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Dynamic Coordination and Bankruptcy Regulations

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Abstract

Many regulations aim at promoting coordination among creditors in bankruptcy by ex post restricting their ability to exit distressed firms. However, such restrictions may harm creditors' ex ante incentives to stay invested, thereby worsening coordination outcomes. We build a dynamic coordination model to show how this force shapes creditor runs, bankruptcy filings, and regulation designs. Intriguingly, filing for bankruptcy early, thereby preserving more assets for latecomers, can prolong firm life. Furthermore, regulators' clawbacks on pre-bankruptcy repayments can be superior to firms' commitment to early bankruptcy filing. Our analysis generates implications for automatic stay, avoidable preference, bank failures, and seniority structure.

Keywords: Automatic Stay, Avoidable Preference, Bank Failure, Clock Game, Runs

JEL Classification Numbers: D83, G33, G38

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1 Introduction

Coordination failure among creditors can inject chaos into the bankruptcy process, as individual creditors often exhibit a natural inclination to swiftly claim a share of a distressed firm's assets. To manage such situations, bankruptcy laws worldwide incorporate regulations aimed at fostering an orderly resolution, mitigating the first-come-first-serve dynamics inherent in the process. Among these provisions is the well-known "automatic stay" clause, which, upon a debtor's bankruptcy filing, compels creditors to suspend individual debt collections, awaiting a collective resolution in a bankruptcy court. Another widely utilized regulation with similar objectives is the legal treatment of "avoidable preference."[1] This regulation, often invoked in bankruptcy cases, prevents a troubled firm from displaying preferential treatment toward specific creditors by clawing back repayments made shortly before bankruptcy.[2] The proceeds are then shared among all creditors in bankruptcy court. These regulations often coordinate creditors surrounding bankruptcy by restricting their ability to collect debt ex post. What is often ignored, however, is that such ex post restrictions can also reduce creditors' ex ante incentives to stay invested while the firm is still relatively healthy.[3]

In this paper, we analyze how these forces shape creditor runs, the optimal timing for a firm to file for bankruptcy, and the optimal design of bankruptcy regulations including automatic stay and avoidable preference. Creditors face a trade-off when staying invested in a distressed firm. On the one hand, they earn additional interests if the firm does not default; on the other hand, they are more exposed to a potential bankruptcy which can lead to credit losses. How long creditors are willing to keep their investments therefore depends on the likelihood of them being affected by bankruptcy (henceforth, the hazard rate channel) and the recovery payoff they can expect in the case of a default (henceforth, the recovery rate channel). Several theoretical insights emerge from these two channels.

First, firms can extend their survival by committing to filing for bankruptcy protection early. One might intuitively think that, to survive longer, a firm should continue making repayments until all assets are depleted. However, this logic is incomplete. Zero ex post recovery in bankruptcy may trigger more frantic runs ex ante because creditors may choose to exit immediately at the first sign of trouble, pushing the firm into bankruptcy even sooner. Early filing preserves some assets for creditors in bankruptcy, improving their recovery rate. Hence, despite the higher hazard rate as fewer creditors can exit before bankruptcy, the improved recovery rate can make creditors more patient ex ante, delaying the bankruptcy process.

[1] Between 2017 and 2019, among the 595 bankruptcy cases collected by Westlaw legal research service, 290 cases (or 48.74%) cited avoidable preference in the United States. In the past, many well-known bankruptcies, such as WorldCom, General Motors, and Lehman Brothers, resorted to avoidable preference legislation to settle disputes among creditors.

[2] The typical clawback window is between 90 days and one year in the United States (see Chapter 11, Sections 547 and 550), between six months and one year in China (see Enterprise Bankruptcy Law, Articles 31 and 32), and between six months and two years in the United Kingdom (see Insolvency Act 1986, Sections 239 and 240).

[3] Legal professionals also share this concern. See, for example, McCoid (1981) and Countryman (1985).

This insight offers rationales for two provisions in corporate bankruptcy and bank failures. Automatic stay is arguably one of the most important features in corporate bankruptcy. The absence of an automatic stay is akin to a scenario in which early creditors receive full repayments, while latecomers receive nothing. In this case, the low recovery in bankruptcy motivates creditors to run faster, possibly leading to earlier bankruptcy. Automatic stay protection are therefore more valuable when the recovery rate channel is more pronounced. In the model, we show that this case occurs when firms have milder shocks and lower leverage.

Related, bank failures are often triggered by the regulator's decision to seize the bank while considerable assets are still present. Forcing a bank failure early can similarly increase the recovery payoff of (uninsured) depositors, which can in turn alleviate a potential run.[4]

Our second insight demonstrates that regulators' ex post clawback of some pre-bankruptcy repayments can achieve an even better outcome than firm's own commitment to file for bankruptcy early. Intuitively, early bankruptcy disrupts production early at the time of filing. By clawing back repayments made before bankruptcy, the regulator effectively shifts forward the pivotal creditor—the last one who is allowed to exit the firm with a full repayment—without actually disrupting the production process at the time when the pivotal creditor exits. In other words, compared with early bankruptcy filing, avoidable preference can implement the same hazard rate and deliver a higher recovery rate from the preserved production.[5] This explains the frequent use of avoidable preference in bankruptcy court.

To formally deliver these insights, we construct a tractable dynamic coordination framework based on the clock game (e.g., Abreu and Brunnermeier (2003) and Brunnermeier and Morgan (2010)) to study a financially distressed yet still productive firm that is heading for bankruptcy.[6] The firm with a staggered debt structure experiences a random bad shock that reduces its asset growth rate, causing it to fall below the required interest rate on its liabilities.

There are three sets of players: creditors, the manager of the firm, and a regulator. Each creditor gradually becomes aware of the bad shock upon debt maturity and decides privately when to cease rolling over the debt and exit the firm.[7] Waiting allows for additional interest accrual but also increases the risk of bankruptcy before the creditor can exit. The firm's manager can decide when to file for bankruptcy based on the number of exiting creditors, in order to

[4]It is worth noting that deposit insurance is another important mechanism to ensure confidence. In this sense, the model applies to banks with significant amount of uninsured depositors, where the uncertainty about the recovery payoff in a bank failure is more pronounced.

[5]To illustrate the intuition with a simple example, consider the following two cases: (1) the firm files for bankruptcy early when 40% of creditors exit; (2) the firm files for bankruptcy late when 70% of creditors exit, but the repayments made to the final 30% of creditors need to be returned and shared among all remaining 60% (equal to 1 - 70% + 30%) creditors in bankruptcy. Both cases effectively allow only 40% of creditors to exit successfully and, therefore, share the same hazard rates. However, case 2 offers a higher recovery rate because production is only terminated later at the 70% threshold rather than at the 40% threshold.

[6]In real-world scenarios, the majority of bankruptcy filings by large companies fall under Chapter 11 (restructuring) rather than Chapter 7 (liquidation). This choice stems from the idea that these struggling firms still possess a viable business model but require a balance sheet restructuring. Translated into our model, we assume that the firm remains productive despite facing an impending costly bankruptcy.

[7]Private actions imply that, when deciding the waiting time, the creditor lacks knowledge about how many other creditors are aware of the bad shock or have withdrawn their capital.

maximize the troubled firm's life span. We also consider the case when the manager cannot commit to a bankruptcy filing threshold (or, equivalently, a world without automatic stay), and bankruptcy is triggered by the depletion of all assets. Finally, a welfare-maximizing regulator can impose a clawback window and design its length (avoidable preference). Creditors who exit before the clawback window can leave with full repayments, but those who receive payments within this window before bankruptcy must return the money and share these proceeds and remaining assets with other creditors in bankruptcy.

The trade-off between the recovery rate channel and the hazard rate channel naturally emerges from our model. Initiating bankruptcy proceedings earlier or instituting a longer clawback window both conserve additional capital within the bankruptcy process, improving the recovery rate and bolstering coordination. Nonetheless, these actions also subject more creditors to the impact of bankruptcy, escalating the hazard rate and jeopardizing coordination.

With the model, we can compare the manager's choice of bankruptcy threshold that maximizes the firm's life span versus the regulator's choice of bankruptcy threshold that maximizes total welfare. We demonstrate that the regulator prefers to intervene earlier than the manager. To see the intuition behind this, note that welfare loss results from the outflow of productive capital. Maximizing welfare therefore entails two aspects: delaying bankruptcy (similar to the manager's objective) and minimizing the capital outflow given the time of bankruptcy. The second objective, unique to the regulator, motivates early intervention and allowing fewer creditors to exit. This finding highlights the misaligned incentives between the regulator and the manager, and helps demonstrate how regulations such as avoidable preference can realign their incentives.

In addition, we show that avoidable preference is a relatively detail-free regulation in that the optimal clawback window does not depend on the firm's performance characteristics. This is because the recovery rate channel, measuring the proceeds from the clawback of repayments, only depends on the features of debt contracts, whereas the hazard rate channel only relies on the stochastic structure of the bad shock. Neither side of this trade-off depends on the firm's performance characteristics. Furthermore, the simple closed-form solution of the optimal clawback window allows us to conduct clean comparative statics, providing guidance for legal rulings in practice. For example, a higher intensity of the bad shock or a shorter maturity (equivalently, faster information transmission) exacerbates the hazard rate channel, resulting in a longer clawback window and fewer creditors who can exit successfully. A higher interest rate implies that more repayments can be seized, increasing the recovery rate and thereby reducing the duration of the clawback window.

We finally extend the model to incorporate ex ante heterogeneous creditors based on seniority. In this case, junior creditors always exit sooner than their senior counterparts because of their inferior recovery payoff. Interestingly, the coordination outcome is non-monotonic with respect to creditor composition. A small number of senior creditors always worsens coordination because their presence makes junior creditors less patient, leading to earlier bankruptcy. However, as more creditors become senior, they replace juniors and become more pivotal in triggering bankruptcy.

Their willingness to wait can prolong firm life and delay bankruptcy. As a robustness analysis, we also consider the possibility for the firm to recover from the bad shock and different timelines of actions.

Literature Review

Our framework builds on the clock game, which was first introduced in the computer science literature (see Halpern and Moses (1990)). The key idea is that not everyone learns of a piece of news at the same time, but instead some individuals become informed sooner than others. Morris (1995) applies this information structure to a dynamic coordination game in labor economics. Abreu and Brunnermeier (2003), Brunnermeier and Morgan (2010), and Doblas-Madrid (2012) then use the clock game setup to understand the formation of bubbles and their subsequent crashes. In the context of banking, He and Manela (2016) allow investors to actively acquire information and show that such information acquisition may accelerate runs. We contribute to this literature by constructing a tractable framework on bankruptcy and clearly delineating the trade-off between the recovery rate and hazard rate channels. The tractability of the framework also allows us to study the strategic interactions between creditors' exit timing, the firm's bankruptcy threshold, and the regulator's policy design.

A popular alternative approach to study the coordination game is the global game framework introduced by Carlsson and Van Damme (1993). It has also been extensively applied to modeling currency attacks and bank runs (see Morris and Shin (1998, 2004) and Goldstein and Pauzner (2005) for details).

Among this literature, Schilling (2023) and Matta and Perotti (2023) are closely related. In global game settings, these two papers study how many early withdrawals should be allowed (forbearance level in Schilling (2023) and selling illiquid assets in Matta and Perotti (2023)) before imposing costly actions (resolution in Schilling (2023) and suspension and liquidation in Matta and Perotti (2023)). There is a similar trade-off: allowing for more withdrawals makes it more likely for depositors to receive full repayments but simultaneously lowers the payoff the remaining ones receive in a resolution. Both papers therefore can deliver interesting insights on optimal forbearance or liquidation policies.

Like many other global game models, these two papers consider a static setting in which depositors only choose whether to run but not the exact timing. However, debt runs are intrinsically a dynamic concept: the timing of exit affects the asset evolution (recovery rate channel) and the likelihood of a successful exit (hazard rate channel), which in turn affect creditors' incentives to exit. Our dynamic coordination framework built on the clock game can naturally capture this trade-off. Furthermore, there are only two sets of players in both Schilling (2023) and Matta and Perotti (2023): the policy setter and depositors, whereas our framework can comprehensively consider the creditors', the firm's, and the regulator's strategic actions in a unified model, thereby generating dynamic implications on bankruptcy regulations. For instance, the finding that ex post clawback in bankruptcy is more efficient than the firm's own commitment to filing for bankruptcy early crucially relies on the fact that assets can appreciate over the clawback window.

Among the large literature on creditor (depositor) coordination following the seminal work by Bryant (1980) and Diamond and Dybvig (1983), many papers study how various policies can alleviate creditor runs.[8] Recent studies can be broadly classified into two categories: policies that affect information structure (for example, bank stress tests; see Goldstein and Huang (2016); Inostroza and Pavan (2020); Basak and Zhou (2020, 2024)) and policies that directly intervene payoffs (for example, subsidies and insurance; see Sakovics and Steiner (2012); Frankel (2017); Allen et al. (2018); Dávila and Goldstein (2023); Shen and Zou (2024)). This paper belongs to the second category. Different from the literature, we build a model of a dynamic debt run and study a new policy: avoidable preference (repayment clawback). A novel policy insight is that the optimal clawback window is relatively detail-free in that it does not depend on the parameters of the firm's production process. Therefore, such regulation requires less information from the regulator.

Finally, our paper also contributes to a large literature on corporate bankruptcy. For example, Bebchuk (2002) finds that the ex post violation of absolute priority in bankruptcy can exacerbate the moral hazard problem and thereby lower ex ante efficiency. Bolton and Oehmke (2015) investigate the ex ante impact of exempting derivatives from automatic stay in bankruptcy. He and Xiong (2012) study runs on staggered corporate debt and show that a creditor's decision not to roll over maturing debt poses an externality on other creditors whose claims have not yet matured. Donaldson, Gromb and Piacentino (2020*a*,*b*) study the role of collateral and covenants in regulating creditors' payoffs in bankruptcy. Donaldson et al. (2020) compare out-of-court restructuring and a formal bankruptcy procedure. Zhong (2021) studies how creditor dispersion and the resulting coordination friction affect the evolution of creditor structure and the time of bankruptcy. Different from the existing studies, we focus on two novel aspects of bankruptcy regulation—avoidable preference and automatic stay in bankruptcy protection—and focus on how these ex post regulations affect creditors' ex ante incentives to exit the firm. Our study also generates implications on the timing of bankruptcy, the recovery rate, and social welfare.

We structure the paper as follows. Sections 2 and 3 introduce and solve the benchmark model featuring creditors' choices of exit time and the firm's choice of bankruptcy threshold. We consider two cases depending on the firm's ability to commit to a bankruptcy threshold and discuss two applications on automatic stay and bank failure. In Section 4, we integrate the regulation of avoidable preference into our framework and analytically solve the optimal clawback window in addition to the firm's and the creditors' optimization problems. We then apply our

[8] For early papers in the literature, see, for example, Gorton (1985); Chari and Jagannathan (1988); Jacklin and Bhattacharya (1988); Green and Lin (2003); Peck and Shell (2003). There is also a more recent literature on the dynamic global game with a different focus (see, for example, Dasgupta (2007); Angeletos, Hellwig and Pavan (2007); Basak and Zhou (2020)). These papers study how learning the history of play can affect future coordination. In contrast, creditors in our model are willing to stay invested not because they are waiting for more information but rather because of the interest income before bankruptcy.

framework to study seniority among creditors and temporary bad shocks in Section 5. Finally, Section 6 concludes.

2 Benchmark Model

The benchmark model introduced in Subsection 2.1 focuses on characterizing the dynamic patterns of creditor runs and the firm's bankruptcy decisions. We further elaborate some of the model assumptions in Subsection 2.2.

2.1 Model Setup

Time $t \in [0, \infty)$ is continuous, and the discount rate is normalized to zero. To begin, consider a firm with initial assets A and one unit of liability at $t = 0$. For simplicity, both assets and liabilities grow at the same rate g, resulting in the firm maintaining a constant leverage $\frac{e^{gt}}{Ae^{gt}} = \frac{1}{A} \leq 1$. A continuum of risk-neutral creditors, indexed by $i \in [0, 1]$, initially holds unit face value debt contracts. These contracts have staggered maturity η, with creditors repeatedly rolling over their maturing debt. The parameter g represents the promised interest rate. For creditors to prefer the debt investment initially, we normalize the return on capital outside the firm to 0.

We focus on the events surrounding bankruptcy; hence, we model a distressed yet still productive firm that eventually enters costly bankruptcy. At some random time $t_0 > 0$, a bad shock occurs with intensity λ, permanently reducing the firm's asset growth rate to $g' \in (0, g)$.[9] The density function of the exponentially distributed t_0 is expressed as $\phi(t_0) = \lambda e^{-\lambda t_0}$.

Following a bad shock, creditors gradually become aware of it asynchronously over the interval $[t_0, t_0 + \eta]$, when their debt matures. While the learning window η need not align precisely with debt maturity, we assume for simplicity that creditor $i \in [0, 1]$ privately learns of the shock at debt maturity $t_i \equiv t_0 + i\eta$.[10] Upon learning, creditors then select the maturity associated with the final debt rollover $\beta_i(t_i) - t_i$ and exit the firm at $\beta_i(t_i) \geq t_i$. In making the rollover decision, creditors do not observe how many other creditors are aware of the bad shock (or equivalently, its arrival time t_0). Creditors may wish to keep invested for some time despite the bad shock because doing so generates interest income, however, an immediate exit or a refusal to roll over the maturing debt ($\beta_i(t_i) = t_i$) is permissible and sometimes happen in the model.[11]

[9]Throughout most of the paper, we concentrate on the empirically more relevant scenario where the post-shock firm remains productive with $g' > 0$. In practice, many bankruptcy filings involve Chapter 11 reorganization, presupposing the business model's ongoing economic viability. The same mathematical framework can be used to analyze the simple extension with $g' \leq 0$, which is discussed in footnote 27. We also consider recoverable temporary bad shocks in Subsection 5.2 for robustness.

[10]For instance, the due diligence carried out by creditors at the time of debt rollover uncovers the bad shock.

[11]Alternatively, it is also possible to interpret the asynchronous learning setting as a covenant violation. As the firm's fundamental deteriorates, covenants in different debt contracts are gradually violated. Upon violation, creditors do not know how many other covenants are violated, hence the asynchronous structure. The creditors can also decide when to exit the firm, including the possibility of demanding an immediate repayment $\beta_i(t_i) = t_i$.

The conditional density of t_i is given by $f(t_i|t_0) = \frac{1}{\eta}\mathbf{1}_{t_i \in [t_0, t_0+\eta]}$. The posterior belief about t_0 is therefore given by

$$\psi(t_0|t_i) = \frac{f(t_i|t_0)\phi(t_0)}{\int_{t_i-\eta}^{t_i} f(t_i|s)\phi(s)\mathrm{d}s} = \frac{\lambda e^{-\lambda(t_0-t_i)}}{e^{\lambda\eta}-1}, \tag{1}$$

for $t_0 \in [t_i - \eta, t_i]$, and $\psi(t_0|t_i) = 0$ otherwise. Denote by $\Psi(t_0|t_i)$ the corresponding cumulative distribution function.

The density (rate) of creditors exiting at any time $t \geq t_0$ is given by

$$w_t(t_0, \{\beta_i\}) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\beta_i(t_i) \in [t, t+\Delta t]} f(t_i|t_0)\mathrm{d}t_i. \tag{2}$$

The total remaining assets in the firm at time $t \geq 0$, denoted as Y_t, evolve as follows:

$$\mathrm{d}Y_t = \begin{cases} gY_t\mathrm{d}t & \text{if } t \leq t_0 \\ \left(g'Y_t - w_t e^{gt}\right)\mathrm{d}t & \text{if } t > t_0 \end{cases}. \tag{3}$$

The Y_t process is intuitive: assets grow at a rate of g before the bad shock at t_0, and at g' thereafter. Following t_0, at any time t, a mass of $w_t\mathrm{d}t$ creditors receive a total repayment of $w_t e^{gt}\mathrm{d}t$ from the firm.

When a fraction $k \in (0,1)$ of creditors exit, the firm faces bankruptcy. The time of bankruptcy $\hat{t}$ is thus defined as

$$\hat{t}\left(k, t_0, \{\beta_i\}\right) = \inf\left\{u \,\middle|\, \int_{t_0}^{u} w_t\left(t_0, \{\beta_i\}\right)\mathrm{d}t \geq k\right\}. \tag{4}$$

The equilibrium variables in this model are the creditors' exit strategies $\beta^* = \{\beta_i^*(t_i)|i \in [0,1]\}$ and the bankruptcy threshold k^* chosen by the manager of the firm.

The game ends when the firm enters bankruptcy at $\hat{t}$, and all creditors realize their final payoffs.[12] If a creditor exits before bankruptcy (i.e., $\beta_i \leq \hat{t}$), she receives full repayment $e^{g\beta_i}$. Conversely, if bankruptcy precedes the creditor's exit (i.e., $\beta_i > \hat{t}$), she receives the recovery payoff in bankruptcy $\frac{Y_{\hat{t}}}{1-k}$; that is, $1-k$ creditors share the remaining assets $Y_{\hat{t}}$. Hence, given bankruptcy threshold k and other creditors' equilibrium strategy β_{-i}^*, creditor i's strategy $\beta_i^*(t_i|\beta_{-i}^*, k)$ maximizes her expected payoff Π_i:

$$\begin{aligned} \beta_i^*(t_i) &= \arg\max_{\beta_i} \Pi_i(\beta_i|t_i, \beta_{-i}^*, k) \\ &\equiv \underbrace{\int_{\{t_0:\beta_i \leq \hat{t}(t_0)\}} e^{g\beta_i}\psi(t_0|t_i)\mathrm{d}t_0}_{\text{exit before } \hat{t}} + \underbrace{\int_{\{t_0:\beta_i > \hat{t}(t_0)\}} \frac{Y_{\hat{t}}}{1-k}\psi(t_0|t_i)\mathrm{d}t_0}_{\text{exit after } \hat{t}}, \end{aligned} \tag{5}$$

[12] While our formal model concludes with the firm's bankruptcy at $\hat{t}$, one should not narrowly construe this as a mere liquidation. The essence of our payoff structure is that a creditor's payoff from bankruptcy typically falls short of a successful exit. Empirically, the recovery rate is often below 1. The bankruptcy payoff in the model can also be microfounded with a costly renegotiation.

where the posterior belief $\psi(t_0|t_i)$ and the time of bankruptcy $\hat{t}\,(k, t_0, \{\beta_i\})$ are given by (1) and (4), respectively.[13]

The manager chooses the bankruptcy threshold k^* in order to maximize the firm's life span $\hat{t}(k, t_0, \{\beta_i^*\})$, as defined in (4). This preference can be attributed to the manager's desire to prolong their tenure within the firm.[14] Throughout the paper, we do not distinguish between the "firm" and its "manager" and use the two terms interchangeably, although the potential agency friction in this context may be an interesting direction for future work.

We consider two scenarios depending on the manager's ability to commit to a termination threshold k up front. First, in the case of commitment, the manager strategically chooses k_c to influence creditors' exit decisions $\beta_i^*(t_i|\beta_{-i}^*, k_c)$, seeking to maximize the firm's life span

$$k_c \equiv \arg\max_k \hat{t}(k, t_0, \{\beta_i^*|k\}). \tag{6}$$

This commitment may be implemented via holding illiquid assets, convertible into cash only through a prolonged bankruptcy process. In addition, the solution offers a useful benchmark when we later compare with what can be achieved through regulatory intervention in Section 4. There is a natural feasibility constraint in that the remaining assets at the time of committed bankruptcy cannot turn negative: $Y_{\hat{t}} \geq 0$.

Second, without commitment power, the manager maximizes the firm's life span ex post, and bankruptcy is therefore triggered by feasibility—when all assets are depleted. Hence, the bankruptcy threshold without commitment k_{nc} satisfies

$$Y_{\hat{t}(k_{nc}, t_0, \{\beta_i^*|k_{nc}\})} = 0.$$

Finally, we introduce social welfare. As will be verified later, the game ends when the distressed firm goes bankrupt, and equity receives nothing. Hence, the total welfare can be defined as the total equilibrium payoff to creditors:

$$W \equiv \int_{t_0}^{t_0+\eta} \Pi_i \left(\beta_i^*|t_i, \beta_{-i}^*, k\right) f(t_i|t_0)\mathrm{d}t_i. \tag{7}$$

Welfare loss in our model arises from both the productive assets being taken outside the firm and the termination (disruption) of the production process due to bankruptcy. The bankruptcy policies that we consider later in the paper influence creditors' exit strategies β_i^* or the bankruptcy threshold k or both, thereby generating welfare implications.

To prevent infinite values, we introduce a purely technical assumption: if the bad shock occurs at t_0, the game exogenously terminates at $t_0 + T$ irrespective of creditors' actions. In

[13]Note that $\hat{t}$ depends on other creditors' equilibrium exit strategies $\{\beta_{-i}^*\}$ but not on creditor i's strategy β_i because of the infinitesimally small size of any individual creditor.

[14]Furthermore, if the firm can stochastically recover from the bad shock, an extension studied in Subsection 5.2, equity value maximization yields a similar objective: maximizing creditors' waiting time. The economic insights remain robust.

the event of exogenous termination at t, creditors equally share the remaining assets up to the promised value e^{gt}. Throughout the paper, we focus on the scenario where T is sufficiently large and, thus, nonbinding, such that in equilibrium, the firm reaches endogenous bankruptcy (i.e., when k creditors exit the firm).

2.2 Further Discussion of Modeling Assumptions

2.2.1 A Banking Application of the Model

Although our model is mainly framed in the context of corporate bankruptcy, the feature of asynchronous learning among creditors is also pertinent in bank runs. The setting, interpreted slightly differently, can be naturally applied to study bank runs.

The banks and uninsured depositors can be interpreted as the firm and creditors respectively in our model. A bad shock hitting a bank's net interest margin at some random time t_0 resembles the reduction in the growth rate of the assets, a scenario analogous to the interest rate shock in a regional bank crisis. Depositors are uncertain about the bank's exposure to the bad shock and others' awareness of it, reflected in the model by the asynchronously learning window over $[t_0, t_0 + \eta]$. Upon learning about the shock, depositors privately decide to withdraw at $\beta_i(t_i) \geq t_i$. A shorter η reflects faster news dissemination among depositors, akin to the 2023 U.S. regional bank crisis rapidly affecting Silicon Valley Bank within days.

We compare two bank failure thresholds in Section 3.4: the bank's voluntary termination threshold k_c that maximizes the duration of the bank (6) and the regulator's choice of the failure threshold k_W that maximizes welfare in (7).

2.2.2 Renegotiation and New Financing

If all creditors could collectively renegotiate and accept a lower interest rate $g' < g$ following the bad shock, then the coordination problem would obviously dissolve. However, achieving such a coordinated renegotiation in a decentralized manner is practically unattainable, which is arguably one of the reasons for formal bankruptcy procedures. Empirical evidence indicates that when firms disclose adverse news, private renegotiation commonly leads to higher interest rates (see Roberts and Sufi (2009*a*,*b*) and Roberts (2015)).[15] Consequently, endogenizing interest rates could exacerbate the coordination problem by widening the gap in creditors' payoffs between successful exits and bankruptcy due to faster interest accrual and fewer remaining assets. Therefore, the fundamental channel of our model remains robust.

Moreover, the prospect for the firm to secure new equity or debt financing may also be very difficult. A recent prominent instance is the collapse of First Republic Bank during the 2023 regional bank crisis in the U.S. Unlike the swift failure of Silicon Valley Bank over a couple of days, First Republic Bank had more than a month to secure additional financing following the initial bank run. Despite mounting efforts, their financing efforts proved futile as no investors

[15]Related to the covenant interpretation of the model, interest rates often increase upon covenant violations.

were willing to cover the shortfall on the bank's balance sheet. In our model, when the bad shock occurs at t_0, the firm becomes immediately insolvent. The value of debt (assuming no default and paid at natural maturity $t_0 + T$) amounts to $e^{g(t_0+T)}$, while the value of assets post-shock becomes $Ae^{gt_0+g'T}$. If T is significantly large, as assumed in this paper, the firm lacks adequate assets to cover its liabilities. This debt overhang problem substantially impedes the firm's ability to raise new capital.

