which further implies that: $\frac{\partial \Pi_L}{\partial \alpha_L} = n\frac{\partial \Pi_S}{\partial \alpha_S} < 0$ and $\frac{\partial \Pi_{L,C}}{\partial \alpha_L} = n\frac{\partial \Pi_{S,C}}{\partial \alpha_S} < 0$. It then follows that for L-traders, we have:

$$\frac{d\widetilde{\alpha}_L}{dn} = -\frac{1}{n} * \frac{\frac{\partial \Pi_L}{\partial n}}{\frac{\partial \Pi_S}{\partial \alpha_L}} \quad \text{and} \quad \frac{d\widetilde{\alpha}_{L,C}}{dn} = -\frac{1}{n} * \frac{\frac{\partial \Pi_{L,C}}{\partial n}}{\frac{\partial \Pi_c}{\partial \alpha_L}}$$

In contrast, for S-traders, we have:

$$\frac{d\widetilde{\alpha}_S}{dn} = -\frac{\frac{\partial \Pi_S}{\partial n}}{\frac{\partial \Pi_S}{\partial \alpha_L}} \quad \text{and} \quad \frac{d\widetilde{\alpha}_{S,C}}{dn} = -\frac{\frac{\partial \Pi_{S,C}}{\partial n}}{\frac{\partial \Pi_{S,C}}{\partial \alpha_L}}$$

Furthermore, from $\Pi_L = n\Pi_S$, we know that $\frac{\partial \Pi_L}{\partial n} = n\frac{\partial \Pi_S}{\partial n} + \Pi_S$. It follows that

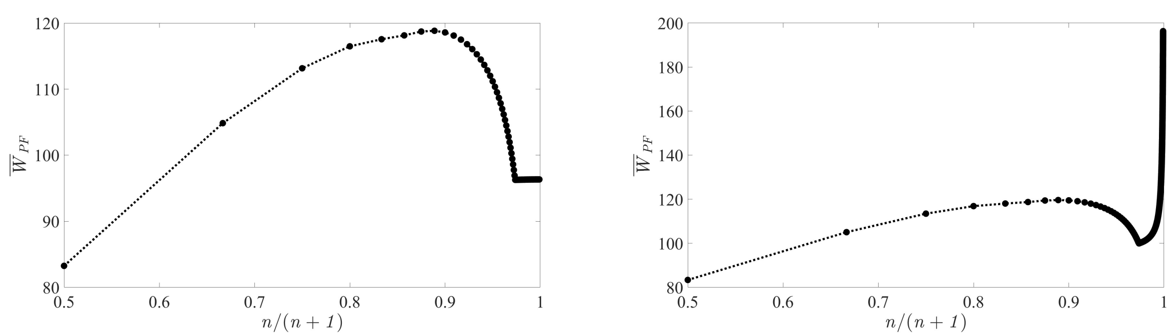
$$\frac{d\widetilde{\alpha}_L}{dn} = \frac{d\widetilde{\alpha}_S}{dn} - \frac{\Pi_S/n}{\frac{\partial \Pi_S}{\partial \alpha_L}} > \frac{d\widetilde{\alpha}_S}{dn}$$

Since $\frac{d\widetilde{\alpha}_S}{dn} < 0$, we have $\left|\frac{d\widetilde{\alpha}_L}{dn}\right| < \left|\frac{d\widetilde{\alpha}_S}{dn}\right|$, when $\frac{d\widetilde{\alpha}_L}{dn} < 0$. Similarly, with cross-asset learning, we also have: $\left|\frac{d\widetilde{\alpha}_{L,C}}{dn}\right| < \left|\frac{d\widetilde{\alpha}_{S,C}}{dn}\right|$, when $\frac{d\widetilde{\alpha}_{L,C}}{dn} < 0$. Thus, intensified market competition will negatively affect S-traders more than L-traders in terms of information production.

