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A Theory of Corporate Communication

by

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Abstract

How should we expect firms to communicate with their shareholders in the presence of uncertainty? This paper studies a model of corporate communication in which cash flow variance is priced and stochastic. The model rationalizes "biases" for reports that are internally consistent and confirm market priors. Managers prioritize *consistency* over *confirmation* when cash flows have higher variance or when the signal space is larger. Ex-ante, managers prefer larger signal spaces to smaller ones. Complementary notions of *dissociation* and *divergence* arise in settings with priced, stochastic covariance (e.g., one-factor SDF, multiple segments, multiple firms). The model makes several cross-sectional predictions.

Keywords: corporate communication, signal jamming, uncertainty
JEL: C11, D83, G14, M41

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1 Introduction

Modern corporate communication encompasses a wide range of media, including regulatory filings, earnings calls, press releases, social media posts, pitchbooks, and prospectuses. In addition to informing investors, regulators, and the media, corporate communication is now extensively used in academic research to measure everything from product market boundaries (Hoberg and Phillips, 2016) to country risk (Hassan et al., 2024) to firm objectives (Rajan et al., 2025).[1] Of course, firms craft messages not only to inform, but also to influence—especially when the exact distribution of the firm's cash flows is uncertain. How then should we expect firms to communicate with their shareholders in the presence of uncertainty?

This paper develops a signal-jamming model of corporate communication under the assumption that cash flow variance (or covariance with other cash flows) is (a) priced and (b) stochastic. The analysis rationalizes four distortions in corporate communication. If cash flow variance is priced, we rationalize *consistency* and *confirmation*. By *consistency*, we mean the reduction of the variance of the information being communicated—perhaps by smoothing data or harmonizing prose. By *confirmation*, we mean the manipulation of the mean to confirm market priors. For example, one might rescale data or strike an optimistic tone. In settings where covariance is priced—such as when an SDF is specified, when there are multiple firms and the CAPM applies, or when the firm operates multiple, correlated segments—we additionally rationalize *dissociation* and *divergence*. *Dissociation* means reducing the covariance while *divergence* means emphasizing differences with the mean.

Importantly, these distortions emerge endogenously in a rational expectations equilibrium and do not require the specification of ad-hoc behavioral biases. The manager inflates the mean and deflates the variance (or covariance), knowing the market will attempt to "de-bias her bias." As in Stein (1989), bias is a form of "window dressing." Biasing is costly to the manager, but she cannot commit not to bias.

The main contribution of the present work is to collect these distortions into a single framework, which admits closed-form solutions and comparative statics. In addition to cataloging the distortions, the model delivers several new results. First, the marginal benefit of consistency (dissociation) is higher than that of confirmation (divergence) when cash flows are expected to have higher variance or when the signal space is larger. Second, any pair of distortions—except for consistency and dissociation—can vary independently. Third, the model predicts how uncertainty in cash flow variance ("variance-of-variance") affects both

[1] See Loughran and McDonald (2016) for a review.

consistency and confirmation. Finally, we characterize managers' ex-ante preferences over the size of the signal space. Although the paper is motivated by corporate communication, one could easily reinterpret the model in other disclosure environments, such as disclosure by traders (e.g. Huddart et al., 2001; Banerjee et al., 2024).

In the first part of the paper, we assume that cash flow variance is priced. The manager observes a sample of draws from the cash flow distribution, which are of purely informational value. She then reports the signals to the market with bias. Following the literature, we assume that the manager incurs an exogenous cost from biasing (e.g., Fischer and Verrecchia, 2000). Assuming a Normal-Gamma prior, the posterior variance is linear in (1) the report sample variance and (2) the squared deviation of the report sample mean from the prior mean. The former creates an incentive for consistency, the latter for confirmation.

In equilibrium, consistency and confirmation are orthogonal: The manager can change the sample variance of signals extracted by the market without changing the squared deviation of the sample mean from the prior (and vice-versa).[2] The difference between consistency and confirmation regards their marginal benefit. We show that the marginal benefit of consistency is higher than that of confirmation when the expected variance of the cash flows is high or when the signal space is large—exactly when the market relies on the new data more than its prior. It follows that if the manager has a choice in the dimension of the signal space, she prefers a large one over a small one. The model has implications for the cross section of reports. In particular, firms whose cash flow variance is more uncertain (i.e., higher *variance-of-variance*), such as younger firms in newer industries, should have more optimistic and consistent communication. The effect of cash flow variance uncertainty on confirmation is more nuanced and is explored in the body of the paper.

In the second part of the paper, we consider priced, stochastic covariance through three extensions. In each extension, the Normal-Gamma prior is replaced with a Normal-Wishart prior and the posterior covariance is linear in (1) the sample covariance and (2) the product of the deviations of the sample means from the prior means. In the first extension, we imagine that the covariance between the firm's cash flows and a priced factor—such as the market portfolio—is uncertain. The manager privately observes signals about the firm's cash flows, while the manager and market observe public signals about the priced factor. In equilibrium, the manager distorts her information to maximize the posterior mean and minimize the posterior covariance. The analysis highlights the twin distortions of dissociation

[2]Mathematically, the manager changes the mean without changing the variance through translation and changes the variance without changing the mean through rotation.

and divergence. Specifically, the manager attenuates the sample covariance and emphasizes differences between sources of cash flows using the sample mean.

In the second extension, we consider a firm whose cash flows are derived from two, correlated segments (e.g., "iron and copper" or "food and beverages") and assume that the variance is priced as in the baseline model. In this example, all four distortions arise in equilibrium—consistency, confirmation, dissociation, and divergence—as the manager tries to reduce both the posterior variances of both segments and the covariance between the two. In contrast to other iterations of the model, *consistency* and "dissociation" are not necessarily orthogonal. In simpler terms, one cannot change the variance without also changing the covariance (and vice-versa).

In the third extension, we consider an economy populated by two firms whose cash flows are correlated. The market is uncertain about the variances of each firms' cash flows and the covariance between them. We construct the price of each firm using the wealth portfolio as in the CAPM. We find that as the expected correlation between the firms increases, the incentives for confirmation and consistency become stronger. This is because the manager's best guess about the other manager's information is linear in her own information. Her problem is "as if" hers was the only firm.

The statistical mechanism underlying the model comes from Bayesian updating when updating beliefs about both the first and second moments. In "Normal-Normal" updating (Normal prior and Normal likelihood), the posterior variance is a deterministic function of the number of observations. As observed by several authors (e.g. O'Hagan, 1979), this is not true for priors with "fat-tails." If, as in this paper, the prior is Normal-Gamma (or Normal-Wishart), the posterior variance (or covariance) depends on the observations themselves. In particular, the posterior variance is linear in the sample standard deviation and the squared deviation of the sample mean from the prior mean; the posterior covariance is linear in the sample covariance and the product of the deviations of the sample means. These terms create incentives for consistency, confirmation, dissociation, and divergence.

This paper is intentionally silent on the real effects of these four distortions. A large literature examines the effect of disclosure on prices and investment.[3] Our objective in this paper is different. Given the widespread consumption of corporate communication, we wish to understand the ways in which communication is distorted so that they are used properly,

[3]See Bertomeu and Cheynel (2016) for an excellent review of the literature on disclosure and cost of capital (notable papers include Lambert et al., 2007; Strobl, 2013). Banerjee et al. (2018) and Goldstein and Yang (2019) show that disclosure can "crowd out" information and cause inefficiencies. Terry et al. (2023) explores the relationship between earnings management and investment.

even if such distortions have no equilibrium impact on price or investment.

2 Literature

Broadly speaking, this paper contributes to the literature on information disclosure in financial markets (Goldstein and Yang, 2017) and communication games in which senders engage in costly manipulation to deceive a receiver (e.g. Frankel and Kartik, 2019). This paper is motivated by empirical work studying the text of corporate communication, which is vast and growing (Loughran and McDonald, 2016). Ahern and Sosyura (2014) show that fixed exchange ratio acquirers use press releases to increase their own valuation. Hanley and Hoberg (2010) and Loughran and McDonald (2013) investigate the effects of the content of IPO filings on underpricing. Huang et al. (2014) explore tone management in earnings press releases. Larcker and Zakolyukina (2012) analyze the "truthfulness" and "deceitfulness" of conference calls. Li (2008) and Loughran and McDonald (2014) examine the effects of 10-K readability. The present manuscript suggests a unified framework through which researchers can interpret the findings of these empirical studies.

This paper specifically builds on the earnings management literature, beginning with the canonical models of Stein (1989), Fischer and Verrecchia (2000), and Dye and Sridhar (2004). A number of papers in earnings management model stochastic variance (Subramanyam, 1996; Kirschenheiter and Melumad, 2002; Beyer, 2009). These papers generally conclude that prices are non-linear in earnings surprises (a fact that emerges in the present paper as well). While the earnings management literature has been extended to look at interactions between multiple firms (Acharya et al., 2011; Gao and Zhang, 2019; Aghamolla et al., 2024) and dynamic settings (Beyer et al., 2019; Fang et al., 2024), less has been written on high-dimensional disclosure.[4] A notable exception is Harbaugh et al. (2016), which is the closest paper in terms of model setup. It too considers a manager who receives an arbitrarily large number of i.i.d. signals about a distribution. They find incentives for consistency and "too good to be true" inferences (i.e., confirmation). The main difference between the models is the biasing technology. In their paper, the mean of the report must equal the mean of the signals and the bias to any particular dimension is costless, but capped by a constraint (that always binds in equilibrium). In contrast, the model in this paper adopts the quadratic cost technology used in the accounting literature. This admits richer predictions regarding the shape of the distribution of reports. In addition, the model in this paper extends seamlessly

[4] Battaglini (2002) examines cheap-talk in higher dimensions.

to covariance uncertainty, which is critical to exploring more realistic pricing functions.

3 Model

3.1 Setup

There are three dates, $t \in \{0, 1, 2\}$. On date $t = 0$, a manager (she) privately observes a signal $x \in \mathbb{R}^n$—for some $n \geq 2$—which is informative about the cash flow d of the firm she manages. On date $t = 1$, she issues a potentially biased report $r \in \mathbb{R}^n$ about x to the market. The market sets the price $p(r)$ of the manager's shares. On date $t = 2$, d is revealed and the game ends.

The main assumption of the model is that there is uncertainty about the mean and variance of d. Let τ be Gamma distributed with shape $\alpha_0 > 0$ and rate $\beta_0 > 0$. $1/\tau$ is the stochastic variance of d. Let $\upsilon = 1/(\alpha_0 - 2)$. For the remainder of the paper, we will assume that $\beta_0 = \alpha_0 - 1$. It follows that $\mathrm{E}(1/\tau) = 1$ and $\mathrm{Var}(1/\tau) = \upsilon$, so that υ represents the variance uncertainty (or *variance-of-variance*). The assumption that $\beta_0 = \alpha_0 - 1$ is not critical for any of the results and merely eases the exposition. Let $e|\tau$ be normally distributed with mean μ_0 and precision $\lambda_0\tau$, where $\lambda_0 > 0$. Equivalently, (e, τ) is distributed according to a Normal-Gamma with parameters μ_0, λ_0, α_0, and β_0. Finally, d is normally distributed with mean e and precision τ. Standard calculations show that $\mathrm{E}(d) = \mu_0$ and $\mathrm{Var}(d) = 1 + \lambda_0^{-1}$.

On date $t = 0$, the manager privately observes a vector of n i.i.d. signals $x = (x_1, x_2, \ldots, x_n)$, where x_i is normally distributed with mean e and precision τ—the exact same distribution as the cash flow d. Equivalently, x_i is distributed according to a t-distribution with location μ_0, scale $\alpha_0^{-1}\beta_0(1 + \lambda_0^{-1})$, and degrees of freedom $2\alpha_0$. Upon observing the signal x, the manager releases a report r of x with bias $b(x) \in \mathbb{R}^n$:

$$r = x + b(x). \tag{1}$$

We assume that investors have mean-variance preferences over the cash flow d and we normalize the risk-free rate to be zero. Therefore, the price is

$$p(r) = \mathrm{E}(d|r) - \gamma \mathrm{Var}(d|r) \tag{2}$$

for some $\gamma > 0$. It is in this sense that the cash flow variance is priced.[5]

The manager's problem is to choose a bias to maximize the share price less a quadratic cost of biasing:

$$b(x) \in \underset{b \in \mathbb{R}^n}{\operatorname{argmax}} \left\{ p(x+b) - \tfrac{c}{2} b'b \right\}, \tag{3}$$

where $c > 0$. The quadratic cost represents explicit costs of biasing, such as distraction or regulatory action (Fischer and Verrecchia, 2000). It transpires that if $c = 0$, then the manager always chooses the same report in equilibrium and hence the report is uninformative. Therefore, consistency and confirmation are by themselves insufficient to keep the manager from biasing communication.

Definition 3.1 (Equilibirum)**.** *An equilibrium is a price function p and bias function b such that p and b jointly satisfy equations (2) and (3).*

3.2 First-Best

Before solving for the equilibrium, we compute the price under truth-telling. Specifically, suppose that $b(x) = 0$, so that $r(x) = x$. We first lay out some additional notation. As they will appear often, let

$$m = n^{-1}\mathbf{1} \tag{4}$$

$$C = n^{-1}(I - n^{-1}\mathbf{1}\mathbf{1}'), \tag{5}$$

where I is the $n \times n$ identity matrix and $\mathbf{1}$ is the n-dimensional vector of ones (and therefore $\mathbf{1}\mathbf{1}'$ is the $n \times n$ matrix of ones). The sample mean and variance of x are

$$\mu_x = x'm \tag{6}$$

$$\sigma_x^2 = x'Cx. \tag{7}$$

Note that $C^2 = n^{-1}C$, $(mm')^2 = n^{-1}mm'$, and $C(mm') = (mm')C = 0$. Next, let

$$\eta_0 = \frac{\lambda_0}{\lambda_0 + n} \tag{8}$$

$$\eta_1 = \frac{n}{\lambda_0 + n} \tag{9}$$

[5] One can interpret the price as containing a premium for *earnings quality* (EQ), defined by Beyer et al. (2019) as $-\text{Var}(d|r)$.

and

$$\theta_0 = \frac{2\beta_0(\lambda_0 + n + 1)}{(2\alpha_0 + n - 2)(\lambda_0 + n)} \tag{10}$$

$$\theta_1 = \frac{n(\lambda_0 + n + 1)}{(2\alpha_0 + n - 2)(\lambda_0 + n)} \tag{11}$$

$$\theta_2 = \frac{\lambda_0 n(\lambda_0 + n + 1)}{(2\alpha_0 + n - 2)(\lambda_0 + n)^2}. \tag{12}$$

Lemma 3.1. *Under truth-telling, the posterior mean and variance of d are*

$$\mathrm{E}(d|x) = \eta_0\mu_0 + \eta_1\mu_x \tag{13}$$

$$\mathrm{Var}(d|x) = \theta_0 + \theta_1\sigma_x^2 + \theta_2(\mu_x - \mu_0)^2. \tag{14}$$

Proofs of all results can be found in Section A. Note that θ_0, θ_1, and θ_2 are strictly positive and $\theta_0, \theta_1, \theta_2 \to 0$ as $n \to \infty$, so the posterior variance goes to zero as $n \to \infty$. Moreover, $\theta_2/\theta_1 = \lambda_0/(\lambda_0 + n) < 1$.

We now consider the price. Let

$$\bar{r} = \mu_0 + \frac{\eta_1}{2\gamma\theta_2} \tag{15}$$

Proposition 3.1 (First-Best)**.** *Under truth-telling, the price is*

$$p_F(x) = \pi_{0,B} + \pi_{1,B}'x + \tfrac{1}{2}x'\pi_{2,B}x, \tag{16}$$

where

$$\pi_{2,B} = -2\gamma(\theta_1 C + \theta_2 mm') \tag{17}$$

$$\pi_{1,B} = -\bar{r}\pi_{2,B}\mathbf{1} = (\eta_1 + 2\gamma\theta_2\mu_0)m \tag{18}$$

$$\pi_{0,B} = \eta_0\mu_0 - \gamma(\theta_0 + \theta_2\mu_0^2). \tag{19}$$

Note that p_F achieves its maximum at $\bar{r}\mathbf{1}$.

3.3 Equilibrium Construction

Motivated by Proposition 3.1, we restrict attention to equilibria in which the bias is linear in the signal x and the price is quadratic in the manager's report r.

Definition 3.2 (LBQP Equilibrium). *A linear-bias, quadratic-price* (LBQP) *equilibrium is an equilibrium of the form*

$$b(x) = \rho_0 + \rho_1 x \tag{20}$$

$$p(r) = \pi_0 + \pi_1' r + \tfrac{1}{2} r' \pi_2 r, \tag{21}$$

where $\pi_0 \in \mathbb{R}$*,* $\rho_0, \pi_1 \in \mathbb{R}^n$*, and* $\rho_1, \pi_2 \in \mathbb{R}^{n \times n}$*.*

Evidently, the LBQP equilibrium nests the standard linear equilibria considered in the literature. x is distributed according to a multivariate t-distribution with location $\mu_0 \mathbf{1}$, scale matrix $\alpha_0^{-1}\beta_0(I + \lambda_0^{-1}\mathbf{11}')$, and degrees of freedom 2α. Therefore, $r(x)$ is also distributed according to a multivariate t-distribution, but with location $\rho_0 + \mu_0\rho_1\mathbf{1}$, scale matrix $\alpha_0^{-1}\beta_0\rho_1'(I + \lambda_0^{-1}\mathbf{11}')\rho_1$, and degrees of freedom 2α.