2.2.3 Observability of Exits and Information Revealed in the Capital Market

We follow the literature (Abreu and Brunnermeier (2003), Goldstein and Pauzner (2005) and He and Manela (2016)) and adopt the assumption that an individual creditor's decision to exit the firm is unobservable to others. Moreover, we abstract from potential public signals, such as the firm's equity or bond price. These assumptions align well with scenarios involving bank runs and lending to private firms, where the disclosure of contractual details is typically rare. Even in the case of public firms, where such signals or information disclosure is more prevalent, they are likely imperfect. For instance, the retirement of maturing debt might stem from a creditor's reluctance to roll over, but it could also result from the firm's deliberate reduction of leverage. Many factors unrelated to the firm's performance could be driving the debt composition of the firm—its capital needs, liquidity management, macroeconomic conditions, mispricing in the capital market, and so on. Introducing a noisy public signal about t_0 would significantly complicate the exposition without altering the central trade-off within the model.

3 Solution to the Benchmark Model

In line with existing literature, our analysis centers on symmetric linear equilibria: each creditor opts to wait for a uniform duration $\tau \geq 0$ after learning about the bad news at t_i and divests at

$$\beta_i(t_i) \equiv \beta(t_i) = t_i + \tau. \tag{8}$$

This focus on symmetric linear equilibria aligns naturally with the fact that all creditors are ex ante identical.[16] Throughout the paper, we make the following parameter assumptions, which ensure the existence and uniqueness of the symmetric linear equilibrium with the game ending in finite time.

Assumption 1 *The parameters satisfy (1)* $1 < A < \overline{A} \equiv \frac{\left(\frac{g}{g-\lambda}\right)^{\frac{g-g'}{\lambda}} - 1}{(g-g')\eta}$*; and (2)* $\eta > \frac{1}{g-\lambda} > 0$.

[16]The existence of these equilibria hinges upon the feature that time itself does not inherently convey information. Following Abreu and Brunnermeier (2003) and He and Manela (2016), we focus on the stable learning phase when $t_0 \geq \eta$. In contrast, when the bad shock occurs prematurely (e.g., $t_0 = 0$), the potential for learning from time arises: the initial creditor who becomes aware of the shock at $t_i = 0$ discerns that all other creditors are still uninformed.

Essentially, both conditions in Assumption 1 combined ensure that the coordination problem is a significant concern among creditors in the sense that if all other creditors exit immediately $\beta(t_j) = t_j$, then the optimal strategy for creditor i is also to exit immediately. The first condition states that there cannot be too many assets relative to existing debt ($A < \overline{A}$). Otherwise, creditors, facing an ample pool of assets, may not necessarily worry about others' exit.[17] The second condition requires that the debt maturity η (or the learning window) is sufficiently long, so that early creditors impose significant externalities on latecomers. Otherwise, even in the extreme case where all creditors learn of the bad shock simultaneously $\eta = 0$ and exit immediately, there are still assets remaining in the firm, and individual creditors therefore will have an incentive to deviate and wait.[18]

We characterize the equilibrium waiting time τ^* in Subsection 3.1, emphasizing the trade-off between the recovery rate a creditor can expect in bankruptcy and the hazard rate that measures the exposure of a creditor to potential bankruptcy. We then compare two managerial choices of bankruptcy thresholds k_c and k_{nc} with and without commitment power in Subsection 3.2. The comparison leads to our insight on the automatic stay clause often imposed in bankruptcy (Subsection 3.3). Finally, in Subsection 3.4, we compare the manager's preferred thresholds against the welfare-maximizing threshold and draw implications on bank failure. The comparison also highlights the conflicting incentives between the firm and regulator, which motivates the avoidable preference regulation in Section 4.

3.1 Recovery Rate, Hazard Rate, and Creditor Waiting Time

In a symmetric linear equilibrium (8), creditors start to leave the firm at $t_0 + \tau$, followed by a $\frac{1}{\eta}$ fraction of creditors exiting per unit time thereafter. Mathematically, the exit rate defined in (2) simplifies to

$$w_t(t_0, \beta) = \begin{cases} 0 & t \le t_0 + \tau \\ \frac{1}{\eta} & t_0 + \tau < t \le t_0 + \eta + \tau \end{cases}. \tag{9}$$

The pivotal (kth) creditor learns of the bad shock at $t_k = t_0 + k\eta$ and subsequently exits at

$$\hat{t}\,(k, t_0, \beta) = t_0 + k\eta + \tau, \tag{10}$$

thereby triggering bankruptcy. The asset dynamics outlined in (3) and the corresponding recovery payoff in bankruptcy can be computed accordingly.

[17] As we discuss later (see Footnote 21), if the first condition is violated, it is possible to select a k to eliminate the coordination friction, and creditors would strictly prefer to wait longer.

[18] When the second condition is violated, several exotic outcomes may emerge, including the nonexistence of a non-commitment equilibrium (see a brief discussion following Proposition 3). While we have investigated these outcomes, for exposition, we assume them away due to their lack of practical relevance and additional economic insights.

Lemma 1 *Under the conjectured equilibrium strategy (8), the total remaining assets Y_t are calculated as follows:*

$$Y_t = \begin{cases} Ae^{gt} & 0 \leq t \leq t_0 \\ Ae^{gt_0+g'(t-t_0)} & t_0 < t \leq t_0+\tau \\ Ae^{gt_0+g'(t-t_0)} - \frac{e^{gt}-e^{g(t_0+\tau)+g'(t-t_0-\tau)}}{(g-g')\eta} & t_0+\tau < t \leq t_0+\tau+k\eta \end{cases}. \tag{11}$$

The recovery payoff to a creditor in bankruptcy is

$$\frac{Y_{\hat{t}}}{1-k} = \frac{Ae^{g'(\tau+k\eta)} - \frac{e^{g(\tau+k\eta)}-e^{g\tau+g'k\eta}}{(g-g')\eta}}{1-k} e^{gt_0} \equiv \alpha(\tau,k)e^{gt_0}. \tag{12}$$

After determining the recovery payoff, we now compute creditors' equilibrium waiting time τ^* for any given bankruptcy threshold k. To avoid confusion in subsequent derivations, we denote the exit time of a generic creditor i as $\beta_i = t_i + \tau_i$, while τ^* represents the waiting strategy in a symmetric equilibrium for other creditors. Utilizing (10) and the conjectured equilibrium strategy, creditor i's payoff described in (5) simplifies to

$$\begin{aligned}\Pi_i(\tau_i|t_i,\tau^*,k) = &\int_{t_i+\tau_i \leq t_0+k\eta+\tau^*} e^{g(t_i+\tau_i)}\psi(t_0|t_i)\mathrm{d}t_0 \\ &+ \int_{t_i+\tau_i > t_0+k\eta+\tau^*} \alpha(\tau^*,k)e^{gt_0}\psi(t_0|t_i)\mathrm{d}t_0. \end{aligned} \tag{13}$$

Taking the first-order condition of (13) with respect to τ_i and using symmetry $\tau_i^* = \tau^*$, we arrive at

$$ge^{g(t_i+\tau^*)}\left[1-\Psi(t_i-k\eta|t_i)\right] = [e^{g(t_i+\tau^*)} - \alpha e^{g(t_i-k\eta)}]\psi(t_i-k\eta|t_i). \tag{14}$$

Condition (14) reveals the trade-off associated with waiting an additional moment Δt. On the left-hand side, the marginal benefit is an increase in the exit payoff by $g\Delta t e^{g(t_i+\tau^*)}$ due to interest accrual at the rate of g. This occurs when the firm does not enter bankruptcy—a case with probability $1-\Psi(t_i-k\eta|t_i)$ in equilibrium. Meanwhile, the right-hand side of condition (14) reflects the marginal costs. With probability $\psi(t_i-k\eta|t_i)\Delta t$, the firm fails during the next Δt instant as creditor i approaches pivotal timing ($t_i \in [t_0+k\eta-\Delta t, t_0+k\eta]$). In this scenario, creditor i forfeits the full repayment from a successful exit and instead receives the recovery payoff in bankruptcy $\alpha(\tau^*,k)e^{gt_0}$ (or, equivalently, $\alpha(\tau^*,k)e^{g(t_i-k\eta)}$ as $t_i \approx t_0+k\eta$ is pivotal).

Define the following hazard rate:

$$h(k) \equiv \frac{\psi(t_0=t_i-k\eta|t_i)}{1-\Psi(t_0=t_i-k\eta|t_i)} = \frac{\lambda e^{\lambda k\eta}}{e^{\lambda k\eta}-1}. \tag{15}$$

A simple manipulation of (14) yields an important decomposition of the equilibrium waiting time τ^* as follows.

Proposition 1 (Decomposition of Creditors' Waiting Time) *Any symmetric equilibrium with $\beta(t_i) = t_i + \tau^*$, where the waiting time $\tau^* > 0$, satisfies*

$$\tau^* = \frac{1}{g} \log \left[\alpha(\tau^*, k) e^{-gk\eta} \right] + \frac{1}{g} \log \left(\frac{h(k)}{h(k) - g} \right). \tag{16}$$

Furthermore, the recovery rate in equilibrium is less than 1:

$$\alpha(\tau^*, k) e^{-g(k\eta + \tau^*)} = \frac{\alpha(\tau^*, k) e^{g t_0}}{e^{g\hat{t}}} < 1. \tag{17}$$

Despite being an implicit function of τ^*, the decomposition (16) clearly illustrates the technical contributions of our framework. It distinctly identifies two key factors influencing creditors' exit decisions in a dynamic run scenario. The first term in (16) captures the recovery rate channel, which is the ratio between the payoff in bankruptcy and the promised repayment

$$\frac{\alpha(\tau^*, k) e^{g t_0}}{e^{g(t_0 + k\eta + \tau^*)}} = \alpha(\tau^*, k) e^{-g(k\eta + \tau^*)}. \tag{18}$$

A higher recovery in bankruptcy incentivizes creditors to prolong their waiting time. Notably, unlike prior dynamic coordination models where the termination payoff $\alpha(\tau^*, k)e^{g t_0}$ is typically considered exogenous,[19] it is endogenously determined by creditors' decisions (τ^*), the firm's strategy (k), and regulations (to be introduced in Section 4) in our model, leading to a fixed-point problem in τ^*. Despite its technical complexity, we analytically characterize the equilibrium waiting time τ^* in Proposition 2.

The second term in (16) unveils the hazard rate channel, which captures the likelihood of a creditor being affected by bankruptcy. A higher hazard rate $h(k)$ indicates a greater likelihood of unsuccessful exits, prompting creditors to exit more rapidly, as reflected by the reduction in waiting time τ^* through $(\frac{h(k)}{h(k)-g})$ in (16).

As will be clear soon, there is generally a trade-off between the two channels: often, strategies or regulatory interventions designed to ameliorate one channel might inadvertently intensify the other. This trade-off underscores the intricate balance that managerial decisions and regulatory interventions need to strike. It emphasizes the need for comprehensive assessments and careful considerations that we develop here. We will revisit this point in Subsections 3.3 and 4.2.

Condition (16) as an implicit function of τ^* has a unique solution, as illustrated by Figure 1. Because of the nature of a symmetric equilibrium, the hazard rate channel is independent of the common waiting time τ^* (see the flat line with circles in Figure 1). The recovery rate is decreasing in τ^*, as the longer the creditors wait, the more disproportionate the amount of capital that the exiting creditors can take with them, leaving fewer assets remaining in bankruptcy (see the decreasing curve with solid dots in Figure 1). We can explicitly solve for τ^* in the following proposition.

[19] For example, in Brunnermeier and Morgan (2010), among other differences, the coefficient of termination payoff $\alpha \equiv 1$.

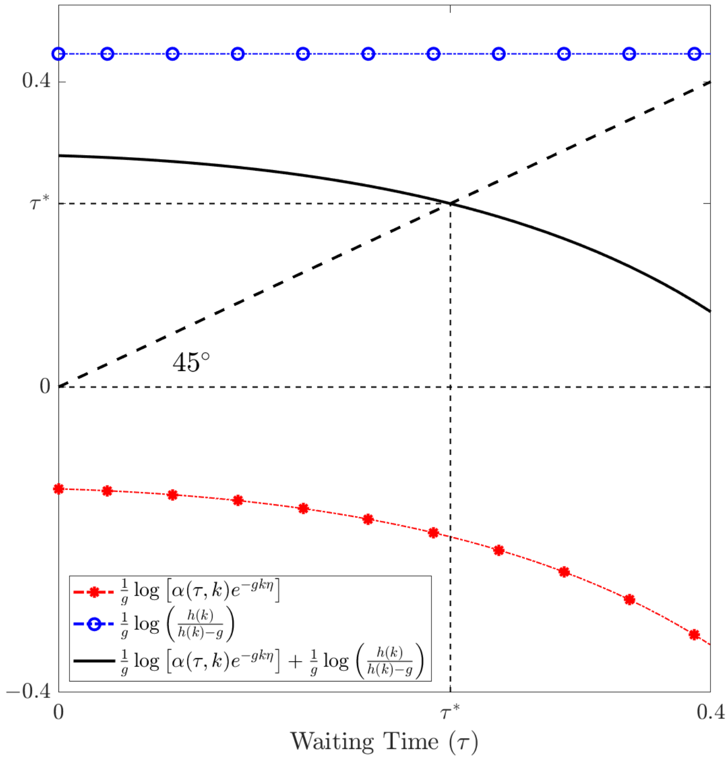


Figure 1: Equilibrium Waiting Time

Note: The figure depicts the effect of waiting time τ on the recovery rate channel and the hazard rate channels, and the determination of τ^* as the fixed-point solution to (16). The decreasing curve with solid dots reflects the recovery rate channel (i.e., $\frac{1}{g}\log\left(\alpha(\tau,k)e^{-gk\eta}\right)$). The flat line with circles plots the hazard rate channel (i.e., $\frac{1}{g}\log\left(\frac{h(k)}{h(k)-g}\right)$). The combined effect given by the solid curve intersects with the 45-degree line, giving the equilibrium waiting time τ^*. The parameters for this figure are $A = 1.1$, $g = 2$, $\lambda = 0.2$, $\eta = 0.6$, $g' = 1$, and $k = 0.5$.

Proposition 2 (Creditors' Equilibrium Strategy) *In the unique equilibrium,*

$$\tau^*(k) = \max\left\{0, \frac{1}{g-g'}\left(\log A - \log v(k)\right)\right\}, \tag{19}$$

where

$$v(k) \equiv \frac{e^{(g-g')k\eta} - 1}{(g-g')\eta} + (1-k)\left[\frac{g}{\lambda}e^{(g-\lambda-g')k\eta} - (\frac{g}{\lambda}-1)e^{(g-g')k\eta}\right]. \tag{20}$$

As a technical remark, it is possible to have all creditors exit immediately upon learning the bad shock; that is, $\tau^* = 0$. Intuitively, regardless of other creditors' strategies, creditor i's best response is always to undercut: $\tau_i < \tau^*$, leading to the corner solution. This case may occur when the hazard rate h is high or when the recovery rate α is low in equilibrium.

3.2 Bankruptcy Threshold k with and without Commitment

Having solved for creditors' exit strategy τ^*, we now turn our attention to the firm's (manager's) choice of bankruptcy threshold k. Comparing the scenarios with and without commitment power reveals an intriguing insight: committing to filing for bankruptcy early even with remaining assets may actually postpone its occurrence.

We first analyze threshold k_{nc} when the manager has no commitment power. As explained in the model setup, bankruptcy is triggered when all assets in the firm are exhausted:

$$Y_{t_0+k_{nc}\eta+\tau^*(k_{nc})} = 0. \tag{21}$$

Consequently, the zero recovery $\alpha(\tau, k_{nc}) = 0$ motivates creditors to exit immediately upon learning the bad shock (i.e., $\tau^*(k_{nc}) = 0$). Formally, we have the following proposition.

Proposition 3 (Bankruptcy Threshold without Commitment) *Without commitment, the bankruptcy threshold is given by*

$$k_{nc} \equiv \frac{1}{(g-g')\eta}\log\left(A(g-g')\eta + 1\right). \tag{22}$$

The corresponding equilibrium waiting time is $\tau^* = 0$.

As a technical remark, we briefly discuss some possible outcomes when the second condition in Assumption 1 is violated. In this case, it is possible that $k_{nc} > 1$, resulting in non-existence of a symmetric equilibrium without commitment. Intuitively, when maturity η (or the learning window) is short, even when all creditors exit immediately ($\tau^* = 0$), early leavers only pose minor externalities on the late comers. As a result, there are still assets left, making it a profitable deviation for creditors to wait longer.[20]

[20]We further argue that there cannot be a no-commitment equilibrium with $\tau^* > 0$ and a corresponding $k_{nc}(\tau^*) < 1$ such that assets are exactly depleted ($\alpha = 0$) at the time of bankruptcy. In fact, one can consider the most likely case where assets can be depleted $k \to 1$. In this case, the equilibrium $\tau^*(k)$ converges to some

Next, we consider the case where the manager can commit to a bankruptcy threshold $k \in (0, 1)$ that maximizes the time of bankruptcy $\hat{t}$. In a linear symmetric equilibrium, this objective (6) can be explicitly written as

$$k_c = \arg\max_k \hat{t} = \arg\max_k k\eta + \tau^*(k). \tag{23}$$

The following proposition reveals the key insight of this subsection: committing to filing for bankruptcy early $k_c < k_{nc}$ may actually postpone its occurrence.

Proposition 4 (Bankruptcy Threshold with Commitment) *With commitment, there exist $g_0 \in (0, g)$ and $A_0 \in (0, \overline{A})$, both defined in the Appendix, such that the manager chooses not to file for bankruptcy early; that is, $k_c = k_{nc}$ if either $g' \leq g_0$ or $g' > g_0$ and $A \in (1, A_0]$. Otherwise, if*

$$g' > g_0 \quad \textit{and} \quad A \in (A_0, \overline{A}), \tag{24}$$

the manager chooses to file for bankruptcy early; that is, the optimal $k_c < k_{nc}$.

A higher bankruptcy threshold k affects the firm's survival time $\tau^* + k\eta$ through three channels. First, by allowing more creditors to exit, the firm can mechanically increase the time to trigger bankruptcy through the $k\eta$ term. Second, the fact that more creditors can exit also alleviates the hazard rate channel in the determination of creditors' equilibrium waiting time τ^*, making them more patient. Finally, the recovery rate channel works in the opposite direction. Allowing more creditors to exit leaves fewer assets for the remaining creditors in bankruptcy and therefore reduces the recovery rate, which in turn motivates creditors to run (lower τ^*). The recovery rate channel creates the possibility that committing to early bankruptcy filing may paradoxically extend the firm's life.

To visualize these effects, we decompose the time of bankruptcy $k\eta + \tau^*$ (the hump shaped curve) into these three components in Figure 2. The straight line with squares, $k\eta$, represents the mechanical effect where a larger threshold k necessitates more exiting creditors. Furthermore, the increasing curve with circles plots the hazard rate channel on τ^*. In contrast, the decreasing dashed curve with solid dots depicts the lower recovery rate as more creditors are allowed to leave.

Proposition 4 identifies the exact conditions under which early termination is beneficial: when the shock is relatively mild (g' is not too small) and leverage ($\frac{1}{A}$) is relatively low.[21]

$\tau^*_{\text{lim}} < \infty$, and the equilibrium recovery rate also converges to some positive number: $\alpha(k) \to \alpha_{\text{lim}} > 0$. Even in this extreme case, assets can only be depleted asymptotically as $k \to 1$ in the sense that $Y_{\hat{t}} = (1-k)\alpha \to 0$, but not exactly $\alpha = 0$ in bankruptcy.

[21] It is also useful to note that we also require A to be not too large as in the first condition of Assumption 1 in order for the game to end in finite time $\tau^* < \infty$. If this condition is violated, then it may be possible to choose a k such that the marginal benefit of waiting (i.e., the left-hand side of equation (14)) always dominates. In this case, creditors have incentives to deviate and wait longer, thereby alleviating the coordination friction. This observation, although is not a full equilibrium construction, is consistent with Proposition 4 that when A is larger, commitment to a termination threshold can delay bankruptcy.

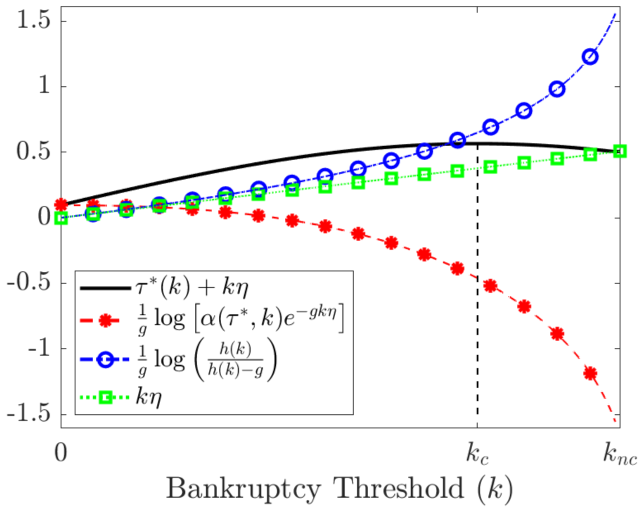


Figure 2: Bankruptcy Threshold and the Firm's Life Span

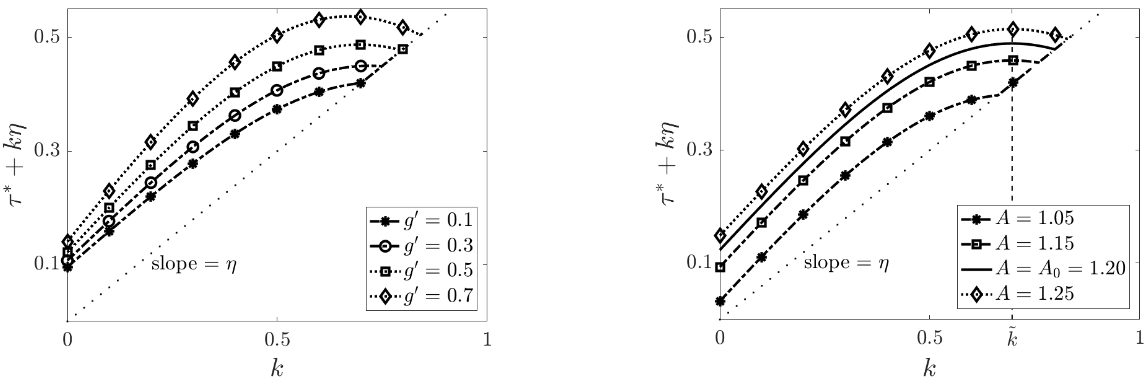
Note: We decompose the equilibrium life span of the firm after the arrival of a bad shock (i.e., $\tau^*(k) + k\eta$, the unmarked curve) into the decreasing recovery rate channel (i.e., $\frac{1}{g}\log\left(\alpha e^{-gk\eta}\right)$, the dashed curve with dots), the increasing hazard rate channel (i.e., $\frac{1}{g}\log\left(\frac{h(k)}{h(k)-g}\right)$, the dashed curve with circles) and the mechanical delay components (i.e., $k\eta$, the the dotted curve with squares) according to (16). The parameters for this figure are $A = 1.1$, $g = 2$, $\lambda = 0.2$, $\eta = 0.6$, and $g' = 1$.

Intuitively, for early termination to prolong firm life, the equilibrium waiting time τ^* must be sufficiently large and sensitive to k in order for its indirect effect through the recovery rate on τ^* to dominate the direct effect through $k\eta$. When the fundamental is not too bad, i.e., relatively larger g' and A, creditors are more willing to wait, making the recovery rate channel more pronounced.

To illustrate this point clearly, we again plot the time to bankruptcy against the threshold k in Figure 3, varying the magnitude of the shock g' in the left panel and asset-to-debt ratio A in the right panel. A general feature among these plots is the kinks in the curves, beyond which the survival time $k\eta + \tau^*$ collapses to a straight line $k\eta$. In other words, creditors immediately exit ($\tau^* = 0$) when k is large, because the recovery in bankruptcy is too low to risk any waiting (recall the intuition from Proposition 3). If this region is too big, such as when g' or A is small, the overall effect is dominated by the mechanical term $k\eta$. The manager simply allows creditors to deplete all assets ($k_c = k_{nc}$) as the recovery rate channel is largely muted.

3.3 Application: Automatic Stay

Our analysis in Propositions 3 and 4 sheds light on the impact of automatic stay, a significant aspect in bankruptcy proceedings mandating creditors to cease individual debt collection and await a collective resolution in a bankruptcy court. In this context, a scenario with automatic stay mirrors the commitment case in our model ($k = k_c$). Conversely, a world without automatic



(a) The dependence of $(\tau^* + k\eta)$ on k and g'

(b) The dependence of $(\tau^* + k\eta)$ on k and A

Figure 3: Bankruptcy Threshold and the Firm's Life Span: g' and A

Note: This figure depicts the firm's life span $\tau^* + k\eta$ against the bankruptcy threshold k. The parameters are $A = 1.2$, $g = 2$, $\lambda = 0.2$, $\eta = 0.6$ (panel 3a), and $g = 2$, $\lambda = 0.2$, $\eta = 0.6$, $g' = 0.5$ (panel 3b).

stay aligns with allowing a first-come-first-serve system for exiting creditors, resulting in no assets left for latecomers, as modeled in the non-commitment case ($k = k_{nc}$).

Early bankruptcy filings, facilitated by the automatic stay feature, ensure the preservation of some assets for remaining creditors during bankruptcy. This incentivizes all creditors to remain invested ex ante ($\tau^* > 0$), fostering coordination and delaying bankruptcy. On the flip side, letting creditors run until all assets are depleted may ex post increase firm life, but the zero recovery in bankruptcy may cause creditors to start exiting distressed firms early on.

Proposition 4 generates testable empirical predictions regarding the value of automatic stay and early bankruptcy filings. The conditions that guarantee $k_c < k_{nc}$ suggest that such legal protection and commitments are more valuable when they can significantly affect the recovery rate in bankruptcy—when the shock is relatively mild (high g') and the firm has relatively low leverage (high A).

3.4 Welfare-Maximizing k and A Banking Application

It is useful to compare the firm's choice of bankruptcy threshold (k_c and k_{nc}) against the one that maximizes welfare (7), formally denoted by k_W. The comparison highlights the different incentives of the firm and a welfare-maximizing regulator: while the firm may prefer to declare bankruptcy early in order to prolong firm life, the regulator prefers to terminate even sooner and preserve more assets in bankruptcy.

We start by explicitly rewriting the welfare function (7). In a linear symmetric equilibrium, the first k depositors exit during $[t_0 + \tau^*, t_0 + k\eta + \tau^*]$ and receive e^{gt}. The remaining creditors share the assets at bankruptcy $Y_{t_0+k\eta+\tau^*}$. Hence,

$$W(k) = \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*} \frac{1}{\eta} e^{gt} \mathrm{d}t + Y_{t_0+k\eta+\tau^*}. \tag{25}$$

Proposition 5 (Welfare-maximizing Bankruptcy Threshold) *The bankruptcy threshold that maximizes welfare is weakly smaller than the one maximizing the firm's life span:*

$$k_W \leq k_c.$$

Furthermore, there exists $\tilde{A} \in (0, \overline{A})$, defined in the Appendix, such that the inequality is strict if $g' > g_0$ and $A \geq \tilde{A}$.

In fact, it can be shown that $A_0 > \tilde{A}$, and therefore Propositions 4 and 5 combined imply that whenever early bankruptcy is valuable $k_c < k_{nc}$, the regulator strictly prefers to terminate even sooner than the firm.