B.5 Formal Analysis for Section 5.4

This section provides a formal analysis for Section 5.4. Specifically, we first present a non-monotonic welfare result and then depict the relationship between competition and total welfare when investor welfare is included. Recall that $\Phi(l)$ is defined in Equation (16), and $l_0 = \inf\{l \in \mathbb{N} : \Phi(l) \geq 1\}$. Define $\tilde{c} = \frac{2bc}{(\bar{A}-MC)^2}$.

Lemma B.5 (Informational Feedback & Over-Competition)**.** *Assume $B(n) = B_0$ for some constant B_0. Suppose that $\Phi(l) - l * \tilde{c} - 1 > 0$ for some $l \geq l_0$. Then, for any $n \geq N(l) > l$, $\overline{W}(\widehat{\alpha}_l, l) > \overline{W}(\widehat{\alpha}_n, n)$ holds for any $c \in [\bar{c}_n, \underline{c}_l)$ with $\bar{c}_n < \underline{c}_l$.*



(a) $B(n) = 1$ (b) $B(n) = 0.1 * n$

Figure 17: Competition & Total Welfare (with Investor Welfare)

Parameters: $\theta = 0.75$, $b = 1.5$, $A_H = 30$, $A_L = 10$, $MC = 3$, and $c = 1.5$.

Figure 17 illustrates the relationship between product competition and total welfare when investor welfare is included in the calculation. Specifically, when the aggregate benefit of liquidity trading is fixed, Figure 17a demonstrates a non-monotonic pattern between competition and total welfare, which is similar to Figure 2. In particular, total welfare first increases and then decreases, and is maximized at $n = 8$. Similarly, Figure 17b illustrates the relationship by specifying the aggregate benefit of liquidity trading as an increasing function of the number of stocks, i.e., $B(n) = 0.1 * n$. The total welfare is also non-monotonic and becomes infinitely large due to the unbounded return from liquidity trading.

B.6 Skipped Proofs in the Online Appendix

B.6.1 Proof of Lemma B.1

Proof. We first state two properties: (a) We compute the following derivatives, including:

$$\frac{\partial \Pi_L(\alpha_L, \alpha_S)}{\partial \alpha_L} = -\frac{n(A_H - A_L)\left(\bar{A} - MC\right)(2\theta - 1)^2\left(2 + n(n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2} < 0;$$
$$\frac{\partial \Pi_S(\alpha_L, \alpha_S)}{\partial \alpha_S} = -\frac{(A_H - A_L)\left(\bar{A} - MC\right)(2\theta - 1)^2\left(2 + n(n-1)(\gamma^{LS})^{n-1}\right)}{2b(n+1)^2} < 0.$$

and (b) Note that $\Pi_L(\alpha_L, \alpha_S) = n\Pi_S(\alpha_L, \alpha_S)$.

Now, we prove the lemma. First, consider $c_L \geq \Pi_L(0,0)$. Obviously, $\widetilde{\alpha}_L = 0$. Meanwhile, since $c_S \geq c_L$ and $\Pi_L(0,0) \geq \Pi_S(0,0)$, $\widetilde{\alpha}_S = 0$.

Second, consider $\Pi_L(\lambda, 0) < c_L < \Pi_L(0,0)$. By the derivative $\frac{\partial \Pi_L(\alpha_L,\alpha_S)}{\partial \alpha_L} < 0$ and continuity, there exists a unique $\widetilde{\alpha}_L$ such that $\Pi_L(\lambda, 0) = c_L$. Furthermore, given $\widetilde{\alpha}_L$, $\frac{\partial \Pi_S(\alpha_L,\alpha_S)}{\partial \alpha_S} < 0$ implies that $\Pi_S(\lambda, 0) > \Pi_S(\lambda, \alpha_S)$ for any $\alpha_S > 0$. Thus, $c_S \geq c_L = \Pi_L(\lambda, 0) \geq \Pi_S > \Pi_S(\lambda, \alpha_S)$ for any $\alpha_S > 0$. Therefore, $\widetilde{\alpha} = 0$.