Lemma 3.2 (Price Formation). *Given a manager's bias function in equation (20), the market extracts from the report* r *the signal*

$$s(r) = (I + \rho_1)^{-1}(r - \rho_0). \tag{22}$$

As a function of the report r*, the price is given by*

$$p(r) = \eta_0\mu_0 + \eta_1\mu_{s(r)} - \gamma\left(\theta_0 + \theta_1\sigma^2_{s(r)} + \theta_2(\mu_{s(r)} - \mu_0)^2\right). \tag{23}$$

Equation (23) illustrates the manager's incentive for consistency and confirmation. First, there is the marginal benefit of increasing $\mu_{s(r)}$ in the first term. However, there is a cost of increasing $\mu_{s(r)}$ related to the squared deviation of the report from the prior, $(\mu_{s(r)} - \mu_0)^2$—what we have called *confirmation*. The manager wants to bias upwards, but not so much that it greatly increases the posterior variance. Second, the manager wishes to minimize the sample standard deviation of the market's signal, $\sigma^2_{s(r)}$—what we have called *consistency*.

In the following proposition, equation (23) is shown to be quadratic in the report r. Additionally, the optimal bias, obtained by maximizing the price in equation (21) minus the quadratic cost, is shown to be linear in the manager's signal x. The following proposition characterizes the equilibrium values of $\rho_0, \rho_1, \pi_0, \pi_1, \pi_2$.

Proposition 3.2 (Equilibrium). *If $c > 8\gamma\theta_1/n$, there are four* LBQP *equilbria:*

$$\pi_2^* = \kappa_1 C + \kappa_2 mm' \tag{24}$$
$$\rho_1^* = (cI - \pi_2^*)^{-1}\pi_2^* = \kappa_1(c - n^{-1}\kappa_1)^{-1}C + \kappa_2(c - n^{-1}\kappa_2)^{-1}mm', \tag{25}$$

where κ_1 and κ_2 are roots of

$$c^2\kappa_1 = -2\gamma\theta_1(c - n^{-1}\kappa_1)^2 \tag{26}$$
$$c^2\kappa_2 = -2\gamma\theta_2(c - n^{-1}\kappa_2)^2 \tag{27}$$

and

$$\pi_1^* = -\bar{r}\pi_2^*\mathbf{1} = -\bar{r}\kappa_2 m \tag{28}$$
$$\rho_0^* = (cI - \pi_2^*)^{-1}\pi_1^* = -\bar{r}\rho_1^*\mathbf{1} = -\bar{r}\kappa_2(c - n^{-1}\kappa_2)^{-1}m. \tag{29}$$

Finally,

$$\pi_0^* = \pi_{0,B} - c^{-1}\pi_{1,B}'\pi_1^* + \tfrac{1}{2}c^{-2}\pi_1^{*\prime}\pi_{2,B}\pi_1^*. \tag{30}$$

Although we will explore many implications of the Proposition 3.2, we begin with the orthogonality of consistency and confirmation.

Corollary 3.1. *In general, changes to the sample mean of the extracted signal are not orthogonal to changes in the sample variance:*

$$\nabla_r\mu_{s(r)} \cdot \nabla_r\sigma_{s(r)}^2 = 2m'(I + \rho_1)^{-1}(I + \rho_1)^{-1\prime}Cs(r). \tag{31}$$

In equilibrium, however, they are orthogonal: $\nabla_r\mu_{s^*(r)} \cdot \nabla_r\sigma_{s^*(r)}^2 = 0$.

As an elementary example, translating all signals by a constant changes the sample mean, but does not change the sample variance. Similarly, rotating all signals about the mean by a constant angle changes the variance, but does not change the mean. This is true equilibrium—where the market calculates the sample mean and variance using using ρ_0^* and ρ_1^*—but is not true for an arbitrary conjecture of ρ_0 and ρ_1.

Since confirmation and consistency are orthogonal, any conflict between the two regards their relative marginal benefit.

Corollary 3.2. *The manager has a greater incentive to reduce the sample variance $\sigma_{s(r)}^2$—consistency—than to reduce the squared deviation $(\mu_{s(r)} - \mu_0)^2$—confirmation—when the size* *of the signal space n is large or when the variance $Var(d) = 1 + \lambda_0^{-1}$ is large.*

Intuitively, the manager focuses more on consistency exactly when the market relies more on new data than on its priors.

3.4 Equilibrium Selection

To sharpen the empirical predictions of the model, we choose one of the equilibria for our analysis. Fortunately, one of the equilibria is Pareto efficient.

Lemma 3.3. *The price distributions in each of the four equilibria are identical to the price distribution under first-best.*

This is the "window dressing" result, which shows that although there is bias in equilibrium, the market anticipates and removes the equilibrium bias.

In what follows, we will refer to the equilibrium in which κ_1 and κ_2 are the larger roots of equations (26) and (27) as the *light-catering equilibrium*. The manager strictly prefers the light-catering equilibrium as it is less costly.

Proposition 3.3. *The market is indifferent across equilibria. However, the manager's expected utility is strictly higher in the light-catering equilibrium than any other equilibrium equlibrium.*

For the remainder of this section, we will operate under the assumption that all equilibrium objects belong to the light-catering equilibrium. The following technical result will be used in the propositions that follow.

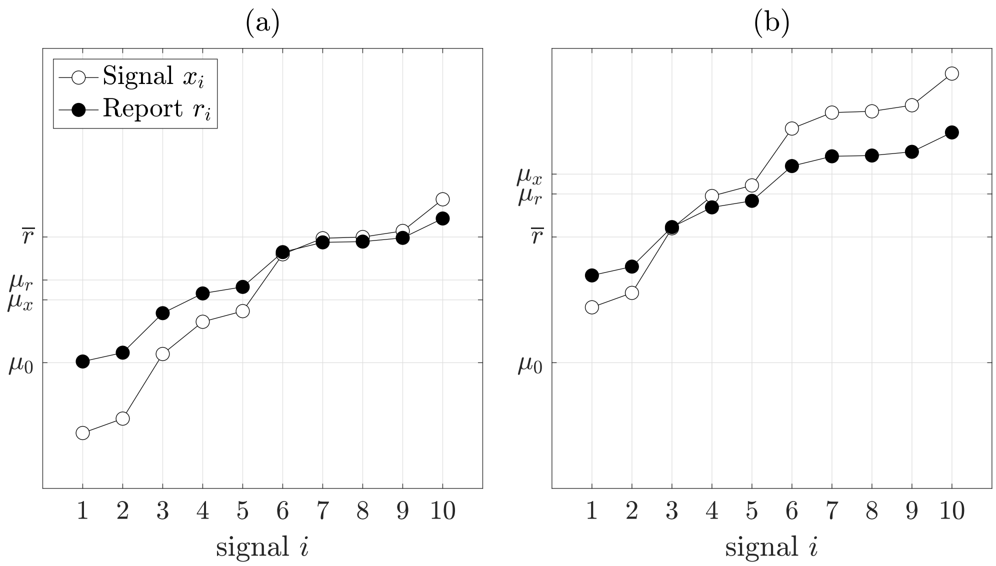
Lemma 3.4. *In the light-catering equilibrium, $\kappa_1, \kappa_2 \in (-nc, 0)$. Moreover, κ_1 and κ_2 are strictly decreasing in γ and υ. Finally, $n^{-1}\kappa_1$ and $n^{-1}\kappa_2$ are strictly increasing in n.*

3.5 Equilibrium Analysis

3.5.1 Reporting

In this section, we prove a number of results that shed light on the properties of the equilibrium selected in the previous section. We first wish to understand the action of the reporting policy under the light-catering equilibrium. The signal-to-report map $x \mapsto r^*(x)$—an example of which is illustrated in Figure 1—pulls signals towards the price maximizing report, $\bar{r}\mathbf{1}$, in the following sense.

Figure 1: **Equilibrium Reporting.** This figure plots two examples of signals and the associated equilibrium reports for two sets of signals. The parameters used are $\mu_0 = 1$, $\lambda_0 = 100$, $\alpha_0 = 3$, $\beta_0 = 2$, $n = 10$, $c = 0.05$, and $\gamma = 0.95cn/8\theta_1$.



Lemma 3.5. *r^* has the fixed-point $\overline{r}\mathbf{1}$, which can be obtained by fixed-point iteration.*

By "pulling" signals towards $\overline{r}\mathbf{1}$, the manager simultaneously reduces sample variance while trading-off the costs and benefits of increasing the sample mean.

Another way to see this is to look at how the map $x \mapsto r^*(x)$ changes the sample mean and sample variance.

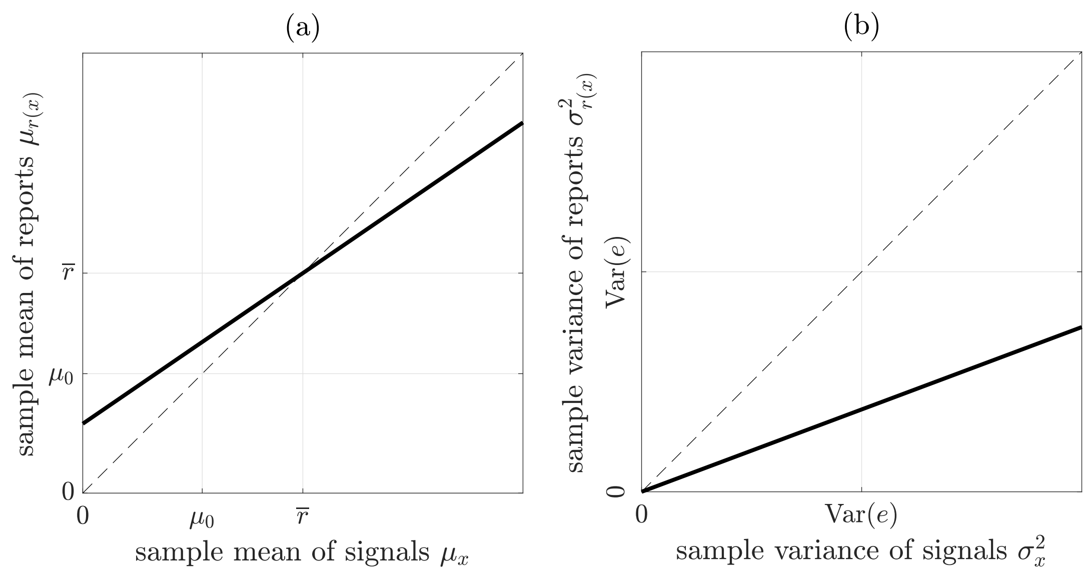
Lemma 3.6. *The mean and variance are*

$$\mu_{r^*(x)} = \overline{r} + c(c - n^{-1}\kappa_2)^{-1}(\mu_x - \overline{r}) \tag{32}$$

$$\sigma^2_{r^*(x)} = c^2(c - n^{-1}\kappa_1)^{-2}\sigma^2_x. \tag{33}$$

These relationships are plotted in Figure 2. From equation (32), the sample mean of the report, $\mu_{r(x)}$, is a convex combination of the sample mean of the signals, μ_x, and the ideal mean $\overline{r}$. The exact combination depends primarily on the cost of biasing—the larger the cost, the less the manager is able to bias, the closer is $\mu_{r(x)}$ to μ_x. From equation (33), the sample variance of the report, $\sigma^2_{r(x)}$, is always less than the sample variance of the signals, σ^2_x. Unlike the sample mean, there is no benefit to increasing the sample variance. As in equation (32), the extent to which the manager can reduce the sample variance depends on

Figure 2: **Sample Mean and Variance.** This figure visualizes the mapping from the sample mean μ_x and variance σ_x^2 of the signals to the sample mean $\mu_{r(x)}$ and variance $\sigma^2_{r(x)}$ of reports (black line). The dashed line is the 45° line and represents the mapping under truth-telling. The parameters used are $\mu_0 = 1$, $\lambda_0 = 100$, $\alpha_0 = 3$, $\beta_0 = 2$, $n = 10$, $c = 0.05$, and $\gamma = 0.95cn/8\theta_1$.



the cost of biasing—the larger the cost, the less the manager is able to bias, the closer is $\sigma^2_{r(x)}$ to σ^2_x.

Our main empirical implications regard how changes in the variance-of-variance υ, affect the distribution of reports. First, note that

Lemma 3.7. $\mathrm{E}(\mu_x) = \mu_0$, $\mathrm{Var}(\mu_x) = 1/n + 1/\lambda_0$, *and* $\mathrm{E}(\sigma_x^2) = 1 - 1/n$.

From Lemmas and 3.7, we immediately obtain the expected report mean, the expected report variance (i.e., consistency), and the expected squared deviation (i.e., confirmation):

$$\mathrm{E}(\mu_{r^*(x)}) = \mu_0 + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}(\mu_0 - \overline{r}) \tag{34}$$

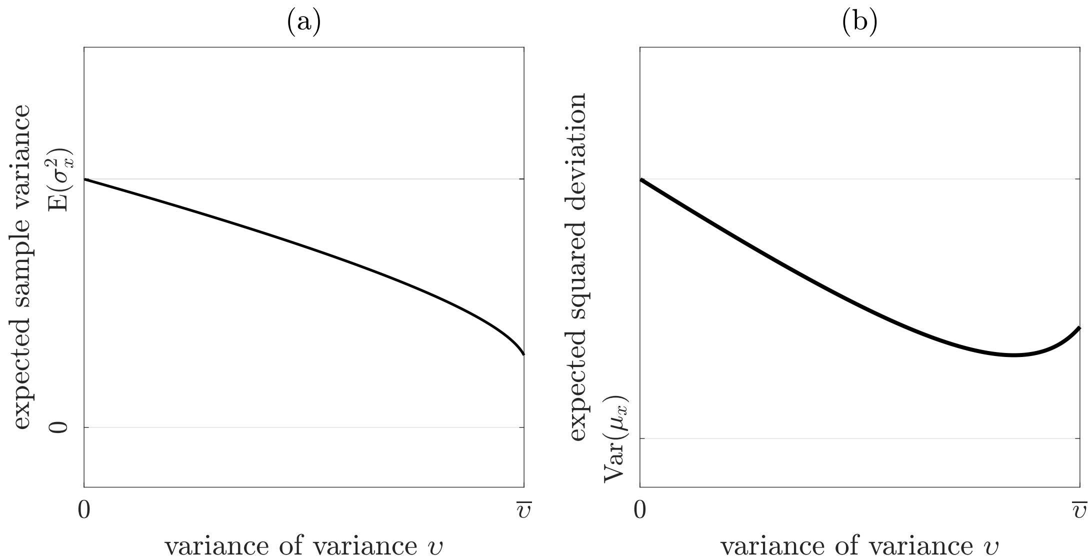
$$\mathrm{E}(\sigma^2_{r^*(x)}) = c^2(c - n^{-1}\kappa_1)^{-2}\mathrm{E}(\sigma_x^2) \tag{35}$$

$$\mathrm{Var}(\mu_{r^*(x)}) = c^2(c - n^{-1}\kappa_2)^{-2}\mathrm{Var}(\mu_x). \tag{36}$$

Consequently, the expected squared deviation is given by

$$\mathrm{E}((\mu_{r^*(x)} - \mu_0)^2) = \mathrm{Var}(\mu_{r^*(x)}) + (\mathrm{E}(\mu_{r^*(x)}) - \mu_0)^2. \tag{37}$$

Figure 3: **Consistency, Confirmation, and the Variance-of-Variance.** This figure shows the relationship between the expected sample variance of reports, $\mathrm{E}(\sigma_{r^*(x)})$ (consistency) and the variance-of-variance υ in panel (a) and the relationship between the expected squared deviation, $\mathrm{E}((\mu_{r^*(x)} - \mu_0)^2)$ (confirmation) and the variance-of-variance υ in panel (b). The parameters used are $\mu_0 = 1$, $\lambda_0 = 100$, $n = 10$, $c = 0.05$, and $\gamma = 10$. $\overline{\upsilon} = c(\lambda_0 + n)/(4\gamma(\lambda_0 + n + 1) - c(\lambda_0 + n))$ is the maximum value of υ for which LBQP equilibria exist.



Evidently, $\mathrm{E}(\mu_{r^*(x)}) > \mu_0$ and $\mathrm{E}(\sigma^2_{r^*(x)}) < \mathrm{E}(\sigma^2_x)$.

Proposition 3.4 (Comparative Statics: Variance)**.** *The expected sample mean* $\mathrm{E}(\mu_{r^*(x)})$ *is strictly increasing in the variance-of-variance* υ*; the variance of the sample mean* $\mathrm{Var}(\mu_{r^*(x)})$ *and expected sample variance* $\mathrm{E}(\sigma^2_{r^*(x)})$ *are strictly decreasing in* υ*. The mean-squared-error of the sample mean* $\mathrm{E}((\mu_{r^*(x)} - \mu_0)^2)$ *may be non-monotonic in* υ*.*

These results are shown in Figure 3. In general, variance uncertainty increases the posterior variance and incentivizes both consistency and confirmation. In the case of consistency, the relationship is clear-cut: the lower the sample variance, the better. According to Proposition 3.4, this is true in expectation. The effect on the expectation of the confirmation term is more complicated. The variance of the sample mean is decreasing in variance uncertainty, but the expected sample mean is increasing in variance uncertainty. As a result, the net effect on the expectation of the confirmation term can be U-shaped, as demonstrated in Figure 3.

3.5.2 Dimensionality

In this section, we explore the manager's ex-ante preferences over the size of the signal space, n. First, the expected price is

$$\mathcal{P}(n) = \mathbb{E}\big[p^*(r^*(x))\big], \tag{38}$$

and the manager's expected profit is

$$\Pi(n) = \mathbb{E}\Big[p^*(r^*(x)) - \tfrac{c}{2}\, b^*(x)' b^*(x)\Big]. \tag{39}$$

Next, the expected price is strictly increasing in n:

Lemma 3.8. $\mathcal{P}(n) = \mu_0 - \gamma(1 + 1/(\lambda_0 + n))$.

Finally, we apply the Envelope Theorem:

Proposition 3.5. *The expected profit* $\Pi(n)$ *is strictly increasing in* n*.*

Our analysis shows that the manager prefers a richer signal space. However, empirical studies document that gathering and processing information is not free. For instance, Blankespoor et al. (2020) find substantial information processing costs borne by investors and analysts as they digest firm disclosures. To capture these frictions, the literature models a constraint on agents' ability to reduce uncertainty—formalized as an entropy-reduction constraint (Veldkamp, 2023). Under such an entropy-reduction constraint, an agent optimally chooses a finite number of signals to acquire. The finite signal-space size n that we assume throughout the paper can be interpreted the outcome of a more complicated game featuring these frictions.