To illustrate why a welfare-maximizing regulator favors an earlier bankruptcy (a smaller $k_W < k_c$), we analyze the two factors affecting welfare. The first factor is the termination of asset growth at $\hat{t} = t_0 + k\eta + \tau^*$; and both the regulator and the firm share the same incentive to delay bankruptcy. The second factor is that capital is more productive inside the firm before bankruptcy: assets in the firm appreciate at the rate of $g' > 0$, whereas the outside return is normalized to 0. Therefore, in addition to a bigger $\hat{t}$, which is the firm's sole objective, the regulator also prefers a delay in starting withdrawal, i.e., a larger $t_0 + \tau^* = \hat{t} - k\eta$. Hence, relative to the firm's preference on k, the regulator suffers an additional marginal dis-utility from the "$-k\eta$" term, which implies a smaller optimal threshold k preferred by the regulator.[22] In the next section, we consider another bankruptcy regulation, avoidable preference, and show that it can effectively realign the incentives between the firm and the regulator.

In addition to serving as a welfare benchmark, the threshold k_W is also particularly relevant in the context of bank runs. Although regulators often do not have the authority to force a distressed firm into bankruptcy, many bank failures are triggered by regulators' decisions to seize a troubled bank based on the magnitude of bank runs. The welfare maximizing k_W therefore sheds light on the optimal timing to force a bank failure. There is an intricate balance. Seizing the bank too early may make it too challenging for depositors to exit, potentially accelerating runs, whereas triggering bankruptcy too late risks having few assets left for latecomers, also incentivizing ex ante runs.

The prediction that the regulator prefers an earlier termination than the bank (i.e., $k_W \leq k_c$ in Proposition 5) also provides a rationale for regulatory intervention rather than relying on bank's self-motivated decision to file for bankruptcy. It echoes the Federal Reserve's stance in the wake of the 2023 regional bank crisis that brought down three mid-sized banks: Signature Bank, Silicon Valley Bank, and First Republic Bank. In a postmortem review of the crisis, the Federal Reserve explicitly calls for a quicker response from the supervisors.[23]

[22] In practice, there may be other reasons for misaligned incentives between the distressed firm and the regulator. For example, there could be a more efficient user of the assets that warrants a sale of the firm as an efficient outcome. The regulator in these situations may also prefer to act sooner than the firm's own choice.

[23] Specifically, in the Review of the Federal Reserve's Supervision and Regulation of Silicon Valley Bank (https://www.federalreserve.gov/publications/files/svb-review-20230428.pdf), one of the key takeaways is that

4 Avoidable Preference

The framework developed in this paper allows us to study a frequently cited regulation in corporate bankruptcy: "avoidable preference." Under this legislation, creditors who receive repayments shortly before bankruptcy, resulting in a more favorable treatment relative to the remaining creditors, need to return those payments. These returned repayments, together with other assets left in the firm, are shared among all creditors in the bankruptcy process.[24]

In the United States, for example, Chapter 11, Section 547 (b) of the Bankruptcy Code states that "the trustee may, ... avoid any transfer of an interest of the debtor in property ... made (A) on or within 90 days before the date of the filing of the [bankruptcy] petition; or (B) between 90 days and 1 year before the date of the filing of the [bankruptcy] petition, if such creditor at the time of such transfer was an insider." While rarely studied in the academic literature of finance, this clause is commonly cited in bankruptcy litigation. Among the 595 bankruptcy cases collected by the Westlaw legal research service between 2017 and 2019, 290 cases (or 48.74%) cite avoidable preference in the United States. For instance, when General Motors filed for bankruptcy in 2009, the bankruptcy trustee sued creditor JPMorgan Chase Bank to recover approximately $28 million in interest and $1.4 billion in principal repayments, citing the avoidable preference clause.

In Subsection 4.1, we introduce avoidable preference to study creditors' exit decisions, the firm's selection of bankruptcy thresholds, and the regulation design in a comprehensive framework. Subsequently, in Subsection 4.2, we characterize the optimal clawback window and show that it is a relatively detail-free regulation in that it does not rely on the parameters of firms' production process. In Subsection 4.3, we present the final key insight that the avoidable preference regulation can deliver a more efficient outcome than what the firm's own commitment to filing for bankruptcy can achieve. Furthermore, this insight is robust to various different timelines of actions, as discussed in Subsection 4.4.

4.1 Introducing Avoidable Preference

We formally model avoidable preference as a clawback window m chosen by a welfare-maximizing regulator. If, at the time of bankruptcy $\hat{t}$, the firm is unable to honor the full repayment of $e^{g\hat{t}}$ to all remaining creditors (i.e., $(1-k)\,e^{g\hat{t}} > Y_{\hat{t}}$), then the avoidable preference clause becomes effective. Specifically, only creditors who exit at least m dates before bankruptcy (i.e., $\beta_i(t_i) < \hat{t} - m$) can keep the full repayment $e^{g\beta_i(t_i)}$. Creditors who originally receive the full repayment $e^{g\beta_i(t_i)}$ during the final m dates prior to bankruptcy (i.e., $\beta_i(t_i) \in \left[\hat{t} - m, \hat{t}\right]$) need to return the money (i.e. subject to clawback) and instead receive the same payoff as the remaining $1-k$ creditors in bankruptcy. In a symmetric linear equilibrium (8), a mass $\frac{1}{\eta}$ creditors withdraw

"[w]hen supervisors did identify vulnerabilities, they did not take sufficient steps to ensure that Silicon Valley Bank fixed those problems quickly enough."

[24] It is useful to distinguish the avoidable preference clause from a related clause known as fraudulent conveyance in Chapter 11, Section 548. While both are implemented by clawing back repayments made prior to bankruptcy, the latter focuses more on the intention of the debtor. While this channel is an interesting one to investigate, our model assumes away any collusion between the firm and (a subset of) creditors.

every instant, and therefore a total of $\frac{m}{\eta}$ creditors are subject to clawback. Collectively, they return total proceeds of

$$\int_{\hat{t}-m}^{\hat{t}} e^{gt}\frac{1}{\eta}\mathrm{d}t = \frac{e^{g(t_0+\tau)}}{g\eta}\left(e^{gk\eta} - e^{g(k\eta-m)}\right).$$

Consequently, the bankruptcy payoff now contains two parts: the remaining assets in the firm $Y_{\hat{t}}$ and the clawback proceeds. Mathematically,

$$\alpha(\tau,k,m)e^{gt_0} = \frac{Ae^{g'(k\eta+\tau)} - \frac{e^{g\tau}}{(g-g')\eta}\left(e^{gk\eta} - e^{g'k\eta}\right) + \frac{e^{g\tau}}{g\eta}(e^{gk\eta} - e^{g(k\eta-m)})}{1-k+\frac{m}{\eta}}e^{gt_0}. \tag{26}$$

Note that if there is no clawback $m = 0$, the bankruptcy payoff naturally degenerates to (12) in the benchmark model.

With the aforementioned adjustment to the bankruptcy payoff $\alpha(\cdot)e^{gt_0}$, the payoff to creditors taking into account the possible clawback becomes

$$\begin{aligned}\Pi_i(\tau_i|t_i,\tau^*,k,m) = &\int_{t_i+\tau_i\leq\hat{t}-m} e^{g(t_i+\tau_i)}\psi(t_0|t_i)\mathrm{d}t_0 \\ &+\int_{t_i+\tau_i>\hat{t}-m} \alpha(\tau^*,k,m)e^{gt_0}\psi(t_0|t_i)\mathrm{d}t_0. \end{aligned}\tag{27}$$

The equilibrium in this three-party (creditors, the firm, and the regulator) game contains three objects:

1. The firm's choice of bankruptcy threshold k^* maximizes its life span. Similar to the benchmark model, we consider two choices of the bankruptcy threshold k^*. With commitment, the management chooses $k^* = k_c^m$ that maximizes the termination time $\hat{t}$ as in (23). Without commitment, the threshold k_{nc}^m is determined by when assets are depleted as in (21).

2. The regulator's choice of the clawback window m^* maximizes welfare in (7).

3. Creditors' exit decision $\tau^*(k,m)$ maximizes their payoff in (27).

The sequence of actions is an intricate aspect as it determines whether a player can internalize the impact of its actions on other players, and therefore may affect the equilibrium outcome. For our main analysis, we focus on the timeline where the firm moves first to set k, followed by the regulator's choice of $m^*(k)$ and then creditors' choice of $\tau^*(k,m)$.[25] Several alternative timelines are discussed in Section 4.4, and our key theoretical insight remains robust in the sense that

[25]For instance, bankruptcy codes often impose different clawback windows depending on the types of repayments and the role of recipients. Creditors who are insiders are often subject to a longer clawback window, whereas trade creditors who receive payments for "ordinary course of business" are often exempt from the avoidable preference clause; see Chapter 11, Section 547 (c). The court can also make ex post judgment calls based on the repayment pattern before bankruptcy.

ex post clawback can be a more efficient policy tool than the firm's own commitment to early bankruptcy filing (Proposition 8 below).

We rule out the uninteresting and unrealistic case (a technical nuance) that all repayments are subject to clawback by restricting our attention to cases with

$$m < k\eta. \tag{28}$$

The essence of condition (28) is to allow some creditors to exit successfully in a symmetric equilibrium.[26]

4.2 Waiting Time $\tau^*(k, m)$ and Clawback Window $m^*(k)$

To solve the three-party game, we calculate equilibrium actions backward. First, we determine creditors' optimal waiting strategy $\tau^*(k, m)$, followed by the regulator's optimal policy response $m^*(k)$ for any given k, and finally the choice of k in the next subsection. The determination of $\tau^*(k, m)$ is similar to the benchmark model. From (27), the first-order condition with respect to τ is

$$\begin{aligned} & g e^{g(t_i+\tau^*)} \left[1 - \Psi(t_i - k\eta + m | t_i)\right] \\ & = [e^{g(t_i+\tau^*)} - \alpha(\tau^*, k, m) e^{g(t_i - k\eta + m)}] \psi(t_i - k\eta + m | t_i), \end{aligned}$$

which is almost identical to (14) except for the modified recovery coefficient $\alpha(\cdot)$ from (26) and the original time frame for successful exit $k\eta$ being replaced by $k\eta - m$. We can already see from this modification that repayment clawback effectively moves the pivotal creditor from k to $k - \frac{m}{\eta}$, a point that we will frequently revisit later.

Similar to (15), we can define the hazard rate associated with the effective pivotal creditor $k - \frac{m}{\eta}$ as

$$h(k, m) \equiv \frac{\psi(t_0 = t_i - k\eta + m | t_i)}{1 - \Psi(t_0 = t_i - k\eta + m | t_i)} = \frac{\lambda e^{\lambda(k\eta - m)}}{e^{\lambda(k\eta - m)} - 1}. \tag{29}$$

Similar to (16), the key equilibrium variable—creditors' waiting time τ^*—can be decomposed into the recovery rate and hazard rate channels:

$$\tau^* = \frac{1}{g} \log \left[\alpha(\tau^*, k, m) e^{-g(k\eta - m)}\right] + \frac{1}{g} \log \left(\frac{h(k, m)}{h(k, m) - g}\right) \tag{30}$$

Next, we characterize the optimal clawback window $m^*(k)$. The ex post redistributive nature of clawback (m) implies that its welfare implication on (25) is only through affecting creditors' exit decision τ^* ex ante. A longer waiting time τ^* increases welfare by delaying the initial repayment at $t_0 + \tau^*$ (and, consequently, all subsequent ones) and the time of bankruptcy at

[26] Otherwise, no creditors can exit in a symmetric equilibrium, and arbitrary outcomes may emerge.

$t_0 + k\eta + \tau^*$. This delay allows the firm to keep more productive assets growing for a longer period of time at the rate of $g' > 0$, thereby enhancing welfare.[27]

Proposition 6 (Equivalent Welfare Measure: Waiting Time) *The total welfare (25) is increasing in τ^*. Furthermore, the regulator's welfare objective is equivalent to maximizing τ^*:*

$$m^* = \arg\max_{m<k\eta} W(k,m) = \arg\max_{m<k\eta} \tau^*(k,m). \tag{31}$$

It is also useful to note that when the policy variable is the clawback window m, the regulator's incentive is aligned with that of the firm, both maximizing the firm's life span $k\eta + \tau^*(k,m)$. However, the regulator and the firm control different choice variables, the former m and the latter k. A key takeaway of the subsequent analysis is that repayment clawback (m) can achieve a more efficient outcome than the firm's own commitment to seek bankruptcy protection (k).

The regulator's choice of m is also shaped by the recovery rate channel and the hazard rate channel (recall the decomposition (30)). We visualize the two channels in Figure 4. On the one hand, a larger m subjects more creditors ($\frac{m}{\eta}$) to clawback, thereby generating more proceeds and improving recovery in bankruptcy $\alpha(\tau,k,m)\,e^{gt_0}$. As a result, creditors are more willing to wait, reflected by the increasing curve with solid dots in Figure 4. On the other hand, a longer clawback window makes it more difficult for creditors to exit successfully because a creditor needs to exit sooner than the $(k-\frac{m}{\eta})$th creditor to be outside of the clawback window. This channel increases the associated hazard rate $h(k,m)$ and thereby causes creditors to run more anxiously ex ante. The decreasing curve with circles in Figure 4 features the hazard rate channel. Combining the two channels, the optimal clawback window m^* that maximizes τ^* can be analytically calculated.

Proposition 7 (Optimal Clawback Window) *The optimal clawback window $m^*(k) \in [0, k\eta)$ that maximizes welfare, or, equivalently, the equilibrium waiting time $\tau^*(k,m)$, is given by*

$$m^*(k) = \max\left\{0, \frac{1}{g-\lambda} - (1-k)\eta\right\} < k\eta. \tag{32}$$

Just like the optimal choice of k_c discussed in Subsection 3.2, the optimal clawback window m^* is also generally interior. This implies that allowing either too many or too few creditors to exit hurts their willingness to remain invested. Allowing too many creditors to exit (a small $m < m^*$) increases the likelihood of a successful exit and reduces the hazard rate h. However, the associated low level of assets in bankruptcy makes it more costly for creditors to risk bankruptcy, leading to a more accelerated exit. Similarly, aiming for more recovery in bankruptcy by clawing back too many creditors (a large $m > m^*$) may also backfire. The difficulty in exiting the firm ex ante captured by the elevated hazard rate may again exacerbate runs.

[27] The simple extension with a negative post-shock growth rate $g' < 0$ is exactly opposite. In this case, the firm is no longer productive, and keeping money inside the firm is costly. Hence, welfare is decreasing in τ^*.

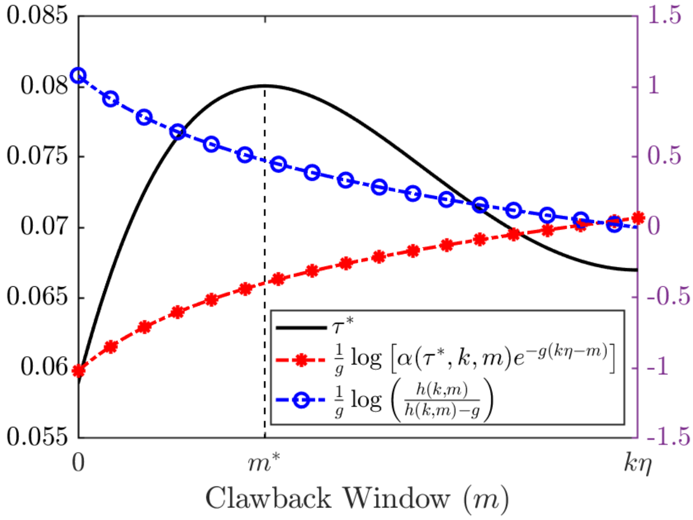


Figure 4: Decomposition of the equilibrium waiting time τ^*.

Note: We decompose the equilibrium waiting time (τ^*) into the payoff channel (i.e., $\frac{1}{g}\log\left(\alpha e^{-g(k\eta-m)}\right)$ with dots) and the hazard rate channel (i.e., $\frac{1}{g}\log\left(\frac{h(k,m)}{h(k,m)-g}\right)$ with circles) according to the two terms in (30). The curves of τ^* correspond to the left y-axis, and the remaining two curves correspond to the right y-axis. The parameters for this figure are $A = 1.1$, $g = 2$, $\lambda = 0.5$, $\eta = 1$, $g' = 0.1$, and $k = 0.5$.

A simple rearrangement of (32) clearly reveals the economic role of the clawback window $m^*(k)$:

$$k - \frac{m^*(k)}{\eta} = 1 - \frac{1}{(g-\lambda)\,\eta}. \tag{33}$$

Effectively, the regulator sets the pivotal creditor to $1 - \frac{1}{(g-\lambda)\eta}$ should the firm's choice of k be different.[28]

One might therefore conjecture that clawback regulation is equivalent to the firm's commitment to bankruptcy threshold $1 - \frac{1}{(g-\lambda)\eta}$. Intriguingly, this conjecture is incorrect. In the next subsection, we show that ex post clawback is always superior to a firm's own commitment to terminate, and the reason, as a preview, is that clawback allows the productive process to continue longer.

We would like to highlight that ex post clawback is a relatively detail-free regulation in that it does not require the regulator to know some of the firm's operating characteristics, such as the initial leverage A and the post-shock growth rate g'.[29] This result is robust to different model specifications because the trade-off associated with the clawback window is independent of the firm's performance characteristics. On the one hand, the higher recovery rate due to clawback

[28]Here, we focus on the interior solution with $m^* > 0$. It is possible that the firm chooses a low threshold $k < 1 - \frac{1}{(g-\lambda)\eta}$, resulting in a corner solution of no clawback: $m^*(k) = 0$.

[29]The optimal m^* in (32) does depend on the pre-shock growth rate g, which is a firm performance parameter. We argue that this is because g also represents the interest rate. Based on the intuition that immediately follows, we conjecture that in a more complicated model where the pre-shock growth rate is different from the interest rate, the former parameter does not affect m^*.

proceeds only depends on the contractual terms of debt, such as the interest rate g, but not on the firm's performance. On the other hand, the hazard rate channel only depends on the stochastic structure of events (shock intensity λ, the endogenous bankruptcy threshold k, and debt maturity parameter η, or equivalently, the speed of information transmission), but again, not on the firm's performance.

Finally, we conclude this subsection with a comparative statics analysis of the optimal clawback window m^*, or equivalently, the number of creditors who are allowed to exit $(k - \frac{m^*}{\eta})$ based on (33). If the debt contracts feature a higher interest rate g, then more proceeds are subject to clawback for any given m. The improved recovery rate reduces the need to have a long clawback window, allowing more creditors to exit in equilibrium. In addition, a higher intensity of the bad shock λ or a shorter maturity η (or faster information transmission) exacerbates the hazard rate channel, resulting a longer clawback window and fewer creditors who can exit successfully.

4.3 Bankruptcy Threshold k under Avoidable Preference

After establishing the optimal policy design in (32), we study how avoidable preference regulation affects the managerial choices of the bankruptcy threshold k with or without commitment. Intriguingly, these two cases coincide and both dominate the best outcome that can be achieved by the firm's own commitment to file for bankruptcy early without ex post clawback (i.e., the case with k_c and $m = 0$ in Proposition 4). We start by characterizing the no-commitment bankruptcy threshold k^m_{nc}.

Proposition 8 (Equilibrium without Managerial Commitment) *When the manager cannot commit to a bankruptcy threshold, the equilibrium outcome with avoidable preference is given by*

$$k^m_{nc} = \frac{1}{g\eta} \log\left(\frac{g^2}{(g-\lambda)\lambda} e^{(g-\lambda)(\eta - \frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda} e^{g(\eta - \frac{1}{g-\lambda})} \right),$$

$$\tau^* = \frac{1}{g-g'} \left[\log A - \log \frac{e^{(g-g')k^m_{nc}\eta} - 1}{(g-g')\eta} \right], \quad \textit{and} \quad m^* = k^m_{nc}\eta - (\eta - \frac{1}{g-\lambda}). \tag{34}$$

Furthermore, when compared against the benchmark cases with no clawback, we have

$$\tau^*(k^m_{nc}, m^*) \geq \tau^*(k_c, 0) \geq \tau^*(k_{nc}, 0) = 0, \tag{35}$$

and both inequalities are strict when $k_c < k_{nc}$.

The result establishes an interesting comparison with the benchmark cases without clawback. With the help of avoidable preference regulation, even if the firm cannot commit to early bankruptcy, creditors are still more patient than in the case with only firm commitment (k_c in Proposition 4). This is because the clawed-back proceeds improve the recovery in bankruptcy, making creditors less eager to run. Clawback allows the regulator to alter the pivotal creditor who is allowed to exit with full repayment, effectively committing to a bankruptcy threshold. From (33) and (34), the pivotal creditor is $k_0 \equiv k_{nc}^m - \frac{m^*}{\eta} = 1 - \frac{1}{(g-\lambda)\eta}$.

More intriguingly, the outcome implemented by optimal clawback is superior to the case if the firm were to commit to terminating at the same level (k_0), even though both cases implement the same hazard rate.[30] The reason is the recovery rate channel. If the firm commits to terminating upon the exit of the pivotal creditor k_0, asset appreciation stops at $t_0 + k_0\eta + \tau$. If implemented by clawback regulation, asset appreciation only stops m periods later at $t_0 + k_{nc}^m\eta + \tau$ when all assets are depleted. This effect prolongs the asset appreciation at the rate of $g' > 0$ over the clawback window $(t_0 + k_0\eta + \tau, t_0 + k_{nc}^m\eta + \tau]$ and therefore increases the total resources available for creditors.

In addition to making creditors more patient (larger τ^*), thereby increasing welfare, the ex post clawback regulation can also attain a longer life span ($k\eta + \tau^*$) for the firm relative to the case with the firm's own commitment power. The intuition is essentially the same. If the firm commits to triggering bankruptcy early at some $k_c^m < k_{nc}^m$ while some assets still remain at the time of bankruptcy, increasing the committed threshold to k_{nc}^m can always increase the life span of the firm. First, the higher threshold mechanically delays bankruptcy through the $k\eta$ term in (23). Second, creditors are more patient under the larger bankruptcy threshold k_{nc}^m as discussed. Consequently, it is no longer optimal for firms to commit to early bankruptcy filing. Formally, we have the following result.

Proposition 9 (Redundancy of Managerial Commitment) *Under avoidable preference, the firm's ability to commit to a bankruptcy threshold does not change the equilibrium outcome; that is, $k_c^m = k_{nc}^m$, m^*, and τ^* are identical to those in (34).*

Essentially, when ex post clawback can be perfectly and freely enforced, the optimal regulation (m^*) renders the firm's commitment power inconsequential. Any threshold k_c^m that can be committed by the firm can be better implemented by letting creditors deplete all assets and then making ex post adjustments to their proceeds. In other words, when the regulator controls the ex post regulation m, the regulator's incentive is aligned with that of the firm's, and both maximize the firm's life span $k\eta + \tau^*(k, m)$. Unlike early termination through the firm's commitment on k, the regulator's ex post action through m does not generate a deadweight loss from terminating asset growth early and is therefore superior.

In practice, clawing back prior repayments often involves a lengthy and costly legal battle. In contrast, automatic stay upon bankruptcy filing is arguably a cheaper coordination device. Our analysis highlights the extra benefit of avoidable preference in preserving valuable production, but is silent on the legal costs of enforcing such a clawback. Hence, a direct implication from the cost consideration is that when such legal costs are lower or the value of uninterrupted production

[30] Note that the hazard rates are the same because of the same pivotal creditor.

is higher, avoidable preference is more prevalent. Future structural analysis may shed light on the economic magnitude of these forces and their impact on firms, creditors, and regulation designs.

4.4 Sequence of Actions

So far in this section, we focus on the case where the firm chooses termination threshold k first, and the regulator then responds with clawback window m. In this subsection, we consider several alternative timelines and show that our main theoretical insight that ex post clawback regulation can achieve a superior outcome relative to the firm's own commitment (Proposition 8) remains robust.

Since the firm's ability to commit to a termination threshold k proves to be inconsequential from the analysis in Section 4.3, the equilibrium outcome (without firm's commitment) can be readily interpreted in some alternative timelines. For example, the regulator first sets the clawback window m^*; creditors then choose their waiting time $\tau^*(m)$, anticipating the pivotal creditor k^m_{nc} who depletes all assets; and finally the firm implements the non-committed threshold k^m_{nc}.[31]

In addition, if the two actions are chosen simultaneously—that is, k^* and m^* constitute a Nash equilibrium—the equilibrium outcome $(m^*(k^m_{nc}), k^m_{nc})$ characterized in Proposition 8 is still an equilibrium under the new concept. To see this, note that the optimal clawback window $m^*(k)$ given by Proposition 7 is the best response function of the regulator. In addition, the firm's best response to the clawback window $m^*(k^m_{nc})$ is indeed k^m_{nc}. To see this, note that for any other $k' < k^m_c = k^m_{nc}$, we have

$$\begin{aligned} k'\eta + \tau^*(k', m^*(k^m_{nc})) &\le k'\eta + \tau^*(k', m^*(k')) \\ &< k^m_c\eta + \tau^*(k^m_c, m^*(k^m_c)) = k^m_{nc}\eta + \tau^*(k^m_{nc}, m^*(k^m_{nc})), \end{aligned}$$

where the first inequality follows from the fact that $m^*(k')$ is the optimal choice of m that maximizes the waiting time τ^*, given k'; the second inequality follows from the definition of k^m_c, which maximizes the firm's survival time $k\eta + \tau^*(k, m^*(k))$; and the final equality applies Proposition 9; that is, $k^m_c = k^m_{nc}$.

Finally, if the regulator moves first to decide regulation m and the firm chooses $k^*(m)$ based on the regulatory environment, the game is substantially more complicated. The welfare criterion in this case is no longer equivalent to τ^* maximization (Proposition 6), and the optimal m^* no longer has an explicit expression as in Proposition 7. However, the theoretical insight that clawback regulation dominates the firm's commitment to early bankruptcy filing remains robust. In fact, the regulator can always set $m^* = m^*(k^m_{nc})$, and the firm's best response is the no-commitment solution, as previously discussed in the case of Nash equilibrium. Therefore, the regulator's

[31] A subtle detail is whether the regulator treats the expected k^m_{nc} as given or considers its policy's impact on k^m_{nc} through equilibrium τ^*. The proof of Proposition 8 shows that the two equilibrium outcomes also coincide.

ability to choose a different m that can potentially influence $k(m)$ to achieve higher welfare must be weakly superior.

5 Extensions and Robustness

5.1 Heterogeneous Creditors: Seniority

The analysis in Section 4 suggests that treating creditors differently ex post (some are subject to clawback, whereas others are not) can improve welfare and delay bankruptcy. A natural question that follows is whether ex ante heterogeneity among creditors can also improve welfare. In this section, we extend the model to consider one such heterogeneity: seniority. Interestingly, numerical analysis suggests that having both senior and junior creditors may harm welfare.

Suppose, among creditors, an ω-fraction is senior, and the remaining $(1-\omega)$-fraction is junior, where $\omega \in [0,1]$. Denote by $\alpha_S e^{gt_0}$ and $\alpha_J e^{gt_0}$ the recovery payoffs in bankruptcy for senior and junior creditors, respectively. For simplicity, we model absolute priority, which implies that if senior creditors do not receive full repayment ($\alpha_S e^{gt_0} < e^{g\hat{t}}$, where $\hat{t}$ denotes the time of bankruptcy), then nothing is recovered for junior creditors (i.e., $\alpha_J = 0$). Or, equivalently, if junior creditors get a strictly positive payoff, then senior creditors must receive full repayment, that is, $\alpha_S e^{gt_0} = e^{g\hat{t}}$ if $\alpha_J > 0$. The less extreme relative priority, featuring $\alpha_S \geq \alpha_J$, delivers similar economic insights.

As before, we focus on the linear symmetric equilibrium and denote by τ_S and τ_J the waiting times for junior and senior creditors, respectively. Bankruptcy is triggered by the exit of the kth creditor. It is possible that only one type or both types of creditors have started exiting at the time of bankruptcy $\hat{t}(\tau_J, \tau_S)$—a term that is modified accordingly:

$$\hat{t}(\tau_J, \tau_S) \equiv \inf \left\{ t_0 + u \Big| \int_0^u \left[\frac{1-\omega}{\eta} \cdot \mathbf{1}\{\tau_J \leq \delta \leq \tau_J + \eta\} + \frac{\omega}{\eta} \cdot \mathbf{1}\{\tau_S \leq \delta \leq \tau_S + \eta\} \right] \mathrm{d}\delta \geq k \right\}. \tag{36}$$

Senior and junior creditors optimize their waiting times τ_S and τ_J, respectively, by solving (5), with their type-specific recovery payoffs and the time of bankruptcy $\hat{t}(\tau_J, \tau_S)$ defined above.