Third, consider $c_L < \Pi_L(\lambda, 0)$ and $c_S \geq \Pi_S(\lambda, 0)$. Obviously, $(\widetilde{\alpha}_L, \widetilde{\alpha}_S) = (\lambda, 0)$. Furthermore, this is also the unique equilibrium. If not, consider any equilibrium $(\widetilde{\alpha}_L, \widetilde{\alpha}_S)$ with $\alpha_S > 0$. Note that by property (b), we can infer: $\Pi_L(\widetilde{\alpha}_L, \widetilde{\alpha}_S) > \Pi_S(\widetilde{\alpha}_L, \widetilde{\alpha}_S) \geq c_S \geq c_L$, which implies that $\widetilde{\alpha}_L = \lambda$, which in turn implies that $\widetilde{\alpha}_S = 0$.

Fourth, consider $c_L < \Pi_L(\lambda, 0)$ and $\Pi_S(\lambda, 1 - \lambda) < c_S < \Pi_S(\lambda, 0)$. We have shown above that if $\alpha_S > 0$, then $\alpha_L = \lambda$. Given that $c_L < \Pi_L(\lambda, 0)$, we can infer that $\widetilde{\alpha}_L = \lambda$. Given this and the assumed condition $\Pi_S(\lambda, 1-\lambda) < c_S < \Pi_S(\lambda, 0)$, by the monotonicity and continuity of $\Pi_S(\alpha_L, \alpha_S)$, there is a unique $\widetilde{\alpha}_S \in (0, 1 - \lambda)$ such that $\Pi_S(\lambda, \widetilde{\alpha}_S) = c_S$.

Fifth, consider $c_L < \Pi_L(\lambda, 0)$ and $c_S \leq \Pi_S(\lambda, 1 - \lambda)$. Obviously, by the facts $c_S \geq c_L$ and $\Pi_L \geq \Pi_S$, we have: $\widetilde{\alpha}_L = \lambda$ and $\widetilde{\alpha}_S = 1 - \lambda$. The proof concludes. □

B.6.2 Proof of Lemma B.2

Proof. **Case 1:** $\widetilde{\alpha}_L = \lambda$. We can rewrite $\widetilde{W}(\widetilde{\alpha}_L, \widetilde{\alpha}_S, n)$ and $\Pi_S(\alpha_L, \alpha_S)$ as:

$$\widetilde{W}(\widetilde{\alpha}_S, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A} - MC)^2 + (1 - {\gamma_S}^n)(A_H - A_L)^2\right),$$
$$\Pi_S(\widetilde{\alpha}_S, n) = \frac{\gamma_S(2\theta - 1)(A_H - A_L)(\bar{A} - MC)\left(2 + (\gamma_S)^{n-1}(n-1)\right)}{2b(n+1)^2}$$

where $\gamma_S = 1 - (\lambda + \widetilde{\alpha}_S)(2\theta - 1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_S} = \frac{n^2(n+2)\gamma_S^{n-1}(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}}{\partial n} = \frac{n(n+2)\gamma_S^n(A_H-A_L)^2\ln(1/\gamma_S)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\gamma_S^n(A_H-A_L)^2}{4b(n+1)^3}$$
$$\frac{\partial \Pi_S}{\partial \widetilde{\alpha}_S} = -\frac{(2\theta-1)^2\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2+n(n-1){\gamma_S}^{n-1}\right)}{4b(n+1)^2}$$
$$\frac{\partial \Pi_S}{\partial n} = -\frac{(2\theta-1)\left((A_H-MC)^2-(A_L-MC)^2\right)\left(4\gamma_S+\gamma_S^n\left(n-3-(n^2-1)\ln\gamma_S\right)\right)}{4b(n+1)^3}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_S}{\partial n} = -\frac{\partial \Pi_S/\partial n}{\partial \Pi_S/\widetilde{\alpha}_S} = -\frac{(\gamma_S)^n \times \left(\left(4\gamma_S^{1-n}+(n-3)\right)/(n+1)+(n-1)\ln(1/\gamma_S)\right)}{(2\theta-1)\left(2+n(n-1)\hat{\gamma}_S^{n-1}\right)}$$

which further implies:

$$\frac{d\widetilde{W}\left(\widetilde{\alpha}_{S,C},n\right)}{dn} = \frac{\partial \widetilde{W}}{\partial n} + \frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_S}\frac{\partial \widetilde{\alpha}_S}{\partial n} = \frac{\left(A_H-A_L\right)^2\left(G_1-g_S\left(\widetilde{\alpha}_S,n\right)\right)}{8b(n+1)^3},$$