4 Covariance

In this section, we explore three extensions in which the covariance of the firm's cash flows is priced. In the first extension, we imagine that instead of a variance discount, there is the usual discount due to the covariance of the firm's cash flows with the SDF. In the second extension, we imagine that the firm's cash flows are derived from the cash flows of two, correlated segments. Finally, we imagine an economy in which there are two firms and derive the price using the CAPM. Mathematically, each of these extensions extends the previous model by introducing a second source of uncertainty (an SDF, another cash flows source, or another firm). The one-dimensional Gamma distribution becomes a two-dimensional Wishart distribution.

While the variance is bounded below, the covariance has no lower bound. In our applications, the manager will wish to make the cash flows (or segments contributing to cash flows) appear as different as her cost function permits. The notion of *consistency* becomes "dissociation," whereby the manager wishes to make the sample covariance as negative as possible. Similarly, the notion of *confirmation* becomes *divergence*. If one cash flow is unusually low, she wishes to make the other cash flow appear unusually high (and vise-versa).

In the previous model, one of the main variables of interest was the stochastic precision, τ, which was distributed according to a Gamma distribution with parameters α_0 and β_0. In the extensions that follow, we consider two variables with an unknown precision matrix. Specifically, we consider a random, 2×2 precision matrix Λ, distributed according to a Wishart distribution with a known, 2×2 scale matrix Ψ_0^{-1} and degrees of freedom $\nu > 0$. It follows that the covariance matrix, Λ^{-1}, is distributed according to an Inverse-Wishart (just as $1/\tau$ was distributed according to an Inverse-Gamma). Now $\mathrm{E}(\Lambda^{-1}) = \Psi_0/(\nu - 3)$. Let ψ_0, ϕ_0, and ω_0 be such that

$$\Psi_0 = \begin{pmatrix} \psi_0 & \omega_0 \\ \omega_0 & \phi_0 \end{pmatrix} \tag{40}$$

so that ψ_0 represents the expected variance of the first variable, ϕ_0 the expected variance of the second variable, and ω_0 the expected covariance. Similar to equations (10), (11), and (12), let

$$\zeta_0 = \frac{\lambda_0 + n + 1}{(\nu_0 + n - 3)(\lambda_0 + n)} \tag{41}$$

$$\zeta_1 = \frac{n(\lambda_0 + n + 1)}{(\nu_0 + n - 3)(\lambda_0 + n)} \tag{42}$$

$$\zeta_2 = \frac{\lambda_0 n(\lambda_0 + n + 1)}{(\nu_0 + n - 3)(\lambda_0 + n)^2}. \tag{43}$$

In the examples that follow, the posterior covariance matrix will depend on these three parameters. While in the previous section, the variance-of-variance υ was the primary variable of interest, in this section, the expected covariance ω_0 will be the main variable of interest.

We continue to use the sample mean and sample variance in equations (6) and (7). In addition, equation (7) can be used to compute the sample covariance:

$$\sigma_{x,y} = x'Cy. \tag{44}$$

Note that while most changes to x and y are orthogonal, there are two important exceptions:

$\nabla_x \sigma_x^2 \cdot \nabla_x \sigma_{x,y} = \nabla_y \sigma_y^2 \cdot \nabla_y \sigma_{x,y} = 2n^{-1} x' C y$. In words, one cannot change the variance without also changing the covariance. This fact will be relevant when we study firms with multiple segments or in economies with multiple firms.

4.1 One Factor

Unless the firm's cash flows are truly undiversifiable, the price in equation (2) is misspecified. In this section, we assume that the firm's cash flows have an unknown covariance with a priced factor f. The two main assumptions that we make are that (1) the manager cannot manipulate data about the factor and (2) the variance of the factor is known (although the covariance with the firms' cash flows are not).

Let $(e, f)|\Lambda$ be normally distribution with mean (μ_0, ξ_0) and precision $\lambda_0 \Lambda$, where $\lambda_0 > 0$ is as before. Therefore, $((e, f), \Lambda)$ is Normal-Wishart with parameters (μ_0, ξ_0), λ_0, Ψ_0^{-1}, and ν. The cash flow (d, w) is normally distributed with mean (e, f) and precision $\lambda \Lambda$. The manager observes i.i.d. signals $(x_1, y_1), \ldots, (x_n, y_n)$, which are drawn from the same distribution as the cash flow. Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ so that x represents signals about the firm and y data about the factor.

Now given the stochastic discount factor $1 - \gamma(w - \mathrm{E}(w|r))$, the price is

$$p(r) = \mathrm{E}(d|r) - \gamma \mathrm{Cov}(d, w|r). \tag{45}$$

Naturally, the manager can bias data about her firm, but cannot bias information about the factor. The manager's problem is to choose a bias

$$b(x, y) \in \operatorname*{argmax}_{b \in \mathbb{R}^n} \left\{ p(x + b, y) - \tfrac{c}{2} b' b \right\}. \tag{46}$$

Conjecture again that the manager's bias is linear in signals and that the price is quadratic in the report:

$$b(x, y) = \rho_0 + \rho_1 x + \rho_2 y \tag{47}$$

$$p(r, y) = \pi_0 + \pi_1' r + \pi_2' y + \tfrac{1}{2} r' \pi_3 r + r' \pi_4 y + \tfrac{1}{2} y' \pi_5 y, \tag{48}$$

where $\pi_0 \in \mathbb{R}$, $x, y, r, \rho_0, \pi_1, \pi_2 \in \mathbb{R}^n$, and $\rho_1, \rho_2, \pi_3, \pi_4, \rho_5 \in \mathbb{R}^{n \times n}$.

Lemma 4.1 (Price Formation: One Factor). *Given the manager's bias function in equation*

(47), the market extracts from the report r the signal

$$s(r, y) = (I + \rho_1)^{-1}(r - \rho_0 - \rho_2 y) \tag{49}$$

from which it forms the price

$$p(r, y) = \eta_0\mu_0 + \eta_1\mu_{s(r)} - \gamma\left(\zeta_0\omega_0 + \zeta_1\sigma_{s(r),y} + \zeta_2(\mu_{s(r)} - \mu_0)(\mu_y - \xi_0)\right). \tag{50}$$

Equation (50) illustrates the manager's incentive for dissociation and divergence. First, there is the marginal benefit of increasing $\mu_{s(r)}$ in the first term. However, there is an additional incentive from the product $(\mu_{s(r)} - \mu_0)(\mu_y - \xi_0)$—what we have called *divergence*. If the factor is below average ($\mu_y < \xi_0$), the manager wishes to make her firm's cash flows look above average ($\mu_{s(r)} > \mu_0$) and vice-versa. Second, the manager wishes to minimize the sample covariance of the market's signal with the factor signal, $\sigma_{s(r),y}$—what we have called *dissociation*.

We now turn to equilibrium construction. Let

$$\overline{y} = \xi_0 + \frac{\eta_1}{\gamma\zeta_2}. \tag{51}$$

Note the similarity between $\overline{y}$ in equation (51) and $\overline{r}$ in equation (15).

Proposition 4.1 (Equilibrium: One Factor)**.** *There is a unique equilibrium in which*

$$\rho_2^* = -\gamma c^{-1}(\zeta_1 C + \zeta_2 m m') \tag{52}$$

$$\rho_1^* = 0 \tag{53}$$

$$\rho_0^* = -\overline{y}\rho_2^*\mathbf{1} = c^{-1}(\eta_1 + \gamma\zeta_2\xi_0)m. \tag{54}$$

We omit the parameters for the equilibrium price function $\pi_0^*, \ldots, \pi_5^*$ (they can be found in the proof). It transpires that while the price depends on χ_0 (specifically, π_0^*), the bias function does not.

Corollary 4.1 (Orthogonality: One Factor)**.** *In equilibrium, changes to the sample mean of the extracted signal are orthogonal to changes in the sample variance:*

$$\nabla_r\mu_{s^*(r,y)} \cdot \nabla_r\sigma_{s^*(r,y),y} = 0. \tag{55}$$

Intuitively, the manager can change the sample mean without changing the sample covariance (and vice-versa). Analogous to the baseline model, the manager has a greater incentive to reduce the sample covariance $\sigma_{s(r,y),y}$ (i.e., *dissociation*) than to reduce the product $(\mu_{s(r,y)} - \mu_0)(\mu_y - \xi_0)$ (i.e., *divergence*) when the size of the signal space n is large or when the variance $\text{Var}(d) = 1 + \lambda_0^{-1}$ is large (since $\zeta_2/\zeta_1 = \lambda_0/(\lambda_0 + n)$).

Following our analysis of the baseline model, we examine how the sample statistics of the signals available to the manager map to the sample statistics of the reports she dispatches to the market.

Corollary 4.2 (Sample Stats: One Factor)**.** *The mean and covariance are*

$$\mu_{r(x,y)} = \mu_x + c^{-1}n^{-1}(\eta_1 + \gamma\zeta_2\xi_0) - \gamma c^{-1}n^{-1}\zeta_2\mu_y \tag{56}$$

$$\sigma_{r(x,y),y} = \sigma_{x,y} - \gamma c^{-1}n^{-1}\zeta_1\sigma_y^2 \tag{57}$$

Since $\mu_{r(x,y)} = \mu_x + \mu_{b(x,y)}$, Corollary 4.2 shows that the mean bias should strictly decrease in the factor mean. Intuitively, the manager wishes to bias signals upward, but less so if the mean of the factor is already high. Similarly, $\sigma_{r(x,y),y} = \sigma_{x,y} + \sigma b(x,y), y$ and so Corollary 4.2 shows that the manager always wants to decrease the covariance ("disassociation").

4.2 Two Segments

In this section, we imagine that a single firm operates two segments. One of the key insights is that in a multi-segment firm, *consistency* will be in conflict with "dissociation," as the manager cannot reduce the variance regarding one segment without changing the covariance between the two segments. Whether attempts to increase consistency and dissociation are complimentary or substitutable depends on the sample covariance.

$\boldsymbol{e}|\Lambda$ is normally distributed with mean $\boldsymbol{\mu}_0 = \mu_0\mathbf{1}_2$ and precision $\lambda\Lambda$. Therefore, $(\boldsymbol{e}, \Lambda)$ is distributed according to a Normal-Wishart with parameters $\boldsymbol{\mu}_0$, λ_0, Ψ_0^{-1}, and ν. Finally, the cash flows $\boldsymbol{d}$ are normally distributed with mean $\boldsymbol{e}$ and precision $\lambda\Lambda$. The aggregate cash flow is the sum of the cash flows from the two segments:

$$d = \mathbf{1}_2'\boldsymbol{d} = d_1 + d_2. \tag{58}$$

Under mean-variance pricing, the price is

$$p(r) = \text{E}(d|r) - \gamma\text{Var}(d|r) = \sum_{j\in\{1,2\}} (\text{E}(d_j|r) - \gamma\text{Var}(d_j|r)) - 2\gamma\text{Cov}(d_i, d_j|r). \tag{59}$$

The manager observes i.i.d. signals $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$, which are drawn from the same distribution as the cash flows. The manager's problem is to choose a bias

$$b(\boldsymbol{x}) \in \underset{b \in \mathbb{R}^{n\times 2}}{\operatorname{argmax}} \left\{ p(\boldsymbol{x} + b) - \tfrac{c}{2} \operatorname{tr}(b'b) \right\}. \tag{60}$$

Conjecture again that the manager's bias is linear in signals and that the price is quadratic in the report:

$$b_j(\boldsymbol{x}) = \rho_0 + \rho_1 \boldsymbol{x}_j + \rho_2 \boldsymbol{x}_{-j} \tag{61}$$

$$p(\boldsymbol{r}) = \pi_0 + \pi_1' \boldsymbol{r}_1 + \pi_1' \boldsymbol{r}_2 + \frac{1}{2} \boldsymbol{r}_1' \pi_2 \boldsymbol{r}_1 + \boldsymbol{r}_1' \pi_3 \boldsymbol{r}_2 + \frac{1}{2} \boldsymbol{r}_2' \pi_2 \boldsymbol{r}_2, \tag{62}$$

where $\boldsymbol{x}, \boldsymbol{r} \in \mathbb{R}^{n\times 2}$ ($\boldsymbol{r}_j$ and $\boldsymbol{x}_j$ denote the j-th columns of $\boldsymbol{r}$ and $\boldsymbol{x}$ respectively), $\pi_0 \in \mathbb{R}$, $\rho_0, \pi_1 \in \mathbb{R}^n$, and $\rho_1, \rho_2, \pi_2, \pi_3 \in \mathbb{R}^{n\times n}$. Moreover, we conjecture that $\pi_2 + \pi_3$ and $\pi_2 - \pi_3$ are symmetric negative definite.[6] Equations (61) and (62) can be written as

$$\operatorname{vec}(b(\boldsymbol{x})) = \mathbf{1}_2 \otimes \rho_0 + (I_2 \otimes \rho_1 + J_2 \otimes \rho_2) \operatorname{vec}(\boldsymbol{x}) \tag{63}$$

$$p(\boldsymbol{r}) = \pi_0 + (\mathbf{1}_2' \otimes \pi_1') \operatorname{vec}(\boldsymbol{r}) + \frac{1}{2} \operatorname{vec}(\boldsymbol{r})' (I_2 \otimes \pi_2 + J_2 \otimes \pi_3) \operatorname{vec}(\boldsymbol{r}), \tag{64}$$

where $b(\boldsymbol{x}) \in \mathbb{R}^{n\times 2}$, $b_j(\boldsymbol{x})$ in equation (61) is the j-th column of $b(\boldsymbol{x})$, I_2 is the 2×2 identity matrix and J_2 is the 2×2 exchange matrix.

Lemma 4.2 (Price Formation: Two Segments)**.** *Given the manager's bias function in equation (63), the market extracts from the report $\boldsymbol{r}$ the signal*

$$s(\boldsymbol{r}) = (I_2 \otimes (I + \rho_1) + J_2 \otimes \rho_2)^{-1} (\operatorname{vec}(\boldsymbol{r}) - \mathbf{1}_2 \otimes \rho_0), \tag{65}$$

from which it forms the price

$$\begin{aligned} p(\boldsymbol{r}) = \sum_{j \in \{1,2\}} & \left(\eta_0 \mu_0 + \eta_1 \mu_{s_j(\boldsymbol{r})} - \gamma \left(\zeta_0 \psi_0 + \zeta_1 \sigma^2_{s_j(\boldsymbol{r})} + \zeta_2 (\mu_{s_j(\boldsymbol{r})} - \mu_0)^2 \right) \right) \\ & - 2\gamma \left(\zeta_0 \omega_0 + \zeta_1 \sigma_{s_1(\boldsymbol{r}), s_2(\boldsymbol{r})} + \zeta_2 (\mu_{s_1(\boldsymbol{r})} - \mu_0)(\mu_{s_2(\boldsymbol{r})} - \mu_0) \right), \end{aligned} \tag{66}$$

where $s_j(\boldsymbol{r})$ denotes the j-th column of $s(\boldsymbol{r})$.

In equation (66), we see the incentives for consistency and confirmation in each segment in the first line and incentives for dissociation and divergence in the second.

[6] This guarantees that $I_2 \otimes \pi_2 + J_2 \otimes \pi_3$, which appears in equation (64), is symmetric negative definite.

We now turn to equilibrium construction. Let

$$\overline{r}_{\mathrm{S}} = \mu_0 + \frac{\eta_1}{4\gamma\zeta_2}. \tag{67}$$

and

$$\pi_{0,S} = 2\eta_0\mu_0 - \gamma(2\zeta_0(\omega_0 + \psi_0) + 4\zeta_2\mu_0^2) \tag{68}$$

$$\pi_{1,S} = -2\overline{r}_{\mathrm{S}}\pi_{2,S}\mathbf{1} = (\eta_1 + 4\gamma\zeta_2\mu_0)m \tag{69}$$

$$\pi_{2,S} = \pi_{3,S} = -2\gamma(\zeta_1 C + \zeta_2 mm'). \tag{70}$$

$\pi_{0,S}$, $\pi_{1,S}$, and $\pi_{2,S}$ are analogous to $\pi_{0,B}$, $\pi_{1,B}$, and $\pi_{2,B}$ in equations (19), (18), and (17).

Proposition 4.2 (Equilibrium: Two Segments)**.** *If $c > 16\gamma\zeta_1/n$, there are four equilibria of the form given by equations (61) and (62):*

$$\pi_2^* = \pi_3^* = \kappa_1 C + \kappa_2 mm' \tag{71}$$

$$\rho_1^* = \rho_2^* = \kappa_1(c - 2n^{-1}\kappa_1)^{-1}C + \kappa_2(c - 2n^{-1}\kappa_2)^{-1}mm' \tag{72}$$

where κ_1 and κ_2 are roots of

$$c^2\kappa_1 = -2\gamma\zeta_1(c - 2n^{-1}\kappa_1)^2 \tag{73}$$

$$c^2\kappa_2 = -2\gamma\zeta_2(c - 2n^{-1}\kappa_2)^2 \tag{74}$$

Next, $\rho_1^* = -2\overline{r}_S\rho_2^*\mathbf{1}$ and $\rho_0^* = -2\overline{r}_S\rho_1^*\mathbf{1}$. Finally,

$$\pi_0^* = \pi_{0,S} - 2c^{-1}\pi_{1,S}'\pi_1^* + 2c^{-2}\pi_1^{*\prime}\pi_{2,S}\pi_1^*. \tag{75}$$

According to Proposition 4.2, the manager's bias for any particular segment only depends on the total cash flows, not the individual segment cash flows. As the correlation between the two segments increases, the manager biases more. In particular, since $\pi_2^* = \pi_3^*$, the price can be written in terms of the sum of the reports, $\boldsymbol{r}\mathbf{1}_2$:

$$p(\boldsymbol{r}) = \pi_0^* + \pi_1^{*\prime}\boldsymbol{r}\mathbf{1}_2 + \frac{1}{2}(\boldsymbol{r}\mathbf{1}_2)'\pi_2^*(\boldsymbol{r}\mathbf{1}_2). \tag{76}$$

The only variable that depends on the correlation is π_0^*. Increasing the covariance between segments, ω_0, increases prices, but does not affect the equilibrium report.