The analysis is substantially complicated by the many cases where different types of creditors (jointly) trigger bankruptcy (36). In the next result, we briefly summarize the two types of equilibrium outcomes depending on the fraction of senior creditors while relegating the comprehensive equilibrium characterization to Appendix B.

Proposition 10 (Seniority) *In the model with heterogeneous creditors, there is a unique equilibrium. In equilibrium, junior creditors run faster than their senior counterparts (i.e., $\tau_J \leq \tau_S$). There is a cutoff composition $\tilde{\omega} \in (0, 1-k)$ of senior creditors, such that*

1. *when $\omega \leq \tilde{\omega}$, senior creditors do not exit before bankruptcy, that is,*

$$\tau_S^* > \tau_J^* + \frac{k\eta}{1-\omega} = \hat{t} - t_0;$$

they receive full repayment in bankruptcy, that is,

$$\alpha_S(\tau_J^*, \tau_S^*)e^{gt_0} = e^{g\hat{t}(\tau_J^*, \tau_S^*)};$$

and bankruptcy is triggered purely by junior creditors' exits;

2. *when $\omega > \tilde{\omega}$, the senior creditors do not receive full repayment, and bankruptcy is triggered by the exits of both groups.*

Since junior creditors recover less in bankruptcy than their senior counterparts, waiting is more costly for juniors, and they are therefore less patient. In addition, when there is only a small fraction of senior creditors, they can expect full repayment even in bankruptcy because their total claim is small relative to the total assets available in bankruptcy. Consequently, they do not have an incentive to exit early. As there are more senior creditors, their claims are no longer guaranteed, and therefore they start to exit before bankruptcy.

With the general characterization above, we can numerically demonstrate how the senior composition ω affects the coordination outcome—captured by the firm's survival time following the bad shock $\hat{t} - t_0$ in this extended model.[32] We plot equilibrium waiting times (τ_J^* and τ_S^*) and the life span of the firm ($\hat{t} - t_0$) as functions of ω in Figure 5. An immediate observation is that seniors are more patient than juniors (see Figures 5a and 5c), as predicted by Proposition 10. In addition, we note two more interesting and robust patterns.

First, when the share of senior creditors is small, bankruptcy always occurs sooner than in the benchmark model with homogeneous creditors. In the figures, the benchmark level is depicted by the dashed line when $\omega = 0$ (or equivalently, $\omega = 1$, both featuring homogeneous creditors). The initial declines in both the junior waiting time (Figures 5a and 5c) and the firm's life span (Figures 5b and 5d) demonstrate this result. Economically, when ω is small, the equilibrium outcome follows case 1 in Proposition 10. Seniors receive full repayments because of their small size and therefore do not exit, and bankruptcy is purely driven by junior creditors. Juniors exit sooner than in the benchmark model because the presence of seniors reduces the recovery payoff in bankruptcy, leading to earlier bankruptcy. Despite the difficulties in proving an analytical result,[33] this pattern is robust to all parameter choices that we have experimented with.

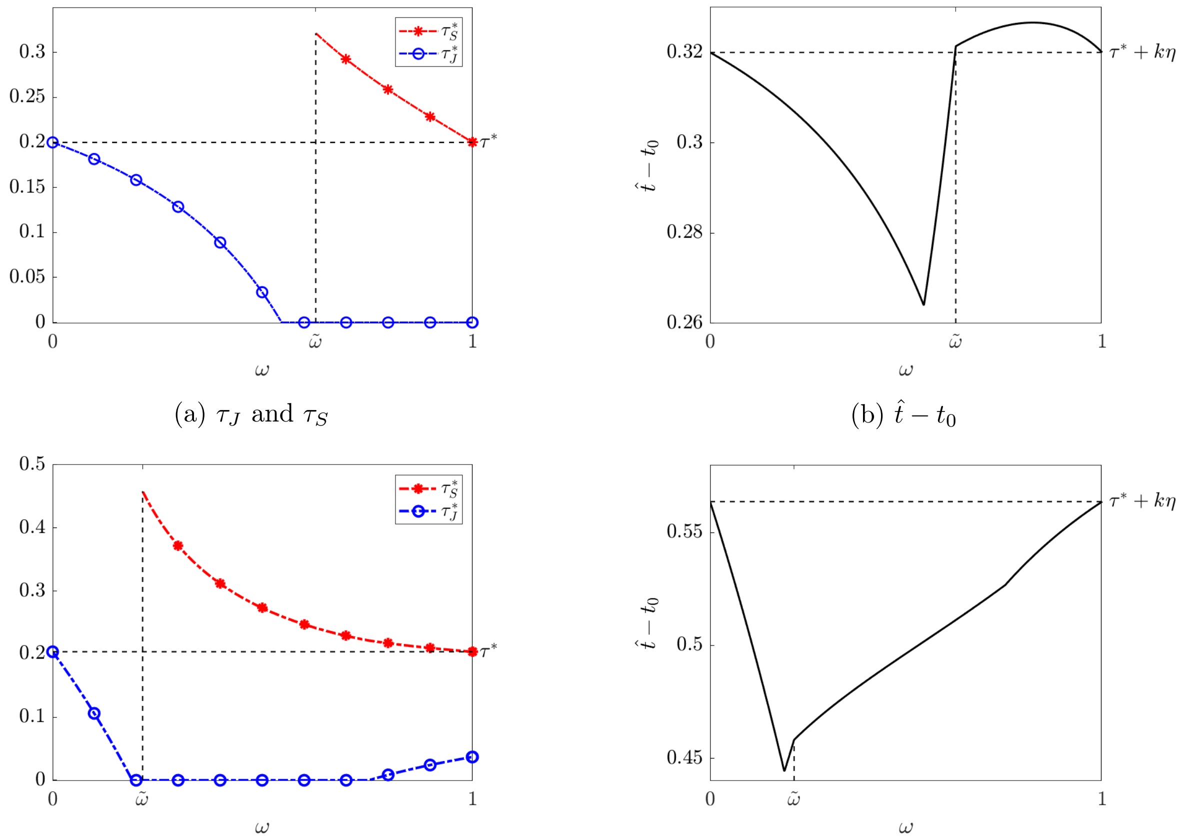
Second, as the fraction of senior creditors further increases, the time of bankruptcy can be delayed. In this case, the equilibrium outcome switches to case 2 in Proposition 10: senior creditors also exit before bankruptcy. As more seniors become pivotal in triggering bankruptcy, they replace their junior counterparts in the exit process, and their willingness to wait may in turn delay the time of bankruptcy. This is revealed by the increase in the firm's life span after $\omega > \tilde{\omega}$ in both Figures 5b and 5d. Intriguingly, depending on the parameters, the life span of the

[32] Note that the measure $\hat{t} - t_0$ equals $k\eta + \tau$ in the baseline model. It no longer has such a simplified representation here because of heterogeneous creditors.

[33] The counteracting force that may delay bankruptcy is that there are fewer junior creditors, leading to a smaller capital outflow per unit of time. Numerically, this channel is always dominated by the smaller waiting time τ_J^*.

distressed firm may even be longer than that in the benchmark model (the case in Figure 5b). However, it is also possible that any ex ante heterogeneity always worsens the outcome, as in Figure 5d.[34]

In summary, there is a non-monotonic relation between the firm's life span and the fraction of senior creditors. A small number of senior creditors always worsens coordination because their presence makes junior creditors less patient, leading to earlier bankruptcy. As more creditors become senior, they replace juniors and become more pivotal in triggering bankruptcy. Their willingness to wait can prolong firm life and delay bankruptcy.



(a) τ_J and τ_S

(b) $\hat{t} - t_0$

(c) τ_J and τ_S

(d) $\hat{t} - t_0$

Figure 5: Equilibrium Waiting and Endogenous Life Span with heterogeneous creditors

Note: This figure depicts the equilibrium waiting time (τ_J, τ_S) and the endogenous life span of the firm, $\hat{t} - t_0$ (defined in (36)). The parameters are $A = 1.1$, $g = 2$, $\lambda = 0.2$, $\eta = 0.6$, $g' = 1$, $k = 0.2$ (panel 5a and 5b), and $k = 0.6$ (panel 5c and 5d).

[34] One interesting observation associated with this case is that junior waiting time τ_J^* may even increase when senior composition approaches 1 (Figure 5c). This is because, despite the recovery rate α_J staying at 0, the increase in the firm's life span (Figure 5d) incentivizes juniors to wait longer because of the reduced hazard rate.

5.2 Recovery of Growth Rate

Throughout the paper, we have assumed that the bad shock is permanent in order to succinctly model a distressed yet productive firm that is heading for bankruptcy. However, in practice, many bad shocks are temporary. We can modify the benchmark model to allow for the possibility of a growth rate recovery. Specifically at some random time $t_0 + \tilde{\tau}_R \geq t_0$, the growth rate g' reverts back to g. The intensity of the Poisson time $\tilde{\tau}_R$ is λ_R. For simplicity, we assume that growth recovery is publicly observable and cannot occur if a positive mass of creditors exit the firm ($\tilde{\tau}_R \leq \tau^*$) or when assets are insufficient to cover all debt; that is,

$$Ae^{gt_0 + g'\tilde{\tau}_R} \geq e^{g(t_0+\tilde{\tau}_R)} \quad \Longleftrightarrow \quad \tilde{\tau}_R \leq \frac{1}{g-g'} \log A.$$

Upon growth recovery, it is clearly optimal for creditors to stay invested until the project naturally matures at $t_0 + T$ and each receives $e^{g(t_0+T)}$. The total firm value at that point is

$$Ae^{gt_0 + g'\tilde{\tau}_R + g(T-\tilde{\tau}_R)} = Ae^{g(t_0+T)-(g-g')\tilde{\tau}_R},$$

and the equity is the residual claimant and receives

$$e^{g(t_0+T)}[Ae^{-(g-g')\tilde{\tau}_R} - 1] \geq 0.$$

Creditor i's payoff is given by

$$\tilde{\Pi}_i(\tau_i|t_i, \tau^*, k, m) \equiv \mathbb{P}\left(\tilde{\tau}_R > \tau^*\right) \Pi_i(\tau_i|t_i, \tau^*, k, m) + \mathbb{P}\left(\tilde{\tau}_R \leq \tau^*\right) e^{g(t_0+T)},$$

where $\Pi_i(\tau_i|t_i, \tau^*, k, m)$ is given by (27). It is clear that the optimization problem regarding the waiting time τ_i is unchanged because the action of an individual creditor does not affect whether growth recovery may occur.

The regulator designs the clawback regulation m to maximize welfare $\tilde{W}$, which, in this case, captures both the outcomes with and without recovery:

$$\tilde{W} \equiv \max_m \mathbb{P}\left(\tilde{\tau}_R > \tau^*\right) W\left(k, m\right) + \int_{\tilde{\tau}_R \leq \tau^*} Ae^{g(t_0+T)-(g-g')\tilde{\tau}_R} \lambda_R e^{-\lambda_R \tilde{\tau}_R} \mathrm{d}\tilde{\tau}_R,$$

where both the welfare $W(m)$, defined in (25), and $\tau^*(k, m)$ depend on the clawback window m. Relative to the regulator's objective in (31), a potential growth recovery introduces an additional benefit of a longer waiting time τ^*: more patient creditors give the firm more time to potentially recover from the bad shock, thereby avoiding a run completely. As such, in the modified model, the regulator's objective remains to be maximizing τ^*, as established by Proposition 6. Therefore, the recovery of growth rate does not affect the creditors' and the regulator's optimization problems.

It is worth noting that the equity value in this case is

$$\tilde{E} \equiv \int_{\tilde{\tau}_R \leq \tau^*} e^{g(t_0+T)}[Ae^{-(g-g')\tilde{\tau}_R} - 1]\lambda_R e^{-\lambda_R \tilde{\tau}_R} \mathrm{d}\tilde{\tau}_R,$$

which is also equivalent to maximizing τ^*, just like the regulator's objective. This is different from the manager's objective $k\eta + \tau^*$ in the benchmark model. Despite the coinciding objective, the two players have different choice variables: the regulator controls m, and the firm controls k. One can show that the main economic insight from Section 4 remains robust: ex post clawback achieves an outcome superior to that of the firm's commitment to trigger bankruptcy because the former can effectively implement an earlier bankruptcy without actually terminating the production process at that point.[35]

6 Conclusion

In this paper, we build a tractable dynamic coordination framework that highlights two channels affecting creditors' decision to stay invested: the recovery rate channel and the hazard rate channel. We apply our framework to study two bankruptcy regulations—avoidable preference and automatic stay—as well as the firm's decision to file for bankruptcy. Those two channels often imply a trade-off: policies aiming at a higher recovery rate in bankruptcy may affect the hazard rate by subjecting more creditors to the impact of bankruptcy, leading to more frantic runs ex ante. Similarly, lenient policies allowing many creditors to exit before bankruptcy lower recovery in bankruptcy, which may again worsen coordination ex ante.

We find that firms' commitment to file for bankruptcy early, together with the automatic stay feature in bankruptcy, can potentially delay bankruptcy because more assets are preserved in bankruptcy, thereby improving recovery rates and motivating creditors to stay invested. Intriguingly, when enforced perfectly and costlessly, avoidable preference regulation through the ex post clawback of some pre-bankruptcy payments can deliver an even superior outcome. It preserves asset appreciation while controlling the pivotal creditor who can exit the firm with full repayment. The optimal clawback window is also detail-free in that it does not rely on the firm's production parameters.

Furthermore, our model sheds light on bank runs and regulatory intervention. We show that regulators in general would like to seize a bank sooner than the bank's self-interested threshold, highlighting the conflicting incentives between the regulator and the bank (or the firm in general). Our analysis also shows how ex post clawback realigns their incentives.

Finally, we consider a different seniority structure and show that ex ante heterogeneity among creditors may not help with coordination. We believe that the analytical framework developed in this paper is general enough to study dynamic coordination problems associated with other securities (e.g., repos and mutual funds), as well as the related policies and industry practices

[35] Proposition 5 no longer applies because both the firm and the regulator share the same objective.

affecting investors' payoffs (e.g., redemption gates, fees, and swing pricing). One important difference in these settings is that the stakeholders' payoff is no longer debt-like, as in our model. Hence, the payoff structure needs to be modified, and the coordination problem needs to be microfounded through externalities from early redemption or other mechanisms closer to these applications. We look forward to additional research in these areas.

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Appendix A Omitted Proofs

Proof of Lemma 1 Under the symmetric exiting strategy $\beta(t) = t + \tau$, $w_t = 0$ for $t \leq t_0 + \tau$. Based on the dynamic evolution of the total asset value (3) and $Y(0) = A$, we have

$$Y_t = \begin{cases} Ae^{gt} & 0 \leq t \leq t_0 \\ Ae^{gt_0 + g'(t-t_0)} & t_0 < t \leq t_0 + \tau \end{cases}.$$

When $t > t_0 + \tau$, $w_t = \frac{1}{\eta}$ and $\mathrm{d}Y_t = \left(g'Y_t - \frac{1}{\eta}e^{gt}\right)\mathrm{d}t$. Thus, we have $\mathrm{d}\left(Y_t e^{-g't}\right) = e^{-g't}\left(\mathrm{d}Y_t - g'Y_t \mathrm{d}t\right) = -\frac{e^{(g-g')t}}{\eta}\mathrm{d}t$. Solving the above differential equation with the initial condition $Y_{t_0+\tau}e^{-g'(t_0+\tau)} = Ae^{gt_0+g'\tau}e^{-g'(t_0+\tau)} = Ae^{(g-g')t_0}$, we have

$$\int_{u=t_0+\tau}^{t} \mathrm{d}\left(Y_u e^{-g'u}\right) = Y_t e^{-g't} - Ae^{(g-g')t_0} = \int_{u=t_0+\tau}^{t} -\frac{e^{(g-g')u}}{\eta}\mathrm{d}u.$$

Hence, for $t \in (t_0 + \tau, t_0 + \tau + k\eta]$,

$$Y_t = Ae^{gt_0+g'(t-t_0)} - \frac{1}{(g-g')\,\eta}\left[e^{gt} - e^{g(t_0+\tau)+g'(t-t_0-\tau)}\right].$$

Given this dynamic process of Y_t, at time $\hat{t} = t_0 + \tau + k\eta$, $Y_{\hat{t}} = Ae^{gt_0+g'(\tau+k\eta)} - \frac{1}{(g-g')\eta}\left[e^{g(t_0+\tau+k\eta)} - e^{g(t_0+\tau)+g'k\eta}\right]$. Hence, each remaining creditor receives $\alpha(\tau,k)e^{gt_0}$; that is, $\alpha(\tau,k) = \frac{Y_{\hat{t}}}{1-k}e^{-gt_0} = \frac{Ae^{g'(k\eta+\tau)} - \frac{e^{g(\tau+k\eta)} - e^{g\tau+g'k\eta}}{(g-g')\eta}}{1-k}$. ■

Proof of Proposition 1 We can rewrite the equilibrium condition (14) as

$$ge^{g(t_i+\tau^*)} = h(k)[e^{g(t_i+\tau^*)} - \alpha e^{g(t_i-k\eta)}]. \tag{A.1}$$

Eliminating e^{gt_i} on both sides of (A.1), we have $e^{g\tau^*} = \alpha\frac{h(k)}{h(k)-g}e^{-gk\eta}$. Hence, for any $\tau^* > 0$ that satisfies the first-order condition, we have

$$\tau^* = \frac{1}{g}\log\left[\alpha(\tau^*,k)e^{-gk\eta}\right] + \frac{1}{g}\log\left(\frac{h(k)}{h(k)-g}\right),$$

thereby completing the proof.

Next, we prove that condition (17) holds true. A simple rearrangement of (16) yields

$$\frac{h(k)-g}{h(k)} = \alpha(\tau^*,k)e^{-g(k\eta+\tau^*)} = \alpha(\tau^*,k)e^{gt_0}e^{-g\hat{t}}.$$

Since $\frac{h(k)-g}{h(k)} < 1$, the above condition implies (17). ■

Proof of Proposition 2 First, we claim that any reasonable bankruptcy threshold k satisfies (for the definition of k_{nc}, see (22))

$$k \leq k_{nc} = \frac{1}{(g-g')\eta} \log[A(g-g')\eta + 1]. \tag{A.2}$$

This is because, for $k > k_{nc}$, the firm will not have enough assets to honor full payments to the k share of withdrawing creditors, regardless of their waiting strategy τ^*. To see this, consider any $k > k_{nc}$. In an equilibrium with any $\tau^* \geq 0$, we have

$$\begin{aligned}
\alpha(\tau^*, k) &= \frac{Ae^{g'(\tau^*+k\eta)} - \frac{e^{g(\tau^*+k\eta)} - e^{g\tau^*+g'k\eta}}{(g-g')\eta}}{1-k} \\
&= \frac{e^{g\tau^*+g'k\eta}}{1-k}\left(Ae^{-(g-g')\tau^*} - \frac{e^{(g-g')k\eta}-1}{(g-g')\eta}\right) \leq \frac{e^{g\tau^*+g'k\eta}}{1-k}\left(A - \frac{e^{(g-g')k\eta}-1}{(g-g')\eta}\right).
\end{aligned}$$

Thus, by definition of k_{nc}, for any $k > k_{nc}$, $A - \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} < 0$, and, therefore, $\alpha(\tau^*, k) < 0$ for any $\tau^* \geq 0$. This contradicts the fact that k is a bankruptcy threshold that determines the time of bankruptcy. As a result, any reasonable bankruptcy threshold must satisfy $k \leq k_{nc}$.

The following lemma will be helpful in proving the existence and uniqueness of equilibrium.

Lemma A.1 *Under Assumption 1,*

$$k_{nc} = \frac{1}{(g-g')\eta} \log[A(g-g')\eta + 1] < \frac{1}{\lambda\eta} \log \frac{g}{g-\lambda} < 1. \tag{A.3}$$

Proof of Lemma A.1. First, since $A < \overline{A} = \frac{\left(\frac{g}{g-\lambda}\right)^{\frac{g-g'}{\lambda}}-1}{(g-g')\eta}$, we have $\frac{1}{(g-g')\eta}\log[A(g-g')\eta+1] < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$. Then, we prove that, under the condition $\eta > \frac{1}{g-\lambda} > 0$ (Assumption 1),

$$0 < \frac{1}{\lambda\eta} \log \frac{g}{g-\lambda} < 1. \tag{A.4}$$

To see this, we first have $\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} > 0$ and $\frac{\lambda}{g} \in (0,1)$ based on the fact that $g > \lambda > 0$. Then, consider the function $\Omega(x) \equiv \frac{x}{1-x} - \log\frac{1}{1-x}$ for $x \in [0,1)$. It is easy to check that $\Omega(0) = 0$ and $\frac{\partial\Omega(x)}{\partial x} = \frac{x}{(1-x)^2} > 0$. Therefore, for any $x \in (0,1)$, $\Omega(x) > 0$, or $\frac{1}{1-x} > \frac{1}{x}\log\frac{1}{1-x}$. Accordingly, since $\frac{\lambda}{g} \in (0,1)$, $\frac{1}{1-\frac{\lambda}{g}} > \frac{1}{\frac{\lambda}{g}}\log\frac{1}{1-\frac{\lambda}{g}}$, which implies $1 > \frac{g-\lambda}{\lambda}\log\frac{g}{g-\lambda}$. Finally, because $\eta > \frac{1}{g-\lambda} > 0$, we have $\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} < \frac{g-\lambda}{\lambda}\log\frac{g}{g-\lambda} < 1$. ■

Next, we prove that there is a unique symmetric equilibrium with $k \leq k_{nc} < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} < 1$. Plugging $\alpha(\tau, k)$ (see (12)), $\Psi(t_0|t_i)$ and $\psi(t_0|t_i)$ (see (1)) into the first-order derivative of (13) with respect to τ_i at $\tau_i = \tau$, we have

$$\left.\frac{\partial \Pi_i(\tau_i|t_i,\tau,k)}{\partial \tau_i}\right|_{\tau_i=\tau} = ge^{g(t_i+\tau)}[1 - \Psi(t_i - k\eta|t_i)]$$

$$- e^{g(t_i+\tau)}[1-\alpha(\tau,k)e^{-g(\tau+k\eta)}]\psi(t_i-k\eta|t_i) \tag{A.5}$$

$$\propto \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda k\eta} + \frac{Ae^{-(g-g')(\tau+k\eta)} - \frac{1-e^{-(g-g')k\eta}}{(g-g')\eta}}{1-k} \equiv \Lambda(\tau). \tag{A.6}$$

Since $\Lambda'(\tau) = -\frac{A(g-g')}{1-k}e^{-(g-g')(\tau+k\eta)} < 0$, $\Lambda(\tau)$ is decreasing in τ. Moreover, as $k < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$ (see (A.3)), we have

$$\lim_{\tau\to\infty}\Lambda(\tau) = \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda k\eta} - \frac{1-e^{-(g-g')k\eta}}{(g-g')(1-k)\eta} < -\frac{1-e^{-(g-g')k\eta}}{(g-g')(1-k)\eta} < 0. \tag{A.7}$$

Therefore, an equilibrium with $\tau^* = 0$ holds if and only if $\Lambda(0) \leq 0$ (which implies that $\Lambda(\tau) \leq 0$ for all $\tau \in [0,\infty)$), and an equilibrium with $\tau^* > 0$ holds if and only if $\Lambda(0) > 0$ (which implies the existence and uniqueness of $\tau^* > 0$ such that $\Lambda(\tau^*) = 0$). It is easy to check that the condition $\Lambda(0) \leq 0$ is equivalent to[36]

$$A \leq \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} + (1-k)\left[\frac{g}{\lambda}e^{(g-g'-\lambda)k\eta} - \frac{g-\lambda}{\lambda}e^{(g-g')k\eta}\right] \equiv v(k). \tag{A.8}$$

As a result, under the condition that $v(k) \geq A$, we have $\tau^* = 0$ as the unique equilibrium. On the other hand, when $\Lambda(0) > 0$, or, equivalently, $A > v(k)$, rearranging $\Lambda(\tau^*) = 0$ yields $\tau^* = \frac{1}{g-g'}(\log A - \log v(k)) > 0$, which is the unique equilibrium in this case. Therefore, the unique equilibrium is $\tau^*(k) = \max\left\{0, \frac{1}{g-g'}(\log A - \log v(k))\right\}$. ■

Proof of Proposition 3 First, we show that any $\tau^* > 0$ cannot hold as an equilibrium. To see this, consider any $\tau^* > 0$. Suppose that other creditors play this strategy $\tau^* > 0$. Then, the bankruptcy threshold k must satisfy $\alpha(\tau^*, k) = 0$, which implies

$$k = \frac{1}{(g-g')\eta}\log[A(g-g')\eta e^{-(g-g')\tau^*}+1] < \frac{1}{(g-g')\eta}\log[A(g-g')\eta+1] < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda},$$

in which the last inequality holds according to Lemma A.1. The fact that $k < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$ implies that creditor i would strictly prefer a waiting time less than τ^*. This is because the marginal net payoff of waiting at $\tau_i = \tau^*$ is proportional to $\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda k\eta}$, which is negative under $k < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$. As a result, any $\tau^* > 0$ cannot constitute an equilibrium.

Next, we prove that $\tau^* = 0$ and $k = k_{nc}$ can hold as an equilibrium. If every creditor chooses $\tau^* = 0$, then the manager's choice would be k_{nc} such that (21) holds (i.e., $k_{nc} = \frac{1}{(g-g')\eta}\log(A(g-g')\eta+1)$). Given k_{nc} and other creditors choose $\tau^* = 0$, creditor i's best response is also $\tau^* = 0$ because $\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda k_{nc}\eta} < 0$ since $k_{nc} < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$. Therefore, without commitment, $\tau^* = 0$, $k = k_{nc}$ holds as the unique equilibrium. ■

[36] Under the condition $k \leq k_{nc} < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$ (see (A.2) and (A.3)), by definition, $v(k) > 0$. To see this, it is easy to check that $\frac{g}{\lambda}e^{(g-g'-\lambda)k\eta} - \frac{g-\lambda}{\lambda}e^{(g-g')k\eta} = e^{(g-g')k\eta}\left(\frac{g}{\lambda}e^{-\lambda k\eta} - \frac{g-\lambda}{\lambda}\right) > 0$ following $k < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$. As a result, $v(k) > \frac{e^{(g-g')k\eta}-1}{(g-g')\eta} > 0$.

Proof of Proposition 4 The parameter g_0 and A_0 are defined as

$$g_0 \equiv g - \lambda - \frac{\log \frac{1}{g\left(\eta - \frac{1}{\lambda}\log\frac{g}{g-\lambda}\right)}}{\frac{1}{\lambda}\log\frac{g}{g-\lambda}} \quad \text{and } A_0 \equiv \frac{v(\tilde{k})}{e^{(g-g')\tilde{k}\eta} - (g-g')\eta v(\tilde{k})},$$

where $\tilde{k}$ is the unique solution to

$$(1-\tilde{k})g\eta + \frac{g}{\lambda} - \frac{g-\lambda}{\lambda}e^{\lambda\tilde{k}\eta} - e^{-(g-\lambda-g')\tilde{k}\eta} = 0. \tag{A.9}$$

When $g' > g_0$, the cutoff $A_0 \in (0, \overline{A})$.

Next, the following lemma regarding τ^* will be helpful to derive the optimal bankruptcy threshold.