Thus, $\frac{d\widetilde{W}\left(\widetilde{\alpha}_{S,C},n\right)}{dn} < 0$ if and only if $g_S\left(\widetilde{\alpha}_S,n\right) > G_1$.

Case 2: $\widetilde{\alpha}_S = 0$**.** We can rewrite $\widetilde{W}(\widetilde{\alpha}_L,\widetilde{\alpha}_S,n)$ and $\Pi_L(\alpha_L,\alpha_S)$ as:

$$\widetilde{W}(\widetilde{\alpha}_L,n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2+(1-(\gamma_L)^n)(A_H-A_L)^2\right),$$
$$\Pi_S(\widetilde{\alpha}_L,n) = \frac{\gamma_S(2\theta-1)(A_H-A_L)(\bar{A}-MC)\left(2+(\gamma_L)^{n-1}(n-1)\right)}{2b(n+1)^2}$$

where $\gamma_L = 1-\widetilde{\alpha}_L\times(2\theta-1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_L} = \frac{n^2(n+2)\gamma_L^{n-1}(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}}{\partial n} = \frac{n(n+2)\gamma_L^n(A_H-A_L)^2\ln(1/\gamma_L)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2+(A_L-MC)^2\right)-\gamma_L^n(A_H-A_L)^2}{4b(n+1)^3}$$

$$\frac{\partial \Pi_L}{\partial \widetilde{\alpha}_L} = -\frac{n(2\theta-1)^2\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2+n(n-1){\gamma_L}^{n-1}\right)}{4b(n+1)^2}$$
$$\frac{\partial \Pi_L}{\partial n} = -\frac{(2\theta-1)\left((A_H-MC)^2-(A_L-MC)^2\right)\left(2(1-n)\gamma_L+\gamma_L^n\left((3n-1)+n(n^2-1)\ln\gamma_L\right)\right)}{4b(n+1)^3}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_L}{\partial n} = -\frac{\partial \Pi_L/\partial n}{\partial \Pi_L/\widetilde{\alpha}_L} = \frac{2\gamma_L\times(1-n)+(\gamma_L)^n\left((3n-1)-n(n^2-1)\ln(1/\gamma_L)\right)}{n(n+1)(2\theta-1)\left(2+n(n-1)\gamma_L^{n-1}\right)}$$

which further implies:

$$\frac{d\widetilde{W}\left(\widetilde{\alpha}_L, n\right)}{dn} = \frac{\partial \widetilde{W}}{\partial n} + \frac{\partial \widetilde{W}}{\partial \widetilde{\alpha}_L}\frac{\partial \widetilde{\alpha}_L}{\partial n} = \frac{n\left(A_H - A_L\right)^2\left(G_1 - g_L\left(\widetilde{\alpha}_L, n\right)\right)}{8bn(n+1)^3},$$

Thus, $\frac{d\widetilde{W}(\widetilde{\alpha}_L,n)}{dn} < 0$ if and only if $g_L\left(\widetilde{\alpha}_L, n\right) > G_1$. The proof concludes. □