Corollary 4.3 (Orthogonality: Two Segments)**.** *In equilibrium, changes to the sample variance are not orthogonal to changes in the sample covariance:*

$$\nabla_{\boldsymbol{r}_j}\sigma^2_{s^*_j(\boldsymbol{r})}\cdot\nabla_{\boldsymbol{r}_j}\sigma_{s^*_j(\boldsymbol{r}),s^*_{-j}(\boldsymbol{r})} = 2c^{-2}n^{-1}(c-n^{-1}\kappa_1)\left((c-n^{-1}\kappa_1)\sigma_{s^*_j(\boldsymbol{r}),s^*_{-j}(\boldsymbol{r})} - n^{-1}\kappa_1\sigma^2_{s^*_j(\boldsymbol{r})}\right) \quad (77)$$

Of course, changes to the means of different segments and the means and variances of segments are also orthogonal.

Corollary 4.4 (Sample Stats: Two Segments)**.** *The mean, variance, and covariance are*

$$\mu_{r^*(x_j)} = \mu_{x_j} + n^{-1}\kappa_2(c-2n^{-1}\kappa_2)^{-1}(\mu_{x_j} - \overline{r}_S) \quad (78)$$

$$\sigma^2_{r^*(x_j)} = c^2(c-2n^{-1}\kappa_1)^{-2}\sigma^2_{x_j} \quad (79)$$

$$\sigma_{r^*(x_j),r^*(x_{-j})} = c^2(c-2n^{-1}\kappa_1)^{-2}\sigma_{x_j,x_{-j}}. \quad (80)$$

Although the manager would like to make the sample covariance as small as possible, she cannot do so without negatively affecting the variance.

4.3 Two Firms

In this section, we imagine that there are two firms in the economy. The main assumption is that each manager observes signals about her own firm, but not the signals observed by the other manager. As a result, the manager makes a best-guest about what the manager observed based on what she herself observed.

Just as in the two segment case, $\boldsymbol{e}|\Lambda$ is normally distributed with mean $\boldsymbol{\mu}_0 = \mu_0\mathbf{1}_2$ and precision $\lambda\Lambda$. Finally, the cash flows $\boldsymbol{d}$ are normally distributed with mean $\boldsymbol{e}$ and precision $\lambda\Lambda$. Signals $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n$ are drawn from the same distribution as d. In contrast to the two segment case, the manager only observes the n signals regarding her firm, which we collect in the vector x_j. Importantly, we must compute firm j's expectation of $-j$'s signal given its own. Let

$$\chi_0 = \frac{\omega_0}{\psi_0}. \quad (81)$$

Because the two firms are symmetric, χ_0 is the correlation between the firms' cash flows.

Lemma 4.3. $\mathrm{E}(x_{-j}|x_j) = \mu_0\mathbf{1} + \chi_0(x_j - \mu_0\mathbf{1})$.

Lemma 4.3 is related to the standard results regarding conditional distributions of multivariate t-distributions. If the managers' signals are expected to be perfectly correlated (i.e.,

$\omega_0 = \psi_0$), then $\mathrm{E}(x_{-j}|x_j) = x_j$; if they are expected to be uncorrelated, (i.e., $\omega_0 = 0$), then $\mathrm{E}(x_{-j}|x_j) = 0$.

Assuming that the supply of shares is fixed, the payoff of the wealth portfolio is $d_1 + d_2$. Given the stochastic discount factor

$$1 - \gamma((d_1 - \mathrm{E}(d_1|r_1, r_2)) + (d_2 - \mathrm{E}(d_2|r_1, r_2))) \tag{82}$$

the price of firm j is

$$p_j(r) = \mathrm{E}(d_j|r) - \gamma(\mathrm{Var}(d_j|r) + \mathrm{Cov}(d_j, d_{-j}|r)). \tag{83}$$

Manager j's problem is to choose a bias

$$b_j(x_j) \in \underset{b_j \in \mathbb{R}^n}{\operatorname{argmax}} \left\{ \mathrm{E}\left(p_j(x_j + b_j, x_{-j} + b_{-j}) - \tfrac{c}{2} b_j' b_j \right) \right\}. \tag{84}$$

Conjecture a symmetric equilibrium in which

$$b(x_j) = \rho_0 + \rho_1 x_j \tag{85}$$

$$p(r_j, r_{-j}) = \pi_0 + \pi_1' r_j + \pi_2' r_{-j} + \tfrac{1}{2} r_j' \pi_3 r_j + r_j' \pi_4 r_{-j} + \tfrac{1}{2} r_{-j}' \pi_5 r_{-j}. \tag{86}$$

Lemma 4.4 (Price Formation: Two Firms)**.** *Given the manager's bias function in equation (85), the market extracts from the report r_j the signal*

$$s(r_j) = (I + \rho_1)^{-1}(r_j - \rho_0) \tag{87}$$

from which it forms the price

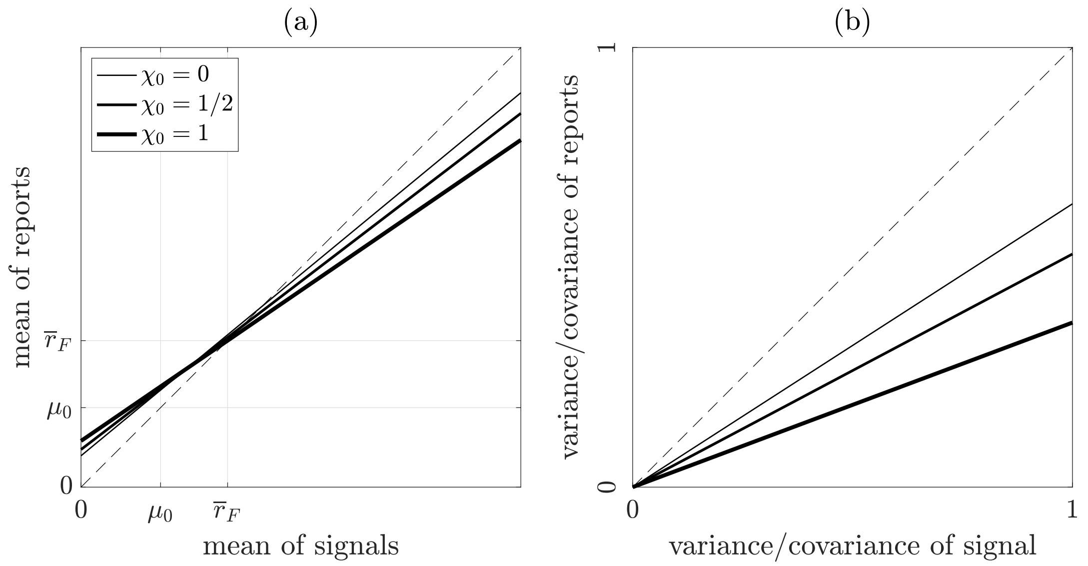
$$\begin{aligned} p_j(r_j, r_{-j}) = \eta_0 \mu_0 + \eta_1 \mu_{s(r_j)} - \gamma(\zeta_0 \psi_0 + \zeta_1 \sigma^2_{s(r_j)} + \zeta_2 (\mu_{s(r_j)} - \mu_0)^2) \\ - \gamma\left(\zeta_0 \omega_0 + \zeta_1 \sigma_{s(r_j), s(r_{-j})} + \zeta_2 (\mu_{s(r_j)} - \mu_0)(\mu_{s(r_{-j})} - \mu_0)\right). \end{aligned} \tag{88}$$

Equation (88) illustrates the manager's incentives to bias. Now under the equilibrium conjecture, $s(r_{-j}) = x_{-j}$. Therefore,

$$\mathrm{E}(\mu_{x_{-j}}|x_j) = \mu_0 + \chi_0(\mu_{x_j} - \mu_0) \tag{89}$$

$$\mathrm{E}(\sigma_{s(r_j), x_{-j}}|x_j) = \chi_0 \sigma_{s(r_j), x_j} \tag{90}$$

Figure 4: **Sample Mean and Variance: Two Firms.** This figure visualizes the mapping from the sample mean μ_{x_j} and variance $\sigma^2_{x_j}$ (or covariance $\sigma_{x_j,x_{-j}}$) of the signals to the sample mean $\mu_{r(x_j)}$ and variance $\sigma^2_{r(x_j)}$ (or covariance $\sigma_{r(x_j),r(x_{-j})}$) of reports for three different values of χ_0. The dashed line is the 45° line and represents the mapping under truth-telling. The parameters used are $\mu_0 = 1$, $\lambda_0 = 100$, $\nu_0 = 3$, $n = 10$, $c = 0.05$, and $\gamma = 0.95cn/12\zeta_1$.



Taking expectations yields

$$\begin{aligned} \mathrm{E}(p_j(r_j, r_{-j})|x_j) = \eta_0\mu_0 + \eta_1\mu_{s(r_j)} - \gamma(\zeta_0\psi_0 + \zeta_1\sigma^2_{s(r_j)} + \zeta_2(\mu_{s(r_j)} - \mu_0)^2) \\ - \gamma\left(\zeta_0\omega_0 + \zeta_1\chi_0\sigma_{s(r_j),x_j} + \zeta_2\chi_0(\mu_{s(r_j)} - \mu_0)(\mu_{x_j} - \mu_0)\right). \end{aligned} \tag{91}$$

Define

$$\overline{r}_F = \mu_0 + \frac{\eta_1}{3\gamma\zeta_2} \tag{92}$$

and

$$\pi_{0,F} = \eta_0\mu_0 - \gamma\left(\zeta_0(\psi_0 + \omega_0) + 2\zeta_2\mu_0^2\right) \tag{93}$$

$$\pi_{1,F} = -3\overline{r}_F\pi_{4,F}\mathbf{1} = (\eta_1 + 3\gamma\zeta_2\mu_0)m \tag{94}$$

$$\pi_{2,F} = \gamma\zeta_2\mu_0 m \tag{95}$$

$$\pi_{3,F} = -2\gamma(\zeta_1 C + \zeta_2 mm') \tag{96}$$

$$\pi_{4,F} = -\gamma(\zeta_1 C + \zeta_2 mm'). \tag{97}$$

$\pi_{0,F}, \ldots, \pi_{4,F}$ are analogous to $\pi_{0,B}$, $\pi_{1,B}$, and $\pi_{2,B}$ in equations (19), (18), and (17).

Proposition 4.3 (Equilibrium: Two Firms)**.** *If $c > 4\gamma(2+\chi_0)\zeta_1/n$, the low-cost equilibrium is of the form*

$$\pi_5^* = 0 \tag{98}$$

$$\pi_4^* = (2+\chi_0)^{-1}(\kappa_1 C + \kappa_2 m m') \tag{99}$$

$$\pi_3^* = 2\pi_4^* \tag{100}$$

and hence

$$\rho_1^* = \kappa_1(c - n^{-1}\kappa_1)^{-1}C + \kappa_2(c - n^{-1}\kappa_2)^{-1}mm', \tag{101}$$

where κ_1 and κ_2 are roots of

$$c^2\kappa_1 = -\gamma(2+\chi_0)\zeta_1(c - n^{-1}\kappa_1)^2 \tag{102}$$

$$c^2\kappa_2 = -\gamma(2+\chi_0)\zeta_2(c - n^{-1}\kappa_2)^2. \tag{103}$$

Corollary 4.5 (Orthogonality: Two Firms)**.** *In equilibrium, changes to the sample variance are not orthogonal to changes in the sample covariance:*

$$\nabla_{r_j}\sigma^2_{s^*(r_j)} \cdot \nabla_{r_j}\sigma_{s^*(r_j),x_j} = 2c^{-2}n^{-1}(c - n^{-1}\kappa_1)^2\sigma_{s^*(r_j),x_j}. \tag{104}$$

In this extension, we are principally interested in the role of the correlation between the firms' cash flows as measured by χ_0.

Corollary 4.6. *In the light-catering equilibrium, $\kappa_1, \kappa_2 \in (-nc, 0)$. Moreover, κ_1 and κ_2 are strictly decreasing in χ_0.*

Following our analysis in previous iterations of the model, we investigate the mapping from signal moments to report moments.

Corollary 4.7 (Sample Stats: Two Firms)**.** *The mean, variance, and covariance are*

$$\mu_{r^*(x_j)} = \mu_{x_j} + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}(\mu_{x_j} - \widetilde{r}) \tag{105}$$

$$\sigma_{r^*(x_i),r^*(x_j)} = c^2(c - n^{-1}\kappa_1)^{-2}\sigma_{x_i,x_j} \tag{106}$$

where $\widetilde{r} = -m'\rho_1^{*-1}\rho_0^*$.

Together with 4.6, we see that when the firms' cash flows are less correlated, the covariance discount in the price is smaller, and so the manager has to bias less. This effect is illustrated in Figure 4 In contrast to previous analyses, where there was a clear expression for $\overline{r}$, here it is more complicated. $\widetilde{r}$ is a highly non-linear function of χ_0.

5 Conclusion

In this paper, we answer the question of how firms optimally shape their messages under uncertainty by developing a unified signal-jamming framework that admits closed-form solutions and comparative statics. Under priced, stochastic variance (and covariance), managers endogenously distort their reports along four dimensions—consistency, confirmation, dissociation, and divergence. We obtain these distortions without appealing to ad-hoc behavioral biases. These distortions emerge in rational expectations equilibrium in which the manager trades off the benefit of influencing the market's posterior against the cost of biasing and cannot credibly commit to truthful reporting.

Our comparative-statics results reveal that when cash flows are more volatile or the signal space is richer, the marginal benefit to consistency (and, in covariance-priced settings, dissociation) exceeds that of confirmation (or divergence). We show that any two distortions except consistency and dissociation can vary independently, and we characterize how variance uncertainty drives shifts in confirmation and consistency. Extending to multiple segments and multiple firms under CAPM pricing, the model highlights how correlation structures shape a firm's incentive to bias.

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A Proofs

Proof of Lemma 3.1. Standard calculations show that the posterior $(e, \tau)|x$ is Normal-Gamma with parameters

$$\mu_n = \eta_0 \mu_0 + \eta \mu_x \tag{107}$$

$$\lambda_n = \lambda_0 + n \tag{108}$$

$$\alpha_n = \alpha_0 + n/2 \tag{109}$$

$$\beta_n = \beta_0 + (n/2)\left(\sigma_x^2 + \eta_0(\mu_x - \mu_0)^2\right). \tag{110}$$

Therefore, the posterior mean and variance of d are

$$\mathrm{E}(d|x) = \mathrm{E}(e|x) = \mu_n = \eta_0 \mu_0 + \eta_1 \mu_x \tag{111}$$

and

$$\mathrm{Var}(d|x) = \mathrm{E}(1/\tau|x) + \mathrm{Var}(e|x) = \frac{(\lambda_n + 1)\beta_n}{\lambda_n(\alpha_n - 1)} = \theta_0 + \theta_1 \sigma_x^2 + \theta_2(\mu_x - \mu_0)^2 \tag{112}$$

respectively. ■

Proof of Proposition 3.1. Note that

$$\begin{aligned}(\mu_x - \mu_0)^2 &= (m'(x - \mu_0 \mathbf{1}))^2 && (113)\\ &= (x - \mu_0 \mathbf{1})' m m' (x - \mu_0 \mathbf{1}) && (114)\\ &= x' m m' x - 2\mu_0 \mathbf{1}' m m' x + \mu_0^2 \mathbf{1}' m m' \mathbf{1} && (115)\\ &= x' m m' x - 2\mu_0 m' x + \mu_0^2. && (116)\end{aligned}$$

From equations (2), (13), (14), (6), and (7), we have

$$p(r) = \eta_0 + \eta_1 m'(x - \mu_0 \mathbf{1}) - \gamma(\theta_0 + \theta_1 x' C x + \theta_2(x' m m' x - 2\mu_0 m' x + \mu_0^2)) \tag{117}$$

from which equations (19), (18), and (17) follow. ■

Proof of Lemma 3.2. Consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1)$. The manager's report is

$$r = x + b^*(x) = x + (\rho_0 + \rho_1 x) = \rho_0 + (I + \rho_1)x \tag{118}$$

from which the market extracts the signal in equation (22). Standard calculations show that the posterior $(e, \tau)|s(r)$ is Normal-Gamma with parameters

$$\mu_n = \eta_0\mu_0 + \eta_1\mu_{s(r)} \tag{119}$$

$$\lambda_n = \lambda_0 + n \tag{120}$$

$$\alpha_n = \alpha_0 + n/2 \tag{121}$$

$$\beta_n = \beta_0 + (n/2)\left(\sigma^2_{s(r)} + \eta_0(\mu_{s(r)} - \mu_0)^2\right). \tag{122}$$