Lemma A.2 *Under Assumption 1, there exists $k_v \in (0, k_{nc})$ that uniquely solves $v(k_v) = A$, that is,*

$$\frac{e^{(g-g')k_v\eta} - 1}{(g-g')\eta} + (1-k_v)\left[\frac{g}{\lambda}e^{(g-\lambda-g')k_v\eta} - (\frac{g}{\lambda} - 1)e^{(g-g')k_v\eta}\right] = A. \tag{A.10}$$

Moreover, there exists $k_1 \in (0, k_v)$ such that $\tau^*(k)$ is strictly increasing in $k \in (0, k_1]$, strictly decreasing in $k \in (k_1, k_v)$, in which k_1 uniquely solves

$$-\frac{g}{\lambda}e^{-\lambda k_1\eta} + \frac{g}{\lambda} + (1-k_1)\left(\frac{g(g-g'-\lambda)\eta}{\lambda}e^{-\lambda k_1\eta} - \frac{(g-\lambda)(g-g')\eta}{\lambda}\right) = 0, \tag{A.11}$$

and $\tau^*(k) = 0$ for $k \in [k_v, k_{nc}]$.

Proof of Lemma A.2. Since $\tau^*(k) = \max\left\{\frac{1}{g-g'}\left[\log A - \log v(k)\right], 0\right\}$, the shape of $\tau^*(k)$ depends on the shape of $v(k)$. The first-order derivative of $v(k)$ w.r.t. k is given by

$$\begin{aligned}
v'(k) &= e^{(g-g')k\eta} - \frac{g}{\lambda}e^{(g-g'-\lambda)k\eta} + \frac{g-\lambda}{\lambda}e^{(g-g')k\eta} \\
&\quad + (1-k)\left[\frac{g(g-g'-\lambda)\eta}{\lambda}e^{(g-g'-\lambda)k\eta} - \frac{(g-\lambda)(g-g')\eta}{\lambda}e^{(g-g')k\eta}\right] \\
&= e^{(g-g')k\eta}\left[-\frac{g}{\lambda}e^{-\lambda k\eta} + \frac{g}{\lambda} + (1-k)\left(\frac{g(g-g'-\lambda)\eta}{\lambda}e^{-\lambda k\eta} - \frac{(g-\lambda)(g-g')\eta}{\lambda}\right)\right] \\
&\equiv e^{(g-g')k\eta}v_1(k).
\end{aligned}$$

$v'(k)$ shares the same sign as $v_1(k)$. Taking the first-order derivative of $v_1(k)$ yields

$$\begin{aligned}
v_1'(k) &= g\eta e^{-\lambda k\eta} - \frac{g(g-g'-\lambda)\eta}{\lambda}e^{-\lambda k\eta} + \frac{(g-\lambda)(g-g')\eta}{\lambda} - (1-k)g(g-g'-\lambda)\eta^2 e^{-\lambda k\eta} \\
&= e^{-\lambda k\eta}\left[g\eta - \frac{g(g-g'-\lambda)\eta}{\lambda} + \frac{(g-\lambda)(g-g')\eta}{\lambda}e^{\lambda k\eta} - (1-k)g(g-g'-\lambda)\eta^2\right] \equiv e^{-\lambda k\eta}v_2(k).
\end{aligned}$$

where $v_1'(k)$ shares the same sign as $v_2(k)$. If $g - g' - \lambda < 0$, $v_2(k) > 0$, and thus $v_1'(k)$ is always positive. If $g - g' - \lambda > 0$, $v_2(k)$ is increasing in k, so $v_1'(k)$ can be (1) always positive, (2) always negative, or (3) initially negative and then positive. Combining the two cases, $v_1(k)$ can be (1) increasing in k, (2) decreasing in k, or (3) initially decreasing and then increasing in k. As $v_1(0) = -g'\eta < 0$, $v(k)$ is either (1) decreasing in k or (2) initially decreasing and then increasing in k. Since $v(k = 0) = 1$ and

$$v(k_{nc}) = A + (1 - k_{nc}) \left[\frac{g}{\lambda} e^{(g-g'-\lambda)k_{nc}\eta} - \frac{g-\lambda}{\lambda} e^{(g-g')k_{nc}\eta} \right] > A > 1, \tag{A.12}$$

in which the first inequality holds because $k_{nc} < \frac{1}{\lambda\eta} \log \frac{g}{g-\lambda} < 1$ (Lemma A.1), the only possibility is that $v(k)$ is initially decreasing and then increasing in k. Formally, there exists $k_1 \in (0, k_{nc})$ such that $v(k)$ is decreasing in k when $k \in (0, k_1]$ and increasing in k when $k \in (k_1, k_{nc}]$, in which k_1 uniquely solves $v_1(k_1) = 0$ (see (A.11)).

Based on the continuity of $v(k)$ and (A.12), there exists $k_v \in (k_1, k_{nc})$ such that $v(k) \in (0, A)$ when $k \in (0, k_v)$ and $v(k) \geq A$ when $k \in [k_v, k_{nc}]$, in which k_v uniquely solves $v(k_v) = A$. Therefore,

$$\tau^*(k) = \begin{cases} \frac{1}{g-g'} [\log A - \log v(k)] > 0 & \text{if } k \in (0, k_v) \\ 0 & \text{if } k \in [k_v, k_{nc}] \end{cases}.$$

From the monotonicity of $v(k)$, $\tau^*(k)$ is increasing in $k \in (0, k_1]$ and decreasing in $k \in (k_1, k_v)$, and stays at 0 when $k \in [k_v, k_{nc}]$. ■

According to Lemma A.2, the survival time $\tau^*(k) + k\eta$ is given by

$$\tau^*(k) + k\eta = \begin{cases} \frac{1}{g-g'} [\log A - \log v(k)] + k\eta & \text{if } k \in (0, k_v) \\ k\eta & \text{if } k \in [k_v, k_{nc}] \end{cases}.$$

To derive the optimal bankruptcy threshold, we need to determine the monotonicity of $\tau^*(k) + k\eta$ for $k \in (0, k_{nc})$. When $k \in [k_v, k_{nc}]$, $\tau^*(k) + k\eta$ is increasing linearly in k. When $k \in (0, k_v)$, the first-order derivative of $\tau^*(k) + k\eta$ w.r.t. k is given by

$$\begin{aligned} \frac{\partial(\tau^*(k) + k\eta)}{\partial k} &= \frac{e^{(g-\lambda-g')k\eta}}{(g-g')v(k)} \left[(1-k)g\eta + \frac{g}{\lambda} - \frac{g-\lambda}{\lambda} e^{\lambda k\eta} - e^{-(g-\lambda-g')k\eta} \right] \\ &\equiv \frac{e^{(g-\lambda-g')k\eta}}{(g-g')v(k)} \Gamma(k), \end{aligned}$$

which shares the same sign as $\Gamma(k)$. Taking the first-order derivative of $\Gamma(k)$ w.r.t. k yields $\Gamma'(k) = -g\eta - (g-\lambda)\eta e^{\lambda k\eta} + (g-\lambda-g')\eta e^{-(g-\lambda-g')k\eta} < 0$, in which the inequality holds because $(g-\lambda-g')\eta e^{-(g-\lambda-g')k\eta} < \max\{0, (g-\lambda-g')\eta e^{\lambda k\eta}\}$. As $\Gamma(0) = g\eta > 0$ and $\lim_{k\to\infty} \Gamma(k) = -\infty$, there exists $\tilde{k} \in (0, \infty)$ such that $\Gamma(k) > 0$ when $k \in (0, \tilde{k})$ and $\Gamma(k) \leq 0$ when $k \in [\tilde{k}, \infty)$.

Therefore, $\frac{1}{g-g'}[\log A - \log v(k)] + k\eta$ is increasing (or decreasing) in k when $k \in (0, \tilde{k})$ (or $k \geq \tilde{k}$).

From the above analysis, the monotonicity of $\tau^*(k) + k\eta$ with respect to k is determined by the relative size of k_v and $\tilde{k}$. When $k_v \leq \tilde{k}$, $\tau^*(k) + k\eta$ is increasing in $k \in (0, k_{nc})$, and thus the optimal bankruptcy threshold takes the corner solution, that is, $k_c = k_{nc}$. When $k_v > \tilde{k}$, $\tau^*(k) + k\eta$ is increasing in $k \in (0, \tilde{k}]$, decreasing in $(\tilde{k}, k_v]$, and increasing in $(k_v, k_{nc}]$. In this case, the optimal bankruptcy threshold depends on the relative size of $\tau^*(\tilde{k}) + \tilde{k}\eta$ and $k_{nc}\eta$. It is strictly interior ($k_c = \tilde{k} < k_{nc}$) when $\tau^*(\tilde{k}) + \tilde{k}\eta > k_{nc}\eta$ and becomes a corner solution ($k_c = k_{nc}$) otherwise.

According to Lemma A.2, $k_v \in (k_1, k_{nc})$ and $v(k)$ is increasing in $k \in [k_1, k_{nc})$, which implies

$$\frac{\partial k_v(A)}{\partial A} = \frac{1}{v'(k_v)} > 0. \tag{A.13}$$

In other words, $k_v(A)$ is increasing in A. As $\tilde{k}$ is independent of A, we are going to discuss the optimal bankruptcy threshold based on different initial assets A. Define $\underline{k} > k_1$ as the unique solution for $v(\underline{k}) = 1$ and $\overline{k}$ as the unique solution for $v(\overline{k}) = \overline{A}$ (we can easily check that $\overline{k} = \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$).

1. When $\overline{k} \leq \tilde{k}$ (i.e., $\tilde{k} \geq \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$), it follows that $k_v < \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} \leq \tilde{k}$ for all $A \in (1, \overline{A})$, so $\tau^*(k) + k\eta$ is increasing in $k \in (0, k_{nc}]$, and thus $k_c = k_{nc}$.

2. When $\underline{k} > \tilde{k}$, it follows that $k_v > \tilde{k}$ for all $A \in (1, \overline{A})$, so k_c depends on the relative size of $\tau^*(\tilde{k}) + \tilde{k}\eta$ and $k_{nc}\eta$, which can be measured by

$$\begin{aligned}\tau^*(\tilde{k}) + \tilde{k}\eta - k_{nc}\eta &= \frac{1}{g-g'}[\log A - \log v(\tilde{k})] + \tilde{k}\eta - \frac{1}{g-g'}\log\left[A(g-g')\eta + 1\right] \\ &= \frac{1}{g-g'}\log\left(\frac{1}{(g-g')\eta + \frac{1}{A}}\right) - \frac{\log v(\tilde{k})}{g-g'} + \tilde{k}\eta.\end{aligned}$$

   The difference between $\tau^*(\tilde{k}) + \tilde{k}\eta$ and $k_{nc}\eta$ is increasing in A. When $A \to \overline{A}$, we have $k_v \to k_{nc}$, and thus

$$\lim_{A\to\overline{A}}(\tau^*(\tilde{k}) + \tilde{k}\eta - k_{nc}\eta) > \lim_{A\to\overline{A}}(\tau^*(k_v) + k_v\eta - k_{nc}\eta) = 0.$$

   The first inequality holds because $\tau^*(k)+k\eta$ is decreasing in $k \in (\tilde{k}, k_v]$. From the continuity of $\tau^*(\tilde{k}) + \tilde{k}\eta - k_{nc}\eta$ with respect to A and $\lim_{A\to 0}(\tau^*(\tilde{k}) + \tilde{k}\eta - k_{nc}\eta) = -\infty < 0$, there exists a unique

$$A_0 = \frac{v(\tilde{k})}{e^{(g-g')\tilde{k}\eta} - (g-g')\eta v(\tilde{k})} \in (0, \overline{A})$$

that solves $\tau^*(\tilde{k}) + \tilde{k}\eta - k_{nc}\eta = 0$, such that $\tau^*(\tilde{k}) + \tilde{k}\eta > k_{nc}\eta$ if and only if $A \in (\max\{A_0, 1\}, \overline{A})$. Therefore, $k_c < k_{nc}$ if and only if $A \in (\max\{A_0, 1\}, \overline{A})$ and $k_c = k_{nc}$ if and only if $A \in (1, \max\{A_0, 1\})$.

3. When $\underline{k} \leq \tilde{k} < \overline{k}$, there exists a unique $A_0' \in (1, \overline{A})$ that solves $k_v(A_0') = \tilde{k}$, such that when $A \in (1, A_0']$, we have $k_v \leq \tilde{k}$, and thus $\tau^*(k) + k\eta$ is increasing in k, which implies $k_c = k_{nc}$; when $A \in (A_0', \overline{A})$, we have $k_v > \tilde{k}$, and thus $\tau^*(k) + k\eta$ is increasing in $k \in (0, \tilde{k}]$, decreasing in $k \in (\tilde{k}, k_v]$, and increasing in $(k_v, k_{nc}]$. From the discussion above, we can derive that $k_c = \tilde{k}$ only if $A > A_0$. Formally, when $A \in (A_0', A_0]$, $\tau^*(\tilde{k}) + \tilde{k}\eta \leq k_{nc}\eta$ and $k_c = k_{nc}$; when $A \in (A_0, \overline{A}]$, $\tau^*(\tilde{k}) + \tilde{k}\eta > k_{nc}\eta$ and $k_c = \tilde{k} < k_{nc}$.

Combining the three cases, when $\tilde{k} \geq \frac{1}{\lambda\eta} \log \frac{g}{g-\lambda}$, that is, $\Gamma(\frac{1}{\lambda\eta} \log \frac{g}{g-\lambda}) \geq 0$, which is equivalent to

$$g' \leq g - \lambda - \frac{\log \frac{1}{g\left(\eta - \frac{1}{\lambda}\log\frac{g}{g-\lambda}\right)}}{\frac{1}{\lambda}\log\frac{g}{g-\lambda}} = g_0,$$

we have $k_c = k_{nc}$ for all $A \in (1, \overline{A})$. When $\tilde{k} < \frac{1}{\lambda\eta} \log \frac{g}{g-\lambda}$, that is, $\Gamma(\frac{1}{\lambda\eta} \log \frac{g}{g-\lambda}) < 0$, which is equivalent to $g' > g_0$, there exists $A_0 = \frac{v(\tilde{k})}{e^{(g-g')\tilde{k}\eta} - (g-g')\eta v(\tilde{k})} \in (0, \overline{A})$ such that $k_c = k_{nc}$ when $A \in (1, \max\{A_0, 1\}]$ and $k_c < k_{nc}$ when $A \in (\max\{A_0, 1\}, \overline{A})$. ■

Proof of Proposition 5 The parameter $\tilde{A}$ is defined as $\tilde{A} \equiv \max\{A_1, v(\tilde{k})\}$, where $\tilde{k}$ is defined in (A.9) and A_1 is the unique solution to

$$\frac{g\eta v(\tilde{k})^{-\frac{g}{g-g'}}\left[\frac{e^{g\tilde{k}\eta}-1}{g\eta} + (1-\tilde{k})\left(\frac{g}{\lambda}e^{-\lambda\tilde{k}\eta} - \frac{g-\lambda}{\lambda}\right)e^{g\tilde{k}\eta}\right]}{[(g-g')\eta + \frac{1}{A_1}]^{\frac{g}{g-g'}} - (\frac{1}{A_1})^{\frac{g}{g-g'}}} = 1. \tag{A.14}$$

Next, we plug $Y_{t_0+k\eta+\tau}$ into (25) and have

$$W(\tau, k) = \left[\frac{e^{g(\tau+k\eta)} - e^{g\tau}}{g\eta} + Ae^{g'(\tau+k\eta)} - \frac{e^{g(\tau+k\eta)} - e^{g\tau+g'k\eta}}{(g-g')\eta}\right] e^{gt_0}. \tag{A.15}$$

First, we discuss the monotonicity of $W(\tau^*(k), k)$ with respect to k. We can check that $W(\tau, k)$ is increasing in τ [37] and k:

$$\frac{\partial W(\tau, k)}{\partial k} = \left[e^{g(\tau+k\eta)} + Ag'\eta e^{g'(\tau+k\eta)} - \frac{ge^{g(\tau+k\eta)} - g'e^{g\tau+g'k\eta}}{g-g'}\right] e^{gt_0} \geq 0, \tag{A.16}$$

in which the inequality holds according to $Ae^{g'(\tau+k\eta)} > \frac{e^{g(\tau+k\eta)} - e^{g\tau+g'k\eta}}{(g-g')\eta}$, implied by $\alpha(\tau, k) \geq 0$. From Lemma A.2, we have $\tau^* = 0$ when $k \in [k_v, k_{nc}]$ and $\tau^* > 0$ otherwise, in which k_v is given

[37] We formally prove this in Proposition 6.

in (A.10). Thus, when $k \in [k_v, k_{nc}]$, it follows that $\frac{\mathrm{d}W(\tau^*(k),k)}{\mathrm{d}k} = \frac{\partial W(\tau,k)}{\partial k}\big|_{\tau=0} \geq 0$, and thus $W(\tau^*(k), k)$ is increasing in $k \in [k_v, k_{nc}]$.

When $k \in (0, k_v)$, the first-order derivative w.r.t. k is given by

$$\begin{aligned}\frac{\mathrm{d}W(\tau^*(k),k)}{\mathrm{d}k} &= \left[\frac{\partial W(\tau,k)}{\partial \tau}\frac{\partial \tau^*(k)}{\partial k} + \frac{\partial W(\tau,k)}{\partial k}\right]\Big|_{\tau=\tau^*(k)} \\ &= \left[\frac{\partial W(\tau,k)}{\partial \tau}\frac{\partial \tau^*(k)}{\partial k} + \frac{\partial W(\tau,k)}{\partial \tau}\cdot\eta - e^{g\tau}(e^{g'k\eta}-1)e^{gt_0}\right]\Big|_{\tau=\tau^*(k)} \\ &= \left[\frac{\partial W(\tau,k)}{\partial \tau}\frac{\partial(\tau^*(k)+k\eta)}{\partial k} - e^{g\tau}(e^{g'k\eta}-1)e^{gt_0}\right]\Big|_{\tau=\tau^*(k)}.\end{aligned}$$

From Proposition 4, given that $g' > g_0$, as long as $k_v > \tilde{k}$, $(\tau^*(k) + k\eta)$ is increasing in k for any $k \in (0, \tilde{k})$ and any $k \in (k_v, k_{nc})$, and it is decreasing in k for $k \in [\tilde{k}, k_v]$, which implies $\frac{\partial(\tau^*(k)+k\eta)}{\partial k}\big|_{k\in[\tilde{k},k_v)} \leq 0$. As $W(\tau, k)$ is increasing in τ, this leads to

$$\frac{\mathrm{d}W(\tau^*(k),k)}{\mathrm{d}k}\Big|_{k\in[\tilde{k},k_v]} \leq -e^{g\tau^*(k)}(e^{g'k\eta}-1)e^{gt_0}\Big|_{k\in[\tilde{k},k_v]} < 0.$$

Also, as $\tau^*(k)$ is increasing in $k \in (0, k_1)$ (see Lemma A.2), it implies that

$$\frac{\mathrm{d}W(\tau^*(k),k)}{\mathrm{d}k}\Big|_{k\in(0,k_1)} = \left[\frac{\partial W(\tau,k)}{\partial \tau}\frac{\partial \tau^*(k)}{\partial k} + \frac{\partial W(\tau,k)}{\partial k}\right]\Big|_{\tau=\tau^*(k),k\in(0,k_1)} > 0.$$

Thus, regarding the monotonicity of $W(\tau^*(k), k)$, we have proven that:

1. $W(\tau^*(k), k)$ is strictly increasing in $k \in (0, k_1)$, in which $k_1 < \tilde{k}$;
2. $W(\tau^*(k), k)$ is strictly decreasing in $k \in [\tilde{k}, k_v]$;
3. $W(\tau^*(k), k)$ is increasing in $k \in (k_v, k_{nc}]$.

As a result, whenever $k_v > \tilde{k}$, the welfare-optimizing bankruptcy threshold k_W can either lie within the interval $[k_1, \tilde{k})$ or be equal to k_{nc}. As $k_v(A)$ is increasing in A (see (A.13)) and $v(k_v) = A$ (see (A.10)), $A > v(\tilde{k})$ implies $k_v > \tilde{k}$. Also, we have

$$\arg\max_{k\in(0,k_{nc}]} W(\tau^*(k), k) > W(\tau^*(\tilde{k}), \tilde{k}).$$

Therefore, to prove that $k_W < k_c$, it is sufficient to show that for all $A \geq A_1$, $W(\tau^*(\tilde{k}), \tilde{k}) \geq W(\tau^*(k_{nc}), k_{nc})$.

Plugging $\tau^*(\tilde{k})$ (see (19)) into $W(\tau^*(\tilde{k}), \tilde{k})$ yields

$$W(\tau^*(\tilde{k}), \tilde{k}) = A^{\frac{g}{g-g'}} v(\tilde{k})^{-\frac{g}{g-g'}} \left[\frac{e^{g\tilde{k}\eta}-1}{g\eta} + (1-\tilde{k})e^{g\tilde{k}\eta}\left(\frac{g}{\lambda}e^{-\lambda\tilde{k}\eta} - \frac{g-\lambda}{\lambda}\right)\right] e^{gt_0}.$$

Similarly, when we plug k_{nc} (see (22)) and $\tau^*(k_{nc}) = 0$ (Proposition 3) into $W(\tau^*(k_{nc}), k_{nc})$, it follows that

$$W(\tau^*(k_{nc}), k_{nc}) = \frac{[A(g-g')\eta + 1]^{\frac{g}{g-g'}} - 1}{g\eta} e^{gt_0}.$$

The ratio between $W(\tau^*(\tilde{k}), \tilde{k})$ and $W(\tau^*(k_{nc}), k_{nc})$ is given by

$$\frac{W(\tau^*(\tilde{k}), \tilde{k})}{W(\tau^*(k_{nc}), k_{nc})} = \frac{g\eta v(\tilde{k})^{-\frac{g}{g-g'}} \left[\frac{e^{g\tilde{k}\eta}-1}{g\eta} + (1-\tilde{k})\left(\frac{g}{\lambda}e^{-\lambda\tilde{k}\eta} - \frac{g-\lambda}{\lambda}\right)e^{g\tilde{k}\eta}\right]}{[(g-g')\eta + \frac{1}{A}]^{\frac{g}{g-g'}} - (\frac{1}{A})^{\frac{g}{g-g'}}}.$$

As $\frac{\partial}{\partial A}\{[(g-g')\eta + \frac{1}{A}]^{\frac{g}{g-g'}} - (\frac{1}{A})^{\frac{g}{g-g'}}\} = \frac{1}{A^2}\frac{g}{g-g'}[-[(g-g')\eta + \frac{1}{A}]^{\frac{g'}{g-g'}} + (\frac{1}{A})^{\frac{g'}{g-g'}}] < 0$ and $v(\tilde{k}) > 0$, $\frac{W(\tau^*(\tilde{k}),\tilde{k})}{W(\tau^*(k_{nc}),k_{nc})}$ is increasing in A. Also, when $A \to \overline{A}$, we have $k_v \to k_{nc}$, and thus $\frac{\mathrm{d}W(\tau^*(k),k)}{\mathrm{d}k}\Big|_{k\in[\tilde{k},k_{nc}]} \leq -e^{g\tau^*(k)}(e^{g'k\eta} - 1)e^{gt_0}\Big|_{k\in[\tilde{k},k_{nc}]} < 0$. As a result, $W(\tau^*(k), k)$ is strictly decreasing in $k \in [\tilde{k}, k_{nc}]$, and, therefore, $\lim_{A\to\overline{A}} \frac{W(\tau^*(\tilde{k}),\tilde{k})}{W(\tau^*(k_{nc}),k_{nc})} > 1$. From the continuity of $\frac{W(\tau^*(\tilde{k}),\tilde{k})}{W(\tau^*(k_{nc}),k_{nc})}$ with respect to A, there exists A_1 that satisfies (A.14), such that $W(\tau^*(\tilde{k}), \tilde{k}) \geq W(\tau^*(k_{nc}), k_{nc})$ if and only if $A \geq A_1$, with the equality holding when $A = A_1$.

Finally, we want to show that $A_0 > \max\{v(\tilde{k}), A_1\}$. Regarding the relative size between A_0 and $v(\tilde{k})$, when $A = v(\tilde{k})$, it implies $k_v = \tilde{k}$, so $(\tau^*(k) + k\eta)$ is strictly increasing in $k \in (0, k_{nc}]$. From Proposition 4, we have $k_c = k_{nc}$ and with $g' > g_0$, it must be $v(\tilde{k}) < A_0$. Regarding A_0 and A_1, when $A = A_0$, we have $\tau^*(\tilde{k}) + \tilde{k}\eta = k_{nc}\eta$, and thus

$$\begin{aligned}
\Big[W(\tau^*(\tilde{k}), \tilde{k}) - W(\tau^*(k_{nc}), k_{nc})\Big]\Big|_{A=A_0} &= \Bigg[\frac{e^{g(\tau^*(\tilde{k})+\tilde{k}\eta)} - e^{g\tau^*(\tilde{k})}}{g\eta} + A_0 e^{g'(\tau^*(\tilde{k})+\tilde{k}\eta)} \\
&\quad - \frac{e^{g(\tau^*(\tilde{k})+\tilde{k}\eta)} - e^{g\tau^*(\tilde{k})+g'\tilde{k}\eta}}{(g-g')\eta} - \frac{e^{gk_{nc}\eta} - 1}{g\eta}\Bigg] e^{gt_0} \\
&= \left[\frac{e^{gk_{nc}\eta} - e^{g\tau^*(\tilde{k})}}{g\eta} + A_0 e^{g'k_{nc}\eta} - \frac{e^{gk_{nc}\eta} - e^{g'k_{nc}\eta+(g-g')\tau^*(\tilde{k})}}{(g-g')\eta} - \frac{e^{gk_{nc}\eta} - 1}{g\eta}\right] e^{gt_0} \\
&= \left[-\frac{e^{g\tau^*(\tilde{k})} - 1}{g\eta} + \frac{e^{(g-g')\tau^*(\tilde{k})} - 1}{(g-g')\eta} e^{g'k_{nc}\eta}\right] e^{gt_0} \\
&= \frac{1}{\eta}\left[-\frac{e^{g\tau^*(\tilde{k})} - 1}{g} + \frac{e^{g\tau^*(\tilde{k})} - e^{g'\tau^*(\tilde{k})}}{g-g'} e^{g'\tilde{k}\eta}\right] e^{gt_0},
\end{aligned}$$

in which the second and last equalities hold because $\tau^*(\tilde{k}) + \tilde{k}\eta = k_{nc}\eta$ and the third equality holds according to $\alpha(\tau^*(k_{nc}) = 0, k_{nc}) = 0$. Regarding this difference, we have

$$\begin{aligned}
\frac{e^{g\tau^*(\tilde{k})} - e^{g'\tau^*(\tilde{k})}}{g-g'} e^{g'\tilde{k}\eta} - \frac{e^{g\tau^*(\tilde{k})} - 1}{g} &> \frac{e^{g\tau^*(\tilde{k})} - e^{g'\tau^*(\tilde{k})}}{g-g'} - \frac{e^{g\tau^*(\tilde{k})} - 1}{g} \\
&= \frac{-ge^{g'\tau^*(\tilde{k})} + g'e^{g\tau^*(\tilde{k})}}{g(g-g')} + \frac{1}{g} > 0,
\end{aligned}$$

where the first inequality holds because $\tilde{k} > 0$ and the second inequality holds because $-ge^{g'\tau^*(\tilde{k})} + g'e^{g\tau^*(\tilde{k})} > -g + g'$, which stems from $\tau^*(\tilde{k}) > 0$ and the fact that $-ge^{g'x} + g'e^{gx}$ is increasing in $x \geq 0$. As a result, $\left[W(\tau^*(\tilde{k}), \tilde{k}) - W(\tau^*(k_{nc}), k_{nc})\right]|_{A=A_0} > 0$, which is equivalent to $\frac{W(\tau^*(\tilde{k}),\tilde{k})}{W(\tau^*(k_{nc}),k_{nc})}|_{A=A_0} > 1$. Since $\frac{W(\tau^*(\tilde{k}),\tilde{k})}{W(\tau^*(k_{nc}),k_{nc})} > 1$ if and only if $A > A_1$, it follows that $A_0 > A_1$. This completes the proof. ■

Proof of Proposition 6 Based on the definition of total welfare W in (25) and the symmetric equilibrium captured by τ^*, we can rewrite the total welfare as

$$\begin{aligned} W(\tau^*) &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} \mathrm{d}t + \int_{t_0+k\eta+\tau^*-m}^{t_0+\eta+\tau^*} \frac{1}{\eta} \alpha(\tau^*, k, m) e^{gt_0} \mathrm{d}t \\ &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} \mathrm{d}t + \frac{Y_{t_0+k\eta+\tau^*} + \int_{t_0+k\eta+\tau^*-m}^{t_0+\tau^*+k\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t}{1-k+\frac{m}{\eta}} \int_{t_0+k\eta+\tau^*-m}^{t_0+\eta+\tau^*} \frac{1}{\eta} \mathrm{d}t. \end{aligned}$$

Based on the definition of α and the fact that $\int_{t_0+k\eta-m}^{t_0+\eta} \frac{1}{\eta} = 1 - k + \frac{m}{\eta}$, we have

$$\begin{aligned} W(\tau^*) &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*-m} \frac{1}{\eta} e^{gt} \mathrm{d}t + \int_{t_0+k\eta+\tau^*-m}^{t_0+\tau^*+k\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t + Y_{t_0+k\eta+\tau^*} \\ &= \int_{t_0+\tau^*}^{t_0+k\eta+\tau^*} \frac{1}{\eta} e^{gt} \mathrm{d}t + Y_{\hat{t}}. \end{aligned}$$

Next, we leverage on the following lemma to prove the monotonicity of $W(\tau^*)$ on τ^*. For convenience, we introduce $Y_{t,\tau}$ and $w_{t,s}$ to denote the firm's asset value and the fraction of exiting creditors at time t, respectively, if the waiting time is τ for each creditor.