B.6.3 Proof of Lemma B.3

Proof. We first state two important properties: (a) $\Pi_{L,C}\left(\alpha_L, \alpha_S\right) = n\Pi_S\left(\alpha_L, \alpha_S\right)$; and (b) we compute the following derivatives, including $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_{L,C}}$ and $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_{S,C}}$. Based on the expressions for trading profits of an informed L-trader and an informed S-trader, we have:

$$\frac{\partial \Pi_{L,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{L,C}} = -\frac{n^2\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A} - MC\right)\left(A_H - A_L\right)}{2(n+1)b} < 0$$

$$\frac{\partial \Pi_{S,C}\left(\alpha_L, \alpha_S\right)}{\partial \alpha_{S,C}} = -\frac{n\left(\gamma^{LS}\right)^{n-1}(2\theta-1)^2\left(\bar{A} - MC\right)\left(A_H - A_L\right)}{2(n+1)b} < 0$$

Next, we prove the lemma. First, consider $c_L \geq \Pi_{L,C}(0,0)$. Obviously, $\widetilde{\alpha}_{L,C} = 0$. Meanwhile, since $c_S \geq c_L$ and $\Pi_{L,C}(0,0) = n\Pi_{S,C}(0,0)$, we can deduce that $\widetilde{\alpha}_{S,C} = 0$.

Second, consider $\Pi_{L,C}(\lambda,0) < c_L < \Pi_{L,C}(0,0)$. By the derivative $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_L} < 0$, and continuity, there exists a unique $\widetilde{\alpha}_{L,C}$ such that $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right) = c_L$. By solving the equation $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right) = c_L$, we have $\widetilde{\alpha}_{L,C} = \nu$. Furthermore, given $\widetilde{\alpha}_{L,C}$, $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_S} < 0$ implies that $\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, 0\right) > \Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \alpha_S\right)$ for any $\alpha_S > 0$. Thus, $c_S \geq c_L = \Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, 0\right) > \Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \alpha_S\right)$ for any $\alpha_S > 0$. Therefore, $\widetilde{\alpha}_{S,C} = 0$.

Third, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $c_S \geq \Pi_{S,C}(\lambda,0)$. Obviously, $(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}) = (\lambda, 0)$. Furthermore, this is also the unique equilibrium. If not, consider any equilibrium $(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C})$ with $\widetilde{\alpha}_{S,C} > 0$. Note that by property (b), we can infer: $\Pi_{L,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right) > \Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right) \geq c_S \geq c_L$, which implies that $\widetilde{\alpha}_{L,C} = \lambda$, which in turn implies that $\widetilde{\alpha}_{S,C} = 0$.

Fourth, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $\Pi_{S,C}(\lambda, 1-\lambda) < c_S < \Pi_{S,C}(\lambda,0)$. We have shown above that if $\widetilde{\alpha}_{S,C} > 0$, then $\widetilde{\alpha}_{L,C} = \lambda$. Given that $c_L \leq \Pi_{L,C}(\lambda,0)$, we can infer that $\widetilde{\alpha}_{L,C} = \lambda$. Given this and the assumed condition $\Pi_{S,C}(\lambda, 1-\lambda) < c_S < \Pi_{S,C}(\lambda,0)$, by the monotonicity and continuity of $\Pi_{S,C}\left(\widetilde{\alpha}_{L,C}, \widetilde{\alpha}_{S,C}\right)$, there is a unique $\widetilde{\alpha}_{S,C} \in (0, 1-\lambda)$ such that $\Pi_S\left(\lambda, \widetilde{\alpha}_{S,C}\right) = c_S$. By solving $\Pi_S\left(\lambda, \widetilde{\alpha}_{S,C}\right) = c_S$, we have $\widetilde{\alpha}_{S,C} = \xi$.

Fifth, consider $c_L \leq \Pi_{L,C}(\lambda,0)$ and $c_S \leq \Pi_{S,C}(\lambda, 1-\lambda)$. Obviously, by the facts $c_S \geq c_L$ and $\Pi_{L,C} > \Pi_{S,C}$, we have: $\widetilde{\alpha}_{L,C} = \lambda$ and $\widetilde{\alpha}_{S,C} = 1-\lambda$. The proof concludes. □