Therefore, the posterior mean and variance of d are

$$\mathrm{E}(d|x) = \mathrm{E}(e|x) = \mu_n = \eta_0\mu_0 + \eta_1\mu_{s(r)} \tag{123}$$

and

$$\mathrm{Var}(d|x) = \mathrm{E}(1/\tau|x) + \mathrm{Var}(e|x) = \frac{(\lambda_n + 1)\beta_n}{\lambda_n(\alpha_n - 1)} = \theta_0 + \theta_1\sigma^2_{s(r)} + \theta_2(\mu_{s(r)} - \mu_0)^2 \tag{124}$$

respectively. ■

Proof of Proposition 3.2. Consider first the manager's problem given $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2)$. The first-order condition is

$$0 = \pi_1 + \pi_2(x + b^*(x)) - cb^*(x). \tag{125}$$

Hence,

$$b^*(x) = \tilde{\rho}_0(\boldsymbol{\pi}) + \tilde{\rho}_1(\boldsymbol{\pi})x \tag{126}$$

where

$$\tilde{\rho}_0(\boldsymbol{\pi}) = (cI - \pi_2)^{-1}\pi_1 \tag{127}$$

$$\tilde{\rho}_1(\boldsymbol{\pi}) = (cI - \pi_2)^{-1}\pi_2. \tag{128}$$

Next, consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1)$. Recall the signal $s(r)$ and price $p(r)$ given in equations (22) and (23) of Lemma 3.2. For ease of exposition, let $P = (I + \rho_1)^{-1}$ so that $s(r) = P(r - \rho_0)$. Now

$$s(r) - \mu_0\mathbf{1} = P(r - \rho_0) - \mu_0\mathbf{1} = Pr - (P\rho_0 + \mu_0\mathbf{1}) \tag{129}$$

and hence

$$\mu_{s(r)} - \mu_0 = m'(s(r) - \mu_0\mathbf{1}) = m'Pr - m'(P\rho_0 + \mu_0\mathbf{1}) \tag{130}$$

and hence

$$(\mu_{s(r)} - \mu_0)^2 = (s(r) - \mu_0\mathbf{1})'mm'(s(r) - \mu_0\mathbf{1}) \tag{131}$$
$$= r'P'mm'Pr - 2(P\rho_0 + \mu_0\mathbf{1})'mm'Pr + (P\rho_0 + \mu_0\mathbf{1})'mm'(P\rho_0 + \mu_0\mathbf{1}). \tag{132}$$

Moreover,

$$\sigma^2_{s(r)} = s(r)'Cs(r) = r'P'CPr - 2\rho_0'P'CPr + \rho_0'P'CP\rho_0. \tag{133}$$

Therefore, the market price is

$$p(r) = \mathrm{E}(e|r) - \gamma\mathrm{Var}(e|r) = \tilde{\pi}_0(\boldsymbol{\rho}) + \tilde{\pi}_1(\boldsymbol{\rho})'r + \tfrac{1}{2}r'\tilde{\pi}_2(\boldsymbol{\rho})r \tag{134}$$

where

$$\tilde{\pi}_0(\boldsymbol{\rho}) = \mu_0 - \eta_1 m'(P\rho_0 + \mu_0\mathbf{1})$$
$$- \gamma(\theta_0 + \theta_1\rho_0'P'CP\rho_0 + \theta_2(P\rho_0 + \mu_0\mathbf{1})'mm'(P\rho_0 + \mu_0\mathbf{1})) \tag{135}$$
$$\tilde{\pi}_1(\boldsymbol{\rho}) = \eta_1 P'm + 2\gamma(\theta_1 P'CP\rho_0 + \theta_2 P'mm'(P\rho_0 + \mu_0\mathbf{1})) \tag{136}$$
$$\tilde{\pi}_2(\boldsymbol{\rho}) = -2\gamma(\theta_1 P'CP + \theta_2 P'mm'P) = P'\pi_{2,B}P. \tag{137}$$

An equilibrium jointly satisfies equations (127), (128), (135), (136), and (137). Now since $\pi_{2,B}$ is symmetric, so too is π_2^*, which is given by

$$c^2\pi_2^* = (cI - \pi_2^*)\pi_{2,B}(cI - \pi_2^*) = -2\gamma(cI - \pi_2^*)(\theta_1 C + \theta_2 mm')(cI - \pi_2^*). \tag{138}$$

Since all terms in equation (138) live in the span of C and mm', so too must the solution π_2^*:

$$\pi_2^* = \kappa_1 C + \kappa_2 mm' \tag{139}$$

for constants κ_1 and κ_2. Therefore,

$$
\begin{aligned}
&c^2(\kappa_1 C + \kappa_2 mm') \\
&\quad = -2\gamma(cI - (\kappa_1 C + \kappa_2 mm'))(\theta_1 C + \theta_2 mm')(cI - (\kappa_1 C + \kappa_2 mm')) && (140) \\
&\quad = -2\gamma(cI - (\kappa_1 C + \kappa_2 mm'))(\theta_1(c - n^{-1}\kappa_1)C + \theta_2(c - n^{-1}\kappa_2)mm') && (141) \\
&\quad = -2\gamma(\theta_1(c - n^{-1}\kappa_1)^2 C + \theta_2(c - n^{-1}\kappa_2)^2 mm'), && (142)
\end{aligned}
$$

from which we obtain the quadratic equations in (26) and (27). Equations (26) and (27) have discriminates

$$\Delta_1 = (c - 8\gamma\theta_1 n^{-1})c^3 \quad (143)$$

$$\Delta_2 = (c - 8\gamma\theta_2 n^{-1})c^3 \quad (144)$$

so real roots exist if and only if $c > 8\gamma \max\{\theta_1, \theta_2\}/n = 8\gamma\theta_1/n$. Note that the eigenvalues of π_2^* are $n^{-1}\kappa_2 < 0$ (with multiplicity one and eigenvector m) and $n^{-1}\kappa_1$ (with multiplicity $n-1$). Therefore, π_2 is symmetric negative definite as conjectured. Next, we have that

$$
\begin{aligned}
\rho_1^* &= (cI - \pi_2^*)^{-1}\pi_2^* && (145) \\
&= (cI - (\kappa_1 C + \kappa_2 mm'))^{-1}(\kappa_1 C + \kappa_2 mm') && (146) \\
&= c^{-1}(I + \kappa_1(c - n^{-1}\kappa_1)^{-1}C + \kappa_2(c - n^{-1}\kappa_2)^{-1}mm')(\kappa_1 C + \kappa_2 mm') && (147) \\
&= \kappa_1(c - n^{-1}\kappa_1)^{-1}C + \kappa_2(c - n^{-1}\kappa_2)^{-1}mm' && (148)
\end{aligned}
$$

and hence

$$P^* = (I + \rho_1^*)^{-1} = I - c^{-1}\kappa_1 C - c^{-1}\kappa_2 mm' \quad (149)$$

In terms of π_1^*, $P^*\rho_0^* = c^{-1}\pi_1^*$. Therefore,

$$
\begin{aligned}
\pi_1^* &= \eta_1 P^* m + 2\gamma(c^{-1}\theta_1 P^* C\pi_1^* + \theta_2 P^* mm'(c^{-1}\pi_1^* + \mu_0\mathbf{1})) && (150) \\
&= (\eta_1 + 2\gamma\theta_2\mu_0)P^* m + 2\gamma c^{-1}P^*(\theta_1 C + \theta_2 mm')\pi_1^* && (151) \\
&= P^*\pi_{1,B} - c^{-1}P^*\pi_{2,B}\pi_1^*. && (152)
\end{aligned}
$$

Solving for π_1^*, we obtain

$$\pi_1^* = (I + c^{-1}P^*\pi_{2,B})^{-1}P^*\pi_{1,B} \tag{153}$$
$$= P^{*2}\pi_{1,B} \tag{154}$$
$$= P^*\pi_{2,B}P^*\pi_{2,B}^{-1}\pi_{1,B} \tag{155}$$
$$= -\bar{r}\pi_2^*\mathbf{1}, \tag{156}$$

where the second line follows from the fact that $P^* = c^{-1}(cI - \pi_2^*)$, the third line from the fact that all of the terms are linear combinations of I, C, and mm' and therefore commute, and the fourth line from the facts that $\pi_2^* = P^*\pi_{2,B}P^*$ and $\pi_{2,B}^{-1}\pi_{1,B} = -\bar{r}\mathbf{1}$. Hence

$$\rho_0^* = (cI - \pi_2^*)^{-1}\pi_1^* = (cI - \pi_2^*)^{-1}\pi_2^*\pi_2^{*-1}\pi_1^* = -\bar{r}\rho_1^*\mathbf{1} \tag{157}$$

and

$$\begin{aligned}\pi_0^* = \mu_0 - \eta_1 m'(P^*\rho_0^* + \mu_0\mathbf{1}) \\ - \gamma(\theta_0 + \theta_1\rho_0^{*\prime}P^*CP^*\rho_0^* + \theta_2(P^*\rho_0^* + \mu_0\mathbf{1})'mm'(P^*\rho_0^* + \mu_0\mathbf{1}))\end{aligned} \tag{158}$$
$$= \pi_{0,B} - c^{-1}\pi_{1,B}'\pi_1^* + \tfrac{1}{2}c^{-2}\pi_1^{*\prime}\pi_{2,B}\pi_1^* \tag{159}$$

having used the fact that $P^*\rho_0^* = c^{-1}\pi_1^*$. ■

Proof of Corollary 3.1. Equation (31) follows from the chain rule for gradients. Using Proposition 3.2 and equation (149), we have

$$\nabla_r\mu_{s^*(r)} \cdot \nabla_r\sigma^2_{s^*(r)} = 2m'(I + \rho_1^*)^{-2}Cs^*(r) \tag{160}$$
$$= 2m'(I - \kappa_1c^{-1}C - \kappa_2c^{-1}mm')^2Cs^*(r) \tag{161}$$
$$= 0 \tag{162}$$

as desired. ■

Proof of Corollary 3.2. The result follows from equation (23) and $\theta_2/\theta_1 = \lambda_0/(\lambda_0 + n)$. ■

Proof of Lemma 3.3. Let

$$\underline{A} = (I + \rho_1^*)\pi_2^*(I + \rho_1^*) \tag{163}$$
$$\underline{B} = (\pi_1^{*\prime} + \rho_0^{*\prime}\pi_2^*)(I + \rho_1^*) \tag{164}$$
$$\underline{C} = \pi_0^* + \pi_1^{*\prime}\rho_0^* + \tfrac{1}{2}\rho_0^{*\prime}\pi_2^*\rho_0^* \tag{165}$$

so that

$$p^*(r^*(x)) = \tfrac{1}{2}x'\underline{A}x + \underline{B}x + \underline{C} \tag{166}$$

(using the fact that π_2^* and ρ_1^* are symmetric). It suffices to show that $\underline{A} = \pi_{2,B}$, $\underline{B} = \pi_{1,B}$, and $\underline{C} = \pi_{0,B}$. Note that

$$\begin{aligned}\underline{A} &= (I + \rho_1^*)\pi_2^*(I + \rho_1^*) &(167)\\ &= c^2(cI - \pi_2^*)^{-1}\pi_2^*(cI - \pi_2^*)^{-1}. &(168)\end{aligned}$$

Pre- and post-multiplication by $cI - \pi_2^*$ together with equation (138) imply that that $\underline{A} = \pi_{2,B}$. Using the facts that $\pi_1^* = -\overline{r}\pi_2^*\mathbf{1}$ and $\rho_0^* = -\overline{r}\rho_1^*\mathbf{1}$ (Proposition 3.2), we have that

$$\begin{aligned}\underline{B} &= -\overline{r}(\mathbf{1}'\pi_2^* + \mathbf{1}'\rho_1\pi_2^*)(I + \rho_1^*) &(169)\\ &= -\overline{r}\mathbf{1}'(I + \rho_1)\pi_2^*(I + \rho_1^*) &(170)\\ &= -\overline{r}\mathbf{1}'\underline{A} &(171)\\ &= \pi_{1,B}. &(172)\end{aligned}$$

Using the results from Propositions 3.1 and 3.2, we have that

$$\begin{aligned}\pi_{1,B}'\pi_1^* &= -(\eta_1 + 2\gamma\theta_2\mu_0)\overline{r}n^{-1}\kappa_2 = -2\gamma\theta_2\overline{r}^2n^{-1}\kappa_2 &(173)\\ \pi_1^{*\prime}\pi_{2,B}\pi_1^* &= -2\gamma\theta_2\overline{r}^2n^{-2}\kappa_2^2 &(174)\\ \pi_1^{*\prime}\rho_0^* &= \overline{r}^2n^{-1}\kappa_2^2(c - n^{-1}\kappa_2)^{-1} = -2\gamma\theta_2\overline{r}^2c^{-2}n^{-1}\kappa_2(c - n^{-1}\kappa_2) &(175)\\ \rho_0^{*\prime}\pi_2^*\rho_0^* &= \overline{r}^2n^{-2}\kappa_2^3(c - n^{-1}\kappa_2)^{-2} = -2\gamma\theta_2\overline{r}^2c^{-2}n^{-2}\kappa_2^2. &(176)\end{aligned}$$

From equation (30),

$$\underline{C} = \pi_{0,B} - c^{-1}\pi'_{1,B}\pi^*_1 + \tfrac{1}{2}c^{-2}\pi^{*\prime}_1\pi_{2,B}\pi^*_1 + \pi^{*\prime}_1\rho^*_0 + \tfrac{1}{2}\rho^{*\prime}_0\pi^*_2\rho^*_0 \tag{177}$$
$$= \pi_{0,B} - 2\gamma\theta_2\overline{r}^2(-c^{-1}n^{-1}\kappa_2 + \tfrac{1}{2}c^{-2}n^{-2}\kappa_2^2 + c^{-2}n^{-1}\kappa_2(c - n^{-1}\kappa_2) + \tfrac{1}{2}c^{-2}n^{-2}\kappa_2^2) \tag{178}$$
$$= \pi_{0,B} \tag{179}$$

as desired. ■

Proof of Proposition 3.3. From Lemma 3.3, $\mathrm{E}(p^*(r^*(x))$ is constant across equilibria. We therefore wish to compute $\mathrm{E}(b^*(x)'b^*(x))$. From equation (25),

$$\rho^{*2}_1 = n^{-1}\kappa_1^2(c - n^{-1}\kappa_1)^{-2}C + n^{-1}\kappa_2^2(c - n^{-1}\kappa_2)^{-2}mm' \tag{180}$$
$$= -2\gamma c^{-2}n^{-1}(\theta_1\kappa_1 C + \theta_2\kappa_2 mm'), \tag{181}$$

where the second line follows from equations (26) and (27). Two facts follow. First,

$$\mathrm{tr}(\rho^{*2}_1) = -2\gamma c^{-2}n^{-1}(\theta_1\kappa_1\,\mathrm{tr}(C) + \theta_2\kappa_2\,\mathrm{tr}(mm')) \tag{182}$$
$$= -2\gamma c^{-2}n^{-1}(\theta_1\kappa_1(1 - n^{-1}) + \theta_2\kappa_2 n^{-1}) \tag{183}$$

having used the fact that $\mathrm{tr}(C) = 1 - n^{-1}$. Second,

$$\mathbf{1}'\rho^{*2}_1\mathbf{1} = -2\gamma c^{-2}n^{-1}\theta_2\kappa_2 \tag{184}$$

From Proposition 3.2, we have $b^*(x) = \rho^*_1(x - \overline{r}\mathbf{1})$. Hence,

$$\mathrm{E}(b^*(x)'b^*(x)) = \mathrm{E}((x - \overline{r}\mathbf{1})'\rho^{*2}_1(x - \overline{r}\mathbf{1})) \tag{185}$$
$$= \mathrm{tr}(\rho^{*2}_1) + (\mu_0 - \overline{r})^2\mathbf{1}'\rho^{*2}_1\mathbf{1} \tag{186}$$
$$= -2\gamma c^{-2}n^{-1}(\theta_1\kappa_1(1 - n^{-1}) + \theta_2\kappa_2 n^{-1} + (\mu_0 - \overline{r})^2\theta_2\kappa_2). \tag{187}$$

Therefore, the manager's expected utility is greatest when κ_1 and κ_2 are the larger roots of equations (26) and (27) respectively. ■

Proof of Lemma 3.4. From equations (26) and (27), we have that for $k \in \{1, 2\}$,

$$c^2\kappa_k = -2\gamma\theta_k(c - n^{-1}\kappa_k)^2. \tag{188}$$

where κ_k is the larger root (following the discussion of Section 3.4). Let

$$Q_k(\kappa) = c^2\kappa + 2\gamma\theta_k(c - n^{-1}\kappa)^2 \tag{189}$$

so that $Q_k(\kappa_k) = 0$. Note that $Q_k(-nc) = -nc^3 + 8\gamma\theta_k c^2 < 0$ (see equations 143 and 144). Since κ_k is the larger root, it follows that $-nc < \kappa_k$ and hence $c^2 > n^{-2}\kappa_k^2$. Differentiating equation (188) with respect to θ_k and rearranging yields

$$\frac{d\kappa_k}{d\theta_k} = -\frac{c^2\kappa_k^2}{2\gamma\theta_k^2(c^2 - n^{-2}\kappa_k^2)} < 0. \tag{190}$$

Evidently, θ_1 and θ_2 are strictly increasing in υ. A similar calculation shows that κ_k is strictly decreasing in γ. Next,

$$\frac{d\kappa_k}{dc} = \frac{2n^{-1}\kappa_k^2}{c(c + n^{-1}\kappa_k)} > 0. \tag{191}$$

We can rewrite equation (188) as

$$c^2 n^{-1}\kappa_k = -2\gamma n^{-1}\theta_k(c - n^{-1}\kappa_k)^2. \tag{192}$$

Differentiating with respect to n and rearranging yields

$$\frac{d(n^{-1}\kappa_k)}{dn} = -\frac{2\gamma(c - n^{-1}\kappa_k)^3}{c^2(c + n^{-1}\kappa_k)} \cdot \frac{d(n^{-1}\theta_k)}{dn} > 0 \tag{193}$$

as desired. ■

Proof of Lemma 3.5. From equation (25), $I + \rho_1^*$ has eigenvalue $c(c - n^{-1}\kappa_2)^{-1}$ (with multiplicity one and eigenvector m) and $c(c - n^{-1}\kappa_1)^{-1}$ (with multiplicity $n - 1$). From Lemma 3.4, these eigenvalues are strictly greater than zero and strictly less than one. The result follows from standard results regarding affine mappings. ■