Lemma A.3 *For any $\tau_2 > \tau_1$,*

$$Y_{t,\tau_2} \geq Y_{t,\tau_1},$$

for any $t \leq t_0 + k\eta + \tau_1$ with equality holding if and only if $t \leq t_0 + \tau_1$.

Proof of Lemma A.3. Given the process of asset value Y_t (see (11)), if $t \leq t_0 + \tau_1$, clearly, $Y_{t,\tau_2} = Y_{t,\tau_1}$. Moreover, when $t_0 + \tau_1 < t \leq t_0 + \tau_2$, we have

$$Y_{t,\tau_2} = Ae^{gt_0+g'(t-t_0)} > Ae^{gt_0+g'(t-t_0)} - \frac{1}{(g-g')\eta}\left[e^{gt} - e^{g(t_0+\tau_1)+g'(t-t_0-\tau_1)}\right] = Y_{t,\tau_1}.$$

Finally, if $t > t_0 + \tau_2$, $Y_{t,\tau_2} > Y_{t,\tau_1}$ because the term $e^{gt} - e^{g(t_0+\tau)+g'(t-t_0-\tau)}$ in the third-case scenario in (11) is strictly decreasing in τ. ■

We can rewrite $Y_{t,\tau}$ in its integral form: $Y_{t,\tau} = Y_{t_0} + \int_{t_0}^{t} (g'Y_{s,\tau} - w_{s,\tau}e^{gs})\mathrm{d}s$, where

$$w_{s,\tau} = \begin{cases} \frac{1}{\eta} & \text{if } s \in [t_0+\tau, t_0+k\eta+\tau] \\ 0 & \text{otherwise} \end{cases}.$$

Consider any $\tau_2 > \tau_1 > 0$. The difference in welfare associated with waiting times τ_1 and τ_2 is given by

$$\begin{aligned} W(\tau_2) - W(\tau_1) &= \int_{t_0+\tau_2}^{t_0+\tau_2+k\eta} \frac{1}{\eta} e^{gs} \mathrm{d}s - \int_{t_0+\tau_1}^{t_0+\tau_1+k\eta} \frac{1}{\eta} e^{gs} \mathrm{d}s \\ &+ \int_{t_0}^{t_0+k\eta+\tau_2} \left(g' Y_{s,\tau_2} - w_{s,\tau_2} e^{gs}\right) \mathrm{d}s - \int_{t_0}^{t_0+k\eta+\tau_1} \left(g' Y_{s,\tau_1} - w_{s,\tau_1} e^{gs}\right) \mathrm{d}s. \end{aligned}$$

For any $s \leq t_0 + \tau_1 < t_0 + \tau_2$, $Y_{s,\tau_2} = Y_{s,\tau_1}$ (Lemma A.3), and $w_{s,\tau_1} = w_{s,\tau_2} = 0$. Therefore, we have

$$\begin{aligned} W(\tau_2) - W(\tau_1) &= \int_{t_0+\tau_2}^{t_0+\tau_2+k\eta} \frac{1}{\eta} e^{gs} \mathrm{d}s - \int_{t_0+\tau_1}^{t_0+\tau_1+k\eta} \frac{1}{\eta} e^{gs} \mathrm{d}s \\ &+ \int_{t_0+\tau_1}^{t_0+k\eta+\tau_2} \left(g' Y_{s,\tau_2} - w_{s,\tau_2} e^{gs}\right) \mathrm{d}s - \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} \left(g' Y_{s,\tau_1} - w_{s,\tau_1} e^{gs}\right) \mathrm{d}s \\ &= \int_{t_0+\tau_1}^{t_0+k\eta+\tau_2} g' Y_{s,\tau_2} \mathrm{d}s - \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} g' Y_{s,\tau_1} \mathrm{d}s \\ &= \int_{t_0+k\eta+\tau_1}^{t_0+k\eta+\tau_2} g' Y_{s,\tau_2} \mathrm{d}s + \int_{t_0+\tau_1}^{t_0+k\eta+\tau_1} g' \left(Y_{s,\tau_2} - Y_{s,\tau_1}\right) \mathrm{d}s. \end{aligned}$$

Since $Y_{t,\tau} > 0$ and $Y_{s,\tau_2} - Y_{s,\tau_1} > 0$ (Lemma A.3), the above expression is strictly positive, thereby completing the proof. ■

Proof of Proposition 7 When $\tau^* > 0$, we can rewrite the equilibrium condition as

$$g e^{g(t_i+\tau^*)} = h(k,m) \left[e^{g(t_i+\tau^*)} - \alpha e^{g(t_i - k\eta + m)} \right],$$

in which $h(k,m) = \frac{\lambda e^{\lambda(k\eta - m)}}{e^{\lambda(k\eta-m)}-1}$. When $m \in (0, k\eta)$, we can plug in $\alpha(\tau^*, k, m)$ and rearrange it to find a unique τ^* that satisfies

$$\tau^*(k,m) = \max\left\{0, \frac{1}{g-g'} \left(\log A - \log v(k,m)\right)\right\},$$

where

$$\begin{aligned} v(k,m) \equiv & \frac{e^{(g-g')k\eta} - 1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta} \\ &+ (1 - k + \frac{m}{\eta}) \left[\frac{g}{\lambda} e^{(g-\lambda)(k\eta - m) - g'k\eta} - (\frac{g}{\lambda} - 1) e^{(g-g')k\eta - gm} \right]. \end{aligned}$$

Since $v_m(k,m) = \frac{g}{\lambda\eta} e^{(g-g')k\eta - gm}(1 - e^{-\lambda(k\eta-m)})[-1 + (\eta(1-k) + m)(g-\lambda)]$ and $\eta > \frac{1}{g-\lambda}$ (Assumption 1), we know that if $k\eta - (\eta - \frac{1}{g-\lambda}) > 0$, $\tau^*(k,m)$ is increasing in m when $m \in [0, k\eta - (\eta - \frac{1}{g-\lambda})]$ and decreasing in m when $m \in [k\eta - (\eta - \frac{1}{g-\lambda}), k\eta)$, and thus $m^* = k\eta - (\eta - \frac{1}{g-\lambda})$;

if $k\eta - (\eta - \frac{1}{g-\lambda}) \leq 0$, $\tau^*(k,m)$ is decreasing in $m \in [0, k\eta)$, and thus $m^* = 0$. Therefore, the optimal m^* is given in (32). ■

Proof of Proposition 8 In this proof, we consider two cases. In Case I, the regulator, when choosing m, does not consider the impact of m on the non-commitment bankruptcy threshold k_{nc}^m through m's impact on τ^*. In Case II, however, the regulator does take such an impact into account. We will show that both cases, despite their subtle differences, will end up having the identical equilibrium outcome as shown in Proposition 8.

Case I. In Case I, the triple $(m^*, k_{nc}^m, \tau^*(k,m))$ constitutes a symmetric equilibrium if (a) given m^* and k_{nc}^m, when other creditors choose $\tau^* = \tau^*(k_{nc}^m, m^*)$, no creditor i has an incentive to deviate from $\tau^* = \tau^*(k_{nc}^m, m^*)$; (b) given k_{nc}^m and $\tau^*(k,m)$, total welfare is maximized at m^*; and (c) given $\tau^* = \tau^*(k_{nc}^m, m^*)$, the firm's assets are exhausted at k_{nc}^m; that is, $Y_{t_0+k_{nc}^m\eta+\tau^*} = 0$. We will find the unique equilibrium that gives rise to the equilibrium outcome given in Proposition 8.

First, according to the proof of Proposition 7, for any given m and k, the equilibrium waiting time is

$$\tau^*(k,m) = \frac{1}{g-g'}\left\{\log A - \log\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}\right.\right.$$
$$\left.\left.+(1-k+\frac{m}{\eta})\left(\frac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta} - \frac{g-\lambda}{\lambda}e^{(g-g')k\eta-gm}\right)\right]\right\}. \tag{A.17}$$

Second, according to Proposition 6 and Proposition 7, given k and $\tau^*(k,m)$, the optimal clawback window that maximizes welfare is given by

$$m^* = m^*(k) = \frac{1}{g-\lambda} - (1-k)\eta. \tag{A.18}$$

Third, given τ^*, k_{nc}^m satisfies that

$$Y_{t_0+\tau^*+k_{nc}^m\eta} = 0. \tag{A.19}$$

Next, we will show that there exists a unique triple $(m^* = m^*(k_{nc}^m),\ k_{nc}^m,\ \tau^* = \tau^*(k_{nc}^m, m^*))$ that solves (A.17), (A.18) and (A.19). To begin with, after plugging $\tau^* = \tau^*(k_{nc}^m, m^*)$ into (A.19), we have

$$Ae^{g'(\tau^*(k_{nc}^m,m^*)+k_{nc}^m\eta)} - \frac{e^{g(\tau^*(k_{nc}^m,m^*)+\eta)} - e^{g\tau^*(k_{nc}^m,m^*)+g'k_{nc}^m\eta}}{(g-g')\eta} = 0.$$

This can be further simplified to

$$\tau^*(k_{nc}^m, m^*) = \frac{1}{g-g'}\left\{\log A - \log\frac{e^{(g-g')k_{nc}^m\eta}-1}{g-g'}\right\}. \tag{A.20}$$

Fixing $k = k_{nc}^m$ and $m = m^*$ in (A.17) and combining it with (A.20), we have

$$-\frac{e^{(g-g')k_{nc}^m\eta}(1-e^{-gm^*})}{g\eta}$$
$$+(1-k_{nc}^m+\frac{m^*}{\eta})\left(\frac{g}{\lambda}e^{(g-\lambda)(k_{nc}^m\eta-m^*)-g'k_{nc}^m\eta}-\frac{g-\lambda}{\lambda}e^{(g-g')k_{nc}^m\eta-gm^*}\right)=0,$$

which is equivalent to

$$-\frac{e^{gm^*}-1}{g\eta}+(1-k_{nc}^m+\frac{m^*}{\eta})\left(\frac{g}{\lambda}e^{-\lambda(k_{nc}^m\eta-m^*)}-\frac{g-\lambda}{\lambda}\right)=0. \tag{A.21}$$

We can then solve for k_{nc}^m by plugging (A.18) into (A.21); that is,

$$k_{nc}^m=\frac{1}{g\eta}\log\left[\frac{g^2}{(g-\lambda)\lambda}e^{(g-\lambda)(\eta-\frac{1}{g-\lambda})}-\frac{g-\lambda}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\right]. \tag{A.22}$$

Then, we can plug (A.22) into (A.18) to solve for m^*; that is,

$$m^*=\frac{1}{g}\log\left[\frac{g^2}{(g-\lambda)\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})}-\frac{g-\lambda}{\lambda}\right]. \tag{A.23}$$

Finally, for the equilibrium outcome, we can derive $\tau^*(k^*, m^*)$ by plugging $k = k_{nc}^m$ and $m = m^*$ into (A.17), and we can easily check that the equilibrium waiting time is the one given in Proposition 8.

Case II. In Case II, the triple $\Big(m^*, \hat{k}(\tau), \tau^*(m)\Big)$ constitutes a symmetric equilibrium if (a) given $\hat{k}(\tau)$ and $\tau^*(m)$, the regulator chooses the clawback window m^* such that total welfare is maximized; (b) for any τ, if all creditors choose to wait for τ units of time, then $k_{nc}^m = \hat{k}(\tau)$ is such that $Y_{t_0+\hat{k}(\tau)\eta+\tau} = 0$; and (c) for any m, the equilibrium waiting time $\tau^*(m)$ is such that, given that other creditors choose $\tau^*(m)$ and therefore the asset is depleted at $k_{nc}^m = \hat{k}(\tau^*(m))$, no creditor i has an incentive to deviate from $\tau^*(m)$.

We solve the game backwards. First, we consider the non-committed bankruptcy threshold for any waiting time τ. For any τ, the bankruptcy threshold $\hat{k}(\tau)$ that depletes all assets is the one with which $Y_{t_0+\tau+\hat{k}(\tau)\eta} = 0$:

$$Ae^{g'(\tau+\hat{k}(\tau)\eta)}-\frac{e^{g(\tau+\hat{k}(\tau)\eta)}-e^{g\tau+g'\hat{k}(\tau)\eta}}{(g-g')\eta}=0, \tag{A.24}$$

which gives

$$\hat{k}(\tau)=\frac{1}{(g-g')\eta}\log\left[A(g-g')\eta e^{-(g-g')\tau}+1\right]. \tag{A.25}$$

Then, we solve for the creditors' equilibrium waiting time for any given m. For $\tau^*(m)$ to hold in an equilibrium, it must be that no creditor has an incentive to deviate from $\tau_i = \tau^*(m)$. That implies $\frac{\partial \Pi_i(\tau_i|\tau^*(m),\hat{k},m)}{\partial \tau_i}\big|_{\tau_i=\tau^*(m)} = 0$, in which $\Pi_i(\tau_i|t_i,\tau^*,k,m)$ is given in (27). This first-order condition implies that[38]

$$\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\hat{k}(\tau^*(m))\eta - m)} + \frac{e^{gm}-1}{(1-\hat{k}(\tau^*(m))+\frac{m}{\eta})g\eta} = 0, \tag{A.26}$$

which is consistent with (A.21).

Next, we take the first-order derivative of (A.26) w.r.t. m and arrive at

$$\begin{aligned}&\frac{1}{\eta}\left(\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\hat{k}\eta-m)}\right) - \left(1-\hat{k}+\frac{m}{\eta}\right)ge^{-\lambda(\hat{k}\eta-m)} + \frac{e^{gm}}{\eta}\\ &+\left[-\left(\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\hat{k}\eta-m)}\right) + \left(1-\hat{k}+\frac{m}{\eta}\right)g\eta e^{-\lambda(\hat{k}\eta-m)}\right]\frac{\mathrm{d}\hat{k}}{\mathrm{d}m} = 0.\end{aligned} \tag{A.27}$$

Multiplying the left-hand side of (A.26) by g and then subtracting the left-hand side of (A.27) from it, we have

$$\begin{aligned}&\frac{g(g-\lambda)}{\lambda\eta}\left(1-e^{-\lambda(\hat{k}\eta-m)}\right)\left[m+(1-\hat{k})\eta - \frac{1}{g-\lambda}\right]\\ &+\left[\left(\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\hat{k}\eta-m)}\right) - \left(1-\hat{k}+\frac{m}{\eta}\right)g\eta e^{-\lambda(\hat{k}\eta-m)}\right]\frac{\mathrm{d}\hat{k}}{\mathrm{d}m} = 0.\end{aligned} \tag{A.28}$$

Note that, as the regulator does not incorporate the effect of m on k in Case I (i.e., $\frac{\mathrm{d}\hat{k}}{\mathrm{d}m} = 0$), then we can derive m^* (see (A.18)) directly from (A.28). However, in this case, the regulator would internalize the effect of m on $\hat{k}$ when choosing m. Nevertheless, we will show that the welfare-maximizing regulator would stick to the same m^* in Case II even though the indirect effect of m on $k^m_{nc} = \hat{k}(\tau^*(m))$ is considered.

In appearance of the indirect effect of m on k, the regulator's objective of maximizing welfare may not be equivalent to maximizing the equilibrium waiting time τ^*. (Recall that this result is proved in Proposition 6 where the firm's choice of k is fixed but not affected by the regulator's choice.) However, the following lemma confirms that internalizing this indirect effect would not change that equivalence. In other words, in this case, the optimal m that maximizes total welfare is equivalent to the choice of m that maximizes $\tau^*(m)$.

Lemma A.4 *When the regulator takes into account of the choice of m on the bankruptcy threshold $k^m_{nc} = \hat{k}(\tau^*(m))$, the welfare-maximizing objective is equivalent to maximizing τ^*:*

$$m^* = \arg\max_{m\geq 0} W(m) = \arg\max_{m\geq 0} \tau^*(m).$$

Proof of Lemma A.4. As the clawback can be considered as a redistribution of assets, the total welfare stays the same as in (A.15). With the equilibrium waiting time $\tau^*(m)$ (see (A.26)) and the bankruptcy threshold $\hat{k}(\tau^*)$ (see (A.25)), the total welfare is given by $W(\tau^*(m), \hat{k}(\tau^*(m)))$.

[38] For simplicity, we write τ^* for $\tau^*(m)$ and $\hat{k}$ for $\hat{k}(\tau^*(m))$.

Taking the first-order derivative of $W(\tau^*(m), \hat{k}(\tau^*(m)))$ w.r.t. m, we have

$$\begin{aligned}&\frac{\mathrm{d}W(\tau^*(m),\hat{k}(\tau^*(m)))}{\mathrm{d}m}\\&=\left[\frac{\partial W(\tau,k)}{\partial \tau}\frac{\mathrm{d}\tau^*(m)}{\mathrm{d}m}+\frac{\partial W(\tau,k)}{\partial k}\frac{\mathrm{d}\hat{k}(\tau^*)}{\mathrm{d}\tau^*}\frac{\mathrm{d}\tau^*(m)}{\mathrm{d}m}\right]\Big|_{\tau=\tau^*(m),k=\hat{k}(\tau^*(m))}.\end{aligned}$$

From the above equation, the clawback window m affects total welfare through two channels. In the direct channel, it influences welfare via the equilibrium waiting time τ^*. In the indirect channel, it affects welfare through the bankruptcy threshold k, which is determined by τ^*. However, the net effect of the indirect channel is proved to be zero; that is,

$$\begin{aligned}&\frac{\partial W(\tau,k)}{\partial k}\Big|_{\tau=\tau^*(m),k=\hat{k}(\tau^*(m))}\\&=e^{gt_0}\left[e^{g(\tau+k\eta)}+Ag'\eta e^{g'(\tau+k\eta)}-\frac{ge^{g(\tau+k\eta)}-g'e^{g\tau+g'k\eta}}{g-g'}\right]\Big|_{\tau=\tau^*(m),k=\hat{k}(\tau^*(m))}\\&=g'\eta e^{gt_0}\left[Ae^{g'(\tau+k\eta)}-\frac{e^{g(\tau+k\eta)}-e^{g\tau+g'k\eta}}{(g-g')\eta}\right]\Big|_{\tau=\tau^*(m),k=\hat{k}(\tau^*(m))}=0,\end{aligned}$$

where the last equation comes from (A.24). Thus, m affects W only through τ^*; that is,

$$\frac{\mathrm{d}W(\tau^*(m),\hat{k}(\tau^*(m)))}{\mathrm{d}m}=\frac{\partial W(\tau,k)}{\partial \tau}\frac{\mathrm{d}\tau^*(m)}{\mathrm{d}m}\Big|_{\tau=\tau^*(m),k=\hat{k}(\tau^*(m))}.$$

As $W(\tau, k)$ is increasing in τ (Proposition 6), it follows that $\frac{\partial W(\tau,k)}{\partial \tau} \geq 0$, and thus $\frac{\mathrm{d}W(\tau^*(m),\hat{k}(\tau^*(m)))}{\mathrm{d}m}$ shares the same sign as $\frac{\mathrm{d}\tau^*(m)}{\mathrm{d}m}$. Therefore, the regulator's welfare objective is equivalent to maximizing τ^*. ■

According to Lemma A.4, maximizing welfare is equivalent to maximizing τ^*, which implies $\frac{\mathrm{d}\tau^*(m)}{\mathrm{d}m}\big|_{m=m^*} = 0$. As a result,

$$\frac{\mathrm{d}\hat{k}}{\mathrm{d}m}=\frac{\mathrm{d}\hat{k}}{\mathrm{d}\tau^*}\frac{\mathrm{d}\tau^*}{\mathrm{d}m}\Big|_{m=m^*}=0,$$

and, accordingly, condition (A.28) can be simplified to

$$m^*+\eta(1-\hat{k}(\tau^*(m^*)))-\frac{1}{g-\lambda}=0\Longleftrightarrow m^*=m^*(\hat{k}(\tau^*)). \tag{A.29}$$

This explains why m^* in this case will be the same as that in Case I. Now, following procedures similar to those in the proof of Case I, the equilibrium objects m^*, $\tau^*(m)$, and $\hat{k}(\tau)$ can be solved through (A.25), (A.26), and (A.29). For example, plugging (A.29) into (A.26) yields the optimal clawback window m^*, the same as in (A.23). Therefore, the unique symmetric equilibrium is as follows: the regulator chooses m^* as in (A.23); for any given τ, the bankruptcy threshold $\hat{k}(\tau)$ is determined as in (A.25); and, for any given m, creditors' equilibrium waiting time $\tau^*(m)$ satisfies the (A.26):

Accordingly, the equilibrium outcome, characterized by the regulator, the creditors, and the firm's choices, is

$$m^* = \frac{1}{g}\log\left[\frac{g^2}{\lambda(g-\lambda)}e^{-\lambda(\eta-\frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda}\right] = m^*(k_{nc}^m),$$

$$\begin{aligned}\tau^*(m^*) &= \frac{1}{g-g'}\left[\log A - \log\frac{\left(\frac{g^2}{(g-\lambda)\lambda}e^{(g-\lambda)(\eta-\frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\right)^{\frac{g-g'}{g}} - 1}{(g-g')\eta}\right] \\ &= \tau^*(k_{nc}^m, m^*), \quad \text{and}\end{aligned}$$

$$\hat{k}(\tau^*(m^*)) = \frac{1}{g\eta}\log\left[\frac{g^2}{(g-\lambda)\lambda}e^{(g-\lambda)(\eta-\frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\right] = k_{nc}^m.$$

This outcome is identical to what we derived in Case I.

Comparison of equilibrium waiting times. Finally, regarding the equilibrium outcome derived from the case without commitment but an optimal clawback policy (both Case I and Case II), we will show that the equilibrium waiting time $\tau^*(k_{nc}^m, m^*) \geq \tau^*(k_c, 0) \geq \tau^*(k_{nc}, 0) = 0$, and the inequalities are strict when $k_c < k_{nc}$.

It is easy to check that when $k_c = k_{nc}$, we have $\tau^*(k_{nc}^m, m^*) \geq \tau^*(k_c, 0) = \tau^*(k_{nc}, 0) = 0$, in which the inequality holds because[39]

$$\tau^*(k_{nc}^m, m^*) = \max_m \tau^*(m) \geq \tau^*(m=0) = \tau^*(k_{nc}, 0) = 0 \quad \text{(Lemma A.4)}.$$

Next, consider the case in which $k_c < k_{nc}$. The value of the remaining assets under k_c is strictly positive when all creditors choose $\tau^*(k_c)$; that is, $Y_{t_0+\tau^*(k_c)+k_c\eta} > 0$. Therefore, we can define

$$k_c' \equiv \hat{k}(\tau^*(k_c)) > k_c, \tag{A.30}$$

such that, when the creditors symmetrically choose the strategy $\tau^*(k)$, the value of the remaining assets reaches 0 after k_c' fraction of creditors exit; that is, $Y_{t_0+\tau^*(k_c)+k_c'\eta} = 0$, where $\hat{k}(\cdot)$ is given in (A.25). Furthermore, we define $m' \equiv (k_c' - k_c)\eta$. We use the following lemma to show that the recovery payoff under policy $(k_c, 0)$ is lower than that under (k_c', m') had the creditors taken the strategy $\tau^*(k_c)$. We will show that this result, together with the fact that m^* is the regulator's

[39]To avoid confusion, we write $\tau^*(k_{nc}^m, m^*)$ for $\tau^*(m^*)$ (the no-commitment case where $m = m^*$), $\tau^*(k_c, 0)$ for $\tau^*(k_c)$ (the commitment case without a clawback policy), and $\tau^*(k_{nc}, 0)$ for $\tau^*(k_{nc})$ (the non-commitment case without a clawback policy).

optimal choice, is sufficient to prove that $\tau^*(m^*) \geq \tau^*(m') > \tau^*(k_c)$, in which $\tau^*(m')$ is the equilibrium waiting time for the non-commitment case with a clawback policy m'.

Lemma A.5 *When creditors choose to wait for τ^*, for any $k' < \hat{k}(\tau^*)$, it must be*

$$\int_{t_0+\tau^*+k'\eta}^{t_0+\tau^*+\hat{k}(\tau^*)\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t > Y_{t_0+\tau^*+k'\eta},$$

in which $\hat{k}(\tau^*)$ is given in (A.25).

Proof of Lemma A.5. As $Y_{t_0+\tau^*+k\eta} = e^{g(t_0+\tau^*)+g'k\eta}\left[Ae^{-(g-g')\tau^*} - \frac{e^{(g-g')k\eta}-1}{(g-g')\eta}\right]$ (see (11)), we can easily check that

$$Y_{t_0+\tau^*+k\eta} > 0 \;\; \text{for any} \;\; k < \hat{k}(\tau^*). \tag{A.31}$$

According to (3), we have $\mathrm{d}Y_t = (g'Y_t - \frac{1}{\eta}e^{gt})\mathrm{d}t$ when $t > t_0 + \tau^*$. Integrating both sides over $[t_0 + \tau^* + k'\eta, t_0 + \tau^* + \hat{k}(\tau^*)\eta]$ yields

$$Y_{t_0+\tau^*+\hat{k}(\tau^*)\eta} - Y_{t_0+\tau^*+k'\eta} = \int_{t_0+\tau^*+k'\eta}^{t_0+\tau^*+\hat{k}(\tau^*)\eta} g'Y_t \mathrm{d}t - \int_{t_0+\tau^*+k'\eta}^{t_0+\tau^*+\hat{k}(\tau^*)\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t.$$

As $Y_{t_0+\tau^*+\hat{k}(\tau^*)\eta} = 0$ by definition, it follows that

$$\int_{t_0+\tau^*+k'\eta}^{t_0+\tau^*+\hat{k}(\tau^*)\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t - Y_{t_0+\tau^*+k'\eta} = \int_{t_0+\tau^*+k'\eta}^{t_0+\tau^*+\hat{k}(\tau^*)\eta} g'Y_t \mathrm{d}t > 0,$$

in which the inequality holds according to (A.31). ■

Given that other creditors wait for $\tau^*(k_c)$, from the construction of (k'_c, m'), (1) the hazard rate under (k'_c, m') remains the same as the one under $(k_c, 0)$; that is, $h(k'_c, m') = h(k_c) = \frac{\lambda e^{\lambda k_c \eta}}{e^{\lambda k_c \eta}-1}$ (see (15) and (29)); and (2) the total value of the assets distributed to each staying creditor (and those who are clawed back) is strictly higher under (k'_c, m'); that is, $\alpha(\tau^*(k_c), k'_c, m') > \alpha(\tau^*(k_c), k_c)$. The second property holds because $\int_{t_0+\tau^*(k_c)+k'_c\eta-m'}^{t_0+\tau^*(k_c)+k'_c\eta} \frac{1}{\eta}e^{gt}\mathrm{d}t > Y_{t_0+\tau^*(k_c)+k_c\eta}$ according to Lemma A.5 and $1 - k'_c + \frac{m'}{\eta} = 1 - k_c$ according to the definition of m'.