B.6.4 Proof of Lemma B.4

Proof. We first state two important properties: (a) $\Pi_{L,C}\left(\alpha_L, \alpha_S\right) = n\Pi_S\left(\alpha_L, \alpha_S\right)$; and (b) we compute the following derivatives, including $\frac{\partial \Pi_{L,C}(\alpha_L,\alpha_S)}{\partial \alpha_{L,C}}$ and $\frac{\partial \Pi_{S,C}(\alpha_L,\alpha_S)}{\partial \alpha_{S,C}}$. Based on the expressions for trading profits of an informed L-trader and an informed S-trader, we have:

$$\frac{\partial \Pi_{L,C}(\alpha_L, \alpha_S)}{\partial \alpha_{L,C}} = -\frac{n^2 \left(\gamma^{LS}\right)^{n-1} (2\theta-1)^2 \left(\bar{A}-MC\right)(A_H - A_L)}{2(n+1)b} < 0$$
$$\frac{\partial \Pi_{S,C}(\alpha_L, \alpha_S)}{\partial \alpha_{S,C}} = -\frac{n \left(\gamma^{LS}\right)^{n-1} (2\theta-1)^2 \left(\bar{A}-MC\right)(A_H - A_L)}{2(n+1)b} < 0$$

Now, we prove the lemma.

Case 1: $\widetilde{\alpha}_{L,C} = \lambda$**.** We can rewrite $\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n, n)$ and $\Pi_{L,C}(\boldsymbol{\alpha}_n)$ as:

$$\widetilde{W}_{LS}(\widetilde{\alpha}_S, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2 + (1-(\gamma_S)^n)(A_H - A_L)^2\right),$$
$$\Pi_{S,C}(\widetilde{\alpha}_S, n) = \frac{(\gamma_S)^n (2\theta-1)(A_H - A_L)(\bar{A}-MC)}{2b(n+1)^2}$$

where $\gamma_S = 1 - (\lambda + \widetilde{\alpha}_S)(2\theta - 1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{S,C}} = \frac{\gamma_S^{n-1} n^2 (n+2)(2\theta-1)(A_H-A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}_{LS}}{\partial n} = \frac{\gamma_S^n n(n+2)(A_H-A_L)^2 \ln(1/\gamma_S)}{8b(n+1)^2} + \frac{2\left((A_H-MC)^2 + (A_L-MC)^2\right) - \gamma_S^n (A_H-A_L)^2}{4b(n+1)^3}$$
$$\frac{\partial \Pi_{S,C}}{\partial \widetilde{\alpha}_{S,C}} = -\frac{n\gamma_S^{n-1}\left(\bar{A}-MC\right)(A_H-A_L)(2\theta-1)^2}{2b(n+1)}$$
$$\frac{\partial \Pi_{S,C}}{\partial n} = -\frac{\gamma_S^n (2\theta-1)\left(\bar{A}-MC\right)(A_H-A_L)\left(1+(n+1)(\ln 1/\gamma_S)\right)}{2b(n+1)^2}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_{S,C}}{\partial n} = -\frac{\partial \Pi_{S,C}/\partial n}{\partial \Pi_{S,C}/\widetilde{\alpha}_{S,C}} = -\frac{\gamma_S\left(1+(\ln(1/\gamma_S))\right)}{n(2\theta-1)}$$

which further implies:

$$\frac{d\widetilde{W}_{LS}(\widetilde{\alpha}_{S,C}, n)}{dn} = \frac{\partial \widetilde{W}_{LS}}{\partial n} + \frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{S,C}}\frac{\partial \widetilde{\alpha}_{S,C}}{\partial n} = \frac{(A_H-A_L)^2\left(G_1 - g_{S,C}(\gamma_S, n)\right)}{8b(n+1)^3}.$$

Thus, $\frac{d\widetilde{W}_{LS}(\widetilde{\alpha}_{S,C}, n)}{dn} < 0$ if and only if $g_S(\gamma_S, n) > G_1$.