Proof of Lemma 3.5.1. Note that

$$\begin{aligned}
\mu_{r^*(x)} &= m'r(x) && (194)\\
&= m'(x + \rho_1^*(x - \overline{r}\mathbf{1})) && (195)\\
&= m'x + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}m'(x - \overline{r}\mathbf{1}) && (196)\\
&= \mu_x + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}(\mu_x - \overline{r}) && (197)
\end{aligned}$$

and

$$\sigma^2_{r^*(x)} = r(x)'Cr(x) \tag{198}$$
$$= (x' + (x' - \overline{r}\mathbf{1}')\rho_1^*)C(x + \rho_1^*(x - \overline{r}\mathbf{1})) \tag{199}$$
$$= x'Cx + 2x'C\rho_1^*(x - \overline{r}\mathbf{1}) + (x' - \overline{r}\mathbf{1}')\rho_1^* C\rho_1^*(x - \overline{r}\mathbf{1}) \tag{200}$$
$$= \sigma_x^2(1 + 2n^{-1}\kappa_1(c - n^{-1}\kappa_1)^{-1} + n^{-2}\kappa_1^2(c - n^{-1}\kappa_1)^{-2}) \tag{201}$$
$$= c^2(c - n^{-1}\kappa_1)^{-2}\sigma_x^2 \tag{202}$$

as desired. ■

Proof of Lemma 3.7. Using $x|e,\tau \sim \mathcal{N}(e\mathbf{1}, \tau^{-1}I)$ and $e|\tau \sim \mathcal{N}(\mu_0, \lambda_0^{-1}\tau^{-1})$, we have

$$\mathrm{E}(\mu_x) = \mathrm{E}(m'x) = \mathrm{E}(\mathrm{E}(m'x|e,\tau)) = \mathrm{E}(e) = \mu_0. \tag{203}$$

Next,

$$\mathrm{Var}(\mu_x) = \mathrm{Var}(m'x) \tag{204}$$
$$= \mathrm{E}(\mathrm{Var}(m'x|e,\tau)) + \mathrm{Var}(\mathrm{E}(m'x|e,\tau)) \tag{205}$$
$$= n^{-1}\mathrm{E}(1/\tau) + \mathrm{Var}(e) \tag{206}$$
$$= n^{-1} + \lambda_0^{-1}. \tag{207}$$

Next,

$$\mathrm{E}(\sigma_x^2) = \mathrm{E}(x'Cx) \tag{208}$$
$$= \mathrm{E}(\mathrm{E}(x'Cx|e,\tau)) \tag{209}$$
$$= \mathrm{E}(\mathrm{tr}(\tau^{-1}C) + e^2\mathbf{1}'C\mathbf{1}) \tag{210}$$
$$= \mathrm{E}(\mathrm{tr}(\tau^{-1}C)) \tag{211}$$
$$= \mathrm{E}(1/\tau)\,\mathrm{tr}(C) \tag{212}$$
$$= 1 - n^{-1} \tag{213}$$

as desired. ■

Proof of Proposition 3.4. First, consider $\mathrm{E}(\mu_{r^*(x)})$ from equation (34). Using equation (27), we have

$$\mathrm{E}(\mu_{r^*(x)}) = \mu_0 + c^{-2}(\lambda_0 + n)^{-1}(c - n^{-1}\kappa_2), \tag{214}$$

which, according to Lemma 3.4, is strictly increasing in υ. Next, consider $\mathrm{E}(\sigma^2_{r^*(x)})$ from equation (35). From Lemma 3.7, $\mathrm{E}(\sigma^2_x)$ is constant with respect to υ. Since $c^2(c - n^{-1}\kappa_1)^{-2}$ is strictly decreasing in υ (Lemma 3.4), so too is $\mathrm{E}(\sigma^2_{r^*(x)})$. An identical argument shows that $\mathrm{Var}(\mu_{r^*(x)})$ from equation (36) is also strictly decreasing in υ. Finally, the non-monotonicity of $\mathrm{E}((\mu_{r^*(x)} - \mu_0)^2)$ in equation (37) follows from the facts that (a) $\mathrm{Var}(\mu_{r^*(x)})$ is strictly decreasing in υ and (b) $\mathrm{E}(\mu_{r^*(x)})$ is strictly greater than μ_0 and strictly increasing in υ. An identical set of arguments deliver the results for γ. ■

Proof of Lemma 3.8. As a consequence of Lemma 3.3, $\mathcal{P}(n) = \mathrm{E}(p_F(x))$. From equations (19), (18), and (17),

$$p_F(x) = \eta_0\mu_0 - \gamma(\theta_0 + \theta_2\mu_0^2) + (\eta_1 + 2\gamma\theta_2\mu_0)\mu_x^2 - \gamma(\theta_1\sigma_x^2 + \theta_2\mu_x^2). \tag{215}$$

Therefore,

$$\mathcal{P}(n) = \mathrm{E}(p_F(x)) \tag{216}$$

$$= \mu_0 - \gamma(\theta_0 + \theta_1(1 - n^{-1}) + \theta_2(n^{-1} + \lambda_0^{-1})) \tag{217}$$

$$= \mu_0 - \frac{\gamma(2\beta_0 + n)(\lambda_0 + n + 1)}{(2\alpha_0 + n - 2)(\lambda_0 + n)}. \tag{218}$$

The result follows from the parameter assumption that $\beta_0 = \alpha_0 - 1$. ■

Proof of Proposition 3.5. By the Envelope theorem, $\Pi'(n) = \mathcal{P}'(n)$. ■

Proof of Proposition **??**. Using equations (15) and (27), we have that

$$\frac{n^{-1}\kappa_2}{c - n^{-1}\kappa_2} \cdot (\mu_0 - \overline{r}) = -\frac{n^{-1}\kappa_2}{c - n^{-1}\kappa_2} \cdot \frac{\eta_1}{2\gamma\theta_2} = n^{-1}c^{-2}(c - n^{-1}\kappa_2)\eta_1. \tag{219}$$

Moreover, equations (26) and (27) reveal that $n^{-1}\kappa_1 \to 0$ and $n^{-1}\kappa_2 \to 0$ as $n \to \infty$. Taking these facts to equations (34), (34), and (37) yield the desired results. ■

Proof of Lemma 4.1. Consider the markt's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1, \rho_2)$. The manager's report is

$$r(x, y) = x + b^*(x, y) = x + (\rho_0 + \rho_1 x + \rho_2 y) = \rho_0 + (I + \rho_1)x + \rho_2 y, \tag{220}$$

from which the market extracts the signal in equation (49). Standard calculations show that the posterior $((e,f),\Lambda)|(s(r),y)$ is Normal-Wishart with parameters

$$\mu_n = \eta_0\mu_0 + \eta_1\mu_{s(r)} \tag{221}$$

$$\xi_n = \eta_0\xi_0 + \eta_1\mu_y \tag{222}$$

$$\lambda_n = \lambda_0 + n \tag{223}$$

$$\nu_n = \nu_0 + n \tag{224}$$

$$\omega_n = \omega_0 + n(\sigma_{s(r,y),y} + \eta_0(\mu_{s(r,y)} - \mu_0)(\mu_y - \xi_0)) \tag{225}$$

Therefore, the posterior mean and variance of d are

$$\mathrm{E}(d|r) = \mathrm{E}(e|r) = \mu_n = \eta_0\mu_0 + \eta_1\mu_{s(r,y)} \tag{226}$$

and

$$\mathrm{Cov}(d,w|r,y) = \frac{(\lambda_n+1)\omega_n}{\lambda_n(\nu_n-3)} = \zeta_0\omega_0 + \zeta_1\sigma_{s(r),y} + \zeta_2(\mu_{s(r)} - \mu_0)(\mu_y - \xi_0) \tag{227}$$

respectively. The result follows from equation (45). ■

Proof of Proposition 4.1. Consider first the manager's problem given $\boldsymbol{\pi} = (\pi_0, \ldots, \pi_6)$. The first-order condition is

$$0 = \pi_1 + \pi_3(x + b^*(x,y)) + \pi_4 y - cb^*(x,y). \tag{228}$$

Hence,

$$b^*(x,y) = \tilde{\rho}_0(\boldsymbol{\pi}) + \tilde{\rho}_1(\boldsymbol{\pi})x + \tilde{\rho}_2(\boldsymbol{\pi})y, \tag{229}$$

where

$$\tilde{\rho}_0(\boldsymbol{\pi}) = (cI - \pi_3)^{-1}\pi_1 \tag{230}$$

$$\tilde{\rho}_1(\boldsymbol{\pi}) = (cI - \pi_3)^{-1}\pi_3 \tag{231}$$

$$\tilde{\rho}_2(\boldsymbol{\pi}) = (cI - \pi_3)^{-1}\pi_4. \tag{232}$$

Next, consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1, \rho_2)$. Recall the signal $s(r,y)$ and price $p(r,y)$ given in equations (49) and (50) of Lemma 4.1. Let $P = (I + \rho_1)^{-1}$ so that $s(r,y) = P(r - \rho_0 - \rho_2 y)$. Now

$$\mu_{s(r,y)} = m's(r,y) = m'P(r - \rho_0 - \rho_2 y) = m'Pr - m'P\rho_0 - m'P\rho_2 y \tag{233}$$

and hence

$$\begin{aligned}(\mu_y - \xi_0)(\mu_{s(r,y)} - \mu_0) &= (m'y - \xi_0)(m's(r,y) - \mu_0) && (234)\\
&= (y - \xi_0\mathbf{1})'mm'(s(r,y) - \mu_0\mathbf{1}) && (235)\\
&= (y - \xi_0\mathbf{1})'mm'(P(r - \rho_0 - \rho_2 y) - \mu_0\mathbf{1}) && (236)\\
&= \xi_0(\mu_0 + m'P\rho_0) - \xi_0 m'Pr + (\xi_0 m'P\rho_2 - \rho_0'P'mm' - \mu_0 m')y\\
&\quad + r'P'mm'y - y'mm'P\rho_2 y. && (237)\end{aligned}$$

Moreover,

$$\sigma_{s(r,y),y} = s(r,y)'Cy = (r - \rho_0 - \rho_2 y)'P'Cy = r'P'Cy - \rho_0'P'Cy - y'\rho_2'P'Cy. \quad (238)$$

Therefore, the market price is

$$\begin{aligned}p(r,y) &= \mathrm{E}(d|r,y) - \gamma\mathrm{Cov}(d,w|r,y) && (239)\\
&= \tilde{\pi}_0(\boldsymbol{\rho}) + \tilde{\pi}_1(\boldsymbol{\rho})'r + \tilde{\pi}_2(\boldsymbol{\rho})'y + \tfrac{1}{2}r'\tilde{\pi}_3(\boldsymbol{\rho})r + r'\tilde{\pi}_4(\boldsymbol{\rho})y + \tfrac{1}{2}y'\tilde{\pi}_5(\boldsymbol{\rho})y, && (240)\end{aligned}$$

where

$$\begin{aligned}\tilde{\pi}_0(\boldsymbol{\rho}) &= \eta_0\mu_0 - \eta_1 m'P\rho_0 - \gamma\zeta_0\omega_0 - \gamma\zeta_2\xi_0(\mu_0 + m'P\rho_0) && (241)\\
\tilde{\pi}_1(\boldsymbol{\rho}) &= \eta_1 P'm + \gamma\zeta_2\xi_0 P'm && (242)\\
\tilde{\pi}_2(\boldsymbol{\rho}) &= -\eta_1 m'P\rho_2 + \gamma\zeta_1 CP\rho_0 - \gamma\zeta_2(\xi_0\rho_2'P'm - mm'P\rho_0 - \mu_0 m) && (243)\\
\tilde{\pi}_3(\boldsymbol{\rho}) &= 0 && (244)\\
\tilde{\pi}_4(\boldsymbol{\rho}) &= -2\gamma P'(\zeta_1 C + \zeta_2 mm') && (245)\\
\tilde{\pi}_5(\boldsymbol{\rho}) &= \gamma\rho_2'P'(\zeta_1 C + \zeta_2 mm'). && (246)\end{aligned}$$

Therefore, $\pi_3^* = 0$ and hence $\rho_1^* = 0$ and hence $P^* = I$. Moreover, $\rho_0^* = c^{-1}\pi_1$ and $\rho_2^* = c^{-1}\pi_4$.

Finally,

$$\pi_0^* = \eta_0\mu_0 - c^{-1}\eta_1 m'\pi_1^* - \gamma\zeta_0\omega_0 - \gamma\zeta_2\xi_0(\mu_0 + c^{-1}m'\pi_1^*) \tag{247}$$

$$\pi_1^* = (\eta_1 + \gamma\zeta_2\xi_0)m \tag{248}$$

$$\pi_2^* = -c^{-1}\eta_1 m'\pi_4^* + c^{-1}\gamma\zeta_1 C\pi_1^* - \gamma\zeta_2(c^{-1}\xi_0\pi_4^{*\prime}m - c^{-1}mm'\pi_1^* - \mu_0 m) \tag{249}$$

$$\pi_3^* = 0 \tag{250}$$

$$\pi_4^* = -\gamma(\zeta_1 C + \zeta_2 mm') \tag{251}$$

$$\pi_5^* = 2c^{-1}\gamma\pi_4^{*\prime}(\zeta_1 C + \zeta_2 mm') \tag{252}$$

from which the result follows. ■

Proof of Corollary 4.1. From Proposition 4.1, the report is

$$r(x,y) = x + c^{-1}(\eta_1 + \gamma\zeta_2\xi_0)m - c^{-1}\gamma(\zeta_1 C + \zeta_2 mm')y. \tag{253}$$

The first result follows from equations (6) and (44). Now computing the gradients of equations (233) and (238) with respect to r, we obtain $\nabla\mu_{s(r,y)} = m$ and $\nabla\sigma^2_{s(r,y),y} = Cy$, from which it follows immediately that $\nabla\mu_{s(r,y)} \cdot \nabla\sigma^2_{s(r,y),y} = 0$. ■

Proof of Corollary 4.2. Note that

$$\mu_{r(x,y)} = m'r(x,y) \tag{254}$$

$$= m'(x + c^{-1}(\eta_1 + \gamma\zeta_2\xi_0)m - c^{-1}\gamma(\zeta_1 C + \zeta_2 mm')y) \tag{255}$$

$$= \mu_x + c^{-1}n^{-1}(\eta_1 + \gamma\zeta_2\xi_0) - \gamma c^{-1}n^{-1}\zeta_2\mu_y \tag{256}$$

and

$$\sigma_{r(x,y),y} = r(x,y)'Cry \tag{257}$$

$$= (x + c^{-1}(\eta_1 + \gamma\zeta_2\xi_0)m - c^{-1}\gamma(\zeta_1 C + \zeta_2 mm')y)'Cy \tag{258}$$

$$= \sigma_{x,y} - \gamma c^{-1}n^{-1}\zeta_1\sigma_y^2 \tag{259}$$

as desired. ■

Proof of Lemma 4.2. Consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1, \rho_2)$. The manager's report is

$$\text{vec}(\boldsymbol{r}) = \text{vec}(\boldsymbol{x}) + b^*(\boldsymbol{x}) \tag{260}$$

$$= \text{vec}(\boldsymbol{x}) + \mathbf{1}_2 \otimes \rho_0 + (I_2 \otimes \rho_1 + J_2 \otimes \rho_2)\text{vec}(\boldsymbol{x}) \tag{261}$$

$$= \mathbf{1}_2 \otimes \rho_0 + (I_2 \otimes (I + \rho_1) + J_2 \otimes \rho_2))\text{vec}(\boldsymbol{x}) \tag{262}$$

from which the market extracts the signal in equation (65). Standard calculations show that the posterior $(\boldsymbol{z}, \Lambda)|s(\boldsymbol{r})$ is Normal-Wishart with parameters

$$\boldsymbol{\mu}_n = \eta_0 \mu_0 + \eta_1 \mu_{s(\boldsymbol{r})} \tag{263}$$

$$\lambda_n = \lambda_0 + n \tag{264}$$

$$\nu_n = \nu_0 + n \tag{265}$$

$$\Psi_n = \Psi_0 + n(\sigma^2_{s(\boldsymbol{r})} + \eta_0(\mu_{s(\boldsymbol{r})} - \mu_0)'(\mu_{s(\boldsymbol{r})} - \mu_0)). \tag{266}$$

The posterior mean and variance of d are

$$\text{E}(d|\boldsymbol{r}) = \mathbf{1}_2'\text{E}(\boldsymbol{z}|\boldsymbol{r}) \tag{267}$$

$$= \mathbf{1}_2'\boldsymbol{\mu}_n \tag{268}$$

$$= \eta_0 \mathbf{1}_2'\boldsymbol{\mu}_0 + \eta_1 \mathbf{1}_2'\mu_{s(\boldsymbol{r})} \tag{269}$$

$$= 2\eta_0\mu_0 + \eta_1 \mathbf{1}_2'(I_2 \otimes m')P(\text{vec}(\boldsymbol{r}) - \mathbf{1}_2 \otimes \rho_0) \tag{270}$$

$$= 2\eta_0\mu_0 - \eta_1 \mathbf{1}_2'(I_2 \otimes m')P(\mathbf{1}_2 \otimes \rho_0) + \eta_1 \mathbf{1}_2'(I_2 \otimes m')P\text{vec}(\boldsymbol{r}) \tag{271}$$

and

$$\text{Var}(d|\boldsymbol{r}) = \mathbf{1}_2'\text{Cov}(\boldsymbol{z}|\boldsymbol{r})\mathbf{1}_2 \tag{272}$$

$$= (\lambda_n + 1)\lambda_n^{-1}(\nu - 3)^{-1}\Psi_n \tag{273}$$

$$= \zeta_0 \mathbf{1}_2'\Psi_0\mathbf{1}_2 + \zeta_1 \mathbf{1}_2'\sigma^2_{s(\boldsymbol{r})}\mathbf{1}_2 + \zeta_2 \mathbf{1}_2'(\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0)'(\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0)\mathbf{1}_2 \tag{274}$$

respectively. The result follows from equation (59). ■

Proof of Proposition 4.2. Consider first the manager's problem given $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2)$. The first-order condition is

$$0 = \mathbf{1}_2 \otimes \pi_1 + (I_2 \otimes \pi_2 + J_2 \otimes \pi_3)(\text{vec}(\boldsymbol{x}) + b) - cb. \tag{275}$$

Hence

$$b^*(\boldsymbol{x}) = (I_2 \otimes (cI - \pi_2) - J_2 \otimes \pi_3)^{-1}(\mathbf{1}_2 \otimes \pi_1 + (I_2 \otimes \pi_2 + J_2 \otimes \pi_3)\text{vec}(\boldsymbol{x})). \tag{276}$$

Note that

$$\begin{aligned}&(I_2 \otimes (cI - \pi_2) - J_2 \otimes \pi_3)^{-1} \\ &\qquad = (I_2 \otimes \pi_3^{-1} + J_2 \otimes (cI - \pi_2)^{-1})(I_2 \otimes ((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})).\end{aligned} \tag{277}$$

Hence, the manager's best response to $\boldsymbol{\pi}$ is

$$b^*(\boldsymbol{x}) = \mathbf{1}_2 \otimes \tilde{\rho}_0(\boldsymbol{\pi}) + (I_2 \otimes \tilde{\rho}_1(\boldsymbol{\pi}) + J_2 \otimes \tilde{\rho}_2(\boldsymbol{\pi}))\text{vec}(\boldsymbol{x}) \tag{278}$$

where

$$\tilde{\rho}_0(\boldsymbol{\pi}) = (\pi_3^{-1} + (cI - \pi_2)^{-1})((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})\pi_1 \tag{279}$$

$$\begin{aligned}\tilde{\rho}_1(\boldsymbol{\pi}) &= \pi_3^{-1}((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})\pi_2 \\ &\qquad + (cI - \pi_2)^{-1}((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})\pi_3\end{aligned} \tag{280}$$

$$\begin{aligned}\tilde{\rho}_2(\boldsymbol{\pi}) &= \pi_3^{-1}((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})\pi_3 \\ &\qquad + (cI - \pi_2)^{-1}((cI - \pi_2)\pi_3^{-1} - \pi_3(cI - \pi_2)^{-1})\pi_2.\end{aligned} \tag{281}$$

Next, consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1, \rho_2)$. Recall the signal $s(\boldsymbol{r})$ and price $p(\boldsymbol{r})$ given in equations (65) and (66). For ease of exposition, let

$$P = (I_2 \otimes (I + \rho_1) + J_2 \otimes \rho_2)^{-1} = I_2 \otimes P_1 + J_2 \otimes P_2, \tag{282}$$

where

$$P_1 = -\rho_2^{-1}\left(\rho_2(I + \rho_1)^{-1} - (I + \rho_1)\rho_2^{-1}\right)^{-1} \tag{283}$$

$$P_2 = (I + \rho_1)^{-1}\left(\rho_2(I + \rho_1)^{-1} - (I + \rho_1)\rho_2^{-1}\right)^{-1} \tag{284}$$

so that

$$s(\boldsymbol{r}) = P(\text{vec}(\boldsymbol{r}) - \mathbf{1}_2 \otimes \rho_0). \tag{285}$$

Using the mixed product property, we have that

$$(\mathbf{1}_2' \otimes m')P = (\mathbf{1}_2' \otimes m')(I_2 \otimes P_1 + J_2 \otimes P_2) = \mathbf{1}_2' \otimes m'(P_1 + P_2) \tag{286}$$

and

$$\begin{aligned} P'(\mathbf{1}_2\mathbf{1}_2' \otimes mm')P &= (I_2 \otimes P_1' + J_2 \otimes P_2')(\mathbf{1}_2\mathbf{1}_2' \otimes mm')(I_2 \otimes P_1 + J_2 \otimes P_2) & (287) \\ &= \mathbf{1}_2\mathbf{1}_2' \otimes (P_1 + P_2)'mm'(P_1 + P_2) & (288) \\ &= (I_2 + J_2) \otimes (P_1 + P_2)'mm'(P_1 + P_2) & (289) \end{aligned}$$

and (similarly)

$$P'(\mathbf{1}_2\mathbf{1}_2' \otimes C)P = (I_2 + J_2)' \otimes (P_1 + P_2)'C(P_1 + P_2). \tag{290}$$

Finally,

$$P(\mathbf{1}_2 \otimes \rho_0) = (I_2 \otimes P_1 + J_2 \otimes P_2)(\mathbf{1}_2 \otimes \rho_0) = \mathbf{1}_2 \otimes (P_1 + P_2)\rho_0. \tag{291}$$

The sample mean, $\mu_{s(\boldsymbol{r})}$, is a column vector of length two given by

$$\begin{aligned} \mu_{s(\boldsymbol{r})} &= s(\boldsymbol{r})'m & (292) \\ &= (I_2 \otimes m')\text{vec}(s(\boldsymbol{r})) & (293) \end{aligned}$$

and hence

$$\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0 = (I_2 \otimes m')(\text{vec}(s(\boldsymbol{r})) - \boldsymbol{\mu}_0 \otimes \mathbf{1}) \tag{294}$$

(note that $\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0$ is column vector of length two). We therefore have

$$\mathbf{1}_2'\mu_{s(\boldsymbol{r})} = (\mathbf{1}_2' \otimes m'(P_1 + P_2))\text{vec}(\boldsymbol{r}) - 2m'(P_1 + P_2)\rho_0 \tag{295}$$

and

$$\begin{aligned} \mathbf{1}_2'(\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0)(\mu_{s(\boldsymbol{r})} - \boldsymbol{\mu}_0)'\mathbf{1}_2 = \; &\text{vec}(\boldsymbol{r})'((I_2 + J_2) \otimes (P_1 + P_2)'mm'(P_1 + P_2))\text{vec}(\boldsymbol{r}) \\ &- (\mathbf{1}_2' \otimes 4\rho_0'(P_1 + P_2)'mm'(P_1 + P_2))\text{vec}(\boldsymbol{r}) \\ &+ 4\rho_0'(P_1 + P_2)'mm'(P_1 + P_2)\rho_0 \\ &- (\mathbf{1}_2' \otimes 4\mu_0 m'(P_1 + P_2))\text{vec}(\boldsymbol{r}) \\ &+ 8\mu_0 m'(P_1 + P_2)\rho_0 + 4\mu_0^2 & (296) \end{aligned}$$

and

$$\mathbf{1}_2'\sigma^2_{s(\boldsymbol{r})}\mathbf{1}_2 = \text{vec}(\boldsymbol{r})'((I_2 + J_2) \otimes (P_1 + P_2)'C(P_1 + P_2))\text{vec}(\boldsymbol{r}) \\ - (\mathbf{1}_2' \otimes 4\rho_0'(P_1 + P_2)'C(P_1 + P_2))\text{vec}(\boldsymbol{r}) \\ + 4\rho_0'(P_1 + P_2)'C(P_1 + P_2)\rho_0. \tag{297}$$

Therefore, the market price is

$$p(\boldsymbol{r}) = \tilde{\pi}_0(\boldsymbol{\rho}) + (\mathbf{1}_2' \otimes \tilde{\pi}_1(\boldsymbol{\rho})')\text{vec}(\boldsymbol{r}) + \frac{1}{2}\text{vec}(\boldsymbol{r})'(I_2 \otimes \tilde{\pi}_2(\boldsymbol{\rho}) + J_2 \otimes \tilde{\pi}_3(\boldsymbol{\rho}))\text{vec}(\boldsymbol{r}), \tag{298}$$

where

$$\tilde{\pi}_0(\boldsymbol{\rho}) = 2\eta_0\mu_0 - 2\eta_1 m'(P_1 + P_2)\rho_0 - \gamma(2\zeta_0(\omega_0 + \psi_0) \\ + 4\zeta_1\rho_0'(P_1 + P_2)'C(P_1 + P_2)\rho_0 + 4\zeta_2(\mu_0 + m'(P_1 + P_2)\rho_0)^2) \tag{299}$$

$$\tilde{\pi}_1(\boldsymbol{\rho}) = \eta_1(P_1 + P_2)'m + 4\gamma(\zeta_1(P_1 + P_2)'C(P_1 + P_2)\rho_0 \\ + \zeta_2(P_1 + P_2)'mm'((P_1 + P_2)\rho_0 + \mu_0\mathbf{1})) \tag{300}$$

$$\tilde{\pi}_2(\boldsymbol{\rho}) = \tilde{\pi}_3(\boldsymbol{\rho}) = -2\gamma(P_1 + P_2)'(\zeta_1 C + \zeta_2 mm')(P_1 + P_2). \tag{301}$$

An equilibrium jointly satisfies equations, (279), (280), (281), (299), (300), and (301). Now since C and mm' are symmetric, so too are π_2^* and π_3^*. Since $\pi_2^* = \pi_3^*$, equations (280) and (281) imply that

$$\rho_1^* = \rho_2^* = (\pi_2^{*-1} + (cI - \pi_2^*)^{-1})((cI - \pi_2^*)\pi_2^{*-1} - \pi_2^*(cI - \pi_2^*)^{-1})^{-1}\pi_2^* = (cI - 2\pi_2^*)^{-1}\pi_2^* \tag{302}$$

and hence

$$P_1^* + P_2^* = \left((I + \rho_1^*)^{-1} - \rho_1^{*-1}\right)\left(\rho_1^*(I + \rho_1^*)^{-1} - (I + \rho_1^*)\rho_1^{*-1}\right)^{-1} = c^{-1}(cI - 2\pi_2^*). \tag{303}$$

Similarly, $\rho_0^* = (cI - 2\pi_2^*)^{-1}\pi_1^*$. Using the fact that

$$\tilde{\pi}_2(\boldsymbol{\rho}) = \tilde{\pi}_3(\boldsymbol{\rho}) = -2\gamma(P_1 + P_2)'(\zeta_1 C + \zeta_2 mm')(P_1 + P_2), \tag{304}$$

we have that

$$c^2\pi_2^2 = -2\gamma(cI - 2\pi_2^2)(\zeta_1 C + \zeta_2 mm')(cI - 2\pi_2^2). \tag{305}$$

The subsequent analysis follows that found in the proof of Proposition 3.2 and is abridged for brevity. The solution of equation (305) is of the form

$$\pi_2^* = \pi_3^* = \kappa_1 C + \kappa_2 mm'. \tag{306}$$

Substitution of equation (306) into equation (305) yields

$$c^2(\kappa_1 C + \kappa_2 mm') = -2\gamma(\zeta_1(c - 2n^{-1}\kappa_1)^2 C + \zeta_2(c - 2n^{-1}\kappa_2)^2 mm'), \tag{307}$$

from which we obtain the quadratic equations in (73) and (74). The restriction $c > 16\gamma\zeta_1/n$ follows by requiring that equations (73) and (74) have real roots. Equations (306) and (302) yield the final expressions for ρ_1^* and ρ_2^*. Following equation (302) and (279), we have that

$$\rho_0^* = (cI - 2\pi_2^*)^{-1}\pi_1^* \tag{308}$$

and hence $(P_1^* + P_2^*)\rho_0^* = c^{-1}\pi_1^*$. Therefore,

$$\pi_1^* = (P_1^* + P_2^*)\pi_{1,S} - 2c^{-1}(P_1^* + P_2^*)\pi_{2,S}\pi_1^*. \tag{309}$$

Solving for π_1^*, we obtain $\pi_1^* = -2\overline{r}_{\mathrm{S}}\pi_2^*\mathbf{1}$ and hence $\rho_0^* = -2\overline{r}_{\mathrm{S}}\rho_1^*\mathbf{1}$ (equation 308). Finally, the expression for π_0^* follows from the fact that $(P_1^* + P_2^*)\rho_0^* = c^{-1}\pi_1^*$. ■

Proof of Corollary 4.3. In equilibrium,

$$P_1^* = (I + \rho_1^*)(I + 2\rho_1^*)^{-1} = (n - c^{-1}\kappa_1)C + (n - c^{-1}\kappa_2)mm' \tag{310}$$

$$P_2^* = -\rho_1^*(I + 2\rho_1^*)^{-1} = -c^{-1}\kappa_1 C - c^{-1}\kappa_2 mm'. \tag{311}$$

Evidently, P_1^* and P_2^* are symmetric. Consequently,

$$CP_1^*P_1^*C = c^{-2}n^{-1}(c - n^{-1}\kappa_1)^2 C \tag{312}$$

$$CP_1^*P_2^*C = -c^{-2}n^{-1}(n^{-1}\kappa_1)(c - n^{-1}\kappa_1)^2 C. \tag{313}$$

Recall that $s_j(\boldsymbol{r}) = P_1(\boldsymbol{r}_j - \rho_0) + P_2(\boldsymbol{r}_{-j} - \rho_0)$. Using the chain rule for gradients,

$$(\nabla_{\boldsymbol{r}_j}\sigma^2_{s_j(\boldsymbol{r})})'(\nabla_{\boldsymbol{r}_j}\sigma_{s_{-j}(\boldsymbol{r}),s_j(\boldsymbol{r})}) = (2P_1^*Cs_j(\boldsymbol{r}))'(P_2^*Cs_j(\boldsymbol{r}) + P_1^*Cs_{-j}(\boldsymbol{r})) \tag{314}$$

$$= 2s_j(\boldsymbol{r})'CP_1^*P_2^*Cs_j(\boldsymbol{r}) + 2s_j(\boldsymbol{r})'CP_1^*P_1^*Cs_{-j}(\boldsymbol{r}). \tag{315}$$

Substitution of equations (312) and (313) yields the desired result. ■

Proof of Corollary 4.4. Note that

$$\sigma_{r(x_{-j}),r(x_j)} = r(x_{-j})'Cr(x_j) \tag{316}$$
$$= (x'_{-j} + (x'_{-j} - \bar{r}\mathbf{1}')\rho_1^*)C(x_j + \rho_1^*(x_j - \bar{r}\mathbf{1})) \tag{317}$$
$$= x'_{-j}Cx + 2x'_{-j}C\rho_1^*(x_j - \bar{r}\mathbf{1}) + (x'_{-j} - \bar{r}\mathbf{1}')\rho_1^*C\rho_1^*(x_j - \bar{r}\mathbf{1}) \tag{318}$$
$$= \sigma_{x_{-j},x_j}(1 + 2n^{-1}\kappa_1(c - 2n^{-1}\kappa_1)^{-1} + n^{-2}\kappa_1^2(c - 2n^{-1}\kappa_1)^{-2}) \tag{319}$$
$$= c^2(c - 2n^{-1}\kappa_1)^{-2}\sigma_{x_{-j},x_j} \tag{320}$$

as desired. ■

Proof of Lemma 4.3. Note that $\boldsymbol{x}|\boldsymbol{e},\Lambda$ is distributed according to a matrix normal distribution with mean matrix $\mathbf{1}\boldsymbol{e}'$, row covariance matrix I (observations are i.i.d.), and column covariance matrix Λ^{-1} (i.e., $\boldsymbol{x}|\boldsymbol{e},\Lambda \sim \mathcal{N}_{n\times 2}(\mathbf{1}\boldsymbol{e}', I, \Lambda^{-1})$). It follows that $\boldsymbol{x}|\Lambda \sim \mathcal{N}_{n\times 2}(\mathbf{1}\boldsymbol{\mu}_0', I + \lambda_0^{-1}\mathbf{1}\mathbf{1}', \Lambda^{-1})$. Integrating out Λ reveals that $\boldsymbol{x} \sim \mathcal{T}_\nu(\mathbf{1}\boldsymbol{\mu}_0', I + \lambda_0^{-1}\mathbf{1}\mathbf{1}', \Psi_0^{-1})$ (where $\mathcal{T}$ is the matrix t-distribution) (see, for example, Chapter 4 of Gupta and Nagar (2000)). Therefore, $\text{vec}(\boldsymbol{x}) \sim t_\nu(\boldsymbol{\mu}_0 \otimes \mathbf{1}, \Psi_0^{-1} \otimes (I + \lambda_0^{-1}\mathbf{1}\mathbf{1}'))$. The result follows from standard results about conditional distributions of the multivariate t-distribution. ■

Proof of Lemma 4.4. Consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1)$. The manager's reports is

$$r_j = x_j + b(x) = \rho_0 + (I + \rho_1)x_j, \tag{321}$$

from which the market extracts the signals

$$s(r_j) = (I + \rho_1)^{-1}(r_j - \rho_0). \tag{322}$$

As in previous derivations, let $P = (I+\rho_1)^{-1} = c^{-1}(cI - (\pi_3 + \chi_0\pi_4'))$ so that $s(r) = P(r - \rho_0)$. Standard calculations show that the posterior $(\boldsymbol{\mu}_0, \Lambda)|$ is Normal-Wishart with parameters

$$\mu_{j,n} = \eta_0\mu_0 + \eta_1\mu_{s(r_j)} \tag{323}$$
$$\lambda_n = \lambda_0 + n \tag{324}$$
$$\nu_n = \nu_0 + n \tag{325}$$
$$\psi_{j,n} = \psi_0 + n(\sigma^2_{s(r_j)} + \eta_0(\mu_{s(r_j)} - \mu_0)^2) \tag{326}$$
$$\omega_n = \omega_0 + n(\sigma_{s(r_1),s(r_2)} + \eta_0(\mu_{s(r_1)} - \mu_0)(\mu_{s(r_2)} - \mu_0)), \tag{327}$$

where, with slight abuse of notation, we use $\psi_{j,n}$ to denote the (j,j) entry of the matrix Ψ.

The posterior mean, variance, and covariance of d are

$$\mathrm{E}(e_j|\boldsymbol{r}) = \mu_n = \eta_0\mu_0 + \eta_1\mu_{s(r_j)}, \tag{328}$$

and

$$\mathrm{Var}(e_j|\boldsymbol{r}) = \frac{\psi_{j,n}}{\lambda_n(\nu_n - 3)} = \zeta_0\psi_0 + \zeta_1\sigma^2_{s(r_j)} + \zeta_2(\mu_{s(r_j)} - \mu_0)^2 \tag{329}$$

and

$$\mathrm{Cov}(e_j|\boldsymbol{r}) = \frac{\omega_n}{\lambda_n(\nu_n - 3)} = \zeta_0\omega_0 + \zeta_1\sigma_{s(r_1),s(r_2)} + \zeta_2(\mu_{s(r_1)} - \mu_0)(\mu_{s(r_2)} - \mu_0) \tag{330}$$

respectively. The result follows from equation (83). ■

Proof of Proposition 4.3. In what follows, let $\boldsymbol{\pi} = (\pi_0, \ldots, \pi_5)$. The firm's objective is

$$F(b_j) = \mathrm{E}\left(p_j(x_j + b_j, x_{-j} + b_{-j}) - \tfrac{c}{2}b_j'b_j|x_j\right) \tag{331}$$

$$\begin{aligned} &= \mathrm{E}\,(\pi_0 + \pi_1'(x_j + b_j) + \pi_2'(x_{-j} + b_{-j}(x_{-j})) \\ &\quad + \tfrac{1}{2}(x_j + b_j)'\pi_3(x_j + b_j) + (x_{-j} + b_{-j}(x_{-j}))'\pi_4(x_j + b_j) \\ &\quad + \tfrac{1}{2}(x_{-j} + b_{-j}(x_{-j}))'\pi_5(x_{-j} + b_{-j}(x_{-j})) - \tfrac{c}{2}b_j'b_j|x_j\right). \end{aligned} \tag{332}$$

The first-order condition is

$$0 = F'(b_j) = \pi_1 + \pi_3(x_j + b_j) + \pi_4'\mathrm{E}(x_{-j} + b_{-j}(x_{-j})|x_j) - cb_j \tag{333}$$

$$= \pi_1 + \pi_3(x_j + b_j) + \pi_4'(\rho_0 + (I + \rho_1)\mathrm{E}(x_{-j}|x_j)) - cb_j \tag{334}$$

$$= \pi_1 + \pi_3(x_j + b_j) + \pi_4'(\rho_0 + (I + \rho_1)(\mu_0\mathbf{1} + \chi_0(x_j - \mu_0\mathbf{1}))) - cb_j. \tag{335}$$

Hence,

$$b_j^*(x_j) = \rho_0(\boldsymbol{\pi}) + \rho_1(\boldsymbol{\pi})x_j, \tag{336}$$

where in equilibrium,

$$\rho_0(\boldsymbol{\pi}) = (cI - \pi_3)^{-1}(\pi_1 + \pi_4'(\rho_0(\boldsymbol{\pi}) + (I + \rho_1(\boldsymbol{\pi}))(1 - \chi_0)\mu_0\mathbf{1})) \tag{337}$$

$$\rho_1(\boldsymbol{\pi}) = (cI - \pi_3)^{-1}(\pi_3 + \chi_0\pi_4'(I + \rho_1(\boldsymbol{\pi}))). \tag{338}$$

Solving for ρ_0 and ρ_1, we obtain

$$\rho_0(\boldsymbol{\pi}) = (cI - (\pi_3 + \pi_4'))^{-1}(\pi_1 + (1-\chi_0)\mu_0\pi_4'(I + \rho_1(\boldsymbol{\pi}))\mathbf{1}) \tag{339}$$
$$\rho_1(\boldsymbol{\pi}) = (cI - (\pi_3 + \chi_0\pi_4'))^{-1}(\pi_3 + \chi_0\pi_4'). \tag{340}$$

Next, consider the market's problem given $\boldsymbol{\rho} = (\rho_0, \rho_1)$. Recall the signal $s(r_j)$ and price $p_j(r_j, r_{-j})$ given in equations (87) and (88). As before, let $P = (I + \rho_1)^{-1}$ so that $s(r_j) = P(r_j - \rho_0)$. Now

$$s(r_j) - \mu_0\mathbf{1} = P(r_j - \rho_0) - \mu_0\mathbf{1} = Pr_j - (P\rho_0 + \mu_0\mathbf{1}) \tag{341}$$

and hence

$$\mu_{s(r_j)} - \mu_0 = m'(s(r_j) - \mu_0\mathbf{1}) = m'Pr_j - m'(P\rho_0 + \mu_0\mathbf{1}) \tag{342}$$

and hence

$$(\mu_{s(r_j)} - \mu_0)^2 = (s(r_j) - \mu_0\mathbf{1})'mm'(s(r_j) - \mu_0\mathbf{1}) \tag{343}$$
$$= r_j'P'mm'Pr_j - 2(P\rho_0 + \mu_0\mathbf{1})'mm'Pr_j$$
$$+ (P\rho_0 + \mu_0\mathbf{1})'mm'(P\rho_0 + \mu_0\mathbf{1}). \tag{344}$$

Therefore,

$$\sigma^2_{s(r_j)} = s(r_j)'Cs(r_j) = r_j'P'CPr_j - 2\rho_0'P'CPr_j + \rho_0'P'CP\rho_0. \tag{345}$$

and

$$\sigma_{s(r_j),s(r_{-j})} = s(r_j)'Cs(r_{-j}) = r_j'P'CPr_{-j} - \rho_0'P'CP(r_j + r_{-j}) + \rho_0'P'CP\rho_0 \tag{346}$$

and

$$(\mu_{s(r_1)} - \mu_0)(\mu_{s(r_2)} - \mu_0) = (s(r_j) - \mu_0\mathbf{1})'mm'(s(r_{-j}) - \mu_0\mathbf{1}) \tag{347}$$
$$= (Pr_j - (P\rho_0 + \mu_0\mathbf{1}))'mm'(Pr_{-j} - (P\rho_0 + \mu_0\mathbf{1})) \tag{348}$$
$$= r_j'P'mm'Pr_{-j} - (P\rho_0 + \mu_0\mathbf{1})'mm'P(r_j + r_{-j})$$
$$+ (P\rho_0 + \mu_0\mathbf{1})'mm'(P\rho_0 + \mu_0\mathbf{1}). \tag{349}$$

Therefore, firm j's market price is

$$p_j(\boldsymbol{r}) = \mathrm{E}(e_j|\boldsymbol{r}) - \gamma(\mathrm{Var}(e_j|\boldsymbol{r}) + \mathrm{Cov}(e_j|\boldsymbol{r})) \tag{350}$$

$$= \tilde{\pi}_0 + \tilde{\pi}_1' r_j + \tilde{\pi}_2' r_{-j} + \tfrac{1}{2} r_j' \tilde{\pi}_3 r_j + r_j' \tilde{\pi}_4 r_{-j} + \tfrac{1}{2} r_{-j}' \tilde{\pi}_5 r_{-j}, \tag{351}$$

where

$$\tilde{\pi}_0(\boldsymbol{\rho}) = \mu_0 - \eta_1 m'(P\rho_0 + \mu_0 \mathbf{1})$$
$$- \gamma(\zeta_0(\psi_0 + \omega_0) + 2\zeta_1 \rho_0' P' C P \rho_0 + 2\zeta_2 (P\rho_0 + \mu_0 \mathbf{1})' m m' (P\rho_0 + \mu_0 \mathbf{1})) \tag{352}$$

$$\tilde{\pi}_1(\boldsymbol{\rho}) = \eta_1 P' m + 3\gamma(\zeta_1 P' C P \rho_0 + \zeta_2 P' m m' (P\rho_0 + \mu_0 \mathbf{1})) \tag{353}$$

$$\tilde{\pi}_2(\boldsymbol{\rho}) = \gamma(\zeta_1 P' C P \rho_0 + \zeta_2 P' m m' (P\rho_0 + \mu_0 \mathbf{1})) \tag{354}$$

$$\tilde{\pi}_3(\boldsymbol{\rho}) = -2\gamma(\zeta_1 P' C P + \zeta_2 P' m m' P) \tag{355}$$

$$\tilde{\pi}_4(\boldsymbol{\rho}) = -\gamma(\zeta_1 P' C P + \zeta_2 P' m m' P) \tag{356}$$

$$\tilde{\pi}_5(\boldsymbol{\rho}) = 0. \tag{357}$$

Since C and mm' are symmetric, so too are $\pi_3(\boldsymbol{\pi})$ and $\pi_4(\boldsymbol{\pi})$. Let

$$k^* = \pi_3^* + \chi_0 \pi_4^{*\prime}. \tag{358}$$

It follows that

$$c^2 k^* = -\gamma(2 + \chi_0)(cI - k^*)(\zeta_1 C + \zeta_2 m m')(cI - k^*). \tag{359}$$

The subsequent analysis follows that found in the proof of Proposition 3.2 and is abridged for brevity. The solution of equation (359) is of the form

$$k^* = \kappa_1 C + \kappa_2 m m'. \tag{360}$$

Substitution of equation (360) into equation (359) yields

$$c^2(\kappa_1 C + \kappa_2 m m') = -\gamma(2 + \chi_0)(\zeta_1 (c - n^{-1}\kappa_1)^2 C + \zeta_2 (c - n^{-1}\kappa_2)^2 m m'), \tag{361}$$

from which we obtain the quadratic equations in (102) and (103). The restriction $c > 4\gamma(2 + \chi_0)\zeta_1/n$ follows by requiring that equations (102) and (103) have real roots.

$$\rho_1^* = \kappa_1 (c - n^{-1}\kappa_1)^{-1} C + \kappa_2 (c - n^{-1}\kappa_2)^{-1} m m' \tag{362}$$

Equations (340) and (360) yield the final expression for ρ_1^*. Using the facts that π_3^* and π_4^* are symmetric and $\pi_3^* = 2\pi_4^*$, equations (339) and (340) become

$$\rho_1^* = (2+\chi_0)(cI - (2+\chi_0)\pi_4^*)^{-1}\pi_4^* \tag{363}$$

$$\rho_0^* = (1-\chi_0)(cI - 3\pi_4^*)^{-1}\pi_4^*(I+\rho_1^*)\mu_0\mathbf{1} + (cI - 3\pi_4^*)^{-1}\pi_1^*. \tag{364}$$

and hence

$$P^* = c^{-1}(cI - (2+\chi_0)\pi_4^*). \tag{365}$$

Moreover,

$$\pi_4^* = (2+\chi_0)^{-1}(\kappa_1 C + \kappa_2 mm'). \tag{366}$$

Using the commutability of P^* with all of the terms in equation (364), we have

$$P^*\rho_0^* = (1-\chi_0)(cI - 3\pi_4^*)^{-1}\pi_4^*\mu_0\mathbf{1} + (cI - 3\pi_4^*)^{-1}P^*\pi_1^*. \tag{367}$$

Equation (353) becomes

$$\pi_1^* = \eta_1 P^* m + 3\gamma(\zeta_1 P^* C P^* \rho_0^* + \zeta_2 P^* mm'(P^*\rho_0^* + \mu_0\mathbf{1})) \tag{368}$$

$$= (\eta_1 + 3\gamma\zeta_2\mu_0)P^* m + 3\gamma P^*(\zeta_1 C + \zeta_2 mm')P^*\rho_0^* \tag{369}$$

$$= P^*(\pi_{1,F} - 3\pi_{4,F}((1-\chi_0)(cI - 3\pi_4^*)^{-1}\pi_4^*\mu_0\mathbf{1} + (cI - 3\pi_4^*)^{-1}P^*\pi_1^*)) \tag{370}$$

$$= P^*(\pi_{1,F} - 3(1-\chi_0)\pi_{4,F}(cI - 3\pi_4^*)^{-1}\pi_4^*\mu_0\mathbf{1}) - 3P^*\pi_{4,F}(cI - 3\pi_4^*)^{-1}P^*\pi_1^*. \tag{371}$$

Solving for π_1^*,

$$\pi_1^* = (I + 3P^*\pi_{4,F}(cI - 3\pi_4^*)^{-1}P^*)^{-1}P^*(\pi_{1,F} - 3(1-\chi_0)\pi_{4,F}(cI - 3\pi_4^*)^{-1}\pi_4^*\mu_0\mathbf{1}) \tag{372}$$

$$= c^{-1}(cI - 3\pi_4^*)P^*(\pi_{1,F} - 3(1-\chi_0)\pi_{4,F}(cI - 3\pi_4^*)^{-1}\pi_4^*\mu_0\mathbf{1}) \tag{373}$$

$$= c^{-1}(cI - 3\pi_4^*)P^*\pi_{1,F} - 3c^{-1}(1-\chi_0)P^*\pi_{4,F}\pi_4^*\mu_0\mathbf{1} \tag{374}$$

$$= -3c^{-1}\overline{r}_F(cI - 3\pi_4^*)P^*\pi_{4,F}\mathbf{1} - 3c^{-1}(1-\chi_0)P^*\pi_{4,F}\pi_4^*\mu_0\mathbf{1} \tag{375}$$

$$= -3c^{-1}P^*(\overline{r}_F(cI - 3\pi_4^*)\pi_{4,F} + \mu_0(1-\chi_0)\pi_{4,F}\pi_4^*)\mathbf{1}. \tag{376}$$

Substitution into equation (364) yields ρ_0^*. π_0^* can be computed using equation (352). ■

Proof of Corollary 4.5. In equilibrium,

$$P^* = I - c^{-1}\kappa_1 C - c^{-1}\kappa_2 mm'. \tag{377}$$

Evidently, P^* is symmetric. Consequently,

$$CP^*P^*C = c^{-2}n^{-1}(c - n^{-1}\kappa_1)^2C. \tag{378}$$

Recall that $s(r_j) = P^*(r_j - \rho_0)$. Using the chain rule for gradients,

$$(\nabla_{r_j}\sigma^2_{s^*(r_j)})'(\nabla_{r_j}\sigma_{s^*(r_j),x_j}) = (2P^*Cs(r_j))'(P^*Cx_j) = 2s(r_j)'CP^*P^*Cx_j. \tag{379}$$

Substitution of equation (378) yields the desired result. ■

Proof of Corollary 4.6. From equations (102) and (103), we have that for $k \in \{1, 2\}$,

$$c^2\kappa_k = -\gamma(2 + \chi_0)\zeta_k(c - n^{-1}\kappa_k)^2. \tag{380}$$

where κ_k is the smaller root in absolute value. Let

$$Q_k(\kappa) = c^2\kappa + \gamma(2 + \chi_0)\zeta_k(c - n^{-1}\kappa)^2 \tag{381}$$

so that $Q_k(\kappa_k) = 0$. Note that $Q_k(-nc) = -nc^3 + 4\gamma(2 + \chi_0)\zeta_kc^2 < 0$ (see the discriminates of equations (102) and (103)). Since κ_k is the smaller root in absolute value, it follows that $-nc < \kappa_k$ and hence $c^2 > n^{-2}\kappa_k^2$. Differentiating equation (380) with respect to χ_0 and rearranging yields

$$\frac{d\kappa_k}{d\chi_0} = \frac{\kappa_k(c - n^{-1}\kappa_k)}{(c + n^{-1}\kappa_k)(2 + \chi_0)} < 0 \tag{382}$$

as desired. ■

Proof of Corollary 4.7. Using the facts that $m'\pi_{4,F} = -\gamma\zeta_2n^{-1}m'$, $m'\pi_4^* = (2+\chi_0)^{-1}n^{-1}\kappa_2m'$, and $m'P^* = (1 - c^{-1}n^{-1}\kappa_2)m'$, it can be shown that

$$m'\pi_1^* = 3\gamma\zeta_2c^{-2}n^{-1}(c-n^{-1}\kappa_2)(\bar{r}_F(c-3(2+\chi_0)^{-1}n^{-1}\kappa_2)+\mu_0(1-\chi_0)(2+\chi_0)^{-1}n^{-1}\kappa_2) \tag{383}$$

and

$$m'(cI - 3\pi_4^*)^{-1} = (c - 3(2 + \chi_0)^{-1}n^{-1}\kappa_2)^{-1}m' \tag{384}$$

and

$$m'\pi_4^*(I + \rho_1^*)\mathbf{1} = (2 + \chi_0)^{-1}n^{-1}\kappa_2m'(I + \rho_1^*)\mathbf{1} \tag{385}$$

$$= (2 + \chi_0)^{-1}n^{-1}\kappa_2c(c - n^{-1}\kappa_2)^{-1}. \tag{386}$$

and hence

$$m'\rho_0^* = m'(cI - 3\pi_4^*)^{-1}((1-\chi_0)\pi_4^*(I+\rho_1^*)\mu_0\mathbf{1} + \pi_1^*) \tag{387}$$

$$= (c - 3(2+\chi_0)^{-1}n^{-1}\kappa_2)^{-1}m'((1-\chi_0)\pi_4^*(I+\rho_1^*)\mu_0\mathbf{1} + \pi_1^*). \tag{388}$$

Next, $m'\rho_1^* = n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}m'$, $m'\rho_1^{*-1} = n\kappa_2^{-1}(c - n^{-1}\kappa_2)m'$, and $m'(I+\rho_1^*) = c(c - n^{-1}\kappa_2)^{-1}m'$. Therefore,

$$\mu_{r^*(x_j)} = m'x_j + (\rho_0^* + \rho_1^* x_j) \tag{389}$$

$$= \mu_{r^*(x_j)} + m'\rho_1^*(x_j + \rho_1^{*-1}\rho_0^*) \tag{390}$$

$$= \mu_{r^*(x_j)} + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}m'(x_j + \rho_1^{*-1}\rho_0^*) \tag{391}$$

$$= \mu_{r^*(x_j)} + n^{-1}\kappa_2(c - n^{-1}\kappa_2)^{-1}(\mu_{r^*(x_j)} - \widetilde{r}\mathbf{1}) \tag{392}$$

and

$$\sigma_{r(x_i),r(x_j)} = r(x_i)'Cr(x_j) \tag{393}$$

$$= (\rho_0^* + (I+\rho_1^*)x_i)'C(\rho_0^* + (I+\rho_1^*)x_j) \tag{394}$$

$$= x_i'(I+\rho_1^*)'C(I+\rho_1^*)x_j \tag{395}$$

$$= c^2(c - n^{-1}\kappa_1)^{-2}x_i'Cx_j \tag{396}$$

$$= c^2(c - n^{-1}\kappa_1)^{-2}\sigma_{r(x_i),r(x_j)}, \tag{397}$$

where the second line follows from the fact that $C\rho_0^* = 0$. ■