With the same hazard rate and higher recovery payoff, creditor i's net marginal payoff of waiting $\tau_i = \tau^*(k_c)$ is strictly positive; that is,

$$\begin{aligned} &\left.\frac{\partial \Pi_i(\tau_i|t_i, \tau^*(k_c), k'_c, m')}{\partial \tau_i}\right|_{\tau_i=\tau^*(k_c)} \\ &\propto -\frac{h(k'_c, m') - g}{h(k'_c, m')} + \alpha(\tau^*(k_c), k'_c, m')e^{-g(\tau^*(k_c)+k'_c\eta-m')} \\ &> -\frac{h(k_c) - g}{h(k_c)} + \alpha(\tau^*(k_c), k_c)e^{-g(\tau^*(k_c)+k_c\eta)} = 0, \end{aligned} \tag{A.32}$$

in which $\Pi_i(\cdot)$ is given in (27). This result implies that when the manager commits to k'_c and the regulator chooses m', and given that other creditors choose $\tau^*(k_c)$, creditor i has an incentive to wait longer than $\tau^*(k_c)$. Next, we will show that the equilibrium waiting time $\tau^*(m')$ in the non-commitment case under clawback policy m' is indeed higher than that in the commitment case with k_c but no clawback, or $\tau^*(k_c)$.

Plugging $h(k'_c, m')$ (based on(29)) and $\alpha(\tau^*(k_c), k'_c, m')$ (based on (26)) into (A.32), we have

$$\frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda(k'_c\eta - m')} + \frac{e^{gm'} - 1}{(1 - k'_c + \frac{m'}{\eta})g\eta} > 0. \tag{A.33}$$

Under the clawback policy m' in the non-commitment case, the equilibrium waiting time $\tau^*(m')$ satisfies (see (A.26))

$$\frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda(\hat{k}(\tau^*(m'))\eta - m')} + \frac{e^{gm'} - 1}{(1 - \hat{k}(\tau^*(m')) + \frac{m'}{\eta})g\eta} = 0. \tag{A.34}$$

Comparing (A.34) with (A.33), we have

$$\begin{aligned} &\frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda(k'_c\eta - m')} + \frac{e^{gm'} - 1}{(1 - k'_c + \frac{m'}{\eta})g\eta} \\ &> \frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda(\hat{k}(\tau^*(m'))\eta - m')} + \frac{e^{gm'} - 1}{(1 - \hat{k}(\tau^*(m')) + \frac{m'}{\eta})g\eta}. \end{aligned} \tag{A.35}$$

It is easy to check that the function $\frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda(k\eta - m')} + \frac{e^{gm'}-1}{(1-k+\frac{m'}{\eta})g\eta}$ is increasing in k, so (A.35) implies $k'_c > \hat{k}(\tau^*(m'))$. Recall that, by definition (see (A.30)), $k'_c = \hat{k}(\tau^*(k_c))$. As $\hat{k}(\tau^*)$ is strictly decreasing in τ^* (see (A.25)), it follows that $\tau^*(k_c) < \tau^*(m')$. According to Lemma A.4, the regulator chooses m to maximize $\tau^*(m)$. As a result,

$$\tau^*(m^*) = \max_m \tau^*(m) \geq \tau^*(m') > \tau^*(k_c).$$

Moreover, as $k_c = \arg\max_k(\tau^*(k) + k\eta)$ and $k_c < k_{nc}$, we have

$$\tau^*(k_c) = (\tau^*(k_c) + k_c\eta) - k_c\eta > (\tau^*(k_{nc}) + k_{nc}\eta) - k_c\eta > \tau^*(k_{nc}).$$

This completes the proof. ■

Proof of Proposition 9 We first prove that k_c^m must satisfy $Y_{t_0+\tau^*(k_c^m, m^*(k_c^m))+k_c^m\eta} = 0$, in which $m^*(k)$ is the optimal clawback window given in Proposition 7.[40]

Suppose $Y_{t_0+\tau^*+k_c^m\eta} > 0$; then there exists $\breve{k} > k_c^m$ such that $Y_{t_0+\tau^*+\breve{k}\eta} = 0$. Define $\breve{m} \equiv m^*(k_c^m) + (\breve{k} - k_c^m)\eta$ such that, given that other creditors wait for τ^*, if creditor i

[40]For simplicity, we write $\tau^*(k_c^m, m^*(k_c^m))$ for τ^*.

chooses $\tau_i = \tau^*$, (1) the hazard rate under policy $(\breve{k}, \breve{m})$ remains the same as that under policy $(k_c^m, m^*(k_c^m))$; and (2) the total value of the assets distributed to each staying creditor (and those who are clawed back) is strictly higher under $(\breve{k}, \breve{m})$; that is, $\alpha(\tau^*, \breve{k}, \breve{m}) > \alpha(\tau^*, k_c^m, m^*(k_c^m))$. The second property holds because

$$\begin{aligned}
&\left(Y_{t_0+\tau^*+\breve{k}\eta} + \int_{t_0+\tau^*+\breve{k}\eta-\breve{m}}^{t_0+\tau^*+\breve{k}\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t\right) \\
&- \left(Y_{t_0+\tau^*+k_c^m\eta} + \int_{t_0+\tau^*+k_c^m\eta-m^*(k_c^m)}^{t_0+\tau^*+k_c^m\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t\right) \\
&= \int_{t_0+\tau^*+k_c^m\eta}^{t_0+\tau^*+\breve{k}\eta} \frac{1}{\eta} e^{gt} \mathrm{d}t - Y_{t_0+\tau^*+k_c^m\eta} > 0
\end{aligned}$$

according to Lemma A.5 and $1 - \breve{k} + \frac{\breve{m}}{\eta} = 1 - k_c^m + \frac{m^*(k_c^m)}{\eta}$ according to the definition of $\breve{m}$.

With the same hazard rate and higher recovery payoff, we can compare the equilibrium waiting time under policy $(k_c^m, m^*(k_c^m))$, denoted as τ^*, and the one under policy $(\breve{k}, \breve{m})$, denoted as $\breve{\tau} \equiv \tau^*(\breve{k}, \breve{m})$. The first-order conditions yield

$$\begin{aligned}
&- \frac{h(\breve{k}, \breve{m}) - g}{h(\breve{k}, \breve{m})} + \alpha(\breve{\tau}, \breve{k}, \breve{m}) e^{-g(\breve{\tau}+\breve{k}\eta-\breve{m})} = 0 \\
&= -\frac{h(k_c^m, m^*(k_c^m)) - g}{h(k_c^m, m^*(k_c^m))} + \alpha(\tau^*, k_c^m, m^*(k_c^m)) e^{-g(\tau^*+k_c^m\eta-m^*(k_c^m))}.
\end{aligned}$$

Based on the above analysis, we have $h(k_c^m, m^*(k_c^m)) = h(\breve{k}, \breve{m})$, $\alpha(\tau^*, k_c^m, m^*(k_c^m)) < \alpha(\tau^*, \breve{k}, \breve{m})$, and $\breve{k}\eta - \breve{m} = k_c^m\eta - m^*(k_c^m)$, thus yielding

$$\alpha(\breve{\tau}, \breve{k}, \breve{m}) e^{-g\breve{\tau}} = \alpha(\tau^*, k_c^m, m^*(k_c^m)) e^{-g\tau^*} < \alpha(\tau^*, \breve{k}, \breve{m}) e^{-g\tau^*}.$$

We can easily check that $\alpha(\tau, k, m)e^{-g\tau} = \frac{Ae^{-(g-g')\tau+g'k\eta} + \frac{e^{gk\eta}-e^{g'k\eta}}{(g-g')\eta} + \frac{e^{gk\eta}-e^{g(k\eta-m)}}{g\eta}}{1-k+\frac{m}{\eta}}$ is decreasing in τ, and thus it must be that $\breve{\tau} > \tau^*$. From Proposition 6, the regulator chooses m to maximize $\tau^*(k, m)$. Thus, with the bankruptcy threshold $\breve{k}$, it follows that

$$\tau^*(\breve{k}, m^*(\breve{k})) = \max_m \tau^*(\breve{k}, m) \geq \tau^*(\breve{k}, \breve{m}) = \breve{\tau} > \tau^*. \tag{A.36}$$

Moreover, the bankruptcy threshold k_c^m maximizes the firm's life span, and, therefore, $\max_k \tau^*(k, m^*(k)) + k\eta \geq \tau^*(\breve{k}, m^*(\breve{k})) + \breve{k}\eta$. As a result, we have

$$\max_k \tau^*(k, m^*(k)) + k\eta \geq \tau^*(\breve{k}, m^*(\breve{k})) + \breve{k}\eta > \tau^* + \breve{k}\eta > \tau^* + k_c^m\eta,$$

in which the second inequality holds from (A.36) and the third inequality comes from the definition of $\breve{k}$. This contradicts the fact that k_c^m is the optimal bankruptcy threshold. Therefore, for the optimal threshold k_c^m, it must be that $Y_{t_0+\tau^*(k_c^m, m^*(k_c^m))+k_c^m\eta} = 0$.

Then, we can solve for the optimal bankruptcy threshold k_c^m. As $Y_{t_0+\tau^*(k_c^m,m^*(k_c^m))+k_c^m\eta} = 0$, we have $e^{gt_0}\left[Ae^{g'(\tau+k\eta)} - \frac{e^{g(\tau+k\eta)}-e^{g\tau+g'k\eta}}{(g-g')\eta}\right]\Big|_{k=k_c^m,\tau=\tau^*(k_c^m,m^*(k_c^m))} = 0$ (see (11)), which implies

$$Ae^{-(g-g')(\tau^*(k_c^m,m^*(k_c^m))+k_c^m\eta)} - \frac{1-e^{-(g-g')k_c^m\eta}}{(g-g')\eta} = 0. \tag{A.37}$$

Plugging $m^*(k_c^m) = \frac{1}{g-\lambda} - (1-k_c^m)\eta$ and

$$\tau^*(k_c^m, m^*(k_c^m)) = \frac{1}{g-g'}\left\{\log A - \log\left[\frac{e^{(g-g')k\eta}-1}{(g-g')\eta} - \frac{e^{(g-g')k\eta}(1-e^{-gm})}{g\eta}\right.\right.$$
$$\left.\left. +(1-k+\frac{m}{\eta})\left(\frac{g}{\lambda}e^{(g-\lambda)(k\eta-m)-g'k\eta} - \frac{g-\lambda}{\lambda}e^{(g-g')k\eta-gm}\right)\right]\right\}\Big|_{k=k_c^m,m=m^*(k_c^m)}$$

(see the proof of Proposition 7) into (A.37), it follows that

$$\left(1-\frac{1}{(g-\lambda)\eta}\right)\left(\frac{g}{\lambda}e^{-\lambda(\eta-\frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda}\right) - \frac{e^{g\left[k_c^m\eta-(\eta-\frac{1}{g-\lambda})\right]}-1}{g\eta} = 0$$

since $e^{-g\left[k_c^m\eta-(\eta-\frac{1}{g-\lambda})\right]} > 0$. Therefore, k_c^m can be explicitly solved as

$$k_c^m = \frac{1}{g\eta}\log\left[\frac{g^2}{(g-\lambda)\lambda}e^{(g-\lambda)(\eta-\frac{1}{g-\lambda})} - \frac{g-\lambda}{\lambda}e^{g(\eta-\frac{1}{g-\lambda})}\right],$$

and thus $k_c^m = k_{nc}^m$, which is given in (34). We can easily check that the equilibrium outcome is the same as that in the non-commitment case; that is, $m^*(k_c^m) = m^*$ and $\tau^*(k_c^m, m^*(k_c^m)) = \tau^*(m^*)$.
■

Appendix B Seniority

In this section, we give a complete characterization of equilibrium for the case with heterogeneous creditors in Section 5.1. The result of $\tau_J \leq \tau_S$ in Proposition 10 is superseded by Lemma B.1, and the properties of the unique equilibrium in Proposition 10 are superseded by Proposition B.1. In addition, Proposition B.1 gives a detailed characterization of the unique equilibrium under different parameter conditions.

Lemma B.1 *Junior creditors run faster than their senior counterparts (i.e., $\tau_J \leq \tau_S$).*

Proof of Lemma B.1 Denote the firm's life span under strategy profile (τ_J, τ_S) as $\Delta(\tau_J, \tau_S) \equiv \hat{t}(\tau_J, \tau_S) - t_0$. According to (36), we have

$$\Delta(\tau_J, \tau_S) = \inf\left\{u \middle| \int_0^u \left[\frac{1-\omega}{\eta} \cdot \mathbf{1}\{\tau_J \le \delta \le \tau_J + \eta\} + \frac{\omega}{\eta} \cdot \mathbf{1}\{\tau_S \le \delta \le \tau_S + \eta\}\right] \mathrm{d}\delta \ge k\right\}. \tag{B.1}$$

Suppose there exists some equilibrium where $\tau_J > \tau_S$. In other words, a junior creditor j with type $t_j = \tilde{t}$ withdraws at $b_{J,j} = \tilde{t} + \tau_J$, and a senior creditor i with the same type $t_i = \tilde{t}$ withdraws at $b_{S,i} = \tilde{t} + \tau_S$, in which $b_{J,j} > b_{S,i}$. As such, (τ_J, τ_S) can hold as an equilibrium, and the net marginal payoff of waiting for junior creditor j at $b_{J,j} = \tilde{t} + \tau_J$ is supposed to be 0; that is,

$$\begin{aligned} &e^{gb_{J,j}}\left[g[1 - \Psi(b_{J,j} - \Delta(\tau_J, \tau_S)|\tilde{t})] - \psi(b_{J,j} - \Delta(\tau_J, \tau_S)|\tilde{t})\right] \\ &+ \alpha_J(\tau_J, \tau_S)e^{g(b_{J,j} - \Delta(\tau_J, \tau_S))}\psi(b_{J,j} - \Delta(\tau_J, \tau_S)|\tilde{t}) = 0, \end{aligned}$$

which implies

$$\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\Delta(\tau_J,\tau_S)-\tau_J)} + \alpha_J(\tau_J, \tau_S)e^{-g\Delta(\tau_J,\tau_S)} = 0. \tag{B.2}$$

Plugging (B.2) into the first-order derivative with respect to $\tau_{S,i}$ for senior creditor i, we can obtain the net marginal value of waiting for the senior creditor i at $\tau_{S,i} = \tau_S$:

$$\begin{aligned} &e^{gb_{S,i}}\left[g[1 - \Psi(b_{S,i} - \Delta(\tau_J, \tau_S)|\tilde{t})] - \psi(b_{S,i} - \Delta(\tau_J, \tau_S)|\tilde{t})\right] \\ &+ \alpha_S(\tau_J, \tau_S)e^{g(b_{S,i} - \Delta(\tau_J, \tau_S))}\psi(b_{S,i} - \Delta(\tau_J, \tau_S)|\tilde{t}) \\ \propto &\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\Delta(\tau_J,\tau_S)-\tau_S)} + \alpha_S(\tau_J, \tau_S)e^{-g\Delta(\tau_J,\tau_S)} \\ > &\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\Delta(\tau_J,\tau_S)-\tau_J)} + \alpha_J(\tau_J, \tau_S)e^{-g\Delta(\tau_J,\tau_S)} \\ &+ [\alpha_S(\tau_J, \tau_S) - \alpha_J(\tau_J, \tau_S)]e^{-g\Delta(\tau_J,\tau_S)} = [\alpha_S(\tau_J, \tau_S) - \alpha_J(\tau_J, \tau_S)]e^{-g\Delta(\tau_J,\tau_S)} \ge 0, \end{aligned}$$

in which the first inequality holds because $\tau_J > \tau_S$, and the second inequality holds because $\alpha_S \ge \alpha_J$ according to the rule of seniority. Thus, senior creditor i with $\tau_{S,i} = \tau_S$ has an incentive to wait longer, so such equilibrium cannot hold. Therefore, in any possible equilibrium, we must have that $\tau_J \le \tau_S$. ■

Lemma B.2 *No senior creditors exit before $\hat{t}$ if and only if all staying senior creditors receive full repayment; that is, $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$ if and only if $t_0 + \tau_S \ge \hat{t}(\tau_J, \tau_S)$ in equilibrium.*

Proof of Lemma B.2 First, we will prove that $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$ implies $t_0 + \tau_S \ge \hat{t}(\tau_J, \tau_S)$. Suppose there exists some equilibrium in which $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$ and $t_0 + \tau_S < \hat{t}(\tau_J, \tau_S)$. As senior creditor i will receive full repayment when the firm closes, given other creditors choose strategy (τ_J, τ_S), the expected payoff for creditor i at $\tau_{S,i} < \hat{t}(\tau_J, \tau_S) - t_0$ is

$$\begin{aligned}
&\Pi_i(\tau_{S,i}|t_i, \tau_J, \tau_S, k) \\
&= \int_{t_i+\tau_{S,i}<\hat{t}(\tau_J,\tau_S)} e^{g(t_i+\tau_{S,i})}\psi(t_0|t_i)\mathrm{d}t_0 + \int_{t_i+\tau_{S,i}\geq\hat{t}(\tau_J,\tau_S)} e^{g\hat{t}(\tau_J,\tau_S)}\psi(t_0|t_i)\mathrm{d}t_0 \\
&< \int_{t_i+\tau_{S,i}<\hat{t}(\tau_J,\tau_S)} e^{g\hat{t}(\tau_J,\tau_S)}\psi(t_0|t_i)\mathrm{d}t_0 + \int_{t_i+\tau_{S,i}\geq\hat{t}(\tau_J,\tau_S)} e^{g\hat{t}(\tau_J,\tau_S)}\psi(t_0|t_i)\mathrm{d}t_0 \\
&= \int_{t_0} e^{g\hat{t}(\tau_J,\tau_S)}\psi(t_0|t_i)\mathrm{d}t_0 = \Pi_i(\tau_i|t_i, \tau^*, k)\Big|_{t_i+\tau_{S,i}\geq\hat{t}(\tau_J,\tau_S)},
\end{aligned}$$

in which the inequality holds because $\tau_{S,i} < \hat{t}(\tau_J, \tau_S) - t_0$. Thus, in any possible equilibrium with $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$, senior creditor i with $\tau_{S,i} < \hat{t}(\tau_J, \tau_S) - t_0$ always has an incentive to deviate to $\tau_{S,i} \geq \hat{t}(\tau_J, \tau_S) - t_0$ to obtain a full repayment, which contradicts the assumption of $\tau_S < \hat{t}(\tau_J, \tau_S) - t_0$. Therefore, in any possible equilibrium with $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$, we have $t_0 + \tau_S \geq \hat{t}(\tau_J, \tau_S)$.

Next, we can prove that $t_0 + \tau_S \geq \hat{t}(\tau_J, \tau_S)$ implies $\alpha_S(\tau_J, \tau_S)e^{gt_0} = e^{g\hat{t}(\tau_J,\tau_S)}$. Suppose there exists some equilibrium where $\tau_S \geq \Delta(\tau_J, \tau_S)$ and $\alpha_S(\tau_J, \tau_S) < e^{g\Delta(\tau_J,\tau_S)}$, where $\Delta(\tau_J, \tau_S)$ is given in (B.1). Given the other creditors choose strategy (τ_J, τ_S), the net marginal value of waiting for senior creditor i at $\tau_{S,i}$ is given by

$$\begin{aligned}
\frac{\partial\Pi_i(\tau_{S,i}|t_i, \tau_J, \tau_S, k)}{\partial\tau_{S,i}} &= ge^{g(t_i+\tau_{S,i})}\left[1 - \Psi(t_i + \tau_{S,i} - \Delta(\tau_J, \tau_S)|t_i)\right] \\
&\quad - e^{g(t_i+\tau_{S,i})}(1 - \alpha_S(\tau_J, \tau_S)e^{-\Delta(\tau_J,\tau_S))})\psi(t_i + \tau_{S,i} - \Delta(\tau_J, \tau_S)|t_i).
\end{aligned}$$

When $\tau_{S,i} > \Delta(\tau_J, \tau_S)$, we have $1-\Psi(t_i+\tau_{S,i}-\Delta(\tau_J, \tau_S)|t_i) = 0$ and $\psi(t_i+\tau_{S,i}-\Delta(\tau_J, \tau_S)|t_i) = 0$, so $\frac{\partial\Pi_i(\tau_{S,i}|t_i,\tau_J,\tau_S,k)}{\partial\tau_{S,i}}|_{\tau_{S,i}>\Delta(\tau_J,\tau_S)} = 0$. When $\tau_{S,i} = \Delta(\tau_J, \tau_S)$, we have $1 - \Psi(t_i|t_i) = 0$ and $\psi(t_i|t_i) = \frac{\lambda}{e^{\lambda\eta}-1}$ (see (1)). Hence,

$$\begin{aligned}
&\frac{\partial\Pi_i(\tau_{S,i}|t_i, \tau_J, \tau_S, k)}{\partial\tau_{S,i}}|_{\tau_{S,i}=\Delta(\tau_J,\tau_S)} \\
&= -e^{g(t_i+\Delta(\tau_J,\tau_S))}(1 - \alpha_S(\tau_J, \tau_S)e^{-\Delta(\tau_J,\tau_S))})\frac{\lambda}{e^{\lambda\eta} - 1} < 0,
\end{aligned}$$

in which the last inequality holds because $\alpha_S(\tau_J, \tau_S) < e^{g\Delta(\tau_J,\tau_S)}$. As a result, creditor i with $\tau_{S,i} \geq \Delta(\tau_J, \tau_S)$ can obtain a strictly positive increase in the expected payoff by switching to $\tau_{S,i} = \Delta(\tau_J, \tau_S) - \varepsilon$ (for some small $\varepsilon > 0$). Therefore, $\tau_S \geq \Delta(\tau_J, \tau_S)$ cannot be supported in an equilibrium. In any possible equilibrium with $\tau_S \geq \Delta(\tau_J, \tau_S)$, we have $\alpha_S(\tau_J, \tau_S) = e^{g\Delta(\tau_J,\tau_S)}$. This completes the proof. ■

Next, based on Lemma B.1 and Lemma B.2, we give a complete characterization of the equilibrium.

Proposition B.1 *Denote* $\tilde{\omega} \equiv \min\left\{\omega_0, 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}\right\}$, *where* $\omega_0 \in (0, 1-k)$ *that uniquely solves*

$$A - \frac{(1-\omega_0)\left(e^{(g-g')\frac{k\eta}{1-\omega_0}} - 1\right)}{(g-g')\eta} - \omega_0 e^{(g-g')\frac{k\eta}{1-\omega_0}} = 0. \tag{B.3}$$

1. *When* $\omega \le \tilde{\omega}$, *the senior creditor will get full repayment in the unique equilibrium. In detail, the equilibrium waiting time for junior and senior creditors is*

$$\tau_J^* = \max\left\{0, \frac{1}{g-g'}\left[\log A - \log v(k,\omega)\right]\right\}, \ \tau_S^* \ge \tau_J^* + \frac{k\eta}{1-\omega},$$

where

$$v(k,\omega) = \frac{(1-\omega)\left(e^{(g-g')\frac{k\eta}{1-\omega}} - 1\right)}{(g-g')\eta} + \omega e^{(g-g')\frac{k\eta}{1-\omega}} + (1-\omega-k)\left(\frac{g}{\lambda}e^{(g-g'-\lambda)\frac{k\eta}{1-\omega}} - \frac{g-\lambda}{\lambda}e^{(g-g')\frac{k\eta}{1-\omega}}\right).$$

2. *When* $\omega > \tilde{\omega}$, *the senior creditors cannot get full repayment in the unique equilibrium. In more detail, the equilibrium is as follows, in which*

$$\chi(\tau, d, \omega) = \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(k\eta-(1-\omega)d)} + \frac{Ae^{-(g-g')(\tau+\omega d+k\eta)} - \frac{1-(1-\omega)e^{-(g-g')[\omega d+k\eta]} - \omega e^{(g-g')[(1-\omega)d-k\eta]}}{(g-g')\eta}}{\omega(1-k)+(1-\omega)d\frac{\omega}{\eta}}.$$

   (a) *If* $\chi(0,0,\omega) \le 0$, *in the unique equilibrium, we have* $\tau_J^* = 0$ *and* $\tau_S^* = 0$;

   (b) *If* $\chi(0,0,\omega) > 0$ *and* $\chi\left(0, \frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k), \omega\right) \le 0$, *in the unique equilibrium, we have* $\tau_J^* = 0$ *and* $\tau_S^* > 0$ *that uniquely solve* $\chi(0, \tau_S^*, \omega) = 0$;

   (c) *If* $\chi(0,0,\omega) > 0$ *and* $\chi\left(0, \frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k), \omega\right) > 0$, *in the unique equilibrium, we have* $\tau_J^* > 0$ *and* $\tau_S^* > 0$ *given by*

$$\tau_J^* = \frac{1}{g-g'}\left\{\log A - \log\left[\frac{e^{(g-g')\frac{1}{\lambda}\log\frac{g}{g-\lambda}} - (1-\omega) - \omega e^{(g-g')\frac{\eta}{\omega}\left[\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}-k\right]}}{(g-g')\eta} + \left[\omega(1-k) + (1-\omega)\left(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k\right)\right] \cdot \left(-\frac{g-\lambda}{\lambda}e^{(g-g')\frac{1}{\lambda}\log\frac{g}{g-\lambda}} + \frac{g}{\lambda}e^{(g-g'-\lambda)\frac{1}{\lambda}\log\frac{g}{g-\lambda} + \frac{1}{\omega}\log\frac{g}{g-\lambda} - \frac{\lambda k\eta}{\omega}}\right)\right]\right\}$$

$$\tau_S^* = \tau_J^* + \frac{\eta}{\omega}\left(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k\right).$$

Proof of Proposition B.1 We first introduce a lemma of asset dynamics.

Lemma B.3 *For any $\tau_J \geq 0$ and $\omega \in (0, 1-k]$, consider two strategy profiles (τ_J, τ_S^1) and (τ_J, τ_S^2), in which $\tau_S^1 \geq \tau_J + \frac{k\eta}{1-\omega}$ and $\tau_J \leq \tau_S^2 < \tau_J + \frac{k\eta}{1-\omega}$. Given strategy profile (τ_J, τ_S^1) ((τ_J, τ_S^2)), denote Y_t^1 (Y_t^2) as the asset value at time t, and $\hat{t}_1 \equiv t_0 + \tau_J + \frac{k\eta}{1-\omega}$ ($\hat{t}_2 \equiv t_0 + (1-\omega)\tau_J + \omega\tau_S^2 + k\eta$) as the firm's closing time. Then if $Y_{\hat{t}_2}^2 < \left[\omega - (\hat{t}_2 - t_0 - \tau_S)\frac{\omega}{\eta}\right] e^{g\hat{t}_2}$, we must have $Y_{\hat{t}_1}^1 < \omega e^{g\hat{t}_1}$.*

Proof of Lemma B.3. When $t \in (t_0 + \tau_S^2, \hat{t}_2)$, we have $\mathrm{d}Y_t^1 = \left(g'Y_t^1 - \frac{1-\omega}{\eta}e^{gt}\right)\mathrm{d}t$ and $\mathrm{d}Y_t^2 = \left(g'Y_t^2 - \frac{1}{\eta}e^{gt}\right)\mathrm{d}t$. Then we can calculate the asset values at $\hat{t}_2$ under (τ_J, τ_S^1) and (τ_J, τ_S^2) as

$$
\begin{aligned}
Y_{\hat{t}_2}^1 &= e^{g'(\hat{t}_2 - t_0 - \tau_S^2)} Y_{t_0+\tau_S^2}^1 - \frac{(1-\omega)e^{g\hat{t}_2} - (1-\omega)e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta} \\
Y_{\hat{t}_2}^2 &= e^{g'(\hat{t}_2 - t_0 - \tau_S^2)} Y_{t_0+\tau_S^2}^2 - \frac{e^{g\hat{t}_2} - e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta}.
\end{aligned}
$$

Since $Y_{t_0+\tau_S^2}^1 = Y_{t_0+\tau_S^2}^2$, we have

$$
\begin{aligned}
Y_{\hat{t}_2}^1 - Y_{\hat{t}_2}^2 &= e^{g'(\hat{t}_2 - t_0 - \tau_S^2)} \left[Y_{t_0+\tau_S^2}^1 - Y_{t_0+\tau_S^2}^2\right] + \frac{\omega e^{g\hat{t}_2} - \omega e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta} \\
&= \frac{\omega e^{g\hat{t}_2} - \omega e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta},
\end{aligned}
$$

and as $Y_{\hat{t}_2}^2 < \left[\omega - (\hat{t}_2 - t_0 - \tau_S)\frac{\omega}{\eta}\right] e^{g\hat{t}_2}$, we have

$$
\begin{aligned}
Y_{\hat{t}_2}^1 - \omega e^{g\hat{t}_2} &= Y_{\hat{t}_2}^2 + \frac{\omega e^{g\hat{t}_2} - \omega e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta} - \omega e^{g\hat{t}_2} \\
&< -(\hat{t}_2 - t_0 - \tau_S^2)\frac{\omega}{\eta} e^{g\hat{t}_2} + \frac{\omega e^{g\hat{t}_2} - \omega e^{g(t_0+\tau_S^2) + g'(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')\eta} \\
&= (\hat{t}_2 - t_0 - \tau_S^2)\frac{\omega}{\eta} e^{g\hat{t}_2} \left[\frac{1 - e^{-(g-g')(\hat{t}_2 - t_0 - \tau_S^2)}}{(g-g')(\hat{t}_2 - t_0 - \tau_S^2)} - 1\right] < 0.
\end{aligned}
$$

which implies $Y_{\hat{t}_2}^1 < \omega e^{g\hat{t}_2}$. The last inequality holds because it is easy to check that $\frac{1-e^{-x}}{x} < 1$ with $x \in (0, \infty)$. After the shock hits, the asset grows at rate $g' < g$ from $\hat{t}_2$ to $\hat{t}_1$ under (τ_J, τ_S^1), and, thus, $Y_{\hat{t}_1}^1 < Y_{\hat{t}_2}^1 e^{g(\hat{t}_1 - \hat{t}_2)}$. Hence, $Y_{\hat{t}_1}^1 < Y_{\hat{t}_2}^1 e^{g(\hat{t}_1 - \hat{t}_2)} < \omega e^{g\hat{t}_2} \cdot e^{g(\hat{t}_1 - \hat{t}_2)} = \omega e^{g\hat{t}_1}$. ■

Next, we solve for the equilibrium when (1) $\omega \leq \tilde{\omega}$ and (2) $\omega > \tilde{\omega}$, respectively.

Case 1. $\omega \leq \tilde{\omega}$. We first prove that when $\omega \leq \tilde{\omega}$, in any possible equilibrium, it must be that $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$. In other words, senior creditors always wait until the firm closes. Then we solve for the equilibrium in this case. To prove $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$, we first show that in any possible equilibrium with $\tau_J = 0$, senior creditors must choose $\tau_S \geq \frac{k\eta}{1-\omega}$. Then we show that in any possible equilibrium with $\tau_J > 0$, senior creditors also choose $\tau_S \geq \tau_J + \frac{k\eta}{1-\omega}$.

Suppose there exists some equilibrium $(\tau_J = 0, \tau_S < \frac{k\eta}{1-\omega})$. Then we have $\hat{t} = t_0 + \omega\tau_S + k\eta$. According to Lemma B.2, the staying senior creditors cannot receive full repayment; that is, $Y_{\hat{t}} < \left[\omega - (\hat{t} - t_0 - \tau_S)\frac{\omega}{\eta}\right] e^{g\hat{t}}$. According to Lemma B.3, if $(\tau_J = 0, \tau_S < \frac{k\eta}{1-\omega})$ holds as an equilibrium, regarding the alternative strategy profile $(\tau_J = 0, \tau_S' \geq \frac{k\eta}{1-\omega})$, closing time $\hat{t}' = t_0 + \frac{k\eta}{1-\omega}$, and asset dynamics Y_t', we must have $Y_{\hat{t}'}' < \omega e^{g\hat{t}'}$. However, from the definition of ω_0 in (B.3), we can also show that when $\omega \leq \omega_0$, we must have $Y_{\hat{t}'}' \geq \omega e^{g\hat{t}'}$. Denote

$$\rho(\omega) \equiv A - \frac{(1-\omega)\left(e^{(g-g')\frac{k\eta}{1-\omega}} - 1\right)}{(g-g')\eta} - \omega e^{(g-g')\frac{k\eta}{1-\omega}}. \tag{B.4}$$

Taking the first-order derivative of $\rho(\omega)$ with respect to ω, we have

$$\begin{aligned}\rho'(\omega) &= \frac{e^{(g-g')\frac{k\eta}{1-\omega}}}{(g-g')\eta}\left[1 - e^{-(g-g')\frac{k\eta}{1-\omega}} - (g-g')\frac{k\eta}{1-\omega} - (g-g')\eta - \frac{(g-g')^2\eta^2 k\omega}{(1-\omega)^2}\right] \\ &< \frac{e^{(g-g')\frac{k\eta}{1-\omega}}}{(g-g')\eta}\left[-(g-g')\eta - \frac{(g-g')^2\eta^2 k\omega}{(1-\omega)^2}\right] < 0. \end{aligned} \tag{B.5}$$

The first inequality holds because it is easy to check $1 - e^{-x} - x < 0$ for $x \in (0, \infty)$. From (B.5), we know that $\rho(\omega)$ is decreasing in ω, so $\rho(\omega) > 0$ ($\rho(\omega) < 0$) when $\omega < \omega_0$ ($\omega > \omega_0$). As a result, when $\omega \leq \tilde{\omega} \leq \omega_0$,

$$\begin{aligned} Y_{\hat{t}'}' - \omega e^{g\hat{t}'} &= e^{gt_0 + g'\frac{k\eta}{1-\omega}}\left[A - \frac{(1-\omega)\left(e^{(g-g')\frac{k\eta}{1-\omega}} - 1\right)}{(g-g')\eta} - \omega e^{(g-g')\frac{k\eta}{1-\omega}}\right] \\ &= e^{gt_0 + g'\frac{k\eta}{1-\omega}}\rho(\omega) \geq 0, \end{aligned}$$

which implies $Y_{\hat{t}'}' \geq \omega e^{g\hat{t}'}$. This creates a contradiction. Therefore, in any possible equilibrium with $\tau_J = 0$, senior creditors must choose $\tau_S \geq \frac{k\eta}{1-\omega}$.

Then, we can prove that in any possible equilibrium with $\tau_J > 0$, senior creditors must choose $\tau_S \geq \tau_J + \frac{k\eta}{1-\omega}$. Suppose there exists some equilibrium $(\tau_J > 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$. Then we have $\hat{t} = t_0 + (1-\omega)\tau_J + \omega\tau_S + k\eta$. According to Lemma B.2, the staying senior creditors cannot receive full repayment (i.e., $Y_{\hat{t}} < \left[\omega - (\hat{t} - t_0 - \tau_S)\frac{\omega}{\eta}\right] e^{g\hat{t}}$), and thus $\alpha_J(\tau_J, \tau_S) = 0$. As $\tau_S - \tau_J < \frac{k\eta}{1-\omega}$ and $\omega \leq \tilde{\omega} \leq 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$, the first-order derivative for junior creditor j at $\tau_{J,j} = \tau_J$ is proportional to

$$\begin{aligned} &\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\omega(\tau_S - \tau_J) + k\eta)} + \alpha_J(\tau_J, \tau_S)e^{-g((1-\omega)\tau_J + \omega\tau_S + k\eta)} \\ &= \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\omega(\tau_S - \tau_J) + k\eta)} < \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda\frac{k\eta}{1-\omega}} \leq 0, \end{aligned}$$

so $(\tau_J > 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$ cannot hold as an equilibrium. This is a contradiction. Therefore, in any possible equilibrium with $\tau_J > 0$, senior creditors must choose $\tau_S \geq \tau_J + \frac{k\eta}{1-\omega}$.

Therefore, when $\omega < \tilde{\omega}$, in any possible equilibrium, we must have $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$. Now we solve for the equilibrium. Given $\tau_J \geq 0$ and $\tau_S \geq \tau_J + \frac{k\eta}{1-\omega}$, we have $\hat{t} = t_0 + \tau_J + \frac{k\eta}{1-\omega}$, and the asset dynamics are as follows:

$$Y_t = \begin{cases} Ae^{gt} & 0 \leq t \leq t_0 \\ Ae^{gt_0 + g'(t-t_0)} & t_0 < t \leq t_0 + \tau_J \\ Ae^{gt_0 + g'(t-t_0)} - \frac{(1-\omega)\left(e^{gt} - e^{g(t_0+\tau_J)+g'(t-t_0-\tau_J)}\right)}{(g-g')\eta} & \\ \qquad t_0 + \tau_J < t \leq t_0 + \tau_J + \frac{k\eta}{1-\omega} \end{cases}.$$

When the firm closes, each senior creditor receives $e^{g\hat{t}}$ and each junior creditor receives

$$\alpha_J(\tau_J, \tau_S) e^{gt_0} = \frac{Ae^{g'(\tau_J + \frac{k\eta}{1-\omega})} - \frac{(1-\omega)\left(e^{g(\tau_J + \frac{k\eta}{1-\omega})} - e^{g\tau_J + g'\frac{k\eta}{1-\omega}}\right)}{(g-g')\eta} - \omega e^{g(\tau_J + \frac{k\eta}{1-\omega})}}{1 - \omega - k} e^{gt_0}.$$

Solving the first-order condition for junior creditors (i.e., $\frac{g-\lambda}{\lambda} - \frac{g}{\lambda} e^{-\lambda \frac{k\eta}{1-\omega}} + \alpha_J(\tau_J^*, \tau_S^*) e^{-g(\tau_J^* + \frac{k\eta}{1-\omega})} = 0$), junior creditors' equilibrium waiting time is

$$\tau_J^* = \max\left\{0, \frac{1}{g-g'}\left[\log A - \log v(k,\omega)\right]\right\},$$

where

$$\begin{aligned} v(k,\omega) = & \frac{(1-\omega)\left(e^{(g-g')\frac{k\eta}{1-\omega}} - 1\right)}{(g-g')\eta} + \omega e^{(g-g')\frac{k\eta}{1-\omega}} \\ & + (1-\omega-k)\left(\frac{g}{\lambda} e^{(g-g'-\lambda)\frac{k\eta}{1-\omega}} - \frac{g-\lambda}{\lambda} e^{(g-g')\frac{k\eta}{1-\omega}}\right). \end{aligned}$$

From $\omega \leq 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$, we have $\frac{k}{1-\omega} \leq \frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}$, and thus $v(k,\omega) > 0$.

Case 2. $\omega > \tilde{\omega}$. We first prove that when $\omega > \tilde{\omega}$, in any possible equilibrium, we must have $(\tau_J \geq 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$. Then we solve for the equilibrium. To prove $(\tau_J \geq 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$, we consider the equilibrium under $\omega > \omega_0$ and $\omega > 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$, respectively.

We first consider $\omega > \omega_0$. When $\omega > 1-k$, even if all junior creditors withdraw, the bankruptcy cannot be triggered, so in any possible equilibrium with $\tau_J \geq 0$, we must have $\tau_S < \tau_J + \frac{k\eta}{1-\omega}$, and therefore, we only need to consider $\omega_0 < \omega \leq 1-k$. Suppose there exists some equilibrium $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$. Then we have $\hat{t} = t_0 + \tau_J + \frac{k\eta}{1-\omega}$ and with $\omega > \omega_0$,

$$\begin{aligned} Y_{\hat{t}} - \omega e^{g\hat{t}} = & \left[Ae^{-(g-g')\left(\tau_J + \frac{k\eta}{1-\omega}\right)} - \frac{(1-\omega)\left(1 - e^{-(g-g')\frac{k\eta}{1-\omega}}\right)}{(g-g')\eta} - \omega\right] e^{g(t_0 + \tau_J + \frac{k\eta}{1-\omega})} \\ \leq & \left[Ae^{-(g-g')\frac{k\eta}{1-\omega}} - \frac{(1-\omega)\left(1 - e^{-(g-g')\frac{k\eta}{1-\omega}}\right)}{(g-g')\eta} - \omega\right] e^{g(t_0 + \tau_J + \frac{k\eta}{1-\omega})} \end{aligned}$$

$$= e^{g(t_0+\tau_J)+g'\frac{k\eta}{1-\omega}}\rho(\omega) < e^{g(t_0+\tau_J)+g'\frac{k\eta}{1-\omega}}\rho(\omega_0) = 0,$$

in which $\rho(\omega)$ is given in (B.4). The first inequality holds because $\tau_J \geq 0$, and the second inequality holds because $\rho(\omega)$ is decreasing in ω (see (B.5)). Thus, $Y_{\hat{t}} < \omega e^{g\hat{t}}$ and the staying senior creditors cannot receive full repayment. According to Lemma B.2, we have $\tau_S < \tau_J + \frac{k\eta}{1-\omega}$. This is a contradiction. Therefore, when $\omega > \omega_0$, in any possible equilibrium, we must have $(\tau_J \geq 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$.

Then, we consider $\omega > 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$. Suppose there exists some equilibrium $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$. Then we have $\hat{t} = t_0 + \tau_J + \frac{k\eta}{1-\omega}$, and the junior creditors' first-order derivative at $\tau_{J,j} = \tau_J$ is proportional to

$$\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda\frac{k\eta}{1-\omega}} + \alpha_J(\tau_J,\tau_S)e^{-g(\tau_J+\frac{k\eta}{1-\omega})} > \alpha_J(\tau_J,\tau_S)e^{-g(\tau_J+\frac{k\eta}{1-\omega})} \geq 0.$$

The first inequality holds because $\omega > 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$, and the second inequality holds because $\alpha_J(\tau_J,\tau_S) \geq 0$. Thus, junior creditor j has an incentive to wait longer, and $(\tau_J \geq 0, \tau_S \geq \tau_J + \frac{k\eta}{1-\omega})$ cannot hold as an equilibrium. This is a contradiction. Therefore, when $\omega > 1 - \frac{k}{\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}}$, in any possible equilibrium, we must have $(\tau_J \geq 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$.

As a result, when $\omega > \tilde{\omega}$, in any possible equilibrium, we must have $(\tau_J \geq 0, \tau_S < \tau_J + \frac{k\eta}{1-\omega})$. Now we can solve for the equilibrium. For $\tau_S \geq \tau_J \geq 0$, the firm closes at $\hat{t} = t_0 + (1-\omega)\tau_J + \omega\tau_S + k\eta$.[41] The asset dynamics are as follows:

$$Y_t = \begin{cases} Ae^{gt} & 0 \leq t \leq t_0 \\ Ae^{gt_0+g'(t-t_0)} & t_0 < t \leq t_0+\tau_J \\ Ae^{gt_0+g'(t-t_0)} - \frac{(1-\omega)\left(e^{gt}-e^{g(t_0+\tau_J)+g'(t-t_0-\tau_J)}\right)}{(g-g')\eta} & t_0+\tau_J < t \leq t_0+\tau_S \\ Ae^{gt_0+g'(t-t_0)} \\ \quad - \frac{e^{gt}-(1-\omega)e^{g(t_0+\tau_J)+g'(t-t_0-\tau_J)}-\omega e^{g(t_0+\tau_S)+g'(t-t_0-\tau_S)}}{(g-g')\eta} \\ \qquad\qquad t_0+\tau_S < t \leq t_0+(1-\omega)\tau_J+\omega\tau_S+k\eta \end{cases}.$$

[41]We can prove that $t_0 + \tau_J + \eta < \hat{t}(\tau_J,\tau_S)$ cannot hold in any possible equilibrium. Suppose there exists an equilibrium where $\tau_J + \eta < \hat{t}(\tau_J,\tau_S) - t_0 = \Delta(\tau_J,\tau_S)$. For junior creditor j, the net marginal value of waiting at $\tau_{J,j} = \tau_J$ is proportional to

$$\begin{aligned}\frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\Delta(\tau_J,\tau_S)-\tau_J)} + \alpha_J(\tau_J,\tau_S)e^{-\Delta(\tau_J,\tau_S)} &\geq \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{\lambda(\tau_J-\Delta(\tau_J,\tau_S))} \\ &> \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda\eta} > 0.\end{aligned}$$

The last inequality is derived from $\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} < 1$ (see (A.3)). Thus, in any possible equilibrium, we must have $\tau_J + \eta \geq \Delta(\tau_J,\tau_S)$. In other words, there are always some junior creditors that have not withdrawn by the time the firm closes.

When the firm closes, each senior creditor receives

$$\alpha_S(\tau_J,\tau_S)e^{gt_0} = \frac{Y_{\hat{t}}}{\omega - (k\eta - (1-\omega)(\tau_S-\tau_J))\,\frac{\omega}{\eta}}$$

$$= \frac{e^{gt_0}}{\omega - (k\eta - (1-\omega)(\tau_S-\tau_J))\,\frac{\omega}{\eta}} \cdot \Big[Ae^{g'((1-\omega)\tau_J+\omega\tau_S+k\eta)}$$

$$-\frac{e^{g((1-\omega)\tau_J+\omega\tau_S+k\eta)} - (1-\omega)e^{g\tau_J+g'(\omega(\tau_S-\tau_J)+k\eta)} - \omega e^{g\tau_S+g'(k\eta-(1-\omega)(\tau_S-\tau_J))}}{(g-g')\,\eta}\Big]$$

and each junior creditor receives 0. Junior creditor j's first-order derivative at $\tau_{J,j}=\tau_J$ is

$$\left.\frac{\partial \Pi_{J,j}(\tau_{J,j}|\tau_J,\tau_S,t_j)}{\partial \tau_{J,j}}\right|_{\tau_{J,j}=\tau_J}$$

$$\propto \frac{g-\lambda}{\lambda} - \left.\frac{g}{\lambda}e^{-\lambda\left((1-\omega)\tau_J+\omega\tau_S+k\eta-\tau_{J,j}\right)}\right|_{\tau_{J,j}=\tau_J} + \alpha(\tau_J,\tau_S)e^{-g((1-\omega)\tau_J+\omega\tau_S+k\eta)}$$

$$= \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(\omega(\tau_S-\tau_J)+k\eta)}$$

Senior creditor i's first-order derivative at $\tau_{S,i}=\tau_S$ is

$$\left.\frac{\partial \Pi_{S,i}(\tau_{S,i}|\tau_J,\tau_S,t_i)}{\partial \tau_{S,i}}\right|_{\tau_{S,i}=\tau_S}$$

$$\propto \frac{g-\lambda}{\lambda} - \left.\frac{g}{\lambda}e^{-\lambda\left((1-\omega)\tau_J+\omega\tau_S+k\eta-\tau_{S,i}\right)}\right|_{\tau_{S,i}=\tau_S} + \alpha_S(\tau_J,\tau_S)e^{-g((1-\omega)\tau_J+\omega\tau_S+k\eta)}$$

$$= \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(k\eta-(1-\omega)(\tau_S-\tau_J))}$$

$$+\frac{1}{\omega(1-k)+(1-\omega)(\tau_S-\tau_J)\frac{\omega}{\eta}} \cdot \Big[Ae^{-(g-g')(\tau_J+\omega(\tau_S-\tau_J)+k\eta)}$$

$$-\frac{1-(1-\omega)e^{-(g-g')[\omega(\tau_S-\tau_J)+k\eta]} - \omega e^{(g-g')[(1-\omega)(\tau_S-\tau_J)-k\eta]}}{(g-g')\,\eta}\Big]$$

From the first-order conditions, we know that

1. $(\tau_J=0,\tau_S=0)$ holds as an equilibrium if and only if $\frac{\partial \Pi_{J,j}(\tau_{J,j}|\tau_J,\tau_S,t_j)}{\partial \tau_{J,j}}\Big|_{\tau_{J,j}=\tau_J=0} \le 0$ (i.e., $0 \le \frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k)$) and $\frac{\partial \Pi_{S,i}(\tau_{S,i}|\tau_J,\tau_S,t_i)}{\partial \tau_{S,i}}\Big|_{\tau_{S,i}=\tau_S=0} \le 0$;
2. $(\tau_J=0,\tau_S>0)$ holds as an equilibrium if and only if $\frac{\partial \Pi_{J,j}(\tau_{J,j}|\tau_J,\tau_S,t_j)}{\partial \tau_{J,j}}\Big|_{\tau_{J,j}=\tau_J=0} \le 0$ (i.e., $0 < \tau_S \le \frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k)$) and $\frac{\partial \Pi_{S,i}(\tau_{S,i}|\tau_J,\tau_S,t_i)}{\partial \tau_{S,i}}\Big|_{\tau_{S,i}=\tau_S} = 0$;
3. $(\tau_J>0,\tau_S>0)$ holds as an equilibrium if and only if $\frac{\partial \Pi_{J,j}(\tau_{J,j}|\tau_J,\tau_S,t_j)}{\partial \tau_{J,j}}\Big|_{\tau_{J,j}=\tau_J} = 0$ (i.e., $\tau_S-\tau_J = \frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k)$) and $\frac{\partial \Pi_{S,i}(\tau_{S,i}|\tau_J,\tau_S,t_i)}{\partial \tau_{S,i}}\Big|_{\tau_{S,i}=\tau_S} = 0$.

To solve the equilibrium, denote

$$\begin{aligned}\chi(\tau,d,\omega) &\equiv \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(k\eta-(1-\omega)d)} \\ &+ \frac{Ae^{-(g-g')(\tau+\omega d+k\eta)} - \frac{1-(1-\omega)e^{-(g-g')[\omega d+k\eta]}-\omega e^{(g-g')[(1-\omega)d-k\eta]}}{(g-g')\eta}}{\omega(1-k)+(1-\omega)d\frac{\omega}{\eta}} \\ &= \frac{g-\lambda}{\lambda} - \frac{g}{\lambda}e^{-\lambda(k\eta-(1-\omega)d)} + \tilde{\alpha}(\tau,d,\omega).\end{aligned}$$

We can see that with $\tilde{\alpha}(\tau,d,\omega) \geq 0$, $\chi(\tau,d,\omega)$ is decreasing in $\tau \in (0,\infty)$ and decreasing in $d \in [0,\frac{k\eta}{1-\omega})$ as $\frac{\partial\chi(\tau,d,\omega)}{\partial\tau} = -\frac{A(g-g')e^{-\lambda(k\eta-(1-\omega)d)}}{\omega(1-k)+(1-\omega)d\frac{\omega}{\eta}} < 0$ and

$$\begin{aligned}\frac{\partial\chi(\tau,d,\omega)}{\partial d} &= -g(1-\omega)e^{-\lambda(k\eta-(1-\omega)d)} - \frac{(1-\omega)\frac{\omega}{\eta}\tilde{\alpha}(\tau,d,\omega)}{(1-k)\omega+(1-\omega)d\frac{\omega}{\eta}} \\ &+ \frac{-A(g-g')\omega e^{-(g-g')(\tau+\omega d+k\eta)} + \frac{-\omega(1-\omega)[e^{-(g-g')(\omega d+k\eta)}-e^{(g-g')[(1-\omega)d-k\eta]}]}{\eta}}{(1-k)\omega+(1-\omega)d\frac{\omega}{\eta}} \\ &\leq \frac{-A(g-g')\omega e^{-(g-g')(\tau+\omega d+k\eta)} + \frac{-\omega(1-\omega)[e^{-(g-g')(\omega d+k\eta)}-e^{(g-g')[(1-\omega)d-k\eta]}]}{\eta}}{(1-k)\omega+(1-\omega)d\frac{\omega}{\eta}} \\ &\leq \frac{1}{(1-k)\omega+(1-\omega)(\tau_S-\tau_J)\frac{\omega}{\eta}} \cdot \left[-(g-g')\omega\frac{1-(1-\omega)e^{-(g-g')[\omega d+k\eta]}-\omega e^{(g-g')[(1-\omega)d-k\eta]}}{(g-g')\eta}\right. \\ &+ \left.\frac{-\omega(1-\omega)(g-g')e^{-(g-g')(\omega d+k\eta)}+\omega(1-\omega)(g-g')e^{(g-g')[(1-\omega)d-k\eta]}}{(g-g')\eta}\right] \\ &= -\frac{1-e^{-(g-g')[k\eta-(1-\omega)d]}}{\eta(1-k)+(1-\omega)d} \leq 0,\end{aligned}$$

in which the first and second inequalities come from $\tilde{\alpha}(\tau,d,\omega) \geq 0$, and the last inequality comes from $d < \frac{k\eta}{1-\omega}$.

Therefore, as it is easy to check $\lim_{\tau\to\infty}\chi(\tau,d,\omega) < 0$, when $\omega > \tilde{\omega}$, the equilibrium conditions are equivalent to:

1. If $\chi(0,0,\omega) \leq 0$, in the unique equilibrium, we have $\tau_J^* = 0$ and $\tau_S^* = 0$;
2. If $\chi(0,0,\omega) > 0$ and $\chi\left(0,\frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}-k),\omega\right) \leq 0$, in the unique equilibrium, we have $\tau_J^* = 0$ and $\tau_S^* > 0$ that uniquely solve $\chi(0,\tau_S^*,\omega) = 0$;
3. If $\chi(0,0,\omega) > 0$ and $\chi\left(0,\frac{\eta}{\omega}(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}-k),\omega\right) > 0$, in the unique equilibrium, we have

$$\begin{aligned}\tau_J^* = \frac{1}{g-g'}\Bigg\{&\log A - \log\left[\frac{e^{(g-g')\frac{1}{\lambda}\log\frac{g}{g-\lambda}} - (1-\omega) - \omega e^{(g-g')\frac{\eta}{\omega}\left[\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}-k\right]}}{(g-g')\eta}\right. \\ &+ \left.\left[\omega(1-k)+(1-\omega)\left(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda}-k\right)\right]\right]\end{aligned}$$

$$\cdot \left(-\frac{g-\lambda}{\lambda} e^{(g-g')\frac{1}{\lambda}\log\frac{g}{g-\lambda}} + \frac{g}{\lambda} e^{(g-g'-\lambda)\frac{1}{\lambda}\log\frac{g}{g-\lambda} + \frac{1}{\omega}\log\frac{g}{g-\lambda} - \frac{\lambda k \eta}{\omega}} \right) \Bigg] \Bigg] \Bigg\}, \quad \text{and}$$

$$\tau_S^* = \tau_J^* + \frac{\eta}{\omega}\left(\frac{1}{\lambda\eta}\log\frac{g}{g-\lambda} - k\right).$$

Finally, we prove that there exists $\omega_0 < 1-k$ that uniquely solves $\rho(\omega) = 0$ (see (B.4)). From (B.5), we know that $\rho(\omega)$ is decreasing in ω. Thus, we have

$$\lim_{\omega \to 0} \rho(\omega) = A - \frac{e^{(g-g')k\eta} - 1}{(g-g')\eta} > 0, \quad \text{and}$$

$$\begin{aligned}
\lim_{\omega \to 1-k} \rho(\omega) &= A - \frac{k(e^{(g-g')\eta} - 1)}{(g-g')\eta} - (1-k)e^{(g-g')\eta} \\
&< (1-k)e^{(g-g')\eta}\left(\frac{1-e^{-(g-g')\eta}}{(g-g')\eta} - 1\right) < 0.
\end{aligned}$$

The first inequality holds because $k < k_{nc} = \frac{1}{(g-g')\eta}\log[A(g-g')\eta + 1]$ (see (22)). The second inequality holds because $\frac{1}{(g-g')\eta}\log[A(g-g')\eta + 1] < 1$ (see (A.3)). Therefore, there exists a unique $\omega_0 \in (0, 1-k)$ that solves $\rho(\omega_0) = 0$. This completes the proof. ■