Case 2: $\widetilde{\alpha}_{S,C} = 0$**.** We can rewrite $\widetilde{W}_{LS}(\widetilde{\boldsymbol{\alpha}}_n, n)$ and $\Pi_{L,C}(\boldsymbol{\alpha}_n)$ as:

$$\widetilde{W}_{LS}(\widetilde{\alpha}_{L,C}, n) = \frac{n(n+2)}{8b(n+1)^2}\left(4(\bar{A}-MC)^2 + (1-\gamma_L^n)(A_H - A_L)^2\right),$$
$$\Pi_{S,C}(\widetilde{\alpha}_{L,C}, n) = \frac{n\gamma_L^n (2\theta-1)\left(\bar{A}-MC\right)(A_H - A_L)}{2b(n+1)}$$

where $\gamma_L = 1 - \widetilde{\alpha}_L \times (2\theta - 1)$.

Then, we can calculate the following partial derivatives:

$$\frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_L} = \frac{\gamma_L^{n-1} n^2 (n+2)(2\theta-1)(A_H - A_L)^2}{8b(n+1)^2},$$
$$\frac{\partial \widetilde{W}_{LS}}{\partial n} = \frac{n(n+2)\gamma_L^n (A_H - A_L)^2 \ln(1/\gamma_L)}{8b(n+1)^2} + \frac{2\left((A_H - MC)^2 + (A_L - MC)^2\right) - \gamma_L^n (A_H - A_L)^2}{4b(n+1)^3}$$
$$\frac{\partial \Pi_{L,C}}{\partial \widetilde{\alpha}_L} = -\frac{n^2 \gamma_L^{n-1}\left(\bar{A} - MC\right)(A_H - A_L)(2\theta-1)^2}{2b(n+1)}$$
$$\frac{\partial \Pi_{L,C}}{\partial n} = -\frac{\gamma_L^n (2\theta-1)\left(\bar{A} - MC\right)(A_H - A_L)\left(1 - n(n+1) ln\left(1/\gamma_L\right)\right)}{(n+1)^2}$$

By the implicit function theorem, we have:

$$\frac{\partial \widetilde{\alpha}_{L,C}}{\partial n} = -\frac{\partial \Pi_{L,C}/\partial n}{\partial \Pi_{L,C}/\partial \widetilde{\alpha}_{L,C}} = \frac{2\gamma_L \times (1-n) + (\gamma_L)^n \left((3n-1) - n(n^2-1)\ln(1/\gamma_L)\right)}{n(n+1)(2\theta-1)\left(2 + n(n-1)\gamma_L^{n-1}\right)}$$

which further implies:

$$\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{L,C}, n\right)}{dn} = \frac{\partial \widetilde{W}_{LS}}{\partial n} + \frac{\partial \widetilde{W}_{LS}}{\partial \widetilde{\alpha}_{L,C}}\frac{\partial \widetilde{\alpha}_{L,C}}{\partial n} = \frac{4\left((A_H - MC)^2 + (A_L - MC)^2\right) + n\gamma_L^n (A_H - A_L)^2}{8bn(n+1)^3}$$

Obviously, $\frac{d\widetilde{W}_{LS}\left(\widetilde{\alpha}_{L,C}, n\right)}{dn} > 0$. The proof concludes. □

B.6.5 Proof of Lemma B.5

Proof. First, note that $B(n) = B_0$ eliminates the impact of the benefits of liquidity trading and thus we can focus on the information cost. Second, $\Phi(l) - l * \tilde{c} > 0$ holds for some $l \geq l_0$ for $\frac{2b}{(\bar{A}-MC)^2}$ sufficiently small since $\Phi(l) > 1$ for $l \geq l_0 + 1$. Third, note that

$$\frac{\overline{W}\left(\widehat{\alpha}_l, l\right)}{\overline{W}\left(\widehat{\alpha}_n, n\right)} = \frac{\left(1 - \frac{1}{(l+1)^2}\right) * \left(1 + \mu * \left(1 - (2-2\theta)^l\right)\right) - l * \tilde{c}}{\left(1 - \frac{1}{(n+1)^2}\right)}$$

Then, the remaining proof follows from that of Theorem 1. The proof concludes